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Intangible capital and measured productivity ☆

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ABSTRACT

Because firms invest heavily in R&D, software, brands, and other intangible assets—at a rate close to that of tangible assets—changes in GDP, which does not include all intangible investments, understate the actual changes in total output. If labor inputs are more precisely measured, then it is possible to observe little change in measured total factor productivity (TFP) coincidentally with large changes in hours and investment. The output mismeasurement leaves business cycle modelers with large and unexplained labor wedges accounting for most of the fluctuations in aggregate data. To address this issue, I incorporate intangible investments into a multi-sector general equilibrium model and use data from an updated U.S. input and output table to parameterize income and cost shares, with intangible investments reassigned from intermediate to final uses. I employ maximum-likelihood methods and quarterly observations on sectoral gross outputs for the United States to estimate processes for latent sectoral TFPs that have common and sector-specific components. I do not use aggregate hours to estimate TFPs but find that the predicted hours series compares closely with the actual series and accounts for roughly two-thirds of its standard deviation. I find that sector-specific shocks and industry linkages play an important role in accounting for fluctuations and comovements in aggregate and industry-level U.S. data, and I find that at business-cycle frequencies, the model's common component of TFP is not correlated with the standard measures of aggregate TFP used in the macroeconomic literature. Adding financial frictions and stochastic shocks to financing constraints has a negligible impact on the results.

1. Introduction

This paper sheds light on a measurement issue that confounds analyses of key macrodata during economic booms and busts. Because firms invest heavily in R&D, software, brands, and other intangible assets, changes in GDP, which does not include all intangible investments, understate the actual changes in total output. As a result, it is possible to observe large changes in hours and investment coincidentally with little change in *measured* total factor productivity. In other words, innovation by firms—which is fueled in large part by their intangible investments—may be evident "everywhere but in the

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productivity statistics."[1] Here, I use theory and recently revised U.S. national accounts to more accurately estimate U.S. total factor productivity (TFP) at both the aggregate and industry levels.

I develop a dynamic multi-sector general equilibrium model and explicitly incorporate intangible investment. Multiple sectors are needed to account for the vast heterogeneity in intangible investment rates across industries. Firms in the model economy have access to two production technologies: one for producing new tangible goods and services, and another for producing new intangible capital goods and services. Tangible capital is assumed to be a rivalrous input, but intangible capital is assumed to be a nonrivalrous input, since knowledge can be used simultaneously in producing consumer goods and services and in creating new ideas. I explicitly model industry linkages that occur through purchases of intermediate inputs and through purchases of new tangible or intangible investment goods.

Business-cycle fluctuations in the baseline model are assumed to be driven by shocks to industry and aggregate TFP, the impact of which depends on details of the industry input- and capital-use linkages. In an extension, I also allow for stochastic financing shocks, as in Jermann and Quadrini (2012), with firms facing a cost of adjusting dividends and using costly external finance to fund new projects. Both versions of the model can potentially rationalize the large labor wedges found by Chari et al. (2007) when applying their business cycle accounting approach to U.S. data with their no-intangible, no-financial-friction prototype model.[2]

To parameterize income and cost shares, I start with the 2007 benchmark input-output table and take advantage of the fact that the Bureau of Economic Analysis (BEA) now includes expenditures on intellectual-property products—software; R&D; mineral exploration; and entertainment, literary, and artistic originals—as part of investment rather than as part of intermediate inputs. Additionally, I reassign several categories of intermediate inputs that are under consideration for future inclusion in the BEA fixed assets, including computer design services, architectural and engineering services, management consulting services, advertising, and marketing research. In the version of the model with financing frictions, I use industry-level data from Compustat to construct time series for ratios of tangible capital to output and debt to output, both of which are needed to derive estimates of the shocks to the enforcement constraints.

Because the model includes intangible capital stocks that cannot be accurately measured, it is not possible to use observations on factor inputs and outputs to directly measure the TFP series, as has been done in earlier work (see, for example, Horvath, 2000). Instead, I use maximum-likelihood methods to estimate stochastic processes for the latent TFPs, which are assumed to have both sector-specific components and a common component. This is done using quarterly data on gross outputs for major industries from the BEA and per capita hours for several intangible-intensive industries from the Bureau of Labor Statistics (BLS). Using observations not used in the estimation, I run external tests of the theory and derive model predictions for the latent TFP and intangible investment series.

A key test of the theory is its predictive performance for fluctuations in aggregate U.S. hours and sectoral comovements in hours for all major industries, data not used to estimate the model parameters. For the baseline model, I find that the model's predicted aggregate hours track U.S. hours much better than the simplest one-sector model without intangible investments. The model predicts three sizable booms over the 1985–2015 sample period and then a bust. Moreover, the standard deviation of the model's predicted-hours series is 65 percent of the actual series, as compared with 9 percent in the one-sector version without intangible investments. This improvement in the model's prediction is primarily due to fluctuations in intangible investments, which show up as a time-varying labor wedge for Chari et al. (2007).[3] I also find significant comovement of sectoral hours because of the model's input-output linkages. Computing principal components for sectoral hours, I find that the variance that the first component accounts for is 56 percent in U.S. data and 69 percent in the model. For the extended model with financial frictions, I find that the implied labor wedges are smaller and less volatile than the wedge in Jermann and Quadrini's (2012) one-sector model, and as a result, financial shocks have only a small impact on real activity. A key difference here is the inclusion of intangible investments and the assumption that only tangible capital is externally financed.

After verifying that the baseline model effectively predicts U.S. hours, I put it to use to derive theoretically consistent summary statistics and time paths for latent TFP shocks and intangible investments.[4] I first decompose the variances of U.S. data used in the maximum likelihood estimation (MLE) to determine the relative importance of idiosyncratic and common TFP shocks and to assess the role of input-output linkages. I do this decomposition in two ways: by computing the variance decomposition of the ergodic distribution, and by decomposing predicted growth rates in the technology boom of the 1990s and the Great Recession. I find that sector-specific shocks and industry linkages play an important role in accounting for fluctuations in the aggregate and industry-level gross outputs. Then I construct model time series for investments and TFP processes. I find that at business-cycle frequencies, the model's common component of TFP is not correlated with

[1] Robert Solow remarked that the computer age could be seen "everywhere but in the productivity statistics" ("We'd Better Watch Out," *New York Times Book Review*, July 12, 1987, p. 36).

[2] Business-cycle accounting is a method to assess the promise of economic theories. There are two steps. The first is to show that a large class of models is observationally equivalent to a prototype model with time-varying wedges that look like time-varying productivity, labor income taxes, investment taxes, and government consumption. The second is to use the prototype model's data and equilibrium conditions to measure the wedges and to feed them back into the model in order to assess separately and in combinations the impact of each one.

[3] This would be true even in a one-sector model. I use a multi-sector model because most of the U.S. intangible investment is done by firms in just a few major sectors—namely, manufacturing, information, and professional and business services.

[4] Because the financial frictions add little, I could use either version for this inference.

the standard measures of TFP used in the macroeconomic literature. In the case of investment, I find different time-series properties for intangibles and tangibles: intangible investments vary less over the business cycle than tangible ones and lag the cycle by several quarters.

Previous theoretical work related to this paper has either abstracted from intangible capital or been more limited in scope. Long and Plosser (1983) analyzed a relatively simple multi-sector model, arguing that firm- and industry-level shocks could generate realistic aggregate fluctuations. Horvath (1998, 2000) and Dupor (1999) extended Long and Plosser's (1983) model and studied the nature of industry linkages to determine if independent productivity shocks could in fact generate much variation in aggregate variables. Parameterizing the model to match the input-output and capital-use tables for the 1977 BEA benchmark, Horvath (2000) found that the multi-sector model that features only sectoral shocks is able to account for many patterns in U.S. data as well as a one-sector model driven by aggregate shocks. More recently, Foerster et al. (2011) did a full structural-factor analysis of the errors from the same multi-sector model and found that significant variation in quarterly data is explained by sectoral shocks. However, they used industrial-production data, which cover only about 20 percent of total production in the United States. Atalay (2017) extended the analysis to the entire economy and allowed for more general functional forms. None of these authors distinguished tangible and intangible investments. McGrattan and Prescott (2010) did distinguish the different investments but focused only on aggregate data for a specific episode—namely, the technology boom of the 1990s. Furthermore, they did their analysis well before the BEA completed the comprehensive revision introducing the category of intellectual-property products.

Previous empirical work has documented that intangible investments are large and vary with tangible investments over the business cycle. For example, Corrado et al. (2009) estimate that businesses' intangible investments are about as large as their tangible investments.[5] McGrattan and Prescott (2014) use firm-level data and show that intangible investments are highly correlated with tangible investments such as plant and equipment.

This paper is also related to a burgeoning business-cycle literature in search of new sources of shocks and new sources of propagation mechanisms following the Great Recession of 2008–2009.[6] During the downturn, GDP and hours fell significantly, but TFP fell only modestly and quickly recovered, rising in 2009 when real activity was still well below trend. These observations have led many to conclude that the Great Recession was inherently different from other downturns and certainly not consistent with the predictions of the real business cycle (RBC) theories developed in the early 1980s. In RBC theories, resources are efficiently allocated and fluctuations are driven by changes in TFP.[7] My paper shows that a variant of those models—namely, one that takes into consideration the intangible investments of firms and allows for sectoral shocks to TFP—can go a long way in accounting for U.S. business cycles.

The model is described in Section 2. Estimation techniques and parameter estimates are described in Section 3. Section 4 summarizes the results. Section 5 concludes.

2. Model

I start by describing the baseline model without financing constraints. For this version of the model, the driving forces of business cycles are sectoral and aggregate TFP shocks. I then extend the framework to include financing decisions and enforcement constraints. In the extension, the driving forces are TFP shocks and financing shocks.

