Predicting Farms' Donations to Food Banks using the Analytic Hierarchical Process and Dempster Shafer Theory

Kofi Kyei
Department of Computer Science
North Carolina A&T State University
Greensboro, NC, USA
kkyei@aggies.ncat.edu

Albert Esterline
Department of Computer Science
North Carolina A&T State University
Greensboro, NC, USA
esterlin@ncat.edu

Janelle Mason
Department of Computer Science
North Carolina A&T State University
Greensboro, NC, USA
jcmason@aggies.ncat.edu

***Abstract*—We present an analysis of factors contributing to the annual level of donation of sweet potatoes in 2010-2016 to a North Carolina food bank. Our approach follows that of Su et al., who used the Analytic Hierarchy Process (AHP) and Dempster-Shafer theory (DST) to assess annual grain security in China for 1997-2007. We first identified the indices (or factors or criteria) that influence the level of donation and their "directions:" positive (the more the better), negative, or non-directional (average is best). We divided the range of each index into degrees (intervals) then applied AHP to get weights for the indices. To apply DST, we defined a frame of discernment that would generate focal elements that could be assigned to degrees of the indices. Then, using the index weights, we defined a BPA (basic probability function) for each year. Since for each year we had multiple pieces of possibly conflicting evidence, we used Dempster's rule to combine each BPA with itself several times. In the resulting BPA, the focal element with greatest mass was taken as the prediction for the donation level for that year. We partitioned the range of the donation data into degrees to compare observations with the focal elements in the BPA. Predicted donation degrees matched observed degrees reasonably well if degree boundaries are well chosen. Analysis of apparent anomalies suggested a more sophisticated approach and the need to involve other information sources. This approach allows one to experiment in a principled way (and without assumptions about probability distributions) with the relative importance of the multiple factors that affect the predicted quantity and so to understand how these factors together contribute to that quantity. It is suggested for gaining preliminary insight, which may be exploited in the application of more rigid analytic techniques.**

Keywords—Analytic Hierarchical Process, Dempster-Shafer theory, Foodbanks

1. Introduction

Everyone would like to see a community free from food insecurity. Donating nutritious food to food banks can help achieve this goal. Federal food nutrition programs and food access do not reach everyone in need, but food banks help fill the gap. The U.S. Department of Agriculture (USDA) defines food insecurity as a lack of consistent access to enough food for an active, healthy life. Hunger is a physical discomfort, while food insecurity is a lack of available financial resources for food at the household level [1]. Food insecurity is a real problem throughout the United States, yet food is the number one item Americans throw away: each year, up to 40% of the food supply in the United States is never consumed, which is a $218 billion loss annually. The major place of such food waste in the US is the farmers' fields, where the crops that do not meet the specified grade quality are left to rot or to be plowed under [2]. In 2018, it was estimated that 42% of vegetables grown in North Carolina were unharvested, and in 2015-2016, South Carolina experienced an estimated 641,916 tons of food waste [3]. Yet there are more than 2.6 million hungry individuals across the Carolinas [3]. For instance, North Carolina is ranked in the top ten hungriest States in the U. S. at 13.9% of the population being food-insecure [3]. Several factors are responsible for food insecurity across the Carolinas and the United States in general, including unemployment, the market price of produce, and low household income. These reasons are complicated and often interconnected.

This paper presents our analysis of factors contributing to the level of donation of sweet potatoes to a particular food bank in North Carolina for 2010-2016. North Carolina is the top state in the production of sweet potatoes, which are nutritious and generally store well.

The research reported here seeks to understand the factors that allow us to predict sweet-potato donations by farms to foodbanks. Prediction of donations to foodbanks has been extensively studied by Davis's group at North Carolina A&T State University. Okore-Hanson et al. [4] used standard regression as well as stepwise regression analysis to identify key factors that can be used to predict the demand at branches of a foodbank. Nuamah et al. [5] developed a simulation model to determine the expected quantity of food donations received per month in a multi-warehouse distribution network. The simulation model was based on a state-space model for exponential smoothing. Brock and Davis [6] used an artificial neural network to evaluate and approximate the contributions from supermarkets in North Carolina to foodbanks. Davis et al. [7] found new insight into the predictive power of time series modeling to forecast and analyze supply uncertainty of food donations to organizations. Finally, Pugh and Davis [8] applied support vector regression to forecast donations to the foodbanks and analyze the estimation of these donations. Some of the work by Davis's group (e.g., [4]) used economic indicators that include real GDP, unemployment rate, and consumer confidence, which impact conditions on the demand for food for those in need. They thus help predict future demand of food at foodbanks.

978-1-7281-6861-6/20/$31.00 ©2020 IEEE

The approach taken in the research reported here follows the work of Su et al. [9], who used the Analytic Hierarchy Process (AHP) and Dempster-Shafer theory (DST) to assess annual grain security in China for the years 1997 to 2007. It is our contention that their approach allows one to experiment in a principled way with the relative importance of the multiple factors that affect the predicted quantity and so to understand how these factors together contribute to that quantity. Their approach is also independent of any assumptions about the probability distributions of the contributing factors.

