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Modal parameter estimation using free response measured by a continuously scanning laser Doppler vibrometer system with application to structural damage identification

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ABSTRACT

Spatially dense vibration measurement can be obtained by use of a continuously scanning laser Doppler vibrometer (CSLDV) system that sweeps its laser spot along a scan path. For a linear, time-invariant, viscously damped structure undergoing free vibration, a type of vibration shapes called free response shapes was defined and obtained by the authors using a CSLDV system with the demodulation method. To date, application of free response shapes is limited to structural damage identification, and they cannot be directly used for model validation while mode shapes can be. This paper extends the concept of free response shapes by proposing a new modal parameter estimation (MPE) method using a CSLDV system to estimate modal parameters of the structure undergoing free vibration, including natural frequencies, modal damping ratios, and mode shapes; the MPE method is applicable to linear structures without repeated and closely spaced modes. Advantages of the proposed method are: (1) modal damping ratios and mode shapes can be accurately estimated from obtained free response shapes in the least-square sense, (2) the scanning frequency of the CSLDV system can be relatively low, and (3) estimated mode shapes can be used for structural damage identification as if they were measured by stepped scanning of a scanning laser Doppler vibrometer. A baseline-free method is applied to identify structural damage using mode shapes estimated by the proposed MPE method. The method does not require any baseline information of an undamaged structure, such as its complete geometry, material properties, boundary conditions, modal parameters, and operating deflection shapes. In the proposed MPE method, natural frequencies of the structure are identified from free response of certain fixed points on the structure; its modal damping ratios and mode shapes are simultaneously estimated using free response shapes measured by a CSLDV system. Both numerical and experimental investigations are conducted to study the MPE method and its application to baseline-free damage identification with mode shapes estimated by the MPE method.

1. Introduction

Vibration-based damage identification has been a major research topic of structural dynamics in the past few decades [1,2]. Occurrence of damage in a structure undermines its capability of supporting design loads and can result in its excessive defor-

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Table 1
First five (a) natural frequencies in Hz and (b) modal damping ratios in percentage of the damaged beam from its finite element model and the MPE method using its response caused by a single impulse.

Mode	Finite element	MPE method
	(a)	
1	12.64	12.62
2	79.04	79.00
3	222.0	222.0
4	434.8	434.7
5	711.2	711.2
	(b)	
1	0.0032	0.0032
2	0.0199	0.0199
3	0.0558	0.0558
4	0.1093	–
5	0.1787	–

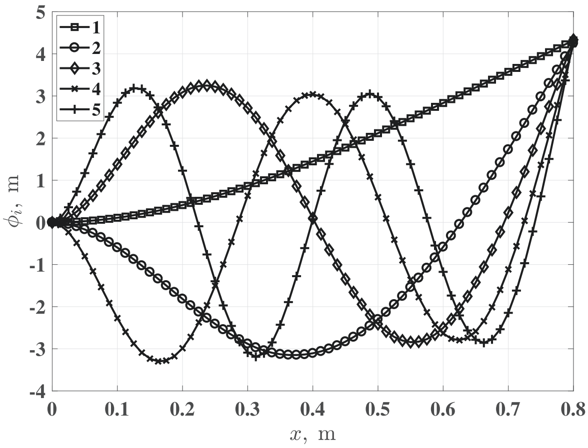

Fig. 1. Mass-normalized mode shapes of the damaged cantilever beam associated with its first five modes from its finite element model.

mation, which is attributed to changes in its structural properties, such as its stiffness. One assumption of a vibration-based damage identification method is that occurrence of damage changes modal parameters of a structure, including natural frequencies, modal damping ratios, and mode shapes, which can be accurately estimated by modal analysis [3]. Accurately estimated modal parameters can also assist model validation and updating.

A continuously scanning laser Doppler vibrometer (CSLDV) system is an ideal instrument for modal parameter estimation (MPE) as it is capable of accurate, non-contact and temporally dense vibration measurement and also capable of spatially dense mode shape measurement [4]. A CSLDV system consists of three key components: a laser Doppler vibrometer, a scanner and a controller [5]. The vibrometer measures the velocity of a point on a test structure where its laser spot is located. The laser beam of the vibrometer is directly shined onto first-surface mirrors of the scanner and the spot is continuously swept along a prescribed scan path on the structure by rotating the mirrors that are controlled by the controller. While the spot is continuously swept, velocity at each measurement point on the scan path is measured at the instant when the spot arrives at the measurement point, and the number of the measurement points can be tens and even hundreds of thousands, depending on the sampling and scan frequencies of the vibrometer. A CSLDV system has been successfully used for modal analysis and measurements of vibration shapes, such as mode shapes [6–8] and operating deflection shapes [9–13], which can be achieved with high accuracy.

A CSLDV system has been used to measure high-fidelity vibration shapes of structures undergoing steady-state vibrations for damage identification [5,14], and the vibration shapes measured by a CSLDV system can be used to identify structural damage as small as notch-size ones [15]. A type of vibration shapes called free response shapes was defined and measured by a CSLDV system when a linear underdamped beam underwent free vibration [16]. Free response shapes were defined to identify structural damage, where damage indices associated with multiple elastic modes of a beam could be obtained. A free response shape is different from a mode shape, since the former is time-varying with decaying amplitudes and the latter is not. So far, application of free response shapes is limited to structural damage identification and they cannot be directly used for model validation and updating due to two reasons. One is that a free response shape has an amplitude that is determined by excitation. Unless one can accurately measure the excitation, a free response shape cannot be used for model validation and updating. Another reason

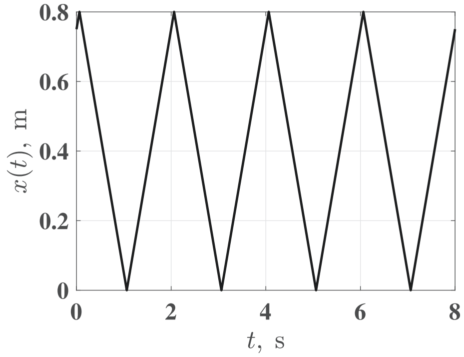

Fig. 2. Position of the laser spot of the simulated CSLDV system on the damaged beam.

