

A Real Options Market-Based Approach to Increase Penetration of Renewables

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Abstract—We propose a framework for trading real options through which flexible sources are incentivized to mitigate the effect of renewable intermittence and show that such options can increase renewable penetration while ensuring the delivery of reliable power and guaranteeing that no market participants are worse-off. We consider that renewable generators are required to bid in a day-ahead market and incur a penalty if, in real-time, they are unable to meet their day-ahead schedule. As a hedge against uncertainty, these non-firm generators purchase options for reserve from sources of flexibility such as natural gas power plants (NGPPs) in an ex-ante options market. Through an option, NGPPs offer to reserve some fuel to be used in case of renewable shortage, while renewable generators purchase the right to request use of that reserve if needed. We solve for the optimal strategies for the generators in the coupled day-ahead and options markets and show that such options increase the amount of generation from renewable sources that is taken by the grid, and provide adequate payment for flexibility providers.

Index Terms—Electricity markets, renewable integration, decision-making, grid flexibility.

NOMENCLATURE

Sets

$\mathcal{I}$	Set of renewable power plants (RPPs).
$\mathcal{J}$	Set of natural gas power plants (NGPPs).
$\mathcal{K}$	Set of coal power plants (CPPs).
$\mathcal{A}_{ri}$	Action space of RPP i in real options market.
$\mathcal{A}_{nj}$	Action space of NGPP j in real options market.
$\mathcal{H}_{ri}$	Information set of RPP i in real options market.
$\mathcal{H}_{nj}$	Information set of NGPP j in real options market.
$\mathcal{H}_C$	Information set of entity that clears real options market.

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Functions

$f_{Ri}(\cdot)$	Probability density function of random renewable production R_i from RPP i .
$F_{Ri}(\cdot)$	Cumulative density function of random renewable production R_i from RPP i .
$I(\cdot)$	Indicator function that equals 1 if argument is positive and equals 0 otherwise.
$u_{ri}(\cdot)$	Expected profit of RPP i .
$u_{nj}(\cdot)$	Expected profit of NGPP j .
$u_{ck}(\cdot)$	Expected profit of CPP k .
$u_{sw}(\cdot)$	Social welfare.
$\kappa_{nj}(\cdot)$	Fuel cost of NGPP j .
$\kappa_{ck}(\cdot)$	Fuel cost of CPP k .

Parameters

λ_{DA}	Per unit day-ahead energy price.
λ_p	Per unit penalty for shortages.
π	Per unit price of reserve in real options market.
μ_{nj}	Per unit variable O&M cost of NGPP j .
μ_{ck}	Per unit variable O&M cost of CPP k .
a_{nj}, b_{nj}, c_{nj}	Parameters of fuel cost function of NGPP j .
a_{ck}, b_{ck}, c_{ck}	Parameters of fuel cost function of CPP k .
$\bar{P}_{ri}$	Maximum capacity of RPP i .
$\bar{P}_{nj}$	Maximum capacity of NGPP j .

Decision Variables

G_{ij}	Reserve amount to RPP i from NGPP j .
G_{ri}	Sum of reserves from all NGPPs to RPP i .
G_{nj}	Sum of reserves to all RPPs from NGPP j .
C_{ri}	Day-ahead commitment of RPP i .
C_{nj}	Day-ahead commitment of NGPP j .
C_{ck}	Day-ahead commitment of CPP k .
$\bar{\pi}_{ri}$	Upper bound on per unit price of reserve set by RPP i .
$\underline{\pi}_{nj}$	Lower bound on per unit price of reserve set by NGPP j .

I. INTRODUCTION

ENSURING reliable operation of the grid has become a bigger challenge as the participation of renewable sources such as wind and solar, that are inherently intermittent, uncertain, and uncontrollable, at various time scales, continues to increase. At the moment, in most electric energy grids, the system operators do not force renewable producers to bid in

the day-ahead market and instead allow them to self-schedule to increase the fraction of renewable energy. To ensure power balance, grid operators have typically relied on increasing their procurement of reserves that can be used to compensate for the variability of renewable supply. Traditionally, the cost of these reserves is socialized and borne by the load serving entities [3]. However, as renewable penetration increases, the total cost that needs to be socialized rises rapidly [4], often leading to opaque subsidies given to specific generators [5]. To ensure that this issue is not a bottleneck in terms of increasing renewable penetration in the grid, market regulations which jettison self-scheduling and instead require utility-scale renewable producers to bid in the day-ahead market and then abide by penalties in case of deviations from their production schedule (similar to traditional generators) have recently been proposed [3], [6], [7]. However, if these renewable producers are penalized for shortfalls, they will bid conservatively [8], leading to a reduction in the share of grid energy supplied by renewable sources. It is important to obtain policy prescriptions that ensure reliable electricity supply while still incentivizing renewable generators.

In this work, we propose a market structure in which the renewable generators are asked to bid in the day-ahead market. However, as a hedge against penalties for shortfalls, they are allowed to purchase options for using reserves from flexible sources with fast ramping capability. Crucially, the cost of this purchase is not socialized by an independent system operator; rather, each non-firm renewable generator and source of flexibility, such as a natural gas power plant (NGPP) or storage, participate in an ex-ante options market to carry out such trades. We analyze the proposed market both in a centralized and in a competitive set-up. This arrangement is transparent to the ISO, who can then treat utility-scale renewable generators as dispatchable sources. Our solution supplements and coexists with the traditional reserve market. Renewable generators can now bid more aggressively (increasing renewable penetration in the energy mix), since they do not have to pay penalty as often in case of a shortfall. On the other hand, neither the load serving entities nor the customers have to suffer the cost of providing reserves in case of renewable shortfall. Finally, the flexible generators have an assured source of revenue for providing *flexibility* rather than *quantity* of energy. The main contribution of this paper is the design and analysis of such an options-based market to show that the intuition discussed above is indeed correct. For concreteness, we focus on natural gas power plants as sources of flexibility. However, the general idea may be extended to having storage owners or other energy sources to provide a similar service.

Related Work: We refer to [4], [9], [10] and the references therein for discussions about system flexibility measures to accommodate renewables. In most deregulated energy grids, a reserve market is set up to deal with any supply shortfall in real-time. Traditionally, such shortfalls are due to unforecasted load variations or equipment malfunctions. Since these system-level errors are rare, the reserve costs are typically socialized irrespective of the contribution of a player to generating a need for such reserves [11]. With increasing renewable penetration, it is generally agreed that the need for operating reserves will

increase; it is not certain, however, how much and what type of extra reserves should be procured in the reserve market¹ [4] and who should pay for these extra reserves. There is a move towards asking renewable power producers (RPPs) to shoulder some or all of the balancing costs to counter their production variability [3], [6], [7], [15]. It has been recognized that this policy may deter renewable integration by forcing renewables to bid conservatively [8]. In our preliminary work, we showed that bilateral contracts between renewable and natural gas power plants can help counter this trend [1]. However, our previous formulation only considers one RPP-NGPP pair. In the present work, we generalize our previous model to a multi-player scenario and develop new theoretical results, such as the comparison between a centralized and a competitive case, the relaxation of the assumption on the penalty value, and the analysis of the probability of shortage in different pricing scenarios.