2.1. Baseline with only TFP shocks

A stand-in household supplies labor to competitive firms and, as the owner of the firms, receives the dividends. A government has certain spending obligations that are financed by various taxes on households and firms. Firms produce final goods for households and the government and intermediate inputs for other businesses. In the baseline model, the only sources of fluctuations in the economy are stochastic shocks to firm productivities.

The economy has J sectors. Firms in sector j maximize the present value of dividends D_j paid to their shareholders. I assume that firms in each sector j produce both *tangible* goods and services, Y_j, and *intangible* investment goods and services, X_{Ij}. The technologies available in period t are as follows:

$$Y_{jt} = (K^1_{Tjt})^{\theta_j}(K_{Ijt})^{\phi_j}(\textstyle\prod_l (M^1_{ljt})^{\gamma_{lj}})(Z^1_{jt}H^1_{jt})^{1-\theta_j-\phi_j-\gamma_j} \tag{2.1}$$

$$X_{Ijt} = (K^2_{Tjt})^{\theta_j}(K_{Ijt})^{\phi_j}(\textstyle\prod_l (M^2_{ljt})^{\gamma_{lj}})(Z^2_{jt}H^2_{jt})^{1-\theta_j-\phi_j-\gamma_j}, \tag{2.2}$$

[5] For more details on measurement of intangible investments in the national accounts, see recent surveys in the BEA's *Survey of Current Business* (U.S. Department of Commerce, 1929–2016). For more details on measurement of R&D investments, see National Science Foundation (1953–2016). For details on entertainment, literary, and artistic originals, see Soloveichik and Wasshausen (2013).

[6] For example, in the recent literature, business cycles are driven by shocks to capital quality (Gertler and Kiyotaki (2010), Gourio (2012), Bigio (2015)), enforcement or collateral constraints (Jermann and Quadrini (2012), Khan and Thomas (2013)), agents' beliefs (Angeletos and La'O (2013)), news about future productivity (Karnizova (2012), Chen and Song (2013)), and second moments (Azzimonti and Talbert (2014), Bachmann and Bayer (2014), Bloom et al. (2018), Schaal (2017)). If cycles are driven by productivity shocks, the source of propagation is different from that in standard real business cycle models. See, for example, Boissay et al. (2016).

[7] The main references, in addition to Long and Plosser (1983), are Kydland and Prescott (1982), Hansen (1985), Prescott (1986), and Cooley (1995).

which depend on inputs of tangible capital K^1_{Tj}, K^2_{Tj}; intangible capital K_{Ij}; intermediate inputs $\{M^1_{lj}\}$, $\{M^2_{lj}\}$; and hours H^1_j, H^2_j. These production technologies are hit in period t by stochastic technology shocks, Z^1_{jt} and Z^2_{jt}, that could have a common component and sector-specific components. The specific choices for the stochastic processes are discussed below.

The maximization problem solved by firms in sector j on behalf of their owners (households) who discount after-tax future earnings at the rate ϱ_t is given by

$$\max\ E_0 \sum_{t=0}^{\infty} (1-\tau_d)\varrho_t D_{jt},$$

subject to

$$\begin{aligned} D_{jt} = {} & P_{jt}Y_{jt} + Q_{jt}X_{Ijt} - W_{jt}H_{jt} - \textstyle\sum_l P_{lt}M_{ljt} - \sum_l P_{lt}X_{Tljt} - \sum_l Q_{lt}X_{Iljt} \\ & -\tau_p\{P_{jt}Y_{jt} + Q_{jt}X_{Ijt} - W_{jt}H_{jt} - (\delta_T+\tau_k)P_{jt}K_{Tjt} \\ & \quad - \sum_l P_{lt}M_{ljt} - \sum_l Q_{lt}X_{Iljt}\} - \tau_k P_{jt}K_{Tjt} \end{aligned} \tag{2.3}$$

$$K_{Tjt+1} = (1-\delta_T)K_{Tjt} + \textstyle\prod_l X^{\zeta_{lj}}_{Tljt} \tag{2.4}$$

$$K_{Ijt+1} = (1-\delta_I)K_{Ijt} + \textstyle\prod_l X^{\nu_{lj}}_{Iljt} \tag{2.5}$$

$$M_{ljt} = M^1_{ljt} + M^2_{ljt}. \tag{2.6}$$

Dividends are equal to gross output $P_jY_j + Q_jX_{Ij}$ less wage payments to workers W_jH_j, purchased intermediate goods $\sum_l P_lM_{lj}$, new tangible investments $\sum_l P_lX_{Tlj}$, new intangible investments $\sum_l Q_lX_{Ilj}$, and taxes. New investment goods and services are purchased from other sectors and used to update capital stocks, as in (2.4) and (2.5). Taxes are levied on accounting profits at rate τ_p and on property at rate τ_k.

Households choose consumption C_t and leisure L_t to maximize expected utility

$$\max\ E_0 \sum_{t=0}^{\infty} \beta^t \left\{[(C_t/N_t)(L_t/N_t)^{\psi}]^{1-\alpha} - 1\right\}/(1-\alpha)N_t \tag{2.7}$$

with the population equal to $N_t = N_0(1+g_n)^t$. The maximization is subject to the following per-period budget constraint:

$$\begin{aligned} & (1+\tau_c)\textstyle\sum_j P_{jt}C_{jt} + \sum_j V_{jt}(S_{jt+1} - S_{jt}) \\ & \quad \le (1-\tau_h)\textstyle\sum_j W_{jt}H_{jt} + (1-\tau_d)\sum_j D_{jt}S_{jt} + \Psi_t, \end{aligned} \tag{2.8}$$

where C_j is consumption of goods made by firms in sector j, which are purchased at price P_j; H_j is labor supplied to sector j, which is paid W_j; and D_j are dividends paid to the owners of firms in sector j with S_j outstanding shares that sell at price V_j. Taxes are paid on consumption purchases (τ_c), labor earnings (τ_h), and dividends (τ_d). Any revenues in excess of government purchases of goods and services are lump-sum rebated to the household in the amount Ψ.

The composite consumption and leisure that enter the utility function are given by

$$C_t = \left[\textstyle\sum_j \omega_j C_{jt}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{2.9}$$

$$L_t = N_t - \textstyle\sum_j H_{jt}. \tag{2.10}$$

Here, notice that I use a constant elasticity of substitution function for consumption and a linear function for hours. As owners of the firm, the household's discount factor is the relevant measure for ϱ_t in (2.3):

$$\varrho_t = \beta^t U_{ct}/[P_t(1+\tau_c)], \tag{2.11}$$

where P_t is the aggregate price index given by $P_t = [\sum_j \omega_j^{\sigma} P_{jt}^{1-\sigma}]^{1/(1-\sigma)}$.

The resource constraints for tangible and intangible goods and services are given as follows:

$$Y_{jt} = C_{jt} + \textstyle\sum_l X_{Tjlt} + \sum_l M_{jlt} + G_{jt} \tag{2.12}$$

$$X_{Ijt} = \textstyle\sum_l X_{Ijlt}, \tag{2.13}$$

where Y_j and X_{Ij} are defined in (2.1) and (2.2), respectively. The model economy is closed; therefore, there is no term for net exports.[8]

[8] In the empirical implementation, net exports will be included with intermediate and final domestic purchases.

I assume that the logs of the sectoral TFP processes are equal to the sum of a sector-specific component $\tilde{Z}^i_{jt}$ and a common component Z_t with factor loading λ_j; that is,

$$\log Z^i_{jt} = \log \tilde{Z}^i_{jt} + \lambda_j \log Z_t \tag{2.14}$$

$$\log \tilde{Z}^i_{jt} = \rho_{ij} \log \tilde{Z}^i_{jt-1} + \eta^i_{jt} \tag{2.15}$$

$$\log Z_t = \rho \log Z_{t-1} + \upsilon_t, \tag{2.16}$$

for $i = 1, 2$ and $j = 1, \ldots, J$, where $E\eta^i_{jt} = 0$, $E\eta^i_{jt}\eta^i_{jt-1} = 0$, $E\eta^i_{jt}\eta^k_{lt} = 0$ if $j \neq l$, $E\upsilon_t = 0$, $E\upsilon_t\upsilon_{t-1} = 0$, and $E\upsilon_t\eta^i_{jt} = 0$. In other words, the shocks to TFP are correlated within a sector but not across sectors, across time, or with the common TFP component.[9]

An approximate equilibrium for the model economy can be found by applying a version of Vaughan's (1970) method to the log-linearized first-order conditions of the household and firm maximization problems. The solution can be summarized as an equilibrium law of motion for the logged and detrended state vector x; namely,

$$x_{t+1} = Ax_t + B\varepsilon_{t+1}, \quad E\varepsilon_t\varepsilon_t' = I, \tag{2.17}$$

where $x_t = [\vec{k}_{Tt}, \vec{k}_{It}, \vec{z}_{1t}, \vec{z}_{2t}, z_t, 1]'$ is a $(4J+2)\times 1$ state vector, $\vec{k}_{Tt}$ is the $J\times 1$ vector of logged and detrended tangible-capital stocks, $\vec{k}_{It}$ is the $J\times 1$ vector of logged and detrended intangible-capital stocks, $\vec{z}_{1t}$ is the $J\times 1$ vector of logged and detrended sectoral TFPs for production of final goods and services, $\vec{z}_{2t}$ is the $J\times 1$ vector of logged and detrended sectoral TFPs for production of new intangible investments, and z_t is the logged and detrended common shock. The variables are detrended by dividing first by the growth in population $(1+g_n)^t$ and then by the growth in technology, which is denoted by $(1+g_z)^t$. The last element of x_t is a 1, which is used for constant terms. The vector ε_t is a $2J+1$ vector of normally distributed shocks. Elements of the vector $B\varepsilon_t$ are the shocks η^i_{jt} and υ_t in (2.15)–(2.16). Thus, the only nonzero off-diagonal elements of B are the parameters governing correlations between TFP shocks to tangible and intangible production within the same sector.