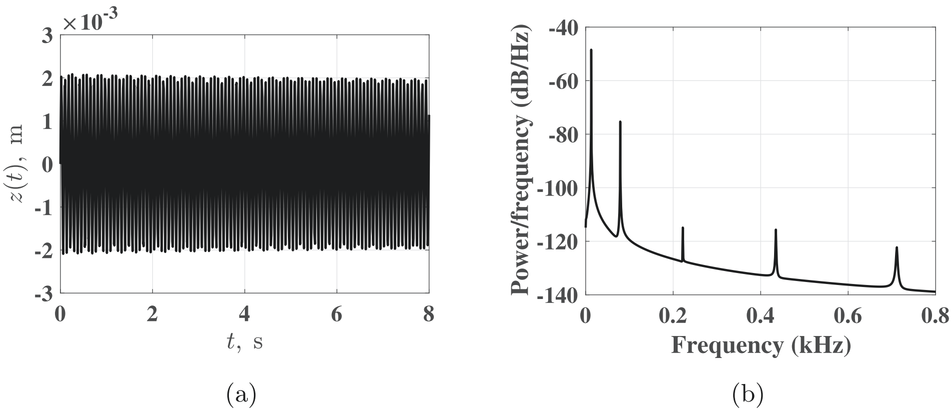

Fig. 3. (a) Response of the damaged cantilever beam at $x = 0.7$ due to the single impact at its free end and (b) the auto-power spectrum of the response in (a).

is that modal damping ratios cannot be estimated from free response shapes that are obtained in the method in Ref. [16]. An experimental modal analysis method was proposed [8], where excitation to a test structure and its free response measured by a CSLDV system yielded pseudo-frequency response functions of the structure, which were used to estimate modal parameters of the structure. In this method, the measured response is lifted to each measurement point as if the response were measured in a pointwise manner. A limitation of the method is that measured mode shapes of modes with relatively high natural frequencies can have low qualities due to speckle noise caused by a relatively high scanning frequency, which is needed since the scanning frequency of the CSLDV system is equal to the sampling frequency of the lifted response at each measurement point.

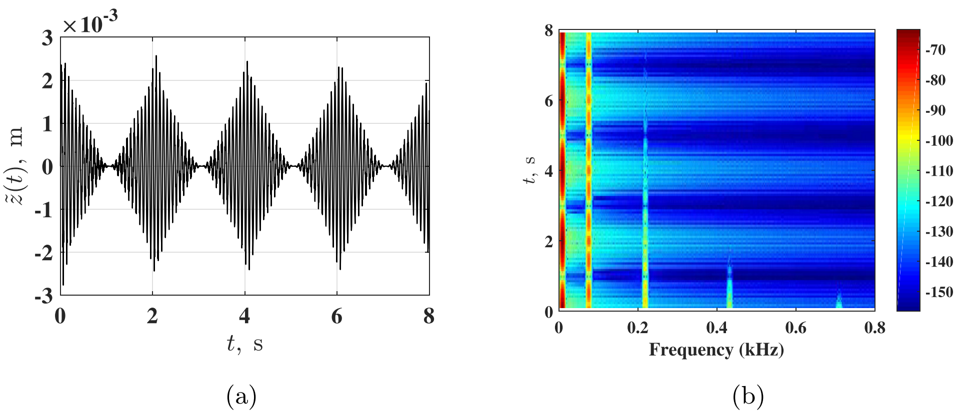

Fig. 4. (a) Response measured by the simulated CSLDV system in the first 8 s and (b) a spectrogram of the response in (a).