There is an extensive literature on the association of renewable sources with conventional generation to mitigate renewable intermittence. For instance, [16], [17] study a wind-hydro joint operation in the electricity market. However, this literature largely assumes joint ownership and operation of the two plants. Another stream of related work is on the use of storage as a palliative for the high variability of renewable production [18]–[20]. However, existing work usually assumes the storage to be owned and operated either by the renewable producer, or by the system operator. In our work, the renewable plant and the source of flexibility (e.g., NGPP) are independent players, which is the more common case in practice.

The real options literature on renewable integration is also very vast. The effect of capacity payments for gas-fired generators in the presence of increasing renewable penetration is analyzed in [21] and the economic viability of hydrogen storage as a flexible source for wind parks is studied in [22]. However, this literature is mostly focused on the decision to invest on projects when there is uncertainty from generation, policy or prices [23]. Conversely, our approach consists on the design of a real options market to incentivize generators that are already operating, aiming to increase renewable penetration even when these generators start being treated as conventional power plants. The work in [24] seems to be the closest to ours. The authors propose a call option market which decreases the volatility of the payments made to market participants in a scenario with increased renewable integration. As opposed to this work, we consider real options corresponding to trades that are not only financial, but also refer to the delivery of a physical good. Further, we consider these trades to occur prior to the day-ahead market, which couples the two markets. Note that the adoption of other types of financial mechanisms such as financial transmission rights (FTRs) has also been considered in electricity markets as a hedging mechanism [25].

Paper Organization: The remainder of this paper is organized as follows. Section II presents the electricity market model considered, and the optimization problems faced by

¹New sources of flexibility such as demand response programs [12]–[14] have been proposed.

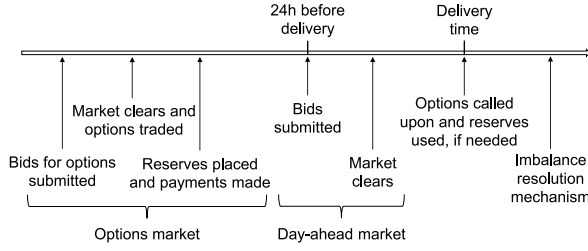


Fig. 1. Proposed electricity market timeline.

the participants. Section III derives and analyzes their optimal strategies in the coupled markets. We present our case studies in Section IV and some avenues for future work in Section V.

II. PROPOSED MARKET STRUCTURE

As illustrated in Fig. 1, we append the proposed options market to a traditional two-settlement market consisting of a day-ahead market and an ex-post imbalance resolution mechanism. We do not model the reserve market here, as its optimal redesign to deal with intermittence due to large-scale renewable integration is not clear. Instead, we transfer the responsibility of ensuring against production shortfall of renewables from the ISO to the RPPs themselves. Thus, our solution supplements and coexists with the reserve market.

A. Day-Ahead Market

The day-ahead (DA) market modeled is operated by an independent system operator (ISO). In this market, all generators (including the RPPs) bid the amount of energy they are willing to commit for delivery in the next operating day and their asking price per unit of energy. The ISO clears the market by scheduling the generators in a least-cost fashion such that the supply meets the demand. In real-time, an imbalance resolution mechanism penalizes the generators that do not supply the amount of energy that they were cleared for. Any generator (including an RPP) pays a penalty per unit of shortfall at price $\lambda_p > \lambda_{DA}$ (this penalty range is relaxed in Section III-E) if it produces below its scheduled level and is curtailed if production exceeds the commitment.

For simplicity, we focus on meeting the load at a particular time, so that issues such as start-up costs and ramp constraints for the commitment problem in the day-ahead market can be ignored. The ensuing unit commitment problem makes the mathematical analysis significantly more complex and is not in the scope of this work. For the same reason, we also ignore transmission line congestion and assume that all the scheduled generators are paid a single market price. However, we discuss the case of a constrained IEEE 14-bus system in a case study in Section IV.

We also assume that the demand is known, which is usually a good assumption [26]. Note that we do not assume the grid-takes-all-renewable paradigm; thus, the uncertainty in the renewable generation is not added to that of the load in order to define a ‘net load’, which usually has a much higher uncertainty. In any case, our results can be extended to the case in which the load is modeled as a random variable.

The DA market is typically composed of a large number of players, and thus each individual generator is considered

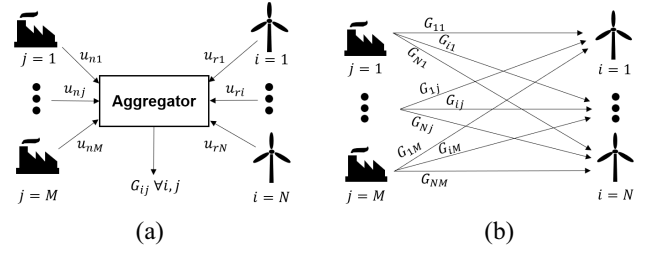


Fig. 2. Options market: centralized (a) and competitive (b) cases.

small relative to the whole market. Therefore, the generators are price takers and the DA energy price can be assumed as fixed and known. We classify the generators into three categories – renewable (which models non-firm and intermittent plants), natural gas (which models power plants with fast ramping capability), and coal (which models inflexible power plants). The RPPs considered are utility-scale solar or wind producers. We index each RPP $i \in \mathcal{I} := \{1, \dots, N\}$, NGPP $j \in \mathcal{J} := \{1, \dots, M\}$, and coal power plant (CPP) $k \in \mathcal{K} := \{1, \dots, P\}$.

We show later that, with the addition of the options market, the participating RPPs increase their DA bids at the same rate as the participating NGPPs decrease theirs. To keep these decisions from affecting the DA energy price, we consider that the subset of generators in the options market are not marginal generators in the DA market.

We assume that both NGPPs and CPPs are always able to meet their commitment and the renewable production is the only source of stochasticity in the problem, which allows us to concentrate on the effect of random generation. We assume that the renewable production R_i of each renewable player i is a random variable with continuous and twice differentiable probability density function $f_{Ri}(r_i)$ and cumulative density function $F_{Ri}(r_i)$. Further, $f_{Ri}(r_i) > 0$ for $r_i \geq 0$.

B. Market to Trade Options

In the proposed options market, each RPP $i \in \mathcal{I}$ seeks options for reserves to be used in real time in case of shortage, while each NGPP $j \in \mathcal{J}$ is a provider of these reserves. Each NGPP can sell an option to every RPP by offering a share of its available capacity to each RPP, as illustrated in Fig. 2. We assume the options are not transferable, in that the reserve provided to one particular RPP is not transferable to any other one. The lower bound on the option price is decided by the NGPPs as at least the minimum value that would make it profitable for them to sell such options. Similarly, RPPs have a maximum price they would be willing to pay for these options.

The option implies that when the renewable production is realized (delivery time in Fig. 1), the RPPs can call upon the NGPPs to realize an energy amount equal to the option purchased. We assume that following an economic dispatch strategy, the options are called upon in increasing order of production cost for the NGPPs until either the shortage is completely covered or the total amount of reserve that the RPP has the right to use is finished, whichever occurs first. If the shortage of a RPP is larger than the sum of all reserves purchased, this player incurs a penalty per unit of shortage

remaining, as established by the ex-post imbalance resolution mechanism. Note that, if a shortage does not occur, the RPP will have paid for the reserve, but the NGPP will not be called upon to produce in real-time, which makes this model resemble an insurance transaction. Without loss of generality, we set the NGPP $j = 1$ to be the least expensive one while $j = M$ to be the most expensive one.