2.2. Extension with financial shocks

The model extension I consider includes capital-market imperfections along the lines of Jermann and Quadrini (2012). I assume, as they do, that firms finance investment using both debt and equity, with debt preferred to equity because of its tax advantage. The main difference is that here I work with a multi-sector version of the model, whereas they work with a representative firm.

In this case, the definition of dividends in (2.3) must be modified to include a new term—namely, $B_{jt+1}/R_{bjt} - B_{jt}$ on the right-hand side, where B_{jt} is the debt of firms in sector j at time t, $R_{bjt} = 1 + r_t(1-\tau_{bj})$ is the effective gross interest rate for firms in sector j, r_t is the net interest rate paid to lending households, and τ_{bj} is the tax benefit. Additionally, firms in Jermann and Quadrini (2012) raise funds to finance working capital, which can be easily diverted. Assume that loans to firms in sector j and time t are denoted by l_{jt}. With probability ξ_{jt}, the lender can recover the loan, implying that the firms are subject to the following enforcement constraints:

$$\xi_{jt}\left(P_{jt+1}K_{Tjt+1} - \frac{B_{jt+1}}{1+r_t}\right) \geq l_{jt}, \tag{2.18}$$

where $P_{jt+1}K_{Tjt+1}$ is the value of the capital that can be partially liquidated in the case of default. If I assume, as Jermann and Quadrini (2012) do, that the size of the loan is equal to current-period output, then I replace l_{jt} by $P_{jt}Y_{jt} + Q_{jt}X_{Ijt}$. This then is an adaptation of the constraint in Jermann and Quadrini (2012), who abstract from multiple sectors and intangible capital.

The enforcement constraint in (2.18) has almost no real impact without an additional feature that Jermann and Quadrini (2012) introduce into their model—namely, a cost for paying dividends over and beyond the payout itself. In other words, D_{jt} in equation (2.3) is replaced by

$$\varphi(D_{jt}) = D_{jt} + \kappa_j(D_{jt} - \bar{D}_j)^2.$$

If $\kappa_j = 0$, shocks to ξ_{jt} can be offset by changes in dividend payouts. Firms would not choose to use costly external finance and pay dividends. If $\kappa_j > 0$, dividend payouts are costly and adjustment is slower, implying that shocks to ξ_{jt} can have a real impact on output, investment, and hours.

In this extension, I add a $J\times 1$ vector of detrended debt levels and a $J\times 1$ vector of financial shocks to the state vector x_t in (2.17).

[9] One exception is the government sector, NAICS 92. I assume that shocks to production in NAICS 92 are independent of all other shocks. If I assume otherwise, then the common shock parameter estimates depend importantly on increases in gross output in this sector during the Great Recession, the source of which is unlikely to be a boom in TFP.

3. Parameters

Next, I describe how to parameterize income and cost shares using the 2007 benchmark BEA input-output use table and how to estimate processes for components of the sectoral TFPs—namely, $\{Z^1_{jt}\}$ and $\{Z^2_{jt}\}$—using data from the BEA and BLS. The remaining parameters, which are also described below, are those related to preferences, growth rates, depreciation, tax rates, and the financing constraints.

3.1. Income and cost shares

The starting point for my analysis is the BEA input-output table, which records intermediate purchases by commodity and industry, final purchases by commodity and final user, and payments to factors by industry. For the analysis below, I use data for the 15 major industries: (1) agriculture, forestry, fishing, and hunting (NAICS 11); (2) mining (NAICS 21); (3) utilities (NAICS 22); (4) construction (NAICS 23); (5) manufacturing (NAICS 31–33); (6) wholesale trade (NAICS 42); (7) retail trade (NAICS 44–45); (8) transportation and warehousing (NAICS 48–49); (9) information (NAICS 51); (10) finance, insurance, real estate, rental and leasing (NAICS 52–53); (11) professional and business services (NAICS 54–56); (12) educational services, health care, and social assistance (NAICS 61–62); (13) arts, entertainment, recreation, accommodation, and food services (NAICS 71–72); (14) other services except government (81); and (15) public administration (NAICS 92).

In the model, intermediate purchases are represented as a $J \times J$ matrix with element (l, j) given by $P_l(M^1_{lj} + M^2_{lj})$ for commodity l purchased by firms in industry j. As a share of gross industry output $P_jY_j + Q_jX_{Ij}$ in industry j, these intermediate purchases are used to parameterize the intermediate shares, $\{\gamma_{lj}\}$, in (2.1) and (2.2).[10] Before computing intermediate shares with the BEA's input-output data, I first recategorize intermediate expenses for several commodities under professional and business services—commodities that national accountants are considering for recategorization—to final uses. Specifically, I move expenses for computer design services, architectural and engineering services, management consulting services, advertising, and marketing research out of the intermediate-inputs matrix and into the capital-use table for intangible investments described below.

In the model, final purchases are computed as the sum of private and public consumption, tangible investments, and intangible investments. In consumption, I include the nondurable goods and services categories from the BEA's personal-consumption expenditures and government consumption. Expenditure shares for these goods and services are governed by the choice of $\{\omega_j\}$ in (2.9), which I set to align the theoretical and empirical shares.[11] In investment, I include the BEA's government investment categories as well as the durable-goods component of personal consumption expenditures, with an imputed service flow for durable and government capital added to consumption services.

Like intermediate purchases, tangible and intangible investments are used by different industries. Tangible-investment purchases are represented as a $J \times J$ capital-use matrix with element (l, j) given by P_lX_{Tlj} for commodity l purchased by firms in industry j. Intangible-investment purchases are also represented as a $J \times J$ capital-use matrix with element (l, j) given by Q_lX_{Ilj} for commodity l purchased by firms in industry j. Detailed investment data from the BEA are used to construct these matrices.[12] I include fixed investment—both public and private—in equipment and structures and changes in inventories with tangible investment, and I include the new BEA category of *intellectual-property products* (IPP)—both public and private—with intangible investment.[13] As mentioned earlier, the IPP category includes expenditures on software; mineral exploration; R&D; and entertainment, literary, and artistic originals. Some of this spending is done in-house by firms (and is what the BEA calls own-account). For this spending, I reassign the commodity source to the own industry, which is more in line with the theory. To the IPP spending, I add the reallocated intermediate expenditures on professional and business services. In the case of consumer durable equipment, I assume it is a manufactured commodity used by the real-estate industry. In the case of consumer durable software and books, I assume these are information commodities used by households. Once I have the capital-use matrices, I can set the parameters ζ_{lj} and ν_{lj} using the spending shares for tangible and intangible investment, respectively.[14]

To compute factor shares, I use the value-added components in the BEA's 2007 input-output table. Three components of value added are reported for industry data: compensation, taxes on production and imports, and gross operating surplus. The labor income share for industry j is compensation W_jH_j divided by industry gross output less taxes on production and imports. For the capital-income shares, I need to infer how much of the operating surplus results from tangible investment and how much from intangible investment. I use total spending on tangible and intangible investments to infer this split by iteratively solving the model and adjusting the shares to ensure a match. When this process is complete, I have estimates for the capital income shares $\{\theta_j, \phi_j\}$.

[10] When estimating the shares, taxes on imports and production are first subtracted from industry value added and final uses to be consistent with the theory.

[11] Consumer spending on the public administration "commodity" is allocated in a pro rata way to spending of all other commodities.

[12] The BEA has not yet published an official capital-use table for the 2007 benchmark input-output accounts. I was able to construct one using detailed investment data available for the BEA fixed-asset tables and the help of David Wasshausen at the BEA.

[13] This category of investment was added in the 2013 comprehensive revision of the accounts.

[14] The economy is closed and does not have a rest-of-world sector. Thus, I reallocate net exports to the domestic categories of intermediates, consumption, and investment. I do so in a pro rata way.

Table 1
Input-output table shares by major industry.[a]

A. Capital and consumption shares

Industry (NAICS)	Capital Shares[b]		Consumption Shares (ω_j)
	Tangible (θ_j)	Intangible (ϕ_j)	
Agriculture (11)	.301	.006	.006
Mining (21)	.546	.024	.000
Utilities (22)	.379	.042	.025
Construction (23)	.167	.082	.000
Manufacturing (31–33)	.162	.196	.146
Wholesale Trade (42)	.126	.149	.048
Retail Trade (44–45)	.130	.078	.110
Transportation & Warehousing (48–49)	.147	.024	.027
Information (51)	.200	.238	.041
Finance, Insurance & Real Estate (52–53)	.412	.036	.250
Professional & Business Services (54–56)	.063	.174	.022
Education, Health & Social Services (61–62)	.076	.032	.201
Leisure & Hospitality (71–72)	.138	.065	.084
Other Services (81)	.132	.053	.039
Public Administration (92)	.137	.048	.001