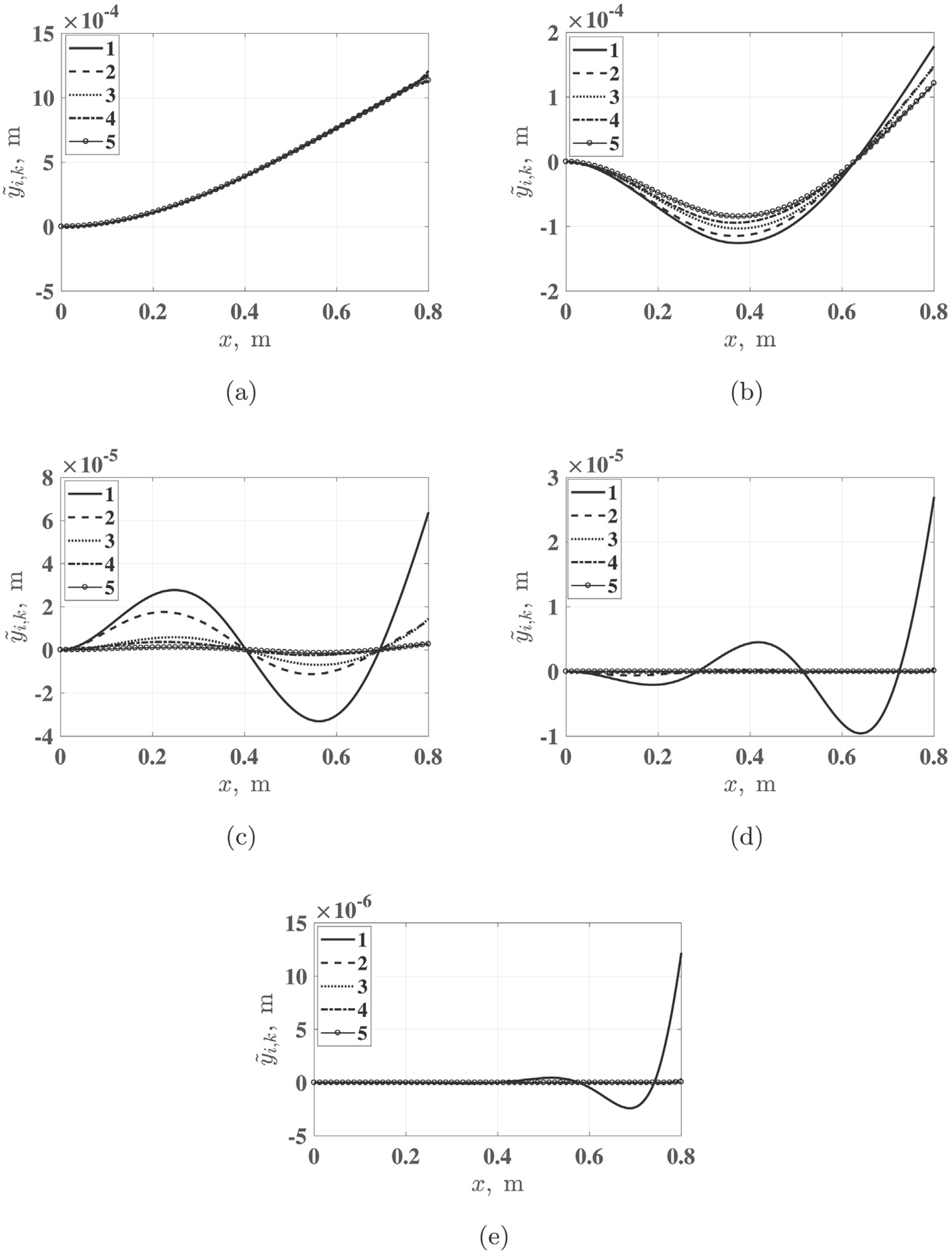

Fig. 5. Free response shapes of the damaged cantilever beam associated with its (a) first, (b) second, (c) third, (d) fourth and (e) fifth modes obtained from the response measured by the CSLDV system in the first five half-scan periods.

In this work, derivation of free response shapes of a linear, time-invariant, viscously damped structure undergoing free vibration is shown. A new MPE method using free response measured by a CSLDV system is proposed to accurately estimate modal parameters of the structure with a step-by-step procedure. The proposed MPE method is novel as it extends the concept of free response shapes of the structure to simultaneously estimate its modal damping ratios and mode shapes and the estimated modal damping ratios and mode shapes can be used for model validation and updating and structural damage detection. A baseline-free non-model-based damage identification method is applied to identify structural damage in a structure. The method does not require any baseline information of an undamaged structure, such as its complete geometry, material properties, boundary conditions, modal parameters, and operating deflection shapes. In the damage identification method, a curvature damage index (CDI) is obtained by comparing a curvature mode shape, which corresponds to a mode shape estimated by the MPE method, with that from a polynomial; the polynomial fits the mode shape estimated by the MPE method. Structural damage can be iden-

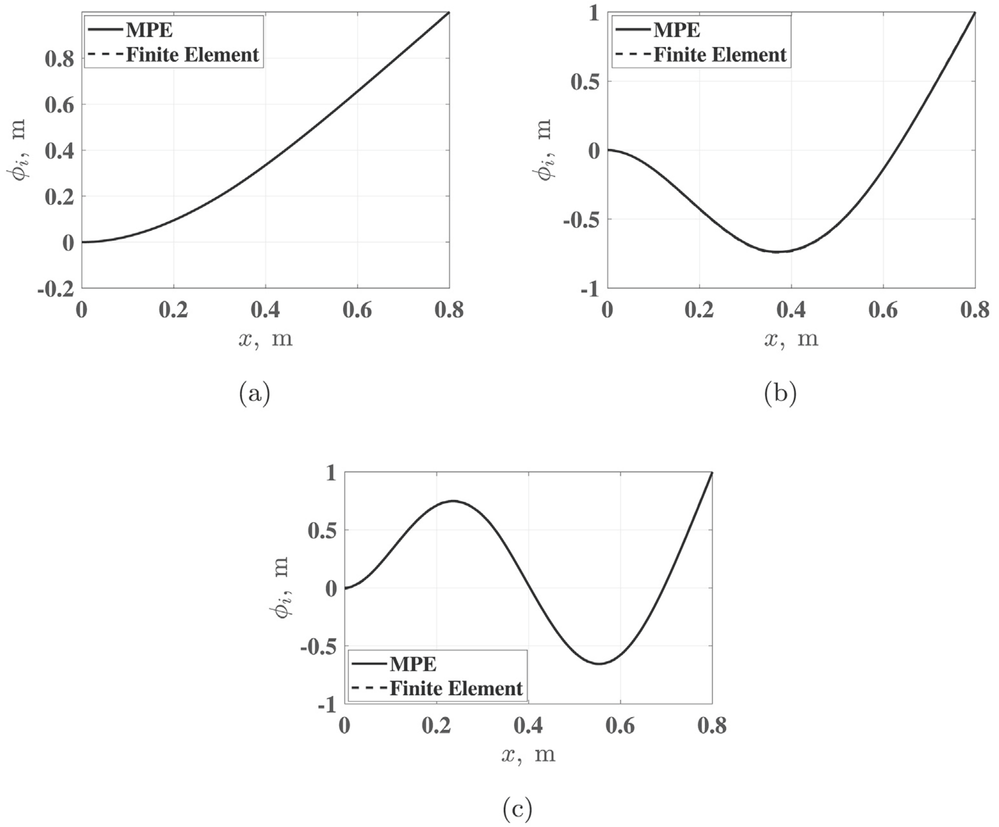

Fig. 6. Comparison between mode shapes of the damaged cantilever beam from the MPE method using the free response shapes and its finite element model associated with its (a) first, (b) second and (c) third modes.

tified in neighborhoods with consistently large CDIs corresponding to multiple modes. A numerical investigation is conducted to study the MPE method and application of the damage identification method. An experimental investigation was also conducted to validate the MPE method and application of the damage identification method by using data from a test for free response shapes in Ref. [16].