The rest of this paper is structured as follows. Sections 2 and 3 provide background on AHP and DST, respectively. Section 3 is a brief review of the literature related to the work of Su et al. and a summary of their work. The next section presents the application of our approach to the data we obtained from the Feeding the Carolinas foodbank. Section 6 discusses our results, and Section 7 concludes.

2. The Analytic Hierarchy Process

The Analytic Hierarchy Process [10], or AHP for short, is for multi-criteria decision making involving possibly both subjective human judgments and objective evaluations. In a classical application, the AHP considers a set of evaluation criteria and a set of options among which the best (according to the criteria) is to be selected. Since the criteria may conflict, the best option need not optimize all the individual criteria; rather, the best option achieves the best trade-off among the criteria. The AHP lets us translate the qualitative and quantitative evaluations of experts into multi-criteria rankings that are applied with simple arithmetic operations.

In outline, the AHP establishes a weight for each criterion based on our pair-wise comparisons of the criteria. More important criteria get higher weights. Next, for each criterion, the AHP assigns a score to each option based on our pair-wise comparisons of the options per that criterion. Options performing better on that criterion get higher scores. Finally, the global score for a given option is the weighted sum of the scores it received per all the criteria, where the weight for a score per a criterion is the weight assigned that criterion. The global scores allow us to rank the options. AHP may be seen as a three-level hierarchy, with the overall goal of the problem at the top, multiple criteria that define alternatives in the middle, and decision alternatives at the bottom. In this work, we stop at finding weights for the criteria.

To compute the weights for the criteria, we first create a *pairwise comparison matrix* **A**, an $n \times n$ real matrix, where n is the number of criteria. Entry a_{jk} of **A** represents the importance of the j^{th} criterion relative to the k^{th} criterion. If $a_{jk} > 1$, then the j^{th} criterion is more important than the k^{th}; if $a_{jk} < 1$, the j^{th} criterion is less important than the k^{th}; and if the two criteria are equally important, then a_{jk} is 1. Clearly, $a_{jj} = 1$ for all j. We require $a_{kj}\ a_{jk} = 1$ so that we need establish only the upper or lower triangle of **A** since **A** is a reciprocal matrix, that is, entries in one triangle are simply the reciprocals of those in the other triangle reflected across the diagonal.

TABLE 1. RELATIVE SCORES

Value of a_{ij}	Interpretation
1	*j* and *k* are equally important
3	*j* is slightly more important than *k*
5	*j* is more important than *k*
7	*j* is strongly more important than *k*
9	*j* is absolutely more important than *k*

The relative importance between two criteria is measured according to a numerical scale from 1 to 9, as shown in Table 1. To be definite, we there assume that the j^{th} criterion is at least as important as the k^{th}. Intermediate values (2, 4, 6, 8) can also be assigned, with the obvious intermediate interpretations. As mentioned, we require $a_{kj}\ a_{jk} = 1$. More generally, a comparison matrix **A** is said to be *consistent* if $a_{ij} \cdot a_{jk} = a_{ik}$ for all i, j, and k, which is a sort of transitivity principle. Such consistency, however, is generally not attained with pair-wise comparisons of criteria. For example, if we have three criteria A, B, and C with A compared to B giving 2 and B compared to C giving 3, we do not require that A compare to C gives 6.

The largest eigenvalue of a consistent comparison matrix, λ_{max}, is equal to the size, n, of the matrix, $\lambda_{max} = n$. As a prelude to defining a measure of inconsistency, we define the *consistency index* as deviation from consistency using the following formula

$$CI = \frac{\lambda_{max} - n}{n-1}$$

It can be shown that, for a reciprocal matrix, $\lambda_{max} \geq n$, so CI is always positive and is 0.0 if (and only if) the comparison matrix is consistent. What is taken as the measure of inconsistency of an $n \times n$ comparison matrix **A** is the *consistency ratio*, CR, defined as CI / RI, where RI, the *random consistency index*, is the average value of CI for random $n \times n$ reciprocal matrices. According to Saaty, a matrix is sufficiently consistent if and only if $CR < 0.1$. RI values for n from 1 to 10 are shown in Table 2.

TABLE 2. RANDOM CONSISTENCY INDEX (RI) VALUES

n	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

3. Dempster Shafer Theory

Arthur P. Dempster introduced Dempster Shafer theory (DST) in 1967 as the theory of evidence or belief functions [11]. In 1976, this theory was enhanced by Glenn Shafer as a new method for the representation of uncertainty [12]. DST provides powerful mechanisms for determining the confidence one may have in evidence combined from several sources. It is a justification-based way of providing a numerical measure of confidence and trust in our hypotheses. DST assumes a *frame of discernment*, Θ, consisting of a set of mutually exclusive and exhaustive hypotheses. It distributes and combines evidence represented as masses assigned to subsets of Θ, with the total mass summing to 1.0. A set with non-zero mass is a *focal element*. The function that maps focal elements to mass is called a mass function or *basic probability assignment* (BPA).

