

## OPTIMAL DESIGN FOR DEPLOYABLE STRUCTURES USING ORIGAMI TESSELLATIONS

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### ABSTRACT

*This work presents innovative origami optimization methods for the design of unit cells for complex origami tessellations that can be utilized for the design of deployable structures. The design method used to create origami tiles utilizes the principles of discrete topology optimization for ground structures applied to origami crease patterns. The initial design space shows all possible creases and is given the desired input and output forces. Taking into account foldability constraints derived from Maekawa's and Kawasaki's theorems, the algorithm designates creases as active or passive. Geometric constraints are defined from the target 3D object. The periodic reproduction of this unit cell allows us to create tessellations that are used in the creation of deployable shelters. Design requirements for structurally sound tessellations are discussed and used to evaluate the effectiveness of our results. Future work includes the applications of unit cells and tessellation design for origami inspired mechanisms. Special focus will be given to self-deployable structures, including shelters for natural disasters.*

### 1. INTRODUCTION

Origami is encountered in nature as an efficient packing strategy. Natural deployable structures include insect wings and collapsing leaves and flowers inside buds. These have inspired innovative parachute packing, as well as the design of solar panels, telescope mirrors, thermal shields, and other unfolding shell-like engineering structures. The growing use of origami in engineering design inspires optimization methods for the creation of optimal patterns for complex mechanisms. Origami provides a creative approach to the design of mechanisms, giving engineering a possibility to advance configurations of structures

between their 2D and 3D states. This allows for the creation of optimized deployable structures. Modern engineering has integrated origami to create innovative packing, adaptive aerospace systems, solar panels, and robotics [1]. Optimality in the packing of these complex structures eases the transportation of such. These techniques have also been used widely in the creation of shelters for the military [2]. Shelters can usually be deployed by soldiers but might range from minutes to a couple of hours of assembly and multiple people to assemble. Reducing the time and soldiers needed to deploy a shelter could give way for better allocation of these resources. Efforts have been made in exploring the design of emergency shelters, especially interesting the multi-layered hemispherical shelters evaluated for emergencies in coal mines [3].

An important focus is the optimization of the design process of origami patterns. There are a number of origami design tools, such as Tree-Maker, that create crease patterns that target given geometric constraints, but it is a challenge to achieve through this process a design within desired engineering performance [4]. The tool utilized contains truss-based analysis derived from topology optimization of ground structures to minimize the amount of crease patterns present in the final origami design [5]. This process ideally minimizes the amount of energy needed to deploy a structure, given the actuation is applied to active fold lines for an origami pattern to be folded.

Fundamental origami theorems are essential to the creation of origami and have been studied extensively to be applied to and evaluated for computational origami [6]. Main theorems include the Huzita-Hatori axiom, Maekawa's theorem (if the structure can be folded completely flat), and the Kawasaki Theorem, which proves the foldability of a crease pattern depending on the sequence of angles surrounding interior vertices. Proposed

tessellation design using Resch's patterns have been discussed before in the use of freeform origami [7]. This method use polyhedral surfaces to achieve desired patterns. Our aim is to create successful tessellations, utilizing optimization technologies to achieve patterns that will require reduced actuation to operate. The target properties for the creation of unit cells for tessellations are studied as well in this project.

## 2. DEPLOYABLE STRUCTURES

There are important characteristics that need to be taken into consideration when creating a self-deployable structures. Highly important deployable structures include emergency shelters. Throughout centuries, emergency shelters have been used to provide aid to those in need of a safe structure to inhabit, for many causes including natural disasters or even during war. In Guoyun's paper about emergency shelters, it is defined as essential to have auxiliary systems such as a ventilation system pumping fresh air and carbon dioxide out, a store room storing water, food, blankets, medicine, and a communication line [3].

Other key components of shelters are widely defined and can be compared to those of houses or buildings. Emergency shelters must provide adequate wind resistance, which means the structure must remain stable and with minimal movement during strong winds. This is a main requirement for a structurally sound design of any structure and provides safety during inclement weather. In relation to wind resistance, rigidity is an important aspect of shelters. The design, material and supports need to be evaluated extensively to provide protection to the people inside the structure.