The remaining part of this paper is outlined as follows. Derivation of free response shapes is presented in Secs. 2.1 and 2.2, the new MPE method using a CSLDV system is proposed in Sec. 2.3, and the structural damage identification method is presented in Sec. 2.4. Numerical and experimental investigations of the MPE method and baseline-free method are presented in Secs. 3 and 4, respectively. Finally, conclusions of this study are presented in Sec. 5.

2. Methodology

2.1. Free response of a damped structure

Free response in the form of the displacement of a linear, time-invariant, viscously damped structure can be obtained by solving its governing partial differential equation:

$$B\left[\frac{\partial^2 z(\mathbf{x},t)}{\partial^2 t}\right]+C\left[\frac{\partial z(\mathbf{x},t)}{\partial t}\right]+L\left[z(\mathbf{x},t)\right]=0,\qquad \mathbf{x}\in D,\qquad t\geq 0 \tag{1}$$

where $B(\cdot)$, $C(\cdot)$ and $L(\cdot)$ are a mass operator, a damping operator and a stiffness operator, respectively, z is the displacement of the structure at the spatial position $\mathbf{x}$ at time t, and D is its spatial domain. Boundary and initial conditions of the structure are known. Note that the initial conditions can be induced by an external force that the structure is subject to when $t<0$. A solution to Eq. (1) can be obtained using the expansion theorem [17]:

$$z(\mathbf{x},t)=\sum_{i=1}^{\infty}\varphi_i(\mathbf{x})\,u_i(t) \tag{2}$$

where $\boldsymbol{\varphi}_i$ is the i-th mass-normalized eigenfunction of the associated undamped structure, whose governing equation

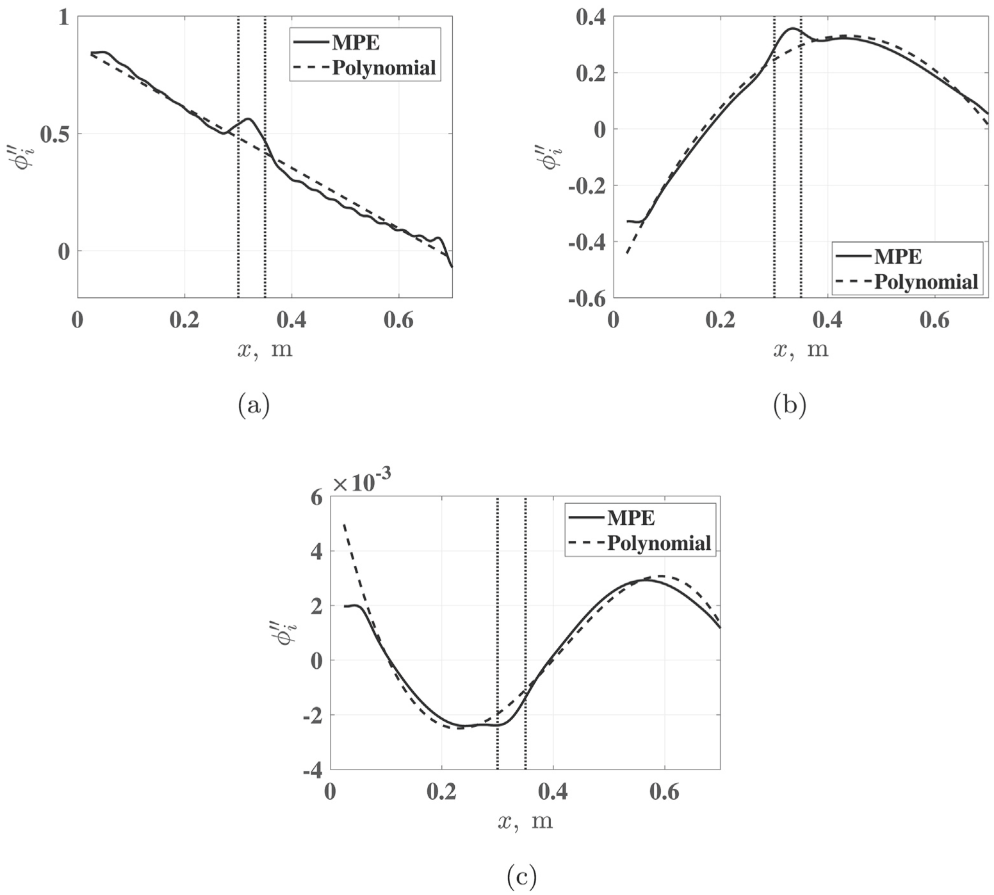

Fig. 7. Comparison between curvature mode shapes of the damaged cantilever beam from the MPE method and polynomial fits associated with its (a) first, (b) second and (c) third modes.