The distribution of reserves from the NGPPs to the RPPs can be accomplished in different ways. We first use an omniscient central planner approach, in which the responsibility to allocate the reserve and set the market price is given to a non-profit seeking operator who maximizes social welfare (which we define as the sum of the expected profits of the generators in the market) with individual rationality of the participants as a constraint. We also analyze a competitive scenario, in which the players make the optimal reserve decision independently and inform the ISO with their choices. In this case, buyers (resp. sellers) are ordered in descending (resp. ascending) asking price and the ISO clears the market.²

The options market decisions occur before the DA market, and thus the outcome of these decisions will be already known when the generators decide on their DA commitments. We denote the reserve amount to the RPP i from the NGPP j as G_{ij} . We also let G_{ri} be the sum of the reserves from each NGPP j to the RPP i , and G_{nj} be the sum of the reserves for each RPP i from the NGPP j , that is $G_{ri} = \sum_{j \in \mathcal{J}} G_{ij}$ and $G_{nj} = \sum_{i \in \mathcal{I}} G_{ij}$.

C. Utility Functions

The utility function of each player is their expected profit. We first present the utility functions for a baseline case in which the players are not allowed to trade options for reserve and then when they are allowed to.

1) *Baseline Case:* The players do not interact outside the DA market. The expected profit of each RPP i is given by

$$u_{ri}(C_{ri}) = \lambda_{DA} C_{ri} - E_{Ri}[I(C_{ri} - R_i) \lambda_p (C_{ri} - R_i)], \quad (1)$$

where C_{ri} is the player's commitment in the DA market, R_i is the renewable production realized in real-time, the expectation is taken over the renewable production R_i , and the indicator function $I(\cdot)$ equals 1 for positive arguments and 0 otherwise. The first term in (1) is the DA market revenue (if its bid is selected), and the second one is the penalty charged in case of shortage. For each NGPP j , the utility function is

$$u_{nj}(C_{nj}) = \lambda_{DA} C_{nj} - \mu_{nj} C_{nj} - \kappa_{nj}(C_{nj}), \quad (2)$$

where C_{nj} is the DA market commitment and μ_{nj} is the O&M cost per unit of production for actual operation of the plant. The function $\kappa_{nj}(\cdot)$ is the fuel cost which varies with the desired production output. Following [25], we assume

$$\kappa_{nj}(P_{nj}) = a_{nj} + b_{nj} P_{nj} + c_{nj} P_{nj}^2. \quad (3)$$

Both the function parameters and the O&M cost are positive. For each CPP k , letting C_{ck} be the player's DA commitment,

$$u_{ck}(C_{ck}) = \lambda_{DA} C_{ck} - \mu_{ck} C_{ck} - \kappa_{ck}(C_{ck}). \quad (4)$$

²As proved in [27], such contingent contracts can be cleared by the ISO without violating network constraints through a coordinated trading process.

The CPP also earns a revenue from committing to the market, which is the first term of (4). The second term is the O&M cost, and the third one is the fuel cost. For a production P_{ck} ,

$$\kappa_{ck}(P_{ck}) = a_{ck} + b_{ck} P_{ck} + c_{ck} P_{ck}^2. \quad (5)$$

2) *With Options Market:* The market structure allows the NGPPs and the RPPs to participate in an ex-ante options market. The expected profit of each RPP i is given by

$$u_{ri}(C_{ri}, G_{ri}, \pi) = \lambda_{DA} C_{ri} - \pi G_{ri} - E_{Ri}[I(C_{ri} - R_i - G_{ri}) \lambda_p (C_{ri} - R_i - G_{ri})], \quad (6)$$

where G_{ri} and π are the reserve purchased and the per unit price of reserve, respectively. Similarly to the baseline case, the first term in (6) is the revenue from the DA market. The second term is the reserve cost, and the third one is the expected penalty in case of shortage remaining after the reserve is fully used. For each NGPP j , the utility function becomes

$$\begin{aligned} u_{nj}(C_{nj}, G_{nj}, \pi) &= (\lambda_{DA} - \mu_{nj}) C_{nj} + \pi G_{nj} \\ &\quad - \kappa_{nj}(C_{nj} + G_{nj}) \\ &\quad - \sum_{i=1}^N E_{Ri} \left[I \left(C_{ri} - R_i - \sum_{m=1}^j G_{im} \right) \mu_{nj} G_{ij} \right] \\ &\quad - \sum_{i=1}^N E_{Ri} \left[I \left(C_{ri} - R_i - \sum_{m=1}^{j-1} G_{im} \right) I \left(\sum_{m=1}^j G_{im} - (C_{ri} - R_i) \right) \right. \\ &\quad \left. \mu_{nj} \left(C_{ri} - R_i - \sum_{m=1}^{j-1} G_{im} \right) \right]. \end{aligned} \quad (7)$$

Each NGPP incurs the cost of purchasing fuel to meet the commitment C_{nj} and to maintain a reserve enough to produce up to G_{nj} in case of renewable shortage. The last two terms in (7) are the expected O&M costs in case of shortage. In the third line, each expectation in the sum indicates the cost incurred by the NGPP j if this player is expected to produce all the reserve that was provided to the RPP i . This will happen if, after completely using the reserves from all NGPPs less expensive than j , the shortage remaining in the RPP i is still greater than or equal to G_{ij} . The last term refers to the expected cost when the NGPP j does not need to produce all of its reserve. For each RPP i , this scenario occurs when the shortage remaining when the NGPP j is called to produce is less than G_{ij} . The participation of the CPPs is still constrained to the DA market, and thus their utility functions remain as in (4).

From this point forward, we abbreviate $u_{ri}(C_{ri}, G_{ri}, \pi)$, $u_{nj}(C_{nj}, G_{nj}, \pi)$, and $u_{ck}(C_{ck})$ as u_{ri} , u_{nj} , and u_{ck} .

a) *Optimization problems:* In the options market, for the case in which a central planner is tasked with making the reserve decisions, we have the problem

$$\mathcal{P}_1: \max_{G_{11}, \dots, G_{1N}, G_{21}, \dots, G_{NM}} u_{sw} = \sum_{i \in \mathcal{I}} u_{ri} + \sum_{j \in \mathcal{J}} u_{nj} \quad (8)$$

$$\text{s.t. } G_{ij} \geq 0 \quad \forall (i, j) \in \mathcal{I} \times \mathcal{J} \quad (9)$$

$$\sum_{i \in \mathcal{I}} G_{ij} \leq G_{maxj} \quad \forall j \in \mathcal{J}, \quad (10)$$

where u_{sw} is the social welfare and G_{maxj} is the maximum reserve that can be offered by the NGPP j . Further, the aggregator must also set a price per unit of reserve that ensures

individual rationality, so that none of the generators are worse off by participating in the market. If each generator makes the reserve decision individually, we have a competitive setting in which each RPP i solves the problem $\mathcal{P}_2$, and each NGPP j solves $\mathcal{P}_3$, both also subject to constraints (9) and (10).

$$\mathcal{P}_2: \max_{G_{r1}, \dots, G_{rM}} u_{ri}, \quad \mathcal{P}_3: \max_{G_{1j}, \dots, G_{Nj}} u_{nj}. \quad (11)$$

Since the options market occurs *ex-ante*, the reserves G_{ri} and G_{nj} are already known when the bids are placed in the DA market, and thus are treated as given. Then, we define the optimization problem for each RPP, NGPP, and CPP in the DA market as

$$\mathcal{P}_4: \max_{C_{ri} \geq 0} u_{ri}, \quad \mathcal{P}_5: \max_{C_{nj} \geq 0} u_{nj}, \quad \mathcal{P}_6: \max_{C_{ck} \geq 0} u_{ck}.$$

III. MAIN RESULTS

For simplicity, we first neglect the upper bound constraints of the generators, which are then considered in Section III-E. We first derive the optimal commitments in the DA market and then the optimal strategies for the options market.