B. Intermediate goods and services (γ_{ij})

From: \ To:	11	21	22	23	31–33	42	44–45	48–49	51	52–53	54–56	61–62	71–72	81	92
11	.205	.000	.000	.001	.033	.001	.001	.000	.000	.000	.000	.000	.007	.000	.001
21	.003	.069	.107	.005	.037	.000	.000	.003	.000	.001	.000	.000	.001	.001	.004
22	.015	.011	.014	.003	.013	.006	.014	.008	.003	.018	.005	.011	.018	.008	.009
23	.007	.017	.019	.000	.002	.001	.003	.005	.002	.025	.001	.001	.003	.006	.019
31–33	.178	.073	.071	.243	.264	.030	.033	.154	.050	.011	.042	.076	.118	.079	.094
42	.071	.015	.016	.044	.047	.029	.017	.030	.012	.003	.008	.021	.022	.016	.014
44–45	.001	.000	.001	.058	.002	.001	.004	.005	.000	.002	.001	.001	.007	.008	.000
48–49	.033	.023	.067	.018	.022	.047	.053	.123	.015	.007	.015	.010	.014	.009	.018
51	.001	.002	.006	.003	.004	.012	.013	.007	.141	.016	.023	.016	.011	.017	.026
52–53	.045	.032	.052	.023	.015	.086	.126	.093	.050	.212	.088	.136	.097	.159	.040
54–56	.010	.040	.045	.011	.042	.085	.059	.046	.040	.068	.088	.068	.082	.039	.038
61–62	.001	.000	.000	.000	.000	.000	.002	.000	.000	.000	.000	.012	.002	.003	.005
71–72	.001	.002	.010	.002	.003	.005	.003	.004	.021	.010	.018	.011	.025	.006	.009
81	.004	.003	.005	.005	.008	.017	.011	.024	.016	.013	.014	.013	.015	.015	.015
92	.000	.000	.002	.000	.001	.011	.006	.022	.003	.002	.003	.003	.007	.003	.003

C. Tangible capital flow shares[b] (ζ_{ij})

From: \ To:	11	21	22	23	31–33	42	44–45	48–49	51	52–53	54–56	61–62	71–72	81	92
11	.084	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	.002	.763	.003	.003	.002	.001	.001	.020	.001	.000	.002	.001	.001	.001	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	.154	.054	.431	.058	.165	.228	.477	.261	.320	.329	.205	.430	.574	.496	.699
31–33	.510	.123	.379	.629	.593	.468	.350	.470	.454	.558	.531	.381	.285	.337	.247
42	.129	.031	.096	.160	.124	.191	.089	.119	.115	.016	.135	.097	.072	.086	.040
44–45	.037	.009	.027	.045	.035	.034	.025	.034	.033	.007	.038	.027	.020	.024	0
48–49	.029	.007	.022	.036	.028	.027	.020	.045	.026	.004	.030	.022	.016	.019	.006
51	.008	.002	.006	.009	.007	.007	.005	.007	.008	.001	.008	.006	.004	.005	0
52–53	0	0	0	0	0	0	0	0	0	.066	0	0	0	0	0
54–56	.049	.012	.036	.060	.047	.045	.033	.045	.043	.020	.051	.036	.027	.032	.008
61–62	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
71–72	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
81	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
92	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

(*continued on next page*)

The results of the calculations are summarized in Table 1. Part A shows the capital income shares, $\{\theta_j, \phi_j\}$, and consumption expenditure shares, $\{\omega_j\}$. Notice that in four industries—manufacturing, wholesale trade, information, and professional and business services—the share of intangible capital in production is larger than the share of tangible capital. Part B shows the implied intermediate input shares, $\{\gamma_{ij}\}$. The first row and column headers indicate the commodity and industry NAICS category, respectively, which in turn correspond to the 15 major industries listed above. These shares provide one measure of the industry linkages. The capital-use tables provide another. Part C shows the shares for the tangible capital-use table, $\{\zeta_{ij}\}$, and Part D shows the shares for the intangible capital-use table, $\{\nu_{ij}\}$. Notice that many rows in Part C have only zeros because the commodities produced are neither structures nor equipment. Commodities categorized under construction (NAICS 23) and manufacturing (NAICS 31–33) are the main sources of tangible investment goods. In the case of intangible

Table 1 (*continued*)

D. Intangible capital flow shares[b] (ν_{ij})

From: \ To:	11	21	22	23	31-33	42	44-45	48-49	51	52-53	54-56	61-62	71-72	81	92
11	.029	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	.191	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	.118	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	.028	0	0	0	0	0	0	0	0	0	0	0
31-33	0	0	0	0	.731	0	0	0	0	0	0	0	0	0	0
42	0	0	0	0	0	.224	0	0	0	0	0	0	0	0	.005
44-45	0	0	0	0	0	0	.093	0	0	0	0	0	0	0	0
48-49	0	0	0	0	0	0	0	.091	0	0	0	0	0	0	.000
51	.112	.148	.107	.024	.027	.047	.086	.094	.621	.192	.044	.047	.197	.065	.030
52-53	0	0	0	0	0	0	0	0	0	.568	0	0	0	0	0
54-56	.859	.661	.778	.948	.247	.734	.824	.817	.386	.240	.956	.613	.793	.669	.794
61-62	0	0	0	0	0	0	0	0	0	0	0	.340	0	0	0
71-72	0	0	0	0	0	0	0	0	0	0	0	0	.011	0	0
81	0	0	0	0	0	0	0	0	0	0	0	0	0	.266	0
92	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.170

[a] The underlying data for the shares in the table are the BEA benchmark input-output table for 2007.
[b] Tangible investments are structures and equipment. Intangible investments are intellectual-property products, as defined by the BEA, and intermediate inputs that are reassigned to final uses. See Appendix B for a list of reassigned categories.

investments, commodities categorized under information (NAICS 51) and professional and business services (NAICS 54–56) are most important. In the BEA data, scientific R&D is listed under NAICS 5417, but much of this is specific to other commodities (e.g., chemical manufacturing) and has been reassigned accordingly (see Appendix A for more details). For this reason, there are nonzero shares on the diagonal of the matrix ν that would be zeros if I were to use the BEA commodity assignments.

The shares in Table 1 are held fixed when estimating TFP processes, which I turn to next.

3.2. Shock processes

Estimates of the parameters governing the shock processes are found by applying maximum likelihood to the following state space system:

$$x_{t+1} = Ax_t + B\varepsilon_{t+1} \tag{3.1}$$

$$y_t = Cx_t, \tag{3.2}$$

where the elements of x_t are defined above (see (2.17)) and assumed to be unobserved, and y_t are quarterly U.S. observations for the period 1985:1–2014:4.[15]

In the baseline model without financial shocks, I assume that there are shocks to TFP in the production of all tangible goods and services and in the production of a subset of intangible goods and services. That is, I assume that Z^2_{jt} is constant for all j except in the cases of manufacturing, information, and professional and business services, where production of intangible goods and services is concentrated. To identify the sectoral TFP shocks to tangible production, Z^1_{jt}, and factor loadings on the common shock, λ_j, I use data on gross outputs for private industries and aggregate gross output.[16] I use gross outputs, rather than data on value added, because there are no issues with the classification of spending as intermediate or final, which has changed over the postwar period.[17] Because the standard deviation of the common TFP shock and the factor loadings are not separately identifiable, I normalize the standard deviation of the common TFP shock and set it equal to 0.01.

For the intangible-intensive sectors, I use additional data to identify the processes for TFP in the production of new intangible investment goods. Specifically, I use hours of work for the following three subsectors: computer and electronic products, broadcasting and telecommunications, and advertising—which are three-digit industries under manufacturing, information, and professional and business and services, respectively.[18] Because the hours in these industries account for only 10 percent, I can use the model's prediction for aggregate hours as an external check on the model. Given the standard one-sector model without intangibles' failure to account for large fluctuations in hours, a comparison of hours is a particularly important test of the new theory.

[15] See Harvey (1989) for details.
[16] Both data and model series are deflated before shocks are estimated. I do not estimate TFP shocks for the public-administration sector (NAICS 92) because stimulus spending during the Great Recession shows up as positive TFP shocks.
[17] As a robustness check, I also worked with IRS business receipts, which are an important source of information for constructing gross outputs and are available from the 1920s onward for many major and minor industries.
[18] Another possible data source is gross outputs for the subsectors. However, measurement issues arise because significant intangible investment may be done in-house and is thus not included in gross output.

The model has a quarterly time period, but time series on gross outputs by industry are only available annually before 2005. Therefore, before estimating parameters for the shock processes, I use a Kalman filter to compute forecasts of quarterly gross outputs.[19] The idea is to use other available quarterly data by industry and construct quarterly forecasts for the series of interest—namely, gross outputs. Specifically, I use quarterly estimates of the BEA's national income by industry, quarterly estimates of the BLS's employment by industry, and *annual* estimates of the BEA's gross outputs. Both the national-income and gross-output data are divided by the GDP deflator.[20] Doing this yields 15 series of quarterly gross outputs for 14 private industries and aggregate gross output. Adding data on hours for the intangible-intensive subsectors implies that the vector y_t in (3.2) has 18 elements, which are used to estimate the 18 TFP processes.

One final step before the TFP processes can be estimated is to set the initial state x_0 in (3.1). Here, I do not use the steady-state values because there are differing growth trends in U.S. industry data. For example, relative to an economy-wide trend, manufacturing has been slowing, and information has been growing. Thus, I choose x_0 in such a way that initial investments do not jump. This is easy to do in two steps. I start by setting x_0 equal to the steady state and then use the model's prediction for the first period state, $\hat{x}_1$, as the new initial condition. Given the observable series, y_t, and initial conditions for the initial state, x_0, I again apply the methods in Harvey (1989) to estimate the parameters of the stochastic TFP processes, which appear in the coefficients A and B in (3.1).

The results of the estimation are shown in Table 2. The four sets of estimates are the factor loadings λ_j; serial correlation coefficients ρ_{ij}; standard deviations of shocks η^i_{jt}; and correlations between tangible shocks η^1_{jt} and intangible shocks η^2_{jt} in the intangible-intensive industries. The factor loadings vary significantly across industries, with a loading of -2.9 for utilities and a loading of 2.2 for finance, insurance, and real estate. Serial correlation coefficients are all high and, in some cases, fixed during estimation at the upper bound of 0.995. Standard deviations of sectoral shocks are all significantly different from zero and, in many cases, are much larger than the standard deviation of the common shocks (which is normalized at 0.01). Finally, the correlations between shocks to tangible production and shocks to intangible production are significantly different from zero in two of the three cases, with a positive correlation in information and a negative correlation in professional and business services.