This approach is very different from the Bayesian one, where we assign probabilities to single events, that is, outcomes of the experiments. Unlike the Bayesian approach, BPAs do not require a set of prior probabilities. And belief in a hypothesis and its negation need not sum to 1.0 as mass can be assigned to Θ to indicate ignorance. Overall, DST is attractive because of its flexibility and ease of use.

DST has functions for lower and upper bounds on a set's likelihood, known as *belief* and *plausibility functions*, respectively. The value of the belief function for a subset θ of Θ, $Bel(\theta)$, is the sum of the masses of its subsets. Hence, $Bel(\emptyset) = 0$, $Bel(\Theta) = 1$, and $0 \le Bel(\theta) \le 1.0$ for all $\theta \subseteq \Theta$. The value of the plausibility function for a subset θ of Θ, $Plaus(\theta)$, is the sum of masses of the sets that overlap with θ. Hence $Bel(\theta) \le Plaus(\theta) \le 1.0$ for all $\theta \subseteq \Theta$ and, where $\bar{\theta}$ is the complement of θ, $Plaus(\bar{\theta}) = 1 - Bel(\theta)$, and $Bel(\bar{\theta}) = 1 - Plaus(\theta)$. Furthermore, the true belief in hypothesis θ lies in the interval $[Bel(\theta), Plaus(\theta)]$, while the degree of uncertainty is represented by the difference $Plaus(\theta) - Bel(\theta)$.

DST provides a rule to combine two independent pieces of evidence in the form of combining the corresponding BPAs while still considering uncertainty. Dempster's rule for combining BPAs divides the mass corresponding to conflict (i.e., non-overlapping focal elements) between the different BPAs evenly among the focal elements of the result to avoid assigning it to the null set, which must have no mass. Dempster's rule calculates a measure of conflict

$$k = \sum_{B \cap C = \emptyset} m_1(B) m_2(C)$$

for a pair of BPAs m_1 and m_2. This is the mass assigned to the null subset and represents contradictory evidence. The combined mass function according to Dempster's rule is

$$m(A) = \frac{\sum_{B \cap C = A} m_1(B) m_2(C)}{1-k}$$

where k is defined above. The intersections represent areas where the BPAs agree. Dividing by $1-k$ normalizes the result.

4. The Approach of Su et al.

Su et al. [9] present a comprehensive early-warning model for evaluating the status of grain security in China for the years 1997-2007 that is based on the AHP method and DST. They review the technical literature on agriculture early-warning in China and find that each of the methods reviewed "puts emphasis on just a single angle and cannot make a concise and quantitative total evaluation." They found, however, that Men et al. [13] sought to rectify this deficiency with a comprehensive model using an enhancement of AHP. The enhancement applies Gray system theory [14] to form a factor analysis method called Gray Comprehensive Evaluation, used to analyze a system with a hierarchical structure. Su et al. point out that the model of Men et al. assumes that the composite index of grain security follows a normal distribution about the mean value. They note, however, that not every factor relating to grain security follows a normal distribution, and they develop a grain security warning system that can handle cases where distributions are non-normal.

Su et al. divided risk assessment into four stages. In Stage 1, they identified the indices (or factors or criteria) that contribute to the level of risk to grain security. They identified eleven, including (for example) grain production, disaster-affected area, and growth rate of the Grain Price Index.

Stage 2 is preprocessing the index data: determining the "directions" of the indices, determining the risk degrees of the grain security, and calculating the risk bounds (for each index, the values dividing the ranges of that index into the degrees). The direction of an index is positive if the more the better (it reduces the risk), negative (if less is better), or neither. For example, grain production is positive, disaster-affected area is negative, and growth rate of the grain price index is neither. Risk degree is divided into five grades: no alarm, light alarm, middle alarm, heavy alarm, and huge alarm. Corresponding to the five degrees, we need four risk bounds. For each index, they calculated the range Δ of the values of that index in the data: they found the maximum value, *max*, and the minimum value, *min*, then $\Delta = max - min$. Then they let $p = \Delta/8$. The no-alarm degree begins at *min*. Thereafter, the four bounds between the five degrees are at $min + p$, $min + 3p$, $min + 5p$, and $min + 7p$. The huge-alarm degree goes up to *max*. They numbered the boundaries 1, 2, 3, 4, from lowest to highest. Note that the final result provided a BPA for each year and that BPA assigned masses to representations of these five degrees. It turned out that, for each year, one of these degrees had significantly more than half the mass. The results matched the facts well.

Stage 3 used the AHP to assign weights to the eleven indices.

In Stage 4, they evaluated the risk of grain security using DST and determined the final risk degree for each year. They defined a BPA, $m_y()$, for each year, $y \in \{1997, 1998, \ldots, 2007\}$, based on the data for the indices. (They used the AHP results in computing $m(f)$ for each focal element f for each year y; for this, they needed a frame of discernment on which they can defined some focal elements.) To describe the five risk degrees for DST, they need at least three elements in the frame of discernment Θ, say, $\Theta = \{a, b, c\}$. They adopted a linear way for focal elements to match with their risk degrees (so not all subsets of Θ are focal elements). The focal elements, then, are a (no alarm), ab (light alarm, actually, the subset $\{a, b\}$ of Θ), b (middle alarm), bc (heavy alarm), and c (huge alarm). They also have as a focal element abc for ignorance.