Ventilation is another main aspect of emergency shelters. Although enclosed and rigid, an emergency shelter has to provide adequate ventilation for its occupants. The flow of fresh air and the exit of carbon dioxide exhaled must be appropriate to be an inhabitable structure, even if its intended use is temporary. This needs to be taken into careful consideration when designing any kind of structure.

For emergency situations, the deployment of the structure needs to be uncomplicated and in a timely manner to provide quick aid. For our research, we have concentrated our efforts in minimizing the time a structure takes to deploy. Further research will include other shelter characteristics, and the comparison of our method to patterns previously used in shelter design. The methodology used focuses on creating a crease pattern based on force inputs and fixed points, and in return reduces the amount of foldlines in the design. The final origami pattern requires a reduced amount of actuation to be deployed.

## 3. TOPOLOGY OPTIMIZATION

Principles of discrete topology optimization and truss-based analysis for ground structures are used in the optimization

process for the origami crease patterns [5]. The size of the design space and the type of grid to be utilized in the tessellation assigned by the user. With these inputs, the code will then assign all possible creases in the design space provided. Fixities, inputs, and desired outputs are applied then to the design space as desired. With the given information the program will evaluate the algorithm through multiple iterations. The amount of iterations the algorithm is allowed to perform affects the complexity of the design. During each iteration, function values are recorded. Each vertex in the preliminary design space is assigned a number as part of a global grid and the foldlines along each vertex are identified.

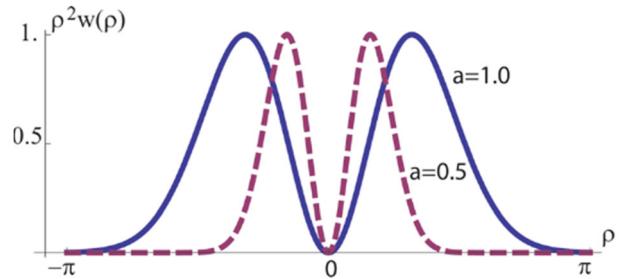


FIGURE 1: ANGLE-DEPENDENT PENALIZATION VALUES.

For each crease available in the original design, with the given force inputs, the algorithm determines if the fold line will perform as a mountain, a valley, or a flat (no fold). All folds are assigned a weight according to their angle ( $\rho$ ), the highest penalization value is assigned to angles away from  $0^\circ$  or  $180^\circ$  (Figure 1). The weight function is as follows:

$$f = \sum_i \rho_i^2 w(\rho_i) \quad (1)$$

$$w(\rho_i) = C e^{-(\frac{\rho_i}{a})^2} \quad (2)$$

$$C = \frac{e}{a^2} \quad a \in (0, \pi) \quad (3)$$

$C$  is a constant defined by Euler's number and the  $a$  defined.  $f$  is an objective function that favors designs with larger number of "off" fold lines. An "off" fold line is a line that remains flat in the folding process, and in turn, can be deleted from the crease pattern. This function forces angles towards zero or large values. The curves in Figure 1 show  $\rho^2 w(\rho)$  for  $a = 0.5$  and  $a = 1$ , and since maximum values occur at  $\rho = \pm a$ , the location can be adjusted by changing  $a$ .

During the evaluation of the entire grid, as with ground structure topology optimization, the creases that are not essential to the integrity of the origami mechanism are eliminated. The final design contains reduced number of creases, with the same structural integrity in its folded state. This pattern is considered optimized once the code has evaluated the function values for the amount of iterations assigned. The optimized design is shown in its folded state along the graph of the function value throughout the iterations of the algorithm.

The goal of this code is to find a pattern that achieves desired deformation for given constraint, like input force. Stiffness coefficients of the folds are key to this objective. The proposed method incorporates gradient based optimization that minimizes the weight function within the given mechanical boundary conditions (Shown below). This particular method minimizes the amount of folds in the crease pattern which reduces the actuation necessary for deployment.

$$\begin{aligned}
 \text{Find} \quad & \rho \in [-\pi, \pi]^N \text{ that} \\
 \text{Maximize} \quad & f(\rho) \\
 \text{Subject to} \quad & g^k = 0 \text{ for } k = 1, 2, \dots, M \\
 & h_f = 0 \text{ for } j = 1, 2, \dots, M \\
 & \rho_r = \rho_r^0
 \end{aligned} \tag{4}$$