is assumed to be self-adjoint, and u_i is the corresponding unknown time function. Orthonormality between $\boldsymbol{\varphi}_i$ and $\boldsymbol{\varphi}_j$ ($j = 1, 2, \ldots, \infty$) with respect to B is expressed by

$$\int_D \varphi_j(\mathbf{x}) B\left[\varphi_i(\mathbf{x})\right] \mathrm{d}\mathbf{x} = \delta_{ij} \tag{3}$$

where δ_{ij} denotes Kronecker delta function, which satisfies $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ if $i \neq j$. Assuming that damping of the structure can be modeled by Kelvin-Voigt viscoelastic model, which leads to a classically damped system [17,18], one can obtain u_i in Eq. (2) by solving an ordinary differential equation:

$$\ddot{u}_i(t) + 2\zeta_i\omega_i\dot{u}_i(t) + \omega_i^2 u_i(t) = 0 \tag{4}$$

where ω_i is the corresponding i-th undamped natural frequency of the structure, ζ_i is the i-th modal damping ratio, which is smaller than 1 for an underdamped structure, and an overdot denotes differentiation with respect to t. The initial conditions $u_i(0)$ and $\dot{u}_i(0)$ can be determined from the initial conditions of Eq. (1). The solution to Eq. (4) can be expressed by [19]

$$\begin{aligned} u_i(t) &= \mathrm{e}^{-\omega_i\zeta_i t}\left[u_i(0)\cos\left(\omega_{i,d}t\right) + \frac{\dot{u}_i(0) + \omega_i\zeta_i u_i(0)}{\omega_{i,d}}\sin\left(\omega_{i,d}t\right)\right] \\ &= A_i \mathrm{e}^{-\omega_i\zeta_i t}\cos\left(\omega_{i,d}t - \gamma_i\right) \end{aligned} \tag{5}$$

where

$$\omega_{i,d} = \omega_i\sqrt{1 - \zeta_i^2} \tag{6}$$

is the i-th damped natural frequency of the structure,

$$A_i = \sqrt{\left[u_i(0)\right]^2 + \left[\frac{\dot{u}_i(0) + \omega_i\zeta_i u_i(0)}{\omega_{i,d}}\right]^2} \tag{7}$$

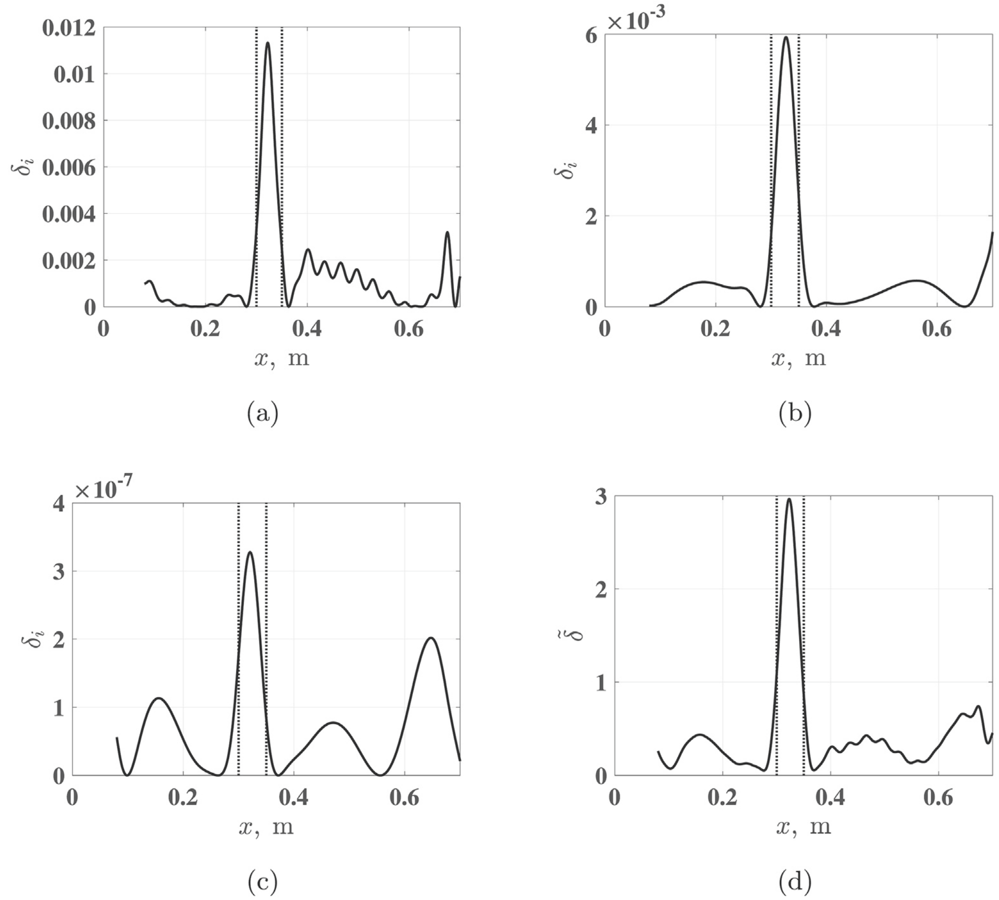

Fig. 8. CDIs of the damaged cantilever beam associated with its (a) first, (b) second and (c) third modes; (d) the auxiliary CDI associated with the curvature damage indices in (a) through (c). Ends of the damage are indicated by two dotted lines.

is an amplitude constant, and

$$\gamma_i = \text{arctan2}\left(\frac{\dot{u}_i(0) + \omega_i\zeta_i u_i(0)}{\omega_{i,d}}, u_i(0)\right) \tag{8}$$

is a phase angle; $\omega_i\zeta_i$ in Eq. (5) is referred to as the decay rate of u_i. Based on Eq. (5), Eq. (2) becomes

$$z(\mathbf{x}, t) = \sum_{i=1}^{\infty} A_i\varphi_i(\mathbf{x})\,\mathrm{e}^{-\omega_i\zeta_i t}\cos\left(\omega_{i,d}t - \gamma_i\right) \tag{9}$$

2.2. Free response shapes

A free response shape associated with the i-th mode of the structure can be defined by

$$y_i(\mathbf{x}, t) = A_i\varphi_i(\mathbf{x})\,\mathrm{e}^{-\omega_i\zeta_i t} \tag{10}$$

and Eq. (9) becomes

$$z(\mathbf{x}, t) = \sum_{i=1}^{\infty} y_i(\mathbf{x}, t)\cos\left(\omega_{i,d}t - \gamma_i\right) \tag{11}$$

The i-th eigenfunction φ_i that is the i-th undamped mode shape of the structure exists in the definition of y_i in Eq. (10). A similarity between φ_i and y_i is that they both correspond to the i-th mode of the structure; however, the former is time-invariant while the latter is time-varying due to the term $\mathrm{e}^{-\omega_i\zeta_i t}$ in Eq. (10).