Given the bounds calculated in Stage 2, how they mapped the data for the indices to focal elements depended on the direction of the indices. For a positive index, they went down, with a starting at *max*, then ab starting at boundary 4, b at boundary 3, bc at boundary 2, and c at boundary 1, down to *min*. For a negative index, they went up, with a starting at *min*, then ab at boundary 1, b at boundary 2, bc at boundary 3, and c at boundary 4, up to *max*. From this, they produced a table whose rows are the eleven years and whose columns are the eleven indices. From the data, they determined what focal element is in the cell for year y and index i given the boundaries in the data calculated for that index. Given this table, they then used the weights for the eleven indices from Stage 3 to construct a table that defined a BPA $m_y()$ for each year $y \in \{1997, 1998, \ldots, 2007\}$. The rows correspond to years, and there is a column for each focal element $f \in \{a, ab, b, bc, c\}$. A cell at row y, column f contains the sum of the weights for all the indices with focal element f for year y (as given by the previous table); this is the value for $m_y(f)$, and can be seen as a sort of average for the weights of the indices with focal element f for year y. Note that the sum of the values in the row for a given year is 1.0.

Note that, for each year, we have eleven pieces of evidence, some possibly conflicting. Murphy noted [15] that, with conflicting evidence, Dempster's rule often produces unacceptable results, and the problem lies with how the masses of focal elements are normalized. She considers several alternatives to Dempster's rule and concludes that averaging the masses of focal elements across the rules best solves the normalization problem. For convergence to an acceptable BPA, one should incorporate the averages into the combining rule. Thus, if there are n pieces of evidences, we use the classical Dempster's rule to combine the weighted averages of the masses $n-1$ times. (Su et al. also reference [16], which modifies Murphy's averaging rule to reduce the contribution of a body of evidence that is highly conflicting with the other bodies of evidence. A measure of the distance between two bodies of evidence [17] is used to determine the weight of each body of evidence.) Su et al. used the average approach to combine these eleven pieces of evidence: the weighted average BPA for each year is combined with itself $11 - 1 = 10$ times to obtain the final results.

Combining BPAs with themselves ten times tended to concentrate the mass on the singletons, a, b, and c. For each year but one, exactly one of these singletons has over 80% of the mass; in the one exception, one singleton had 69% of the mass. The prediction for risk to grain security for a year was taken to be the dominant focal element (always a singleton) for that year. Now, risk to grain security is not a measured quantity but can be gauged from the responses of institutions and agencies. Su et al. maintained that the predicted risk degrees "coincide with the facts well." They found agreement with the result of Men et al. [13] in all but one year, and for that year, their results matched the facts.

5. Calculations to Predict Donations

We followed the four stages of Su et al. outlined above. We here show the results of all calculations.

TABLE 3. DATA FROM 2010 TO 2016, SHOWING INDEX NAMES

Year	Previous-yr. prod. In 1000 lbs. (C_1)	Avg. price/cwt in \$/cwt (C_2)	Percentage of total sales, in %, (C_3)	Unemployment rate, in %, (C_4)
2010	9720000	18	1.8	10.9
2011	12800000	17.7	2	10.3
2012	12400000	13	1.7	9.3
2013	10600000	24.9	2.7	8.0
2014	15840000	22	2.7	6.3
2015	16340000	19.4	2.8	5.7
2016	17100000	18.1	3.2	5.1

5.1 Identification of Indices

The personnel at Feeding the Carolinas identified the factors (or indices) that influence donations of sweet potatoes as the following, where our abbreviation for each index is in parenthesis after its description, and all values are for the state of North Carolina: the production of sweet potatoes in the previous year in thousands of pounds (C_1), the average market price per hundred weight (cwt) of sweet potatoes for the year (C_2), the percentage of total agricultural sales for the year contributed by sweet potatoes in % of value (C_3), and the unemployment rate for the year in % (C_4).

Note that production is for the previous year since generally what is consumed in a given year was produced in the previous year. The data for these indices for the years 2010-2016 is shown in Table 3.

5.2 Preprocessing the Index Data

For Stage 2, we preprocess the index data in two steps: determine the direction of the indices and define the boundaries between the degrees of each index.

5.2.1 Directions to the Indices

We determined the directions of the indices in consulting with our collaborators at Feeding the Carolinas. The previous year's production of sweet potatoes, C_1, is positive as the more there are, the less impact giving a certain amount of sweet potato will have on the overall status of the farm that gives it. The price per cwt, C_2, is negative since, when the price goes up, a donating farmer is giving up more wealth in giving a fixed amount of sweet potato. The percentage of total sales, C_3, is non-directional. If the amount of sweet potato sold is fixed, sales of other products generally have no influence on donation of sweet potatoes unless those sales are particularly large or small; see the discussion below. Finally, the unemployment rate, C_4, is positive since, the higher the unemployment rate, the greater the perceived need. Table 4 summarizes the directions of the indices, with '+' indicating positive, '-' negative, and '0' non-directional.