A. Optimal Commitments

Theorem 1: The optimal commitment strategies for the players in the day-ahead market are

- *Baseline Case:*

$$C_{ri}^*(\lambda_{DA}) = \sigma_i \quad (12)$$

$$C_{nj}^*(\lambda_{DA}) = \frac{\lambda_{DA} - b_{nj} - \mu_{nj}}{2c_{nj}} \quad (13)$$

$$C_{ck}^*(\lambda_{DA}) = \frac{\lambda_{DA} - b_{ck} - \mu_{ck}}{2c_{ck}} \quad (14)$$

- *Options Market Case:*

$$C_{ri}^*(\lambda_{DA}) = G_{ri}^* + \sigma_i \quad (15)$$

$$C_{nj}^*(\lambda_{DA}) = \frac{\lambda_{DA} - b_{nj} - \mu_{nj}}{2c_{nj}} - G_{nj}^* \quad (16)$$

$$C_{ck}^*(\lambda_{DA}) = \frac{\lambda_{DA} - b_{ck} - \mu_{ck}}{2c_{ck}}, \quad (17)$$

where

$$\sigma_i \triangleq F_{Ri}^{-1}(\lambda_{DA}/\lambda_p).$$

Proof: The utility functions are concave in the commitments.

Baseline Case: We take the derivative of (1) with respect to C_{ri} to find the optimal commitment of each RPP.

$$\frac{\partial u_{ri}}{\partial C_{ri}} = \lambda_{DA} - \lambda_p F_{Ri}(C_{ri}) \Rightarrow C_{ri}^* = \sigma_i.$$

Similarly, the first order conditions are also used to find the optimal commitments for the NGPPs and CPPs.

Options Market Case: Taking the derivative of (6) with respect to C_{ri} , we find the optimal RPP commitment.

$$\begin{aligned} \frac{\partial u_{ri}}{\partial C_{ri}} &= \lambda_{DA} - \lambda_p F_{Ri}(C_{ri} - G_{ri}) = 0 \\ \Rightarrow C_{ri}^* &= G_{ri}^* + \sigma_i. \end{aligned} \quad (18)$$

Similarly, for NGPPs, the optimal commitment is derived from (7) while the optimal commitment for CPPs is unchanged. ■

B. Optimal Reserve Distribution With the Central Planner

As the DA commitment of each player must be non-negative, we note from the NGPP commitment (16) that there is an upper bound on the amount of reserve that each NGPP may provide for the RPPs. Then, for each NGPP j ,

$$G_{maxj} = \frac{\lambda_{DA} - b_{nj} - \mu_{nj}}{2c_{nj}}. \quad (19)$$

The following lemma on the concavity of the utility functions is proved in Appendix A.

Lemma 1: The social welfare u_{sw} is concave in $(G_{11}, \dots, G_{NM})$ and attains a maximum at some boundary of the convex set established by constraints in Problem $\mathcal{P}_1$. On the other hand, u_{nj} is convex in $(G_{1j}, \dots, G_{Nj})$ for every NGPP $j \in \mathcal{J}$.

Theorem 2: The optimal reserve distribution that maximizes the social welfare satisfies the rules

$$\begin{aligned} F_{R1}\left(\sigma_1 + \sum_{j=1}^M G_{1j}\right) &= \dots = F_{RN}\left(\sigma_N + \sum_{j=1}^M G_{Nj}\right) \\ F_{R1}\left(\sigma_1 + \sum_{j=2}^M G_{1j}\right) &= \dots = F_{RN}\left(\sigma_N + \sum_{j=2}^M G_{Nj}\right) \\ &\vdots \\ F_{R1}(\sigma_1 + G_{1M}) &= \dots = F_{RN}(\sigma_N + G_{NM}) \end{aligned} \quad (20)$$

Further, all NGPPs provide their maximum reserve, and the RPPs purchase all reserve provided.

Proof: See Appendix B. ■

Remark 1: The optimal reserve distribution in (20) is such that the probability that each NGPP will produce for a certain RPP is the same as that for any other RPP.

Corollary 1: The optimal reserve distribution that maximizes the social welfare when the market has one NGPP and multiple RPPs is such that the probability of shortage (prior to the reserve use) of all RPPs are equal. Therefore,

$$F_{R1}(C_{r1}^*) = F_{R2}(C_{r2}^*) = \dots = F_{RN}(C_{rN}^*). \quad (21)$$

Proof: Proof follows from Theorem 2 for $M = 1$. ■

We now prove the existence of a price that satisfies individual rationality.

Theorem 3: For all $i \in \mathcal{I}$, the range for the reserve price

$$\pi = \left[\lambda_{DA} - \mu_{n1} \left(1 - F_{Ri} \left(\sigma_i + \sum_{j \in \mathcal{J}} G_{ij} \right) \right), \lambda_{DA} \right]$$

satisfies individual rationality for all the generators.

Proof: See Appendix C. ■

Unlike the RPPs, each NGPP will set a different lower bound on the reserve price, which depends on their expected production cost. However, for all NGPPs, this bound is less than or equal to λ_{DA} , which is the upper bound set by the

RPPs. Further, the feasible price range shrinks as the probability that the first NGPP will be called upon to produce in real time increases. If a shortage surely happens, the price interval is reduced to the point $\pi = \lambda_{DA}$, in which case the expected profit of all RPPs and of the NGPPs that will produce for sure will be the same as in the baseline case, while the NGPPs that will not necessarily produce will experience an increase in expected profit. Thus, no generator is ever worse off by joining the options market.

C. Optimal Reserve Distribution - Competitive Case

This section considers a competitive case for the options market in which the players simultaneously decide on the reserve amount that optimizes their individual expected profit, as well as on the per unit price that makes their participation in the market individual rational. Therefore, the action spaces of each RPP i and each NGPP j are defined as

$$\mathcal{A}_{ri} = \{G_{ij} \forall j, \bar{\pi}_{ri}\} \text{ and } \mathcal{A}_{nj} = \{G_{ij} \forall i, \underline{\pi}_{nj}\},$$

respectively. As buyers, each RPP has an upper bound $\bar{\pi}_{ri}$ on how much they are willing to pay for a reserve, while each NGPP j has a lower bound $\underline{\pi}_{nj}$ on how much they are willing to accept to provide one. We remark that these decisions may be in the form of functions that are used by the central entity that clears the options market. Besides the knowledge of their own utility function, the information sets of the players in the market are given by

$$\mathcal{H}_{ri} = \{C_{ri}^*, f_{Ri}\} \text{ and } \mathcal{H}_{nj} = \{C_{nj}^*\}$$

for each RPP i and each NGPP j , respectively, where the optimal commitments are given as in (15) and (16). With that, we consider the NGPPs do not know the optimal commitment expressions for the RPPs, and thus they solve their optimization problem to find their best response to a fixed C_{ri} .