A CSLDV system continuously sweeps its laser spot over a surface of a structure with a specific scan path. The system measures response of a measurement point on the structure with a certain sampling frequency, where its laser spot is located during a scan, and a finite number of modes of the structure are included in free response measured by the system. Let $\widetilde{\mathbf{x}}(t)$ be the position of a laser spot on the surface of the structure at time t, which describes the scan path on the structure as a function of t. Free response of the structure measured by the CSLDV system along $\widetilde{\mathbf{x}}(t)$ can be expressed by

$$\widetilde{z}(t) = \sum_{i=1}^{N} \widetilde{y}_i \left[\widetilde{\mathbf{x}}(t), t\right] \widetilde{u}_i(t) \tag{12}$$

where N is the number of modes included in $\widetilde{z}$, and $\widetilde{y}_i$ and $\widetilde{u}_i$ are the free response shape and time function associated with the i-th mode measured by the system, respectively. The free response shape $\widetilde{y}_i$ in Eq. (12) can be written as

$$\widetilde{y}_i \left[\widetilde{\mathbf{x}}(t), t\right] = A_i \varphi_i \left[\widetilde{\mathbf{x}}(t), t\right] \mathrm{e}^{-\omega_i \zeta_i t} \tag{13}$$

The time function $\widetilde{u}_i$ can be expressed by

$$\widetilde{u}_i(t) = \cos\left(\omega_{i,d} t - \alpha_i - \theta_i\right) \tag{14}$$

where α_i is the difference between the phase determined by the initial conditions and force associated with the i-th mode and that by a mirror feedback signal, and θ_i is a phase variable that controls amplitudes of in-phase and quadrature components of $\widetilde{y}_i$, which can be expressed by

$$\widetilde{y}_{I,i} = \widetilde{y}_i \left[\widetilde{\mathbf{x}}(t), t\right] \cos\left(\alpha_i + \theta_i\right) \tag{15}$$

and

$$\widetilde{y}_{Q,i} = \widetilde{y}_i \left[\widetilde{\mathbf{x}}(t), t\right] \sin\left(\alpha_i + \theta_i\right) \tag{16}$$

respectively [5]. Assume that all damped natural frequencies of the structure are well separated. The demodulation method has been used to obtain $\widetilde{y}_{I,i}$ and $\widetilde{y}_{Q,i}$ corresponding to each half-scan period by the system [16]. The demodulation method to estimate $\widetilde{y}_{I,i}$ and $\widetilde{y}_{Q,i}$ of the i-th mode is applied by multiplying time-domain CSLDV measurement of free vibration in one half-scan period by a sine function and a cosine function at the natural frequency of the mode, respectively, and then applying a low-pass filter to the multiplied measurements so that their sinusoidal components with non-zero frequencies are removed. Resulting filtered multiplied measurements are $\widetilde{y}_{I,i}$ and $\widetilde{y}_{Q,i}$ of the mode in the half-scan period. A half-scan period starts when the laser spot of the system arrives at one end of a scan path and ends when the laser spot arrives at the other end of the scan path. In this work, $\widetilde{y}$ is represented by $\widetilde{y}_{I,i}$, whose amplitude is maximized by adjusting θ_i in Eq. (15). Multiple $\widetilde{y}_i$ can be obtained from free response of the structure measured by the system in one scan. To identify the start and end of a half-scan period, one can refer to mirror feedback signals of the system and determine instants when its laser spot arrives at ends of a scan path.

2.3. MPE method

The amplitude of $\widetilde{y}_i$ in Eq. (13) is time-varying and exponentially decays to zero with t at the decay rate $\omega_i \zeta_i$. In order to obtain a non-zero amplitude of $\widetilde{y}_i$ from the demodulation method, one needs to determine natural frequencies of the structure and instants when the amplitude of $\widetilde{y}_i$ decays to zero. By assuming that the structure does not have repeated or closely spaced modes, the natural frequencies can be determined from the auto-power spectrum of z at a point that is measured by the system, and the instants can be determined using the short-time Fourier transform of $\widetilde{z}$ [20], which is denoted by $\widetilde{V_w}$. Some details of the short-time Fourier transform can be found in Appendix A. Multiple non-zero $\widetilde{y}_i$ can be obtained by using $\widetilde{z}$ of the first $N_{i,0}$ half-scan periods, where $N_{i,0}$ is an integer that is defined by

$$\arg\max_{N_{i,0}} \frac{N_{i,0} T}{2} \leq t_{i,0} - t_1 \tag{17}$$

in which T is the length of a scan period, $t_{i,0}$ is the instant when $\widetilde{V_w}$ at the i-th natural frequency of the structure becomes almost zero, and t_1 is the instant when the first half-scan period starts.