5.2.2 Index Degrees and Boundaries of Degrees

As with Su et al., we categorize the values of each index into five degrees, but what we call the degrees will depend on the direction of the index—see Subsection 5.4.1 below. How the four bounds separating the five degrees are found is shown in Table 5. For a given index, we find the maximum, *max*, and minimum, *min* of its values and calculate the range Δ as $max - min$. Then (following Su et al.) we calculate a step s as $\Delta/8$ and set the bounds at $min + s$, $min + 3s$, $min + 5s$, and $min + 7s$.

5.3 Use of the AHP to Assign Weights to Indices

For the comparison matrix in Stage 3, our collaborators at Feeding the Carolinas judged as follows.

TABLE 4. DIRECTIONS OF INDEX

Index	C_1	C_2	C_3	C_4
Directional	+	-	0	+

TABLE 5. DEGREE BOUNDS

	Bound	C_1	C_2	C_3	C_4
Max		17100	24.9	3.2	10.9
Min		9720	13	1.7	5.1
Range		7380	11.9	1.5	5.8
S		922.5	1.49	0.19	0.73
min+s	**Bound 1**	10642.5	14.49	1.89	5.83
min+3s	**Bound 2**	12487.5	17.47	2.27	7.29
min+5s	**Bound 3**	14332.5	20.45	2.65	8.75
min+7s	**Bound 4**	16177.5	23.43	3.03	10.21

- C_2 is very slightly more important than C_1 but slightly less important than both C_3 and C_4.
- C_1 is slightly less important than both C_3 and C_4.
- C_4 is very slightly more important than C_3.

The result is the matrix

$$A = \begin{bmatrix} 1 & 2 & 0.33 & 0.33 \\ 0.50 & 1 & 0.33 & 0.33 \\ 3 & 3 & 1 & 0.5 \\ 3 & 3 & 2 & 1 \end{bmatrix}$$

The **eig()** function in the linalg (linear algebra) package of NumPy, passed a matrix, returns a two-tuple whose first element is a one-dimensional array of eigenvalues and whose second element is a two-dimensional array whose columns are the eigenvectors, corresponding by position to the eigenvalues in the one-dimensional array. The first element in the array of eigenvalues is the largest, $\lambda_{max} = 4.159$, so the first eigenvector is the principal eigenvector, generally not normalized by linalg. Normalizing (dividing each element by the sum of the elements) gives a vector of weights $\mathbf{w} = [0.150, 0.106, 0.310, 0.435]$, shown in Table 6.

For the consistency ratio (see Section 2), CR, we first calculate the consistency index, CI, as follows, where $n = 4$ is the dimension of array $\mathbf{A}$.

$$CI = \frac{\lambda_{max} - n}{n-1} = \frac{4.1549 - 4}{4-1} = \frac{0.1549}{3} = 0.0516$$

From Table 2, we find the random consistency index, RI, as 0.90, so

$$CR = \frac{CI}{RI} = \frac{0.0516}{0.90} = 0.0574$$

Thus, CR for our comparison matrix $\mathbf{A}$ is significantly less than 0.1, and so $\mathbf{A}$ is considered sufficiently consistent.

5.4 Evaluating the Predicted Donation Degrees

For Stage 4, evaluating the predicted donation degrees, we define the focal elements (Subsection 5.4.1) as subsets of a frame of discernment on a linear scale divided by the bounds calculated earlier; this results in a table with the focal elements for each index for each year. From this table, we construct a BPA for each year (Subsection 5.4.2). Finally, for each year, we combine the BPA with itself three times using Dempster's rule (Subsection 5.4.3).

TABLE 6. INDEX WEIGHTS

Index	C_1	C_2	C_3	C_4
Weight	0.150	0.106	0.310	0.435

5.4.1 Defining the Focal Elements

Following Su et al., we use three elements for our frame of discernment, $\Theta = \{a, b, c\}$, and we adopt a linear way to match focal elements to our prediction degrees (so not all subsets of Θ can be focal elements), and so we have as focal elements *a*, *ab* (actually subset $\{a, b\}$ of Θ), *b*, *bc*, and *c*. We do not have focal element *abc* (that is, Θ) for ignorance. Referring to Fig. 1, for an index with positive direction, focal element *a* indicates very low, *ab* indicates low, *b* indicates medium, *bc* indicates high, and *c* indicates very high. For an index with negative direction, things are reversed: *a* indicates very high, *ab* indicates high, *b* medium, *bc* low, and *c* very low. The reason for the reversal (and that our order is the reverse of that of Su et al) is that we want *a* to be the least conduce level of the index and *c* to be the most conduce, and the English words "low" and "high" are neutral regarding such issues. For the non-directional index C_3, the percentage of sales contributed by sweet potatoes, *a* indicates very low or very high, that is, very extreme, *b* indicates low or high, that is, extreme, and *c* indicates medium; we do not use focal element *ab* or *bc* for this index. The rationale behind these symmetric associations is that, with a fixed amount of sweet potato sold, if there is a lot of other agricultural produce sold that year, there is a wide option of things other than sweet potatoes that could be donated, and if there is not much other agricultural produce that year, farmers will tend to hold on to their sweet potatoes.