3.3. Other parameters

The remaining parameters for the baseline model are those related to preferences, growth in population and technology, depreciation, and taxes.

For preferences, I set $\alpha = 1$, $\sigma = 1$, $\psi = 1.2$, and $\beta = 0.995$. Annual growth in population (g_n) and technology (g_z) are 1 and 2 percent, respectively. Annual depreciation is set at 3.2 percent and assumed to be the same for all types of capital.[21] Tax rates are based on IRS and national account data and are as follows: $\tau_c = 0.065$, $\tau_d = 0.144$, $\tau_h = .382$, $\tau_p = 0.33$, and $\tau_k = 0.003$. For the results below, these rates are held constant.

3.4. Extension with financial shocks

For the extension with financial constraints and shocks, several additional parameters are needed. For all industries j, I set $\kappa_j = 0.146$ and $\tau_{bj} = 0.35$ to be consistent with Jermann and Quadrini's (2012) parameterization. To estimate the financial shocks, I need firm-level data from Compustat for tangible investments, debt, and output. I aggregate these data by industry.[22] Tangible capital stocks are computed using the perpetual inventory method with the Compustat investment data. As in Jermann and Quadrini (2012), I assume the enforcement constraints bind and use equation (2.18) and the Compustat data to derive time paths for the financial shocks ξ_{jt}.[23] I find that the time paths of ξ_{jt} are positive over the sample for only four of the major industries: mining, manufacturing, transportation and warehousing, and leisure and hospitality. These series are added to the vector of observables y_t in (3.2). Thus, I assume that firms in four industries borrow to finance new investment, whereas all others use retained earnings.

[19] See Harvey (1989) for more information on the Kalman filter and Appendix B for details of my application.

[20] To do the forecasting, I first remove trends by applying the filter in Hodrick and Prescott (1997) (with a smoothing parameter of 1600 for the quarterly series and 100 for the annual series). Once I have quarterly estimates, I add the low-frequency Hodrick-Prescott trend back to the forecasted time series.

[21] One issue that arises in models with intangible capital is the lack of identification of all parameters. For example, there are insufficient data to estimate both capital shares and depreciation rates, even in the case of R&D assets that are now included in both the BEA's national income and product accounts (NIPA) and the fixed asset tables. The BEA uses estimates of intangible depreciation rates to calculate the return to R&D investments and the capital service costs, which are used in capitalizing R&D investments for their fixed-asset tables. Unfortunately, as the survey of Li (2012) makes clear, "Measuring R&D depreciation rates directly is extremely difficult because both the price and output of R&D capital are generally unobservable." Li discusses different approaches that have been used to estimate industry-specific R&D depreciation rates, finding a wide range of estimates even within narrow categories. She concludes that "the differences in their results cannot be easily reconciled" (see Li, 2012, Table 2). I conduct sensitivity analysis to ensure that the main results are not affected by the choice.

[22] I updated the data used in Larrain and Yogo (2008). See Appendix A for details.

[23] The assumption that the constraints are always binding can be verified.

Table 2
MLE parameter estimates, 1985:1–2014:4.[a]

Statistic	Parameter estimate	Standard error
Factor Loadings:[b]		
Agriculture	−1.303	0.0025
Mining	−1.197	0.0005
Utilities	−2.874	0.0087
Construction	1.427	0.0147
Manufacturing	0.942	0.0054
Wholesale Trade	0.712	0.0063
Retail Trade	0.761	0.0071
Transportation & Warehousing	1.168	0.0093
Information	1.547	0.0105
Finance, Insurance & Real Estate	2.202	0.0164
Professional & Business Services	0.703	0.0091
Education, Health & Social Services	0.192	0.0123
Leisure & Hospitality	0.542	0.0067
Other Services	0.611	0.0093
Serial Correlation Coefficients:[c]		
Utilities	0.974	0.0124
Retail Trade	0.984	0.0136
Information, Tangible	0.995	0.0020
Information, Intangible	0.989	0.0003
Finance, Insurance & Real Estate	0.987	0.0030
Professional & Business Services	0.995	0.0035
Education, Health & Social Services	0.976	0.0061
Leisure & Hospitality	0.963	0.0163
Other Services	0.957	0.0125

Statistic	Parameter estimate	Standard error
Standard Deviations of Shocks:		
Agriculture	0.138	0.0157
Mining	0.330	0.0386
Utilities	0.326	0.0272
Construction	0.038	0.0046
Manufacturing, Tangible	0.072	0.0129
Manufacturing, Intangible	0.080	0.0044
Wholesale Trade	0.036	0.0042
Retail Trade	0.027	0.0037
Transportation & Warehousing	0.027	0.0038
Information, Tangible	0.055	0.0028
Information, Intangible	0.051	0.0039
Finance, Insurance & Real Estate	0.039	0.0056
Professional & Business Services, Tangible	0.023	0.0009
Professional & Business Services, Intangible	0.015	0.0009
Education, Health & Social Services	0.011	0.0015
Leisure & Hospitality	0.028	0.0020
Other Services	0.037	0.0013
Shock Correlations:		
Manufacturing	−0.138	0.1508
Information	0.154	0.0601
Professional & Business Services	−0.302	0.0941

[a] The table reports estimates of factor loadings λ_j, serial correlation coefficients ρ_{ij}, standard deviations of η^i_j, and correlations between η^1_{jt} and η^2_{jt}. In manufacturing, information, and professional and business services, parameters related to η^1_{jt} and η^2_{jt} are referenced as "tangible" and "intangible," respectively.

[b] In order to identify the factor loadings λ_j, the standard deviation of the common shock υ_t was fixed at 0.01.

[c] An upper bound of 0.995 was imposed on serial correlation coefficients for the common TFP process and for TFP processes in industries not listed.

4. Results

In this section, I present the main empirical findings. First, I find that the model driven by only productivity shocks is successful in generating large fluctuations in aggregate hours and significant comovement of sectoral hours. Second, I find that sector-specific productivity shocks account for a significant fraction of the observed time series and that industry linkages play an important role in generating business cycles. Third, I characterize the cyclical properties of the latent TFP

processes and intangible investments and find important differences between the model's predictions and measures of TFP and investment typically used in the macroeconomic literature. Finally, in an extension of the model that includes financing constraints and financial shocks, I find that the quantitative results are not significantly changed.[24]

4.1. Predictions for hours of work

An important test of any business-cycle model is its ability to generate aggregate fluctuations in hours of work in line with observations. The simplest one-sector real business cycle model without any intangible investment—which is the benchmark model used in the literature—spectacularly fails this test when compared with U.S. data. Here, I find that the multi-sector real business-cycle model with intangible investments does much better in generating aggregate hours that are variable and sectoral hours that comove.

For the benchmark, I set $J = 1$ and $\phi_j = 0$. This version of the model generates results similar to the model of Prescott (1986). In this case, I use the Solow residual as an estimate of the model's one TFP series. The Solow residual is real GDP divided by real fixed assets raised to a power (in this case, one-third) times aggregate hours raised to a power (in this case, two-thirds).[25] I assume the logarithm of the Solow residual is a first-order autoregressive process that can be estimated using ordinary least squares. Given the estimates and an initial condition for the process, I can simulate a path for TFP and feed it into the model's equilibrium decision functions.

The result for the hours decision is plotted in Fig. 1A, along with actual U.S. per capita hours. As the figure shows, the predicted series does not track the U.S. series and varies much less over the business cycle, barely rising during the technology boom and barely falling during the Great Recession. The standard deviation of the predicted series relative to the actual series is 9 percent. Why does it vary so little? The answer is that measured TFP—which in this case is the Solow residual—does not fluctuate very much over the cycle in my sample period.

In the multi-sector model, predictions of the model's state x_t and all decision variables—which are functions of the state—are found by applying a Kalman smoother that conditions on all of the observations, $\{y_t\}$; that is, $\hat{x}_t = E[x_t | y_1, \ldots, y_T]$. Here, the variables of interest are sectoral and aggregate hours, which are not included in the vector y_t when estimating the TFP shocks but are observable. In Fig. 1B, I plot the multi-sector model's predicted per capita hours, along with actual U.S. hours. The figure shows that the predicted hours track actual hours much better than the simplest one-sector model. The model predicts three sizable booms and then a bust, and the standard deviation of the model series is 65 percent of the actual series.

The success of the model can be demonstrated also by applying the business-cycle accounting approach of Chari et al. (2007) to model simulations of aggregate data on hours, consumption, and output. Chari et al. (2007) find that large labor wedges are needed to account for fluctuations in U.S. aggregate data. The labor wedge in the prototype model is the ratio of the marginal rate of substitution between consumption and leisure, $\psi C_t / L_t$, and labor productivity measured as GDP per hour. This wedge is predicted to be a constant in many models but is large and time varying for U.S. data. It is also large and time varying in my model simulations. The reason is that in equilibrium, the marginal rate of substitution is equal to the real wage rate, W_{jt}/P_t, for all j, which in turn is equal to the ratio of total output in sector j—including output in new intangible investments—to total hours of work in sector j. Even if there were only one sector, this measure of labor productivity is not equal to GDP per hour. Fluctuations in intangible investments over the cycle would imply much more variability in labor productivity and would look to Chari et al. (2007) as if there were time variation in labor income taxes.

In Table 3, I report results for predicted hours by sector, which in the case of the model is the sum of hours in tangible and intangible production. The first column compares the correlations of predicted and actual logged hours after applying a Hodrick-Prescott filter to remove low frequencies. With three exceptions, I find positive correlations between the predicted and actual series. If I take a weighted average using industry shares of hours as weights, I find the average is over 50 percent, which is high. In information and professional business and services, the correlations are over 90 percent.[26]

Next, I investigate the model's predictions for the comovement of hours across sectors, which are known to comove positively in U.S. data. As Hornstein and Praschnik (1997) have shown, including input-output linkages can improve the performance of business-cycle models in predicting positive comovements of sectoral labor inputs. The measure of comovement that I use is based on a principal components analysis (PCA). The idea is to transform the data by constructing uncorrelated "components" that are linear combinations of the data, with the first component accounting for the maximal variance. The first component should account for a large fraction of the overall variance if the series positively comove. The coefficients in the linear mapping from data to components are the factor loadings and are bounded between -1 and 1.