One of the folding angles is prescribed to be a fixed angle  $\rho_r^0$  to specify one instant of the folded sheet during the folding process. Constraints  $g^k$  and  $h_f$  express foldability and target properties, where  $M$  is the number of vertices in the initial design, and the definitions of both are design dependent. The optimization problem is solved with a sequential quadratic programming algorithm. With these particular optimization methods and the applied fixities (Figure 2), we can minimize the amount of folds in the crease pattern. For the purpose of deployable structures, a limited number of creases provides a structure that will require a reduced amount of actuation (energy) to be deployed. The type of actuation will also provide added simplicity to the assembly process and the minimization of energy consumption. The type of actuation considered is discussed later in this article.

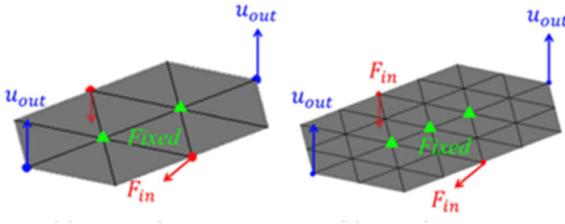


FIGURE 2: EXAMPLE CREASE PATTERN WITH BOUNDARY CONDITIONS.

#### 4. TESSELATION GUIDELINES

A tessellation, in art, is a design made by tiling an identical unit. These units have no gaps between them and, in the case of origami, these building blocks (tiles) are simple crease patterns that can be multiplied. To form a unit tessellation there are guidelines that need to be considered.

A tessellation must be composed of a regular polygon [8]. For origami purposes, the crease pattern must be created in an

original grid that is equilateral to be considered an appropriate unit cell for a tessellation. However, all sides in the polygon must meet to form equal angles, which is why some regular polygons may not be used for the creation of tessellations. The structure must also have, as mentioned previously, no gaps between each building block. Each unit cell must perfectly align on each side with its neighboring unit cell and line up evenly. This is why equal sizes are necessary for the design of the tiles.

The process of creating unique tessellations is one that has proven very important in origami creations. Patterns like the waterbomb and Miura are time and time again utilized in origami design of complex systems, including the creation of auxetic patterns [9].

Following the previous requirements, a common vertex can be achieved. If three or more shapes come together in a vertex to form a  $360^\circ$  angle, then the design is an appropriate tessellation. In addition, the unit cells must contain periodicity in their crease patterns. That is, the design must have a line of symmetry. Once these requirements are evaluated and met, the unit cell and the tessellation are appropriate, and the final design will be successful.

#### 5. OPTIMIZER TESSELATION

Utilizing the program, a unit for tessellation was created, taking into account the requirements for appropriate tessellations. With the user interface provided by the code, the first step is to assign the grid and fixities of the unit. The options are shown in Figure 3, and given the number of units on each axis the initial grid is created. These grid show all possible fold lines in the crease pattern. Once created, fixities can be applied.

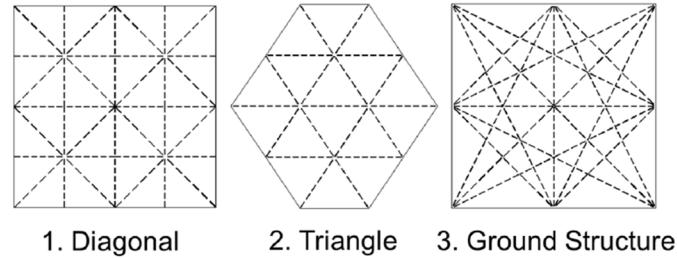


FIGURE 3: GRID TYPES.

An initial consideration when creating the tile is that the fixities must allow no rotational movement, and must be applied considering the periodicity requirement of the tessellation. For this project, the applied fixed points (Figure 4) are located to fix one fourth of the square unit. This is with the intention of this design being multiplied, only after being mirrored along both x and y axes.

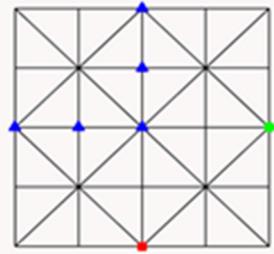


FIGURE 4: INITIAL PATTERN.