Using the bounds given in Table 5 and the focal elements thus bounded for each index as in Fig. 1, we can repeat Table 3 (the data for each index across years 2010-2016) and replace the numbers in the cells with focal elements, giving Table 7.

Fig. 1. Index degree to focal element

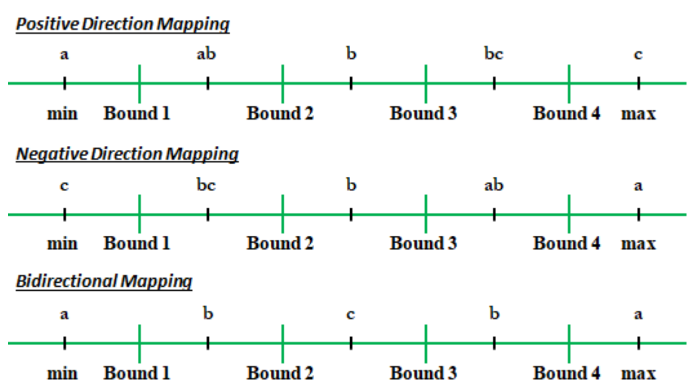

TABLE 7. DATA FROM 2010-2016, SHOWING THE FOCAL ELEMENTS FOR EACH INDEX FOR EACH YEAR

Year	C_1	C_2	C_3	C_4
2010	a	b	a	c
2011	a	b	ab	c
2012	b	c	a	bc
2013	ab	a	bc	b
2014	b	ab	bc	ab
2015	c	b	bc	a
2016	c	b	c	a

5.4.2 Creating a BPA for Each Year

From Tables 6 and 7, we construct a BPA $m_y()$ for each year y, as shown in Table 8. For a given focal element f, the value for $m_y(f)$ is the sum of the weights (given in Table 6) for all the indices with focal element f for year y (as given in Table 7).

For example, letting the name of the index stand for its value, for y = 2010, $m_{2010}(a) = C_1 + C_3 = 0.150 + 0.310 = 0.460$, $m_{2010}(b) = C_2 = 0.106$, $m_{2010}(c) = C_4 = 0.435$, $m_{2010}(ab) = m_{2010}(bc) = 0$.

5.4.3 Determining the Final Results

Following Su et al., we use the average approach and use the classical Dempster's rule to combine the four pieces of evidence (the indices): the weighted average BPD for each year is combined with itself $n - 1 = 4 - 1 = 3$ times to obtain the final result, shown in Table 9.

We take the prediction of donations for the year to be the name of the focal element with the greatest weight. Table 10 shows for each year, the predicted level of donation next to the actual donations.

6. DISCUSSION

Table 10 shows that the predictions increase essentially monotonically across time, from very low to medium to very high. This is broadly in line with reality as conditions did broadly improve during the period under study. In detail, however, the predicted degree of donation appears often to miss the mark.

To see how to align the donation data with the predicted degree, we partition the donation data into three degrees to represent very low, medium, and very high. The minimum (in 2010), *min*, is 177,035, and the maximum (in 2013), *max*, is 978,495. The range is $\Delta = min - max$, which we divide into three equal-length steps with $s = \Delta/3 = 267{,}153$. Thus, boundary 1 is at $min + s = 444{,}188$, and boundary 2 is at $min + 2s = 711341$. We use $y_1 \prec y_2$ to indicate that the donation for year y_1 was less than that for year y_2 and $y_1 \equiv y_2$ to indicate that the donations for the two years were equal. We then order the years as follows, using double vertical bars for degree boundaries:

$$2010 \prec 2012 \equiv 2014 \parallel 2011 \prec 2015 \parallel 2016 \prec 2013$$

The following is how our predictions partition the set of years into degrees. (We assume no order within the same degree.)

$$\{2010, 2011\} \parallel \{2012, 2013, 2014\} \parallel \{2015, 2016\}$$

Table 11 is a version of Table 10 sorted by the donation, omitting the focal-element column, adding a column for actual classification per the three equal-length degrees. The years in bold are those years where the prediction does not agree with the actual classification with three equal-length degrees.

In Table 11, five of the seven predictions are anomalous, but none are off by more than one degree. Our partition was arbitrary, so we consider moving boundaries up or down to get better agreement. Let $don(y)$ be the donation for year y. We begin by considering the location of boundary 1. The fraction of Δ up to $don(2012)$ (= $don(2014)$) is (368,735 - min) / Δ = 0.239. We would have to move boundary 1 down to just below 368,735 (= $don(2012) = don(2014)$) to get 2012 and 2014 into medium, but $don(2011)$ = 448,600, whose prediction was classified as very low, is greater than $don(2012)$ (= $don(2014)$) and so would remain misclassified, as very low. As a fraction of Δ, this is (448600 - 368735) / Δ = 0.010, not very much. Next, $don(2015)$ (= 647,543) is below boundary 2 (= 711,341) and should be medium instead of very high. As a fraction of Δ, the distance it is below is (711341 - 647543) / Δ = 0.080. Boundary 2 would have to be moved down to just below 647,543 to include 2015 in very high. This is quite feasible as it would not change the status of any other year. Table 12 is an updated version of Table 11 where boundary 1 has been moved to just below 368,735, and boundary 2 has been moved to just below 647,543. The two years where the predicted classification and the classification per the current degree