Let

$$Q_i = A_i \mathrm{e}^{-\omega_i \zeta_i t_1} \tag{18}$$

which is a complex constant, and $Q_i \varphi_i$ can also represent the i-th mode shape of the structure. One has $Q_i = A_i$ when $t_1 = 0$, and Eq. (13) with $t \geq t_1$ can be expressed by

$$\widetilde{y}_i \left[\widetilde{\mathbf{x}}(t), t\right] = Q_i \varphi_i \left[\widetilde{\mathbf{x}}(t), t\right] \mathrm{e}^{-\omega_i \zeta_i (t - t_1)} \tag{19}$$

One can estimate $Q_i \varphi_i$ in Eq. (19) if ζ_i is known. Let t_k be the instant when the k-th half-scan period starts and $\widetilde{y}_{i,k}$ be the free response shape associated with the i-th mode in the k-th half-scan period. The term $Q_i \varphi_i$ in Eq. (19) associated with $\widetilde{y}_{i,k}$ can be estimated by eliminating $\mathrm{e}^{-\omega_i \zeta_i (t - t_1)}$ in $\widetilde{y}_{i,k}$; it can be expressed by

$$Q_{i,k} \varphi_{i,k} \left[\widetilde{\mathbf{x}}(t), t\right] = \widetilde{y}_{i,k} \left[\widetilde{\mathbf{x}}(t), t\right] \mathrm{e}^{\omega_i \zeta_i (t - t_1)} \tag{20}$$

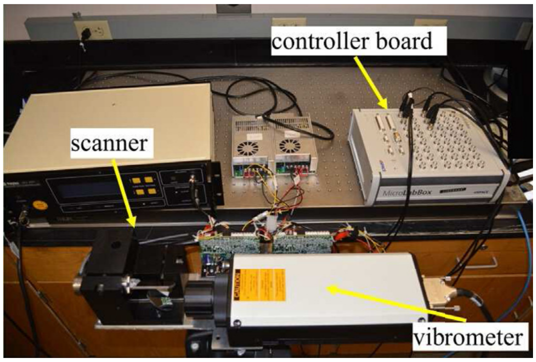

Fig. 9. CSLDV system used in this work.

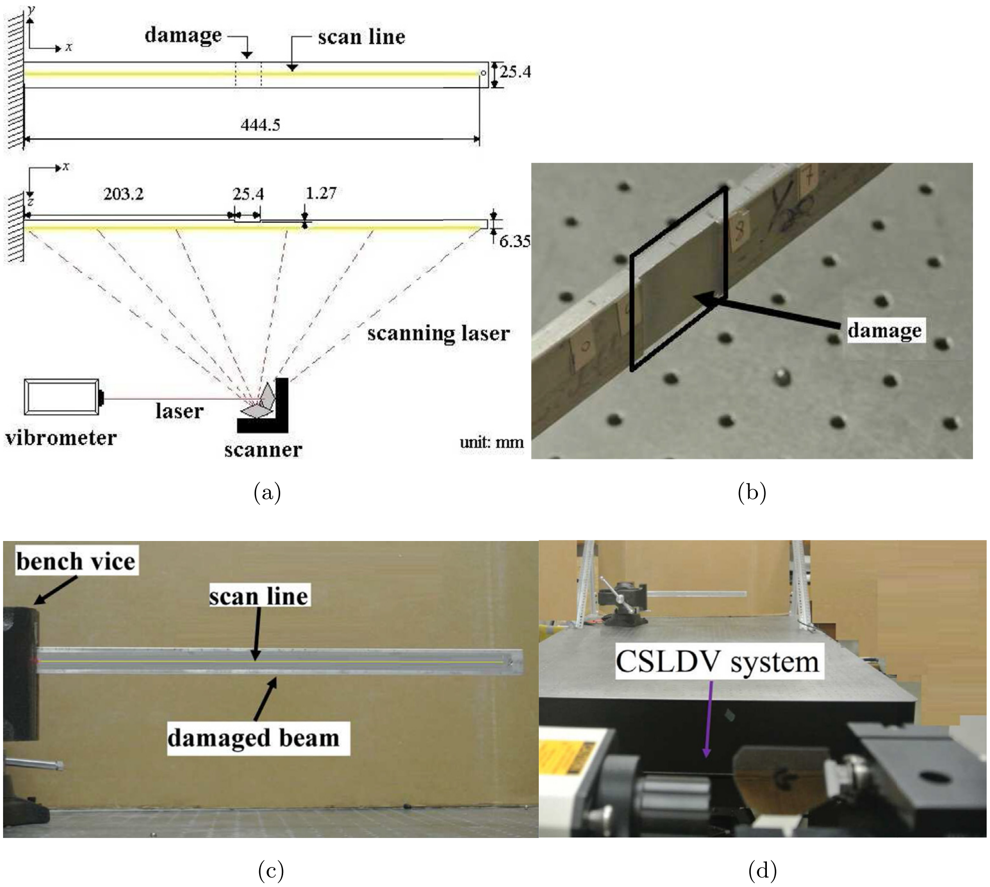

Fig. 10. (a) Schematic of the test setup and dimensions of the damaged aluminum cantilever beam with a region of machined thickness reduction, (b) the region of machined thickness reduction, (c) the beam with its left end clamped by a bench vice and (d) the experimental setup for free response shape measurement of the beam.

where $0 \leq t - t_k \leq \frac{T}{2}$. The mean of $Q_{i,k}\varphi_{i,k}$ with $k \in [1, N_i]$, where $N_i \leq N_{i,0}$, can be defined by

$$\mu_i(\widetilde{\mathbf{x}}) = \frac{\sum_{k=1}^{N_i} Q_{i,k}\varphi_{i,k}(\widetilde{\mathbf{x}})}{N_i} \tag{21}$$

Though $\omega_i\zeta_i$ in Eqs. (19) and (20) is unknown, it can be estimated by solving an associated optimization problem:

$$\overline{\omega_i\zeta_i} = \arg\min_{\omega_i\zeta_i} \sum_{k=1}^{N_i} \left| \mu(\widetilde{\mathbf{x}}) - Q_{i,k}\varphi_{i,k}(\widetilde{\mathbf{x}}) \right| \tag{22}$$

Table 2
Estimated natural frequencies and modal damping ratios of the damaged cantilever beam from the proposed MPE method.