Boundary conditions were applied to both right side and bottom line midpoints. The red pointer belongs to an input force, assigned 500N (the base value given by the program), and the movement given to the origami mechanisms is 1 unit upwards in the y-axis and a negative unit in the z-axis (into the page). The output point in green, is a unit of movement towards the center (along the x-axis) and a positive unit in the z-axis. The results were generated for 100 iterations and fold angle fractions of 0.3, 0.5, 0.8, and 1. Fold angle fractions can be assigned from 0 to one and define the amount of folds allowed in the final structure. The higher the fold angle fraction, more possible folds will make part of the final design.

## 6. RESULTS AND DISCUSSION

Distinct fold angle fractions were utilized in the creation of the optimized crease pattern. As it can be seen in Figure 5, past fold angle fraction 0.5, the optimized design did not exhibit any differences in the final crease pattern. For this project, units (tiles) for the tessellation were created using both 0.3 and 0.5 base patterns.

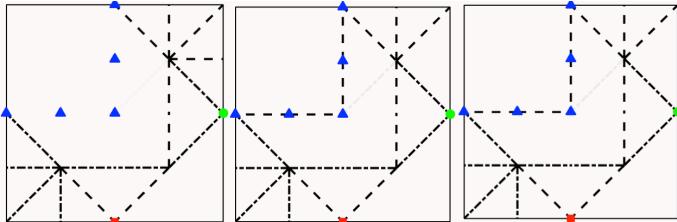


FIGURE 5: CREESE PATTERNS FOR ANGLE FRACTIONS 0.3, 0.5, AND 0.8.

Because of the increased amount of foldlines, one can achieve a higher complexity with a higher foldline without compromising the periodicity of the design or the integrity of the tessellation. The folded state for the 0.3 and 0.5 fold angle fraction crease patterns are shown in Figures 6-7.

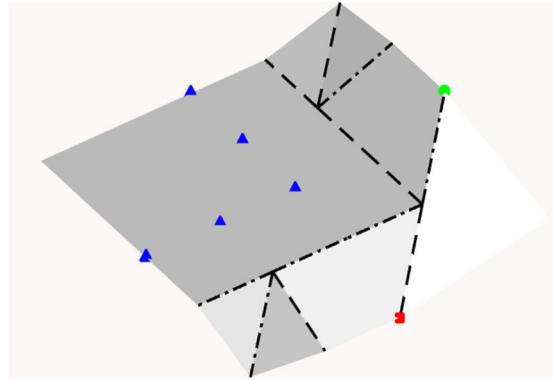


FIGURE 6: FOLDED STATE OF 0.3 ANGLE FRACTION PATTERN.

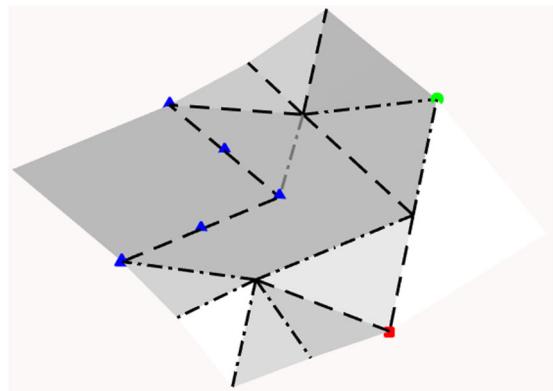


FIGURE 7: FOLDED STATE OF 0.5 ANGLE FRACTION PATTERN.

To complete the tiles for the final design, the optimizer units where mirrored along the x-axis and y-axis. Figure 8 shows both completed patterns.

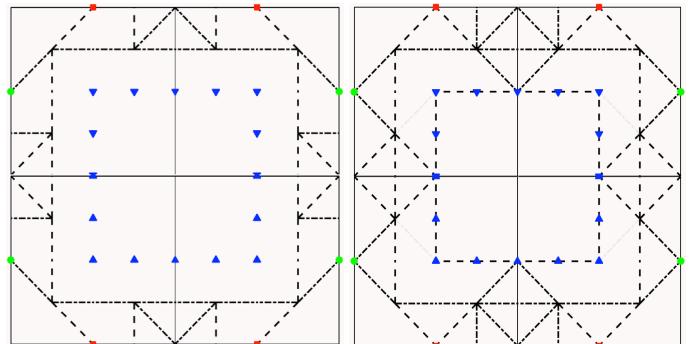


FIGURE 8: FINALIZED TILE FOR ANGLE FRACTION 0.3 AND 0.5.