Table 3 reports the main findings of the analysis. Specifically, I report the factor loadings for the model hours and the U.S. hours by industry along with the percentage of the variance attributed to the first principal component. Not surprisingly, the predicted and actual factor loadings are similar for sectors with a high correlation between the predicted and actual

[24] See Appendix A for data sources used. Materials for replication of all results are available at https://users.econ.umn.edu/~erm. Users can edit the codes to run their own cases.

[25] The NIPA data do include some intangible investments, and the fixed assets do include some intangible capital. Stripping them out does not affect the main results for the one-sector benchmark model.

[26] The high estimates for the intangible-intensive sectors are not a result of including hours in the observer equation, because I include hours of subsectors within these major industries only when estimating the shock processes.

A. One-Sector Model without Intangibles

B. Multi-Sector Model with Intangibles

Fig. 1. Per Capita Hours, 1985:1–2014:4.

hours. What is more surprising is the fact that the model's first component accounts for close to 70 percent of the variance in the model time series, which is even higher than the 56 percent estimate for the U.S. data.

This comovement could be the result of the input-output linkages, or it could be the case that the common component of TFP accounts for most of the variance in the data used to estimate the shock processes. I next turn to a variance decomposition of the observed time series to further investigate the role of the input-output linkages across sectors.

4.2. Variance decompositions

I compute two conceptually different variance decompositions. First, I decompose the variances of the observed time series y_t in (3.2) using the ergodic distribution of the model based on the updated input-output table and the estimated shock processes. Second, I decompose aggregate gross output during the technology boom of the 1990s and the Great Recession of 2008–2009. For both, I find that sectoral shocks and input-output linkages are quantitatively important features of the model.

In Table 4, I report the variance decomposition for the model's ergodic distribution. The rows correspond to the gross outputs for the major private industries and hours for three subsectors of the intangible-intensive industries.[27] The columns in Table 4 correspond to the shocks. The first column is the total variance that is due to sectoral shocks. This variance is

[27] The government sector is not listed, since I imposed restrictions on the shocks in this sector.

Table 3
Cyclical properties of predicted and actual sectoral hours.[a]

Major industry	Correlation, predicted and actual	PCA first factor loadings	
		Predicted	Actual
Agriculture	25	31	10
Mining	65	24	23
Utilities	−8	30	7
Construction	76	9	33
Manufacturing	89	31	33
Wholesale Trade	48	29	33
Retail Trade	55	24	32
Transportation & Warehousing	65	30	33
Information	96	17	24
Finance, Insurance & Real Estate	27	−10	24
Professional & Business Services	95	27	33
Education, Health & Social Services	−4	−30	9
Leisure & Hospitality	−52	−24	31
Other Services	52	27	25
Summary Statistics[b]	51	69	56

[a] For both the model and data, hours series are logged and detrended using the filter of Hodrick and Prescott (1997) and then scaled by their standard deviations. PCA stands for *principal component analysis*.

[b] The summary statistic in the first column is the weighted average correlation for all industries, with weights equal to shares of sector hours in total hours. The second and third columns are the percentage variances of the first principal component in the model and the data, respectively.

Table 4
Variance decomposition of ergodic distribution, 1985:1–2014:4.[a]

Observable	Sector-specific			Common shock
	Total	Own industry	Other industry	
Gross Outputs:				
Agriculture	96.4	61.8	34.6	3.6
Mining	99.9	98.8	1.2	0.1
Utilities	98.8	61.9	37.0	1.2
Construction	77.9	39.2	38.7	22.1
Manufacturing	91.5	75.7	15.8	8.5
Wholesale Trade	81.5	32.5	16.8	18.6
Retail Trade	60.0	27.5	32.5	40.0
Transportation & Warehousing	70.6	29.7	40.9	29.4
Information	74.1	49.4	24.7	25.9
Finance, Insurance & Real Estate	64.7	9.0	55.7	35.3
Professional & Business Services	73.5	57.8	15.7	26.5
Education, Health & Social Services	67.4	8.6	58.9	32.6
Leisure & Hospitality	65.1	10.2	54.9	34.9
Other Services	62.5	20.4	42.2	37.5
Hours:				
Computer & Electronic Products	90.9	80.3	10.6	9.1
Broadcasting & Telecommunications	78.5	49.8	28.8	21.5
Advertising	63.8	42.0	21.8	36.2

[a] These results are based on the estimated state space system in (3.1)–(3.2).

split between own-sector shocks (due to either Z^1_{jt} or Z^2_{jt} for industry j) and other-industry shocks. The last column is the variance that is due to the common TFP shock. Notice first that sectoral shocks are quantitatively important for every industry. In all cases, the variance due to sectoral shocks is at least as high as 60 percent. The industries most affected by the common shock are retail trade and many of the services. Another noteworthy feature of the results is the contribution of other-industry shocks. For many sectors, the contribution is sizable, indicating that input-output linkages are playing an important role in propagating shocks. In fact, in six industries the contribution of other-industry shocks is greater than that of own-industry shocks, and in 10 industries it is greater than the common shock. Only in the case of mining is the variance in gross output nearly all due to own-industry shocks.[28]

One issue with the variance decomposition in Table 4 is the fact that the 1985–2015 sample exhibits significant trends, which will bias these estimates. Most likely, the trends imply more weight on sectoral shocks and less weight on common

[28] Foerster et al. (2011) decompose industrial production data, which cover mining, manufacturing, and some utilities. They find that half of the variation in these data is due to sector-specific shocks.

Table 5
Decomposition of changes in gross output in the technology boom and the Great Recession[a].

TFP shocks	Technology boom	Great recession
Common	7.1	−6.9
Sectoral:		
Agriculture	−0.8	0.0
Mining (21)	0.2	−1.2
Utilities	0.4	−1.0
Construction	0.2	−1.0
Manufacturing	−3.8	−5.9
Wholesale Trade	−0.5	−0.5
Retail Trade	−0.3	−0.4
Transportation & Warehousing	−0.5	−0.2
Information	1.6	−0.2
Finance, Insurance & Real Estate	3.6	−0.8
Professional & Business Services	4.2	0.9
Education, Health & Social Services	0.1	0.4
Leisure & Hospitality	0.2	−0.1
Other Services	0.0	0.0
Total Change (%)	11.5	−16.8

[a] Percent changes are computed over the periods 1991:4–2000:3 and 2007:4–2009:3, respectively.

shocks. Thus, as an alternative summary of the variance decomposition, I decompose the growth rates of gross output in the two episodes mentioned above: the 1990s technology boom and the Great Recession.

The results are shown in Table 5. Here, the rows correspond to the source of shocks. The columns report the change in aggregate gross output growth attributable to shocks from each source. There are two periods and therefore two estimates for each period. The table shows that the common TFP shock accounts for roughly 60 percent of the increase in total gross output over the period 1991:4 to 2000:3 and 40 percent of the decline over the period 2007:4 to 2009:3. As expected, these estimates are higher than the contributions for the ergodic distribution but still imply a large role for sectoral shocks and industry linkages. Which sectors play an important role depends on the episode. In the technology boom, shocks to TFP in information; finance, insurance, and real estate; and professional and business services are important for the business cycle. In the Great Recession, shocks to manufacturing TFP are important.

The variance decompositions of the observed data indicate a clear rejection of the one-sector real business-cycle benchmark model in favor of the new multi-sector model. Next, I investigate the properties of the key latent factors: intangible investments and total factor productivities that are central to this new benchmark model.

4.3. Properties of latent variables

I apply a Kalman smoother to the model in order to construct predictions for the state x_t in (3.1), as well as prices and decisions that are functions of the state. In this section, I discuss the properties of the total factor productivities $\{Z_t, Z^i_{jt}\}$ and the intangible investments X_{Ijt}. I consider the full sample and then look more closely at these time series during the Great Recession.

In Table 6, I report the cyclical properties for the latent variables over the full 1985:1–2014:4 sample after logging and detrending them with the filter of Hodrick and Prescott (1997). The first column reports the standard deviation relative to gross output. The first row shows that the common TFP in the model has a standard deviation that is 80 percent of total output. The sectoral TFPs, which are listed next, vary at least as much over the business cycle as the common TFP. For some industries such as mining and utilities, the variation in sectoral TFP is much larger. Recall from Table 4 that these industries are barely affected by the common shock. The standard deviations relative to gross output for the intangible investments are listed in the last three rows of Table 6 for the intangible-intensive industries. The ratios are in the range of 1.5 to 1.8, which is about half as variable as the predictions for tangible investment in the standard real business cycle model without intangible investments.[29]

Correlations with gross outputs at leads and lags are reported in the last five columns of Table 6. The common TFP and most of the sectoral TFPs are procyclical, with the highest correlation occurring contemporaneously. There are some notable exceptions. TFPs in information and other services are close to acyclical, and TFP in education, health, and social services is countercyclical. Intangible investments are all procyclical, but they lag the cycle by one or two quarters.

A closer examination of the time series during the Great Recession provides further insight into the properties of the latent variables. In Fig. 2, I compare the time series of the model's predicted common-TFP shock with two standard aggregate TFP measures used in the literature. The series are logged and linearly detrended, but other low frequencies are not filtered out. I standardize the series by first subtracting the 2007:4 value and then dividing by the standard deviation of the series over the full sample.

[29] For example, Kydland and Prescott (1982) estimate a ratio of 3.6.