TABLE 8. THE BPA FOR EACH YEAR, 2010-2016

Year	a	b	c	ab	bc	ac
2010	0.46	0.106	0.435	0	0	0
2011	0.15	0.106	0.435	0.31	0	0
2012	0.31	0.15	0.106	0	0.435	0
2013	0.106	0.585	0	0	0.31	0
2014	0	0	0	0.691	0.31	0
2015	0.435	0.106	0.15	0	0.31	0
2016	0.435	0.106	0.46	0	0	0

TABLE 9. FINAL BPAS, 2010-2016

Year	a	b	c	ab	bc	ac
2010	0.983	0.007	0.010	0.001	0	0
2011	0.469	0.197	0.313	0.021	0	0
2012	0.004	0.626	0.305	0	0.065	0
2013	0	1.000	0	0	0	0
2014	0	0.948	0	0.052	0	0
2015	0.313	0.197	0.469	0	0.021	0
2016	0.389	0	0.611	0	0	0

TABLE 10. PREDICTED DEGREE VS. OBSERVED DONATIONS

Year	Highest Focal Elements	Prediction	Donations for the Year
2010	a	Very Low	177035
2011	a	Very Low	448600
2012	b	Medium	368735
2013	b	Medium	978495
2014	b	Medium	368735
2015	c	Very High	647543
2016	c	Very High	967254

TABLE 11. YEARLY PREDICTIONS, ACTUAL DONATIONS, AND CLASSIFICATION OF THE LATTER SORTED ON DONATIONS. ANOMALOUS YEARS IN BOLD.

Year	Prediction	Donations for the Year	Should be
2010	Very Low	177,035	Very Low
2012	**Medium**	**368,735**	**Very Low**
2014	**Medium**	**368,735**	**Very Low**
2011	**Very Low**	**448,600**	**Medium**
2015	**Very High**	**647,543**	**Medium**
2016	Very High	967,254	Very High
2013	**Medium**	**978,495**	**Very High**

boundaries disagree are again in bold.

Table 12 shows two anomalous years, 2011 and 2013. For 2011, the values for the focal elements are *a* (0.469), *b* (0.197), *c* (0.313), *ab* (0.021), *bc* (0). The largest value is for *a*, so we categorized this as very low whereas it should be medium. Note, however, that the value of the dominant focal element, *a*, is less than a half and that *c* is 2/3 as large as *a*. This would suggest a compromise between very low and very high, thus medium (as desired). By the way, 2015 was misclassified until we moved boundary 2 down. Its focal-element values are *a* (0.313), *b* (0.197), *c* (0.469), *ab* (0), *bc* (0.021). The dominant element is *c*, so we categorized this as very high. Again, the value of the dominant element, now *c*, is less than a half and that *a* is 2/3 as large as *c*. This would suggest a compromise between very low and very high, thus medium (as desired with the original degree boundaries).

Note that 2011 and 2015 are the only years where none of the focal elements has a value greater than a half, so categorizing these years according to their dominant elements is questionable to begin with.

The most glaring anomaly is 2013, for which there was the highest amount of donation but which our prediction was medium; in fact, all mass in the final result was for *b*, medium. One index in particular drove the prediction low, and that is index C_2, price per cwt. This has a negative direction and reached its highest value in 2013; it was represented by focal element *a*. A lesser factor was the previous year's production, index C_1, which has a positive direction and experienced a generally increasing trend but had a dip in 2013. (The other indices show nothing remarkable in 2013: unemployment maintained a steady decline, and the percent of sales represented by sweet potatoes was about average.) Evidently, the value of what is donated is not always an effective deterrent. Our model has only four indices (or factors or criteria) and so ignores a great deal of possibly useful information. It is also linear and so does not account for the interaction of indices. 2013 was the peak of a boom in U.S, agriculture[1]. This may have influenced the generosity of farmers and may have motivated donations for tax credits. Also, although unemployment was decreasing, it was still high and coming off levels not seen in 75 years; this may have motivated farmers' altruism.

TABLE 12. YEARLY PREDICATIONS, ACTUAL DONATIONS, AND CLASSIFICATION-WITH ADJUSTED BOUNDARIES-OF THE LATTER SORTED ON DONATIONS. ANOMALOUS YEARS IN BOLD.

Year	Highest Focal Elements	Prediction	Donations for the Year	Should be
2010	a	Very Low	177,035	Very Low
2012	b	Medium	368,735	Medium
2014	b	Medium	368,735	Medium
2011	**a**	**Very Low**	**448,600**	**Medium**
2015	c	Very High	647,543	Very High
2016	c	Very High	967,254	Very High
2013	**b**	**Medium**	**978,495**	**Very High**

7. CONCLUSION

This paper presents our analysis of factors contributing to the annual level of donation of sweet potatoes to the foodbank Feeding the Carolinas for the years 2010 to 2016. The approach taken here follows the work of Su et al. [9], who used the Analytic Hierarchy Process (AHP) and Dempster-Shafer theory (DST) to assess annual grain security in China for the years 1997 to 2007.