Theorem 4: In the competitive scenario, the lower bound set by each NGPP on the reserve price is

$$\underline{\pi}_{nj} = \lambda_{DA} - \mu_{nj} \left(1 - F_{Ri} \left(C_{ri}^* - \sum_{m=1}^j G_{im} \right) \right) \forall j,$$

which is less than the one with a central planner, thus increasing the range of feasible prices.

Proof: See Appendix D. ■

Note that the lower the O&M cost of an NGPP is, the higher is its asking price. This is so because the least expensive generators are called upon in real time with a higher probability and thus require a higher compensation for bearing that risk. Further, the decrease in the lower bound for the reserve price is due to *information asymmetry*. The lack of information about C_{ri}^* implies that the NGPPs are not aware that the RPPs will increase their optimal day-ahead commitment as they purchase more reserve. This is an example of moral hazard, which occurs when an agent behaves in a riskier way because someone else bears the cost of this increased risk.

The players inform their decisions to a central operator whose information set is assumed to be $\mathcal{H}_C = \mathcal{H}_{ri} \cup \mathcal{H}_{nj} \cup \mathcal{A}_{ri} \cup \mathcal{A}_{nj} \cup \{\mu_{nj} \forall j\}$. Then, the sellers (resp. buyers) are ordered

in increasing (resp. decreasing) asking price $\underline{\pi}_{nj}$ (resp. $\bar{\pi}_{ri}$). Given that there exists a range of prices for which the market clears, we present in the next section examples of pricing schemes the operator may utilize.

D. Feasible Pricing Mechanisms for the Options Market

The following pricing schemes can be used to achieve individual rationality, efficiency, and budget balance. Note that the prices are determined based on the bounds established by the generators, and thus reflect their willingness to participate in the market.

Example 1: The prices can be chosen so that the total surplus achieved in the options market is equally divided among the participant players. In this case, each RPP pays a total of $\lambda_{DA} G_{ri} - \frac{S}{N+M}$ and each NGPP receives a total of

$$\left[\lambda_{DA} - \mu_{nj} \left(1 - F_{Ri} \left(\sigma_i + \sum_{m=j+1}^M G_{im} \right) \right) \right] G_{nj} + \frac{S}{N+M},$$

where S is the market surplus, given by

$$S = \sum_{j \in \mathcal{J}} G_{maxj} \mu_{nj} \left(1 - F_{Ri} \left(\sigma_i + \sum_{m=j+1}^M G_{im} \right) \right).$$

We find S by checking how much surplus would be available if every RPP paid the price that makes their change in profit (from the baseline to the options market case) be zero, and every NGPP got paid the value that enforces this zero profit condition. Such prices are the lower (resp. upper) bounds set by each seller (resp. buyer). Note that the share of surplus each player receives tends to zero as the number of players increases. Also, this scheme does not have a single market clearing price.

Example 2: If a single clearing market price is desired, the price can be set at the average of the feasible price range,

$$\pi = \lambda_{DA} - \frac{\mu_{n1} \left(1 - F_{Ri} \left(\sigma_i + \sum_{m=2}^M G_{im} \right) \right)}{2}.$$

E. Bounded Capacity and Flexible Penalty Considerations

In the previous sections, we assumed that the generators had unbounded capacities, and that the penalty for a shortage λ_p was greater than the DA energy price λ_{DA} . For a competitive set-up in the options market, we now evaluate our solution by removing these assumptions. Let $\bar{P}_{ri}$ and $\bar{P}_{nj}$ be the maximum capacity of each RPP $i \in \mathcal{I}$ and NGPP $j \in \mathcal{J}$, respectively. The following result can be proven similarly to Theorems 1 and 4 and summarizes the optimal strategies of these players with the new considerations.

Proposition 1: The optimal day-ahead commitment C_{ri}^* and the optimal reserve G_{ri}^* for each RPP $i \in \mathcal{I}$ are given by

$$C_{ri}^* = \bar{P}_{ri} \quad (22)$$

$$G_{ri}^* = \begin{cases} \bar{P}_{ri} - F_{Ri}^{-1}(\pi/\lambda_p), & \text{if } \pi \leq \min(\lambda_{DA}, \lambda_p) \\ 0, & \text{otherwise.} \end{cases} \quad (23)$$

Further, the optimal day-ahead commitment C_{nj}^* for each NGPP $j \in \mathcal{J}$ is given by (16). Finally, the optimal bid in

TABLE I
SCENARIOS FOR PROBABILITY OF SHORTAGE ANALYSIS

	Scenario 1	Scenario 2	Scenario 3
Penalty per unit of shortage	$\lambda_p = 0$	$\lambda_p > 0$	$\lambda_p > 0$
Options market	Absent	Absent	Present

the options market for this player is such that the maximum reserve is offered, as long as the asking price

$$\pi \geq \lambda_{DA} - \mu_{nj} \left(1 - F_{Ri} \left(\bar{P}_{ri} - \sum_{m=1}^j G_{im} \right) \right) \quad \forall i$$

is accepted. The reserves G_{ij} are split following the rule

$$F_{R1} \left(\bar{P}_{r1} - \sum_{m=1}^j G_{1m} \right) = \dots = F_{RN} \left(\bar{P}_{rN} - \sum_{m=1}^j G_{Nm} \right).$$

Note that when $\lambda_p > \lambda_{DA}$, the RPPs first find their optimal DA commitment as in (15). Then, provided that $\pi \leq \lambda_{DA}$, they procure a reserve amount of $\bar{P}_{ri} - F_{Ri}^{-1}(\lambda_{DA}/\lambda_p)$ to increase their bid up to their maximum capacity (22), and an amount of $F_{Ri}^{-1}(\lambda_{DA}/\lambda_p) - F_{Ri}^{-1}(\pi/\lambda_p)$ to make their expected per unit penalty payment equal their per unit reserve cost, as in (24). On the other hand, when $\lambda_p \leq \lambda_{DA}$, the RPPs bid all their capacity in the DA market, and for any $\pi \leq \lambda_p$, their optimal reserve decision (23) is such that the expected per unit penalty cost equals the per unit price of reserve, i.e.,

$$\lambda_p F_{Ri}(\bar{P}_{ri} - G_{ri}^*) = \pi. \quad (24)$$

In either case, we note that (23) holds.

F. Effect on Renewable Participation

We use the results above to find the probability of shortage remaining after the use of reserve for each RPP, for the three scenarios in Table I. This remaining shortage is the portion that will need to be covered in real-time by peaker plants called upon by the ISO. Scenario 1 is equivalent to a take-all-renewables operation, and RPPs are not penalized for unmet commitments. The second one adopts a penalty for shortage, but does not give RPPs access to the options market. The last one penalizes RPPs if a shortage occurs, but allows them to have an option to procure their own reserve.