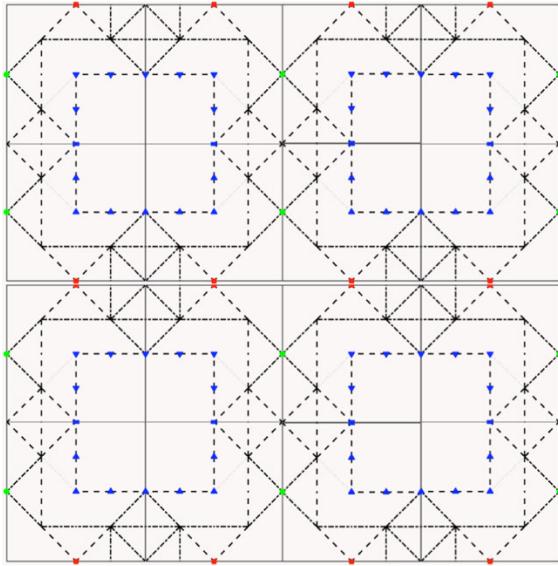


FIGURE 9: FINAL TESSELLATION.

The repetition of this particular unit is the base crease pattern for a tessellation (Figure 8). The base polygon used for the tessellation is a square, which means it qualifies as a regular polygon. All interior angles of a square are equal, hence it provides an appropriate base tile. In addition, there are no gaps between each unit of the tessellation, as shown in Figure 9.

This is the second important requirement for tessellation design. Line of symmetry are met in each of the unit cells created by the optimizer, thus the fold lines of each of the tiles for the tessellation line up evenly and perfectly. Following this visual verification of the design, the final evaluation is to inspect the vertices and validate the vertices meet  $360^\circ$  criteria. All vertices present have angles between foldlines that add up to  $360^\circ$ , which completes the proof of concept that the tessellation created is a valid one. The verification of this design requirement was made by hand and is partially shown in Figure 10.

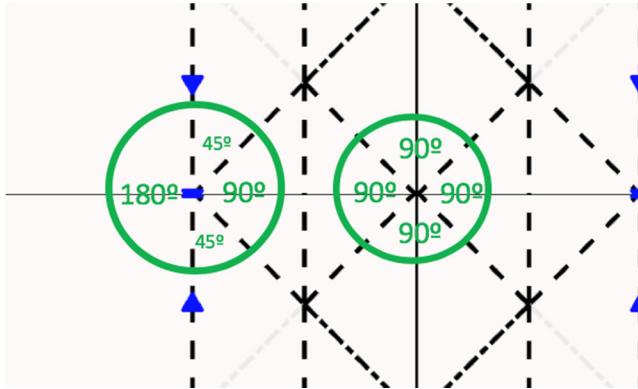


FIGURE 10: VERTICES VERIFICATION.

## 7. FINAL TESSELLATION

The optimized fold pattern was implemented using paper. The folded state of the units thus created are shown in Figure 11. This image shows the folded state of a single unit which matches the computer generated folded state shown in Figure 7. In order to implement such tessellations in a self-deployable structure, the optimized fold pattern needs to be assembled using sturdier material, such as canvas or felt.

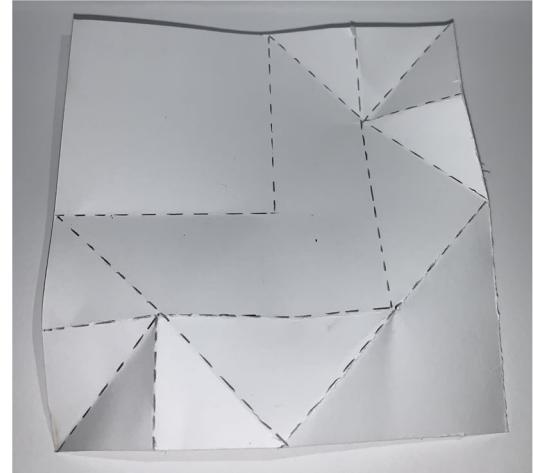


FIGURE 11: FOLDED UNIT.