In our case, we identify four indices (or factors or criteria) that influence the level of donation. They are sweet potato production the previous year (C_1), average price per cwt of sweet potatoes for the year (C_2), the percentage for the year of agricultural sales contributed by sweet potatoes (C_3), and the average unemployment for the year (C_4). C_1 and C_4 have positive directions (the more the better) while C_2 has a negative direction (increase in it leads to decrease in donation) and C_3 is non-directional (normal amounts are conducive to donation, extremes hinder). We found the range of each index and partitioned it into five degrees (intervals).

We then applied AHP to get weights for the indices. Judging how much more important one index is than another, we formed a pairwise comparison matrix, **A**. The principle eigenvector of **A** provides the weights for the indices, and from the largest eigenvalue we computed a consistency ratio for **A** that showed that **A** could be considered sufficiently consistent.

To apply DST, we needed five focal elements corresponding to the five degrees of each index. We get five focal elements with a frame of discernment with three elements, $\Theta = \{a, b, c\}$, and use a linear way for focal elements to match with degrees, having focal elements *a*, *ab* (actually, subset $\{a, b\}$ of Θ), *b*, *bc*, and *c*. How we assign focal elements to degrees of an index depends on the direction of the index; in all cases, *a* is the least conducive and *c* is the most conducive to the amount of donation. We then define a table whose rows are years and columns are indices; a cell at row *y* and column *i* contains the focal element for index *i* for year *y*. From this, we define the basic probability assignment (BPA, or mass function), $m_y()$, for each year *y*: for each focal element *f*, we add the weights of the indices that have *f* for year *y*, giving $m_y(f)$. For each year, we have four pieces of possibly conflicting evidence. Su et al. (who reference work on combining possibly conflicting evidence using averaging) maintain that, if there are *n* pieces of evidences, one can use the classical Dempster's rule to combine the weighted averages of the masses $n-1$ times to converge to an acceptable BPA. For us, $n = 4$, so we used Dempster's rule to combine the BPA for each year with itself $4 - 1 = 3$ times. This tended to concentrate the mass on the elements of the frame of discernment, *a*, *b*, and

[1] "U.S. agriculture has been booming in recent years with record farm incomes and double-digit percentage increases in cropland prices. However, farm income projections suggest a flattening, if not a reversal, of these trends." Quotation from Gary S. Corner, "Agriculture Boom Continued 2013," Federal Reserve Bank of St. Louis, online at

https://www.stlouisfed.org/publications/central-banker/winter-2013/agriculture-boom-continued-2013 *c*. The focal element with greatest mass, the dominant focal element, was interpreted in its positive direction as the prediction or the level of donation of sweet potatoes for the year, resulting in predicted levels of very low, medium, and very high.

The results showed a clear trend of increasing donations, as in fact occurred. To judge our results, we partitioned the range of donations into three degrees (intervals) of equal length and used the same labels as for the predicted degrees: "very low," "medium," and "very high." With these arbitrary boundaries, there were only two of the seven years where the predicted degree was the same as observed degree although nowhere were they off by more than one degree. It was possible, however, to move the boundaries so that there were mismatches for only two years; no further reduction was possible because the donation data imposed a strict order on the years although in one case the difference in the donations was only 1% of the range of donation values.

Of the two remaining anomalous years, the dominant focal element of one (very low) had less than half the mass, and another focal element for that year (very high) had 2/3 of the mass of the dominant one. The observed degree, medium, is the compromise. This indicates that we need another way to assign predicted degrees when there is no clearly dominant focal element. The other anomalous year presented graver difficulties: all the mass is on medium, but the data shows that it was the year with the highest donation level. Suggested possible explanations include missing factors/indices, such as the general state of the agricultural economy and altruism triggers, and interaction among indices (which cannot be handled by this linear approach).

The value of the approach followed here is that it allows one to experiment in a principled way, and without any assumptions about probability distributions, with the relative importance of the multiple factors that affect the predicted quantity and so to understand how these factors together contribute to that quantity. The calculations are straightforward (implemented by us with simple Python code) and are easily modified. This paper has illustrated some of easily implemented choices: selecting and comparing indices (criteria) and defining degrees of real-valued indices and degrees of predicted quantities. Exploratory work with this approach can provide insight for more rigid approaches in data science, including statistical and machine-learning methods.

There are some obvious enhancements to be made in the near future. We need some examples of qualitative indices. An advantage of both AHP and DST is that they handle both quantitative and qualitative data. From the discussion above, it is clear that we need principled ways of handling cases where there is no clearly dominant index in the final result. While our approach is essentially linear, we would like a principle way to progress to something with similar exploratory power that can handle the interaction of indices (another point that came out in the discussion above). Finally, this is quite broad in scope and could be beneficially extended to data gathering, helping to determine what data might provide more insight.

ACKNOWLEDGMENT

This project was supported by NSF National Research Traineeship Project Improving Strategies for Hunger Relief and Food Security using Computational Data Science (Award No. DGE-1735258).

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