Proposition 2: The probability of a RPP having a shortage after using the reserves purchased is non-increasing from Scenario 1 through 3, and strictly decreasing if $\lambda_p > \lambda_{DA} > \pi$.

Proof: The probability of remaining shortage for RPP i is $p_s = F_{Ri}(C_{ri}^* - G_{ri}^*)$. For Scenario 1, from the RPP baseline commitment (12), we find that $C_{ri}^* \rightarrow \bar{P}_{ri}$ as $\lambda_p \rightarrow 0$. In this case, $G_{ri}^* = 0$, and thus $p_s = F_{Ri}(\bar{P}_{ri}) = 1$. Then, the RPP will always bid its entire capacity and will almost surely produce less than that. For Scenario 2, proceeding similarly to Scenario 1, we find

$$p_s = \begin{cases} F_{Ri}(\bar{P}_{ri}) = 1, & \text{if } \lambda_p \leq \lambda_{DA} \\ F_{Ri}(F_{Ri}^{-1}(\lambda_{DA}/\lambda_p)) = \lambda_{DA}/\lambda_p, & \text{otherwise.} \end{cases}$$

We note that low penalties do not help towards decreasing the probability of shortage. However, if $\lambda_p > \lambda_{DA}$, RPPs will

TABLE II
NGPP FUEL FUNCTION PARAMETERS AND RPP PRODUCTION DISTRIBUTIONS (HOUR = 13)

	$a(\text{MBtu}/h)$	$b(\text{MBtu}/\text{MWh})$	$c(\text{MBtu}/(\text{MW})^2 h)$
$j = 1$	130.0021	10.6121	0.0050
	α	β	σ/μ
$i = 1$	0.4316	1.450	1.08
$i = 2$	0.4333	1.249	1.04
$i = 3$	0.4258	1.672	1.13
$i = 4$	0.4548	1.720	1.09

respond by lowering their commitment, leading to $p_s < 1$. For Scenario 3, from Proposition 1 for the cases in which the RPP procures a non-zero reserve (i.e., when $\pi \leq \min(\lambda_{DA}, \lambda_p)$),

$$p_s = F_{Ri}(\bar{P}_{ri} - \bar{P}_{ri} + F_{Ri}^{-1}(\pi/\lambda_p)) = \pi/\lambda_p. \quad \blacksquare$$

Proposition 2 shows that the proposed options market not only enables RPPs to increase their participation without decreasing their expected profit, but also enhances grid reliability by decreasing the probability of shortage of these plants.

IV. CASE STUDY

We analyze a scenario in which a subset of the generators in the electricity market also participates in the options market. The distributions for the renewable productions were estimated from the 2012 data for wind farms in Illinois from the Wind Integration National Dataset Toolkit [28]. For this estimation, we assumed the values to be independent across multiple days. Following [29], we fitted a Beta distribution for the hourly production in each site, for each hour of the day, using a 92-day period (May 1 – July 31).

We first analyze a case in which all generators are paid the same energy price, which is equivalent to having an uncongested network. The parameters in Table II for the NGPP were selected from the IEEE 118-Bus System data [30] and refer to the heat rate curve of a combined cycle unit. To build the fuel cost function from these parameters, we multiply them by \$2.9505/MBtu, which was the average natural gas price sold to the electric power sector in Illinois in 2018 for the months considered [31]. The variable O&M costs for this plant correspond to an advanced combined cycle plant with $\mu_{n1} = \$2/\text{MWh}$ [32].

The DA market prices used are from the Illinois hub, which is within the Midcontinent ISO footprint. As these prices are correlated with natural gas prices, we selected the values in [33] that correspond to 2018. For each hour, λ_{DA} was taken as the average observed for that hour during the 92-day period evaluated. Further, we consider a scenario in which the penalty for shortages for each hour is 1.75 times the corresponding DA price. This situation is likely to occur in hours of peaking demand, when the cost of the resources needed to serve load when a contingency happens is likely to be much higher than the cost of the last unit needed in moderate system conditions [34]. Thus, we proceed to evaluate the real options market from hour 13 to hour 18, when demand peaks in the Illinois hub during the days considered.

Fig. 3 shows the increase in renewable production taken by the grid in real-time for each of the 4 RPPs studied during

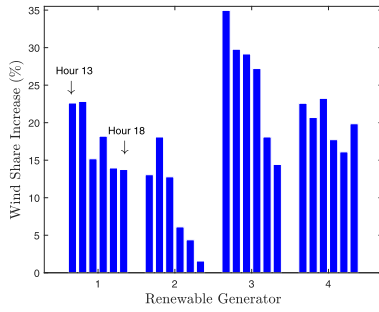


Fig. 3. Average wind production share increase.

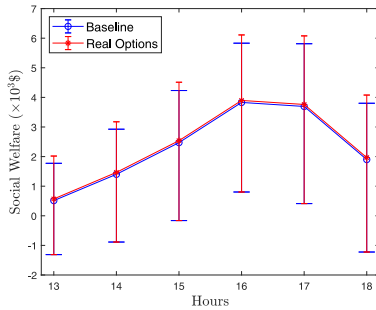


Fig. 4. Realized social welfare.

the hours evaluated. The increase is measured by comparing the results between the options market and the baseline cases, and the values plotted are averages of 2000 simulation runs. We note that the overall renewable participation is improved if these plants purchase real options for reserve. Interestingly, the second RPP experiences the most conservative increase in the average amount of production taken by the grid. A closer look at the data reveals that this RPP is the one with the least variability, having the lowest standard deviation to mean ratio (σ/μ) at every hour. This measure of dispersion attains the lowest values for this RPP in the last two hours, which means that production uncertainty is low and explains why the participation improvement of this RPP is less at these times.

We also analyzed the changes in social welfare due to the adoption of real options. The average values achieved with 2000 runs are plotted in Fig. 4, along with vertical bars that indicate the minimum and maximum values found. It can be observed that the average social welfare is consistently higher in the scenario with real options, and the relative improvements are presented in Table III. We remark that these improvements are achieved without decreasing the expected profits of any individual generator.

The worst-case scenarios generated, corresponding to the lower caps on the error bars in Fig. 4, refer to cases with near-zero renewable production. In such cases, the social welfare may become negative due to the large penalties for deviations. However, the addition of real options does not change the worst-case scenarios, while increasing the maximum achievable social welfare. Similar results are found if we analyze the social welfare as we increase wind production, as can be seen from Fig. 5. The social welfare presented corresponds to the 14th hour of the day and the

TABLE III
AVERAGE SOCIAL WELFARE ($\times 10^3\$$)

Hour	13	14	15	16	17	18
Baseline	0.513	1.401	2.472	3.823	3.694	1.900
Real options	0.570	1.467	2.539	3.897	3.761	1.967
Increase (%)	11.19	4.67	2.70	1.77	1.83	3.52

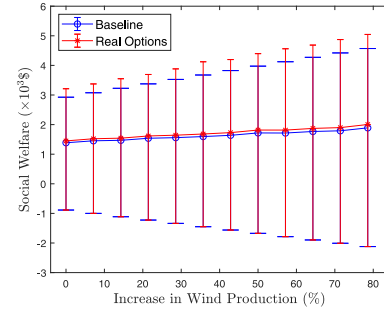


Fig. 5. Realized social welfare with increasing wind production.