A complete tile (paper) of this design shown in a physical folded state is also captured in Figure 12. Periodicity is easily visualized in the figure of this paper prototype. As a final production of the tessellation, given the size of the tile, only four were produced and placed together for the image. This presents the idea of the tessellation design in paper of the folded pattern shown in Figure 9. Additional prototypes can be created using cardboard and a laser cutter.

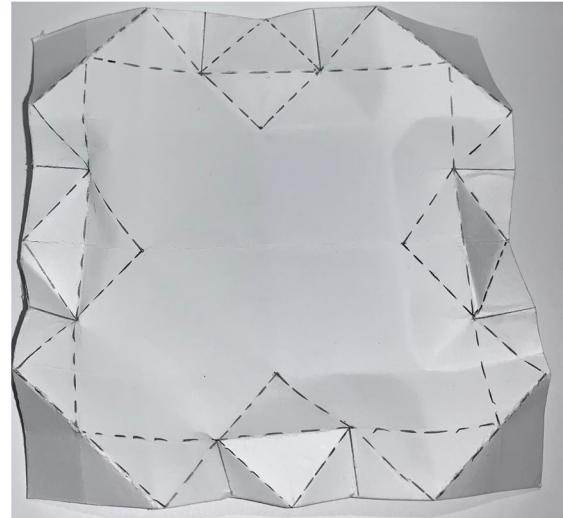


FIGURE 12: TESSELLATION TILE.

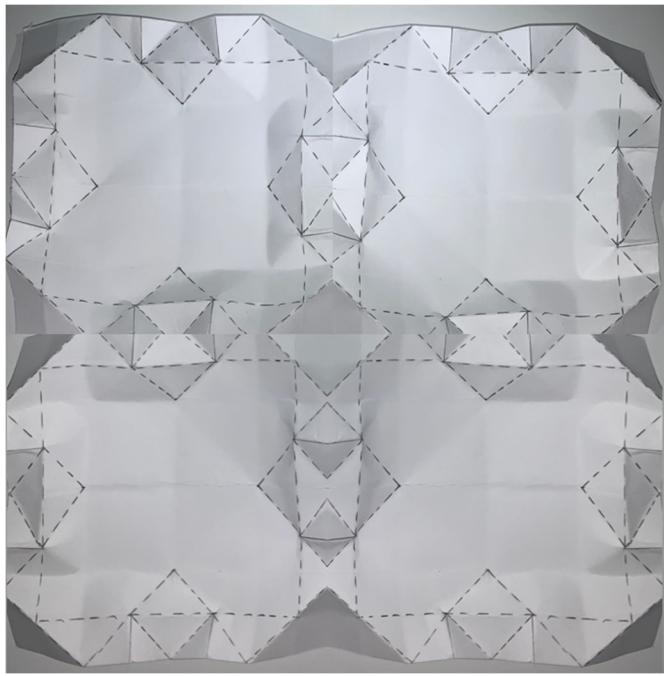


FIGURE 13: 2X2 TESSELLATION OF PROPOSED TILE.

## 8. CONCLUSIONS AND FUTURE WORK

An origami tessellation based structure was designed through optimization of number of folds that would result into minimum actuation energy for a self-deployable shelter. A sequential quadratic programming was used for the optimization process. The resulting fold pattern offered the unit structure that can be replicated to build the deployable shelter. The use of these tessellations will lead to the creation of optimized building blocks for deployable structures. The reduction of fold lines through the optimized code allows for the minimization of actuation necessary to deploy the complete folded structures.

Future work includes the use of these tessellations to create deployable shelter prototypes, taking into consideration the emergency shelter requirements defined above. The prototype(s) can then be compared to current origami patterns used in shelter design, and compared analytically to provide extensive supporting data about our deployment capabilities and structure improvements. Detailed comparison needs to be accomplished for all important characteristics of a habitable emergency shelter, most importantly the deployment of such to numerically evaluate the differences in deployment actuation and time between our design and common origami inspired emergency shelters.

## ACKNOWLEDGMENTS

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