Table 6
Cyclical properties of latent TFPs and intangible investments[a]

Variable	Std. deviation relative to gross output	Cross-correlation with gross output at lag				
		−2	−1	0	1	2
Common TFP	0.8	0.76	0.84	0.86	0.76	0.59
Sectoral TFPs						
Agriculture	8.8	0.09	0.15	0.15	0.10	0.02
Mining	23.8	0.27	0.54	0.69	0.69	0.55
Utilities	16.9	0.16	0.36	0.51	0.57	0.53
Construction	2.1	0.47	0.46	0.44	0.38	0.24
Manufacturing	4.9	0.72	0.86	0.87	0.71	0.45
Computer & Electronic Products	3.2	−0.04	0.04	0.17	0.34	0.49
Wholesale Trade	1.6	0.25	0.42	0.51	0.47	0.33
Retail Trade	1.5	0.48	0.37	0.18	−0.08	−0.32
Transportation & Warehousing	1.5	0.22	0.42	0.60	0.72	0.68
Information	2.1	−0.05	−0.02	−0.00	0.01	0.03
Broadcasting & Telecomm.	2.6	0.25	0.47	0.62	0.64	0.55
Finance, Insurance & Real Estate	2.1	0.68	0.65	0.53	0.38	0.27
Professional & Business Services	1.2	0.00	0.08	0.17	0.26	0.33
Advertising	0.8	−0.04	0.05	0.18	0.38	0.54
Education, Health & Social Services	0.8	−0.59	−0.65	−0.65	−0.62	−0.56
Leisure & Hospitality	0.9	0.16	0.21	0.28	0.33	0.31
Other Services	1.2	−0.29	−0.23	−0.14	−0.05	0.00
Intangible investments:						
Manufacturing	1.8	0.32	0.53	0.70	0.81	0.83
Information	1.5	0.32	0.53	0.70	0.75	0.72
Professional & Business	1.7	0.51	0.67	0.79	0.85	0.83

[a] Series are first logged and detrended using the filter of Hodrick and Prescott (1997). For cross-correlations, the variable at date t is correlated with gross output at date $t-k$, where k is given in the column heading.

The first widely used measure of TFP, which is plotted in panel A of Fig. 2, is the Solow residual—the same series used to generate the hours prediction in Fig. 1A. As the figure shows, the Solow residual falls quickly at the start of the recession and rapidly returns to the long-run trend by mid-2009, exactly when the Great Recession was declared over by the National Bureau of Economic Research. Over the remaining years, there is slower growth, and TFP falls gradually relative to the long-run trend. In contrast, the model predicts that growth in the common TFP slows at the start of the recession and remains on a lower long-run trend.

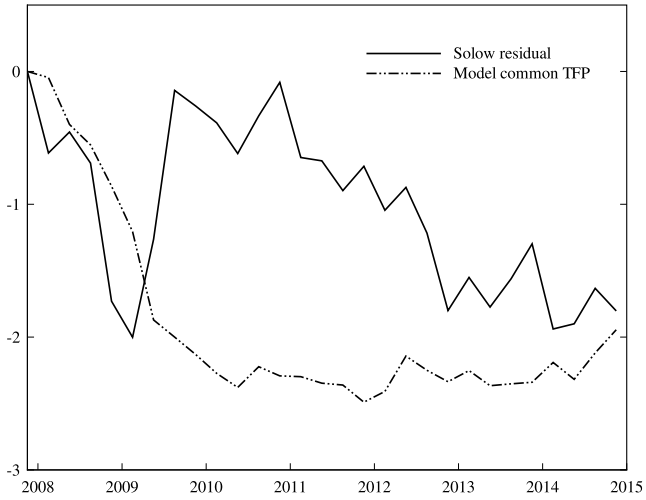
A second widely used measure of TFP is plotted in Fig. 2B, along with the model prediction. Here, I plot the utilization-adjusted TFP series of Fernald (2012), which is based on the methodology of Basu et al. (2006) that uses observed-hours growth to adjust TFP for unobserved variation in labor effort and the workweek of capital. A comparison of the two panels shows that the timing of Fernald's (2012) series and the Solow residual is completely different in 2008 and 2009. The Solow residual falls dramatically below trend and then recovers, whereas Fernald's (2012) series falls modestly and then rises above the long-run trend. After 2010, both gradually fall relative to the long-run trend, but neither resemble the model's prediction over the sample.

Although neither of the two widely used TFP measures behaves like the model's prediction during the Great Recession, over the full sample, they are more correlated at low frequencies. For example, the correlation between the model's common TFP and the Solow residual is 73 percent over the period 1985 to 2015. The correlation between the model's common TFP and Fernald's (2012) TFP is 40 percent over the same period. If I apply instead the filter of Hodrick and Prescott (1997) to all of the series, I find a correlation of 9 percent between the model TFP and the Solow residual and a correlation of 31 percent between the model TFP and Fernald's (2012) TFP.

Fig. 3 shows results for tangible and intangible investment during the Great Recession. In both panels, I plot U.S. tangible investment, which is real gross private domestic investment less investment in intellectual-property products divided by population and geometric growth in technology. This series is not used to estimate the TFP shock processes and therefore provides another external check of the model's predictive capabilities. In panel A, I plot the model's theoretical analogue for the U.S. tangible investment series and in panel B, I plot the model's prediction for intangible investment. To make the data and model series comparable, I set all equal to 100 in 2007:4 (although the model series are similar in magnitude).

Fig. 3A shows that the model does surprisingly well in predicting the sharp reduction in tangible investment during the Great Recession and a slow recovery. The model predicts a more delayed fall in 2008 but by 2008 is roughly 40 percent below trend, which is what was observed in U.S. data. Furthermore, although investment recovers more quickly in the U.S. data, both series are still well below trend by 2015. In contrast, intangible investment shown in Fig. 3B falls more gradually and by only 20 percent by 2015. The pattern of decline for intangible investment is similar to the pattern of decline in the common TFP shown in Fig. 2.

A. Model Prediction vs. Solow Residual



B. Model Prediction vs. Fernald's (2012) TFP

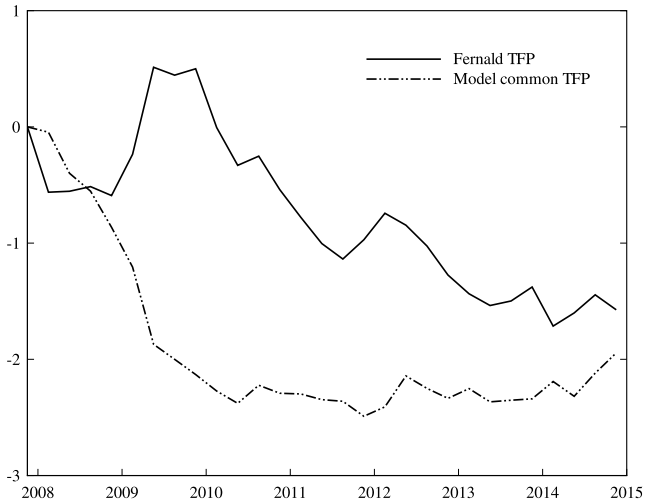


Fig. 2. Aggregate TFP, 2007:4–2014:4.

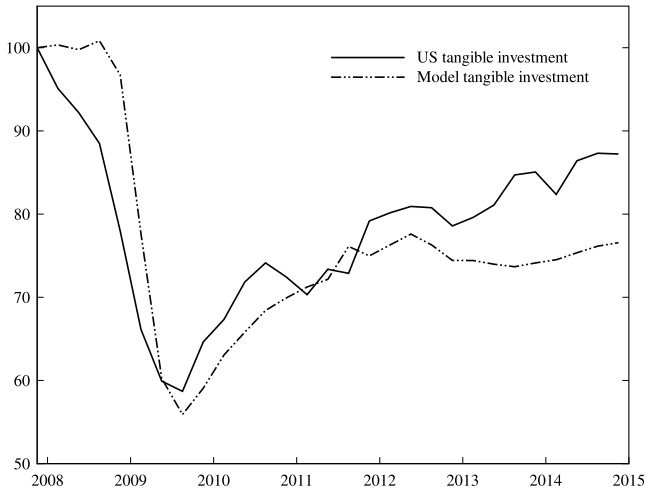
4.4. Extension with financial shocks

The results thus far assume that resources are efficiently allocated and fluctuations are driven by changes in total factor productivities. Next, I introduce financial shocks and rerun all results from Sections 4.1 to 4.3. The main finding is that there is almost no difference in the results shown in Tables 3–6 and Figs. 1–3.

To understand why, it helps to look at the implied labor wedges, which in this model are equal to the multipliers on (2.18) times the derivatives of the full dividend payment $\varphi'(D_j)$ for all industries j with external financing.[30] The impact on real activity depends on how tightly the enforcement constraint in (2.18) binds over the cycle, which is measured by fluctuations in the constraint's multiplier. From the perspective of firms maximizing dividends, this multiplier puts a wedge between the wages paid to workers and their marginal product, because the wages must be financed through borrowing. In equilibrium, this wedge has the same effect as a time-varying tax on labor—that is, time variation in τ_h in (2.8). A tightening of the constraint in recessions is isomorphic to increasing the tax rate. In the spirit of business-cycle accounting, the financial friction manifests itself as a time-varying labor wedge (see Chari et al., 2007). A time-varying labor wedge that comoves with the business cycle is needed to help reconcile the difference between predicted and actual hours shown in Fig. 1A.

[30] Most of the variation in the wedges is due to changes in the multiplier, not changes in the dividend payments.