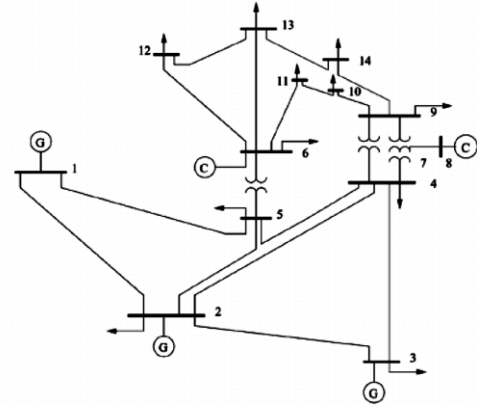


Fig. 6. IEEE 14-bus test system.

increase in wind production is with respect to the wind data used in the previous analysis. With higher wind production, we also observe an increase in production variability, leading to a more variable social welfare. Nonetheless, we still achieve a steady increase in the average social welfare and the worst-case remains unchanged.

Lastly, we evaluate how the decisions of the generators change on a modified IEEE 14-bus system. We consider a case with two RPPs and one flexible generator. The conventional generator data was adopted from MATPOWER [35]. We add two wind generators with 32MW capacity, assume all transmission lines have a 75MW transmission capacity, and let the generator in node 3 be the one that offers real options for reserve.

In this constrained case, the generators decide on their bounds on the reserve price based on the locational marginal price (LMP) they are subject to, and thus participating in the real options market might not always be individually rational for all generators. To exemplify this, we solve for the economic dispatch of the generators in this network for the 13th hour of the day, as we vary the location of the wind power plants. We simulate a scenario in which the RPPs are scheduled based on their expected production, as well as in a case in which they

increase their offer up to their capacity with the addition of real options. As in the previous analysis, we also observe an increase in social welfare and in renewable participation when the real options market clears. Further observations made by evaluating the individual rationality of each generator are:

- It is individually rational for all three generators to trade real options if they are co-located. This is in accordance to our theoretical results and previous numerical analysis.
- A RPP located at bus 1 will not be cleared in the options market. The congestion in this network is such that the LMP at node 1 is much lower than that at node 3, and thus the upper bound set on the reserve price by this RPP is too low to be accepted by the generator at node 3.
- A RPP located at bus 2 will be cleared in the options market. In this case, the LMP difference between nodes 2 and 3 is such that the bounds set on the reserve price give rise to a non-empty interval that satisfies the individual rationality of both the buyer and the seller.

The possibility of adding extra revenues obtained through FTRs is left as a direction for future work. This income may be used to offset losses when the NGPP is unable to transfer the reserve due to congestion, or when the generators are located at nodes in which the LMPs are such that they may not be cleared in the real options market.

V. CONCLUSION AND FUTURE WORK

We modeled a two-settlement electricity market in which renewable, natural gas, and coal power plants bid to be scheduled for generation in the next operating day. An ex-ante real options market was designed, in which RPPs purchase an option to call upon energy reserves to be used in case of shortage, while NGPPs offer such reserves. We derived expressions for the optimal reserve distribution, and showed that the proposed options market increases the share of renewable energy in the grid while not decreasing the profit of any generator. In future work, we aim to evaluate the effect of network congestion on the options market, as well as to explore other flexible energy sources as providers of reserve.

APPENDIX A PROOF OF LEMMA 1

Substituting the optimal strategies (15) and (16) in (8), we use backwards induction to rewrite the social welfare function. Then, for all $(i, j) \in \mathcal{I} \times \mathcal{J}$, the first and second derivatives of u_{sw} with respect to the decision variables become

$$\begin{aligned} \frac{\partial u_{sw}}{\partial G_{ij}} &= \mu_{nj} - (\mu_{nj} - \mu_{n(j-1)})F_{Ri} \left(\sigma_i + \sum_{m=j}^M G_{im} \right) \\ &\quad - \dots - \mu_{n1}F_{Ri} \left(\sigma_i + \sum_{m=1}^M G_{im} \right) \end{aligned} \quad (25)$$

$$\begin{aligned} \frac{\partial^2 u_{sw}}{\partial G_{ij}^2} &= -(\mu_{nj} - \mu_{n(j-1)})f_{Ri} \left(\sigma_i + \sum_{m=j}^M G_{im} \right) \\ &\quad - \dots - \mu_{n1}f_{Ri} \left(\sigma_i + \sum_{m=1}^M G_{im} \right) \end{aligned} \quad (26)$$

$$\frac{\partial^2 u_{sw}}{\partial G_{ij} \partial G_{nm}} = 0 \quad \forall (i, n, j, m) \in \mathcal{I}^2 \times \mathcal{J}^2, i \neq n \quad (27)$$

$$\frac{\partial^2 u_{sw}}{\partial G_{ij} \partial G_{im}} = \frac{\partial^2 u_{sw}}{\partial G_{i \min(j,m)}^2} \quad \forall (i, j, m) \in \mathcal{I} \times \mathcal{J}^2 \quad (28)$$

Note that all the second derivatives are non-positive. It can be shown that the Hessian of u_{sw} is negative definite. Thus, the social welfare is strictly concave in $(G_{11}, \dots, G_{NM})$. We look for candidates for maximum in the convex set defined by constraints (9) and (10). If $(G_{11}^*, \dots, G_{NM}^*)$ is an interior maximum, then it is a stationary point. However, from (25), we note that the first order conditions only hold if $G_{im}^* \rightarrow \infty$ for all i , which is not true. Thus, the function attains a maximum at some boundary. This concludes the first item in Lemma 1.

For the second item, we consider the NGPP j does not know the expressions for the renewable optimal commitments and participates in the options market. We use backwards induction to substitute the optimal NGPP commitment (16) in the utility function (7). The corresponding first and second order derivatives are

$$\begin{aligned} \frac{\partial u_{nj}}{\partial G_{ij}} &= \pi - \lambda_{DA} \\ &\quad + \mu_{nj} \left(1 - F_{Ri} \left(C_{ri} - \sum_{m=1}^j G_{im} \right) \right) \forall i \in \mathcal{I} \end{aligned} \quad (29)$$

$$\frac{\partial^2 u_{nj}}{\partial G_{ij}^2} = \mu_{nj} f_{Ri} \left(C_{ri} - \sum_{m=1}^j G_{im} \right) \forall i \in \mathcal{I} \quad (30)$$

$$\frac{\partial^2 u_{nj}}{\partial G_{ij} \partial G_{nj}} = 0 \quad \forall (i, n) \in \mathcal{I}^2, i \neq n \quad (31)$$

It is readily seen that the Hessian of this function is positive definite for any NGPP j . Thus, u_{nj} is convex in $(G_{1j}, \dots, G_{Nj})$ and attains its maximum at some boundary.

APPENDIX B PROOF OF THEOREM 2

With a central planner, the reserve payments are an internal monetary transfer among the players and cancel out in the social welfare function. We write u_{sw} using backwards induction. Recalling Lemma 1, we use the Karush-Kuhn-Tucker (KKT) conditions to identify the boundary at which u_{sw} is maximum. The Lagrangian of the function is

$$\mathcal{L} = u_{sw} + \sum_{j \in \mathcal{J}} \lambda_j \left(G_{maxj} - \sum_{i \in \mathcal{I}} G_{ij} \right) + \sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{I}} \lambda_{Mi+j} G_{ij}$$

Let $\mathcal{M}_P := \{1, \dots, M(N+1)\}$ denote the set of indices for the multipliers in this central planner case. Using the first order derivatives from Appendix A, the KKT conditions are

$$\frac{\partial \mathcal{L}}{\partial G_{ij}} = \frac{\partial u_{sw}}{\partial G_{ij}} - \lambda_j + \lambda_{Mj+i} = 0 \quad \forall (i, j) \in \mathcal{I} \times \mathcal{J} \quad (32)$$

$$\lambda_j \left(G_{maxj} - \sum_{i \in \mathcal{I}} G_{ij} \right) = 0 \quad \forall j \in \mathcal{J} \quad (33)$$

$$\lambda_{Mj+i} G_{ij} = 0 \quad \forall (i, j) \in \mathcal{I} \times \mathcal{J} \quad (34)$$

$$\lambda_n \geq 0 \quad \forall n \in \mathcal{M}_P \quad (35)$$

The candidates for maximum are at the boundaries. If constraint (10) is tight for all $j \in \mathcal{J}$ while constraint (9) is never tight, that is,

$$G_{maxj} - \sum_{i \in \mathcal{I}} G_{ij} = 0 \quad \forall j \in \mathcal{J}$$

$$G_{ij} > 0 \quad \forall (i, j) \in \mathcal{I} \times \mathcal{J},$$

then we must have $\lambda_j \geq 0 \quad \forall j \in \mathcal{J}$, and $\lambda_{Mj+i} = 0 \quad \forall (i, j) \in \mathcal{I} \times \mathcal{J}$ for conditions (33) and (34) to be satisfied. From this, we use (32) to find that the value of each multiplier λ_j is simply the first derivative of the social welfare function with respect to $G_{ij} \quad \forall i$, which is expressed as in (25) and can be shown to be greater than zero. Therefore, condition (32) imposes the equilibrium condition

$$\frac{\partial u_{sw}}{\partial G_{1j}} = \frac{\partial u_{sw}}{\partial G_{2j}} = \dots = \frac{\partial u_{sw}}{\partial G_{Nj}} \quad \forall j \in \mathcal{J}. \quad (36)$$

For each NGPP j , we can use (25) to simplify this condition, from which the expressions (20) can be found. Thus all KKT conditions are satisfied and this solution is a maximum.

APPENDIX C PROOF OF THEOREM 3

We verify at which price each player accepts to participate in the options market, considering the amount traded is the maximum. For each RPP i , substituting the optimal commitment (15) in its expected profit (6), we find

$$u_{ri} = (\lambda_{DA} - \pi)G_{ri} + \lambda_{DA}\sigma_i - E_{Ri}[I(\sigma_i - R)\lambda_p(\sigma_i - R)], \quad (37)$$

where the last two terms form the utility of the player without a reserve. It follows that the utility of the RPPs remains the same from the baseline to the options market cases if $\pi = \lambda_{DA}$, and is strictly increasing if $\pi < \lambda_{DA}$. Thus, as buyers, the RPPs set $\bar{\pi}_{ri} = \lambda_{DA}$ as the upper bound on the contract price.

Since we assume full information, we can rewrite the utility for the NGPP j using backwards induction and substituting the expression for the RPP commitments (15). Then, we have

$$u_{nj} = (\lambda_{DA} - \mu_{nj})(G_{maxj} - G_{nj}) + \pi G_{nj} - \kappa_{nj}(G_{maxj})$$

$$- \sum_{i \in \mathcal{I}} E_{Ri} \left[I \left(\sigma_i + \sum_{m=j+1}^M G_{im} - R_i \right) \mu_{nj} G_{ij} \right]$$

$$- \sum_{i \in \mathcal{I}} E_{Ri} \left[I \left(\sigma_i + \sum_{m=j}^M G_{im} - R_i \right) I \left(R_i - \sigma_i - \sum_{m=j+1}^M G_{im} \right) \right]$$

$$\mu_{nj} \left(\sigma_i + \sum_{m=j}^M G_{im} - R_i \right). \quad (38)$$

The Lagrangian of this function is

$$\mathcal{L}_j = u_{nj} + \lambda_0 \left(G_{maxj} - \sum_{i \in \mathcal{I}} G_{ij} \right) + \sum_{i \in \mathcal{I}} \lambda_i G_{ij},$$

from which we find the KKT conditions

$$\frac{\partial \mathcal{L}_j}{\partial G_{ij}} = \frac{\partial u_{nj}}{\partial G_{ij}} - \lambda_0 + \lambda_i = 0 \quad \forall i \in \mathcal{I} \quad (39)$$

$$\lambda_0 \left(G_{maxj} - \sum_{i \in \mathcal{I}} G_{ij} \right) = 0 \quad (40)$$

$$\lambda_i G_{ij} = 0 \quad \forall i \in \mathcal{I} \quad (41)$$

$$\lambda_n \geq 0 \quad \forall n \in [0, \dots, N], \quad (42)$$

where

$$\frac{\partial u_{nj}}{\partial G_{ij}} = \pi - \lambda_{DA} + \mu_{nj} \left(1 - F_{Ri} \left(\sigma_i + \sum_{m=j}^M G_{im} \right) \right) \quad \forall i \in \mathcal{I}$$

Let (10) be the only tight constraint for the NGPP j . Then, we must have $\lambda_0 \geq 0$ and $\lambda_i = 0 \quad \forall i$ for conditions (40) and (41) to be satisfied. From this, it is straightforward to show that the remaining KKT conditions also hold if

$$\pi \geq \lambda_{DA} - \mu_{nj} \left(1 - F_{Ri} \left(\sigma_i + \sum_{m=j}^M G_{im} \right) \right) \quad \forall j. \quad (43)$$

Combining the bounds of all NGPPs, the maximum lower bound (supremum) is found when the second term in the right hand side of (43) is minimum. This term is a product of two positive non-decreasing functions from $j = 1$ to $j = M$, and thus it is minimum for the NGPP $j = 1$, who has the lowest μ_{nj} and the highest $F_{Ri}(\sigma_i + \sum_{m=j}^M G_{im})$. With that, we find the price range presented in Theorem 3.

APPENDIX D PROOF OF THEOREM 4

Using backwards induction for the RPPs, we reach the same expression as in (37). Each NGPP j solves problem $\mathcal{P}_3$. From Lemma 1, we know this player maximizes his utility at some boundary. Assuming only condition (10) is tight, we find that all KKT conditions hold if

$$\pi \geq \lambda_{DA} - \mu_{nj} \left(1 - F_{Ri} \left(C_{ri}^* - \sum_{m=1}^j G_{im} \right) \right) \quad \forall j.$$

Substituting C_{ri}^* from (15), we find that the right hand side is lower than that in (43).

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