A. U.S. Tangible Investment vs. Model Prediction



B. U.S. Tangible vs. Model Intangible Investment

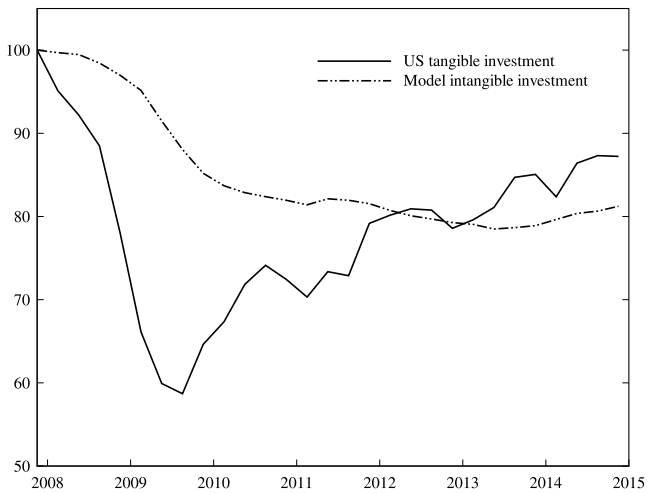


Fig. 3. Tangible and Intangible Investment, 2007:4–2014:4.

Table 7
Properties of implied labor wedges in models with financial shocks.

	Mean	Minimum	Maximum	Standard deviation	Correlation with output
Extended Model[a]					
Mining	0.021	0.008	0.033	0.006	−0.100
Manufacturing	0.006	0.003	0.007	0.001	0.489
Transportation & Warehousing	0.006	−0.002	0.010	0.002	−0.075
Leisure & Hospitality	0.008	0.000	0.016	0.003	0.040
Jermann and Quadrini (2012)					
Original Parameterization	0.039	0.013	0.074	0.013	−0.389
Alternative Parameterization[b]	0.015	0.008	0.023	0.003	−0.292

[a] The model is described in Section 2.2. The labor wedge is the shadow price of the enforcement constraint times the derivative of the dividend payout.
[b] In the alternative parameterization, the capital share is lower (0.22 versus 0.36) and the mean of the financial shock is higher (0.41 versus 0.16) than in Jermann and Quadrini (2012). These parameters are chosen so that the average capital-output ratio and the average financial shock in the Jermann and Quadrini (2012) model is consistent with the data that they use to construct the financial shocks.

Table 7 reports labor wedges for the extended model and Jermann and Quadrini's (2012) one-sector model. Five statistics are reported: mean, minimum, maximum, standard deviation, and correlation with total output.[31] What is most relevant is the variability of the series, which can be measured by comparing the minimum and maximum of the range or the standard deviation. Significant wedge volatility is needed to account for the high variability of U.S. hours of work. Furthermore, the correlation with output needs to be negative to generate procyclical predictions for hours.

Table 7 shows that in the case of the extended multi-sector model, the industry labor wedges are not simultaneously large, volatile, and countercyclical in any sector, while the implied labor wedge derived from Jermann and Quadrini's (2012) one-sector model is. One possible reason for the difference in properties is the parameterizations used for each model. Jermann and Quadrini (2012) use a high capital share, higher than that implied by the capital-output ratios in the data they use. Higher capital shares imply lower values for the financial shocks in (2.18), which in turn implies that the constraints are looser. To test this idea, I recompute the Jermann and Quadrini model with a lower capital share and a higher mean for the financial shock, which are chosen to be consistent with the data they use. More specifically, I set the capital share equal to 0.22—down from 0.36 in the original parameterization—and I set the mean of the financial shock to 0.41—up from 0.16 in the original parameterization. For this alternative parameterization, the labor wedge is significantly smaller, less volatile, and less correlated with output. In this alternative case, the standard deviation of predicted hours is 24 percent of the standard deviation of U.S. hours, which is significantly lower than the estimate of 47 percent for their original parameterization and closer to the estimate of 9 percent for the one-sector real business-cycle model shown in Fig. 1A.

In summary, the time-varying labor wedges arising from a tightening of firms' financing conditions do not vary sufficiently in the extended model to have much of an impact on real activity, and therefore the results are quantitatively similar to the frictionless baseline.

5. Conclusion

In the recent comprehensive revision of the national accounts, the BEA has greatly expanded its coverage of intellectual-property products. In this paper, I expand the coverage further and use a multi-sector general equilibrium model to quantify the impact of including these products, which I refer to as intangible investments, in both the theory and the measures of TFP. I find that updating both the theory and the data is quantitatively important for analyzing fluctuations in aggregate and industry-level U.S. data and provides a new benchmark model for business-cycle research.

Appendix A. Data appendix

In this appendix, I report all data sources for this project. Original data and replication files are available at my website: users.econ.umn.edu/~erm/data/sr545.

- *Input-output shares*
  - The main source of data for the shares is the BEA. I start with the detailed BEA input-output use table before redefinitions at producer value for the 2007 benchmark, which tracks transactions for 389 commodities. The BEA table has not yet published a capital-use table for this benchmark, so I construct two capital-use tables—one for structures and equipment and another for intellectual-property products—using detailed data underlying the fixed asset tables, which are available by industry and by investment type. I assign all custom and own-software and R&D to the investing industry (rather than to information and professional and business services, as the BEA does). I add intermediate purchases of computer systems design services; architectural, engineering, and related services; specialized design services; management consulting services; environmental and other technical consulting services; advertising, public relations, and related services; and marketing research to the capital-use table for intellectual-property products. I add consumer durables and inventories to the capital-use tables. I include public spending with appropriate categories of private spending. I allocate in a pro rata way net exports to domestic categories. (The code setupio.m replicates construction of the shares.)
- *Time series for maximum likelihood estimation*
  - Gross outputs, all major industries: data for nominal gross outputs are available from the BEA annually for all years of my sample and quarterly after 2005. Series are divided by population and by the GDP deflator, and quarterly forecasts are computed with the procedure outlined in Appendix B for years before 2005. The auxiliary quarterly data used for the forecasting are national incomes by major industry from the BEA's national income and product accounts and employment by major industry from the BLS's Current Employment Survey (CES). Both the national income and gross output data are divided by the GDP deflator.
  - Hours per capita, three minor industries: series are constructed with employment data from the CES and hours-per-employee data from the BLS's labor productivity and cost (LPC) database, which provides data for 817 industries. Per capita hours are total employees times hours per employee divided by the noninstitutional population ages 16 to 64.

[31] In deriving time series for shocks in the one-sector model of Jermann and Quadrini (2012), I follow their procedure of removing linear trends from the capital-to-output and debt-to-output ratios, and I am able to replicate all of their results. In the multi-sector model, I do not remove trends from these ratios, which are assumed to be stationary.

- *Time series for external validation*
  - Sectoral hours per capita, major industries: series are constructed in the same way as the minor industries noted above.
  - Aggregate hours per capita: the series for the aggregate economy is computed using the same procedure in Cociuba et al. (2009), who start with total civilian hours from the BLS's Current Population Survey, add estimates for military hours, and divide by the noninstitutional population ages 16 to 64.
  - Tangible investment: the series is gross private domestic investment less investment in intellectual-property products, deflated by the GDP deflator, and divided by the noninstitutional population ages 16 to 64.
- *Total factor productivity series*
  - Solow residual: the series is the BEA's real GDP measure divided by the BEA's total fixed assets raised to the power 1/3 and total hours defined above raised to the power 2/3. Total fixed assets are annually available and are log-linearly interpolated to construct a quarterly time series for TFP.
  - Fernald's (2012) utilization-adjusted TFP: frequently updated by Fernald and available at his website at the Federal Reserve Bank of San Francisco.
- *Compustat data for extension with financial shocks*
  - Debt-to-output ratio: firm-level data for debt are aggregated to the industry level and divided by industry sales. I follow Larrain and Yogo's (2008) procedure to compute total debt, which is the sum of long-term debt, current liabilities, other liabilities, minority interest, and deferred and investment tax credit. The market value of long-term debt is found by imputing a market structure of bonds for each firm and then a price for each maturity based on the Moody's Baa corporate-bond yield.
  - Capital-to-output ratio: capital is computed using the perpetual inventory method, with gross investment equal to capital expenditures plus acquisitions less sales of property, plant, and equipment, and an annual depreciation rate of 3.2 percent. The series is aggregated to the industry level and divided by industry sales.

Appendix B. Quarterly forecasts

In this appendix, I describe the procedure used to construct quarterly forecasts for time series that are only available annually for part of my sample.

Let Z_t be the variable of interest, which is available annually. Let X_t be variables that are available quarterly and are used to make quarterly forecasts of Z_t, which I will call $\hat{Z}_t$. The first step in deriving a forecast is to estimate A and B of the following state space system via maximum likelihood:

$$x_{t+1} = Ax_t + B\epsilon_{t+1}$$
$$y_t = C_t x_t,$$

where $x_t = [X_t, \hat{Z}_t, X_{t-1}, \hat{Z}_{t-1}, \ldots, X_{t-n}, \hat{Z}_{t-n}]'$ for some choice $n \geq 4$, $y_t = [X_t, Z_t]'$, and ϵ_t are normally distributed shocks. The coefficients in this case are given by

$$A = \begin{bmatrix} a_1 & a_2 & \ldots & a_j \\ I & 0 & \ldots & 0 \\ 0 & I & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & 0 \end{bmatrix}, \quad B = \begin{bmatrix} b \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$C_t = \begin{cases} \begin{bmatrix} I & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & \ldots \\ 0 & 1/4 & 0 & 1/4 & \ldots & 0 & 1/4 & 0 & \ldots \end{bmatrix} & \text{if } t \text{ is 4th quarter} \\ \begin{bmatrix} I & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & \ldots \end{bmatrix} & \text{otherwise.} \end{cases}$$

Once I have parameter estimates, $(\hat{A}, \hat{B})$, I construct forecasts in all quarters given the full sample of data—namely, $\hat{Z}_t = E[Z_t | y_1, \ldots, y_T]$—by first applying the Kalman filter and then applying the Kalman smoother. (See Harvey, 1989 for more details on the Kalman filter and smoother.)

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