Mode	1	2	3	4	5
Natural frequency (Hz)	22.07	135.74	393.36	746.48	1261.07
Modal damping ratio (%)	0.16	0.18	0.12	0.12	0.14

where $|\cdot|$ denotes the L^2-norm of a function in a half-scan period. Note that N_i must be greater than two; otherwise $\omega_i\zeta_i$ cannot be estimated since the optimization problem in Eq. (22) becomes trivial. With estimated $\omega_{i,d}$ and $\overline{\omega_i\zeta_i}$, ζ_i can be estimated by

$$\zeta_i = \frac{\overline{\omega_i\zeta_i}}{\sqrt{\omega_{i,d}^2 + \left(\overline{\omega_i\zeta_i}\right)^2}} \tag{23}$$

The physics behind the optimization problem is to find $\omega_i\zeta_i$, which can compensate decays in $\widetilde{y}_i$ in each half-scan period, and $Q_{i,k}\varphi_{i,k}$ of the same mode will compare well with each other in each half-scan period.

This completes theoretical derivation of the MPE method using free response of the structure measured by the CSLDV system. The procedure of the method is summarized below:

Step 1. Measure $z(t)$ of the structure using the system with its laser spot staying at at least one fixed point of the structure.
Step 2. Estimate $\omega_{i,d}$ of the structure using the auto-power spectrum of z measured in Step 1.
Step 3. Measure $\widetilde{z}[\mathbf{x}(t), t]$ using the system along a scan path with certain scan and sampling frequencies.
Step 4. Estimate $\widetilde{y}_i[\mathbf{x}(t), t]$ associated with measured modes in N_i half-scan periods using the demodulation method.
Step 5. Estimate ζ_i based on Eq. (23) using $\overline{\omega_i\zeta_i}$ obtained by solving the optimization problem in Eq. (22).
Step 6. Express $Q_i\varphi_i(\widetilde{\mathbf{x}})$ as $\widetilde{y}_{i,1}(\widetilde{\mathbf{x}})\, e^{\overline{\omega_i\zeta_i}(t-t_1)}$ with $0 \le t - t_1 \le T$ and $\overline{\omega_i\zeta_i}$ obtained in Step 5.

2.4. Baseline-free structural damage identification

Local damage of a structure can cause prominent anomalies in its curvature mode shapes in neighborhoods of the damage, and the damage can be identified by comparing the curvature mode shapes with those of the associated undamaged structure [21]. However, the curvature mode shapes of the undamaged structure that can be considered as baselines are usually unavailable in practice. When the undamaged structure is geometrically smooth and made of materials without mass and stiffness discontinuities, the curvature mode shapes of the undamaged structure can be well approximated by those from polynomials that fit mode shapes of the damaged structure with properly determined orders. In previous works [14,15,22], a curvature damage index (CDI) was proposed, which consists of the difference between a curvature mode shape of a damaged structure and that from a polynomial fit:

$$\delta_i(\mathbf{x}) = \left[\varphi_i''(\mathbf{x}) - \varphi_i^{p\prime\prime}(\mathbf{x})\right]^2 \tag{24}$$

where a prime denotes spatial differentiation with respect to the arc length s of a scan path at $\mathbf{x}$, and φ_i^p is the corresponding mode shape from the polynomial that fits φ_i. The curvature mode shapes φ_i'' and $\varphi_i^{p\prime\prime}$ are numerically calculated in this work using a multi-resolution finite difference scheme with a derivative interval spanning 256 measurement points [22]. Since mode shapes corresponding to multiple modes can be measured in one scan, CDIs corresponding to multiple modes can be obtained in the scan, and damage regions can be identified in neighborhoods with consistently large CDI values associated with the measured modes. Note that use of δ_i corresponding to rigid-body modes of a structure should be excluded in damage identification as their curvature mode shapes are zero, and one should use δ_i corresponding to elastic modes of the structure in damage identification. An auxiliary CDI associated with δ_i corresponding to various measured modes can be defined to assist identification of the neighborhoods; it can be expressed by

$$\widetilde{\delta}(\mathbf{x}) = \sum \widehat{\delta}_i(\widetilde{x}) \tag{25}$$

where $\widehat{\delta}_i$ is the normalized CDI associated with the i-th mode of the structure so that its maximum amplitude is one and $\sum$ denotes summation of $\widehat{\delta}_i$ over all measured modes. Since boundary distortions would occur in curvature free response shapes of a structure associated with its free response shapes obtained from the demodulation method [16], similar distortions would occur in curvature mode shapes here. Hence, boundary regions are excluded in normalization of δ_i in $\widetilde{\delta}$ and presenting them. Neighborhoods with consistently large values of δ_i associated with measured modes can be identified in those with large values of $\widetilde{\delta}$.

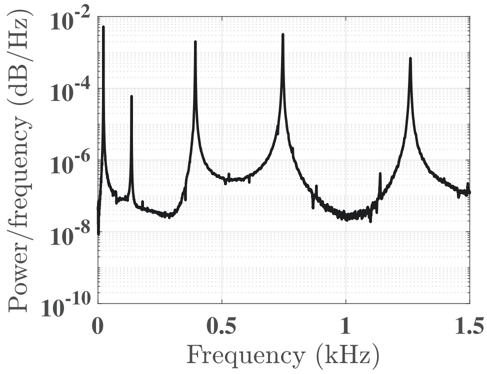

Fig. 11. Auto-power spectrum of the measured free response of the damaged cantilever beam.

Let s be a one-dimensional parameter corresponding to $\mathbf{x}$, and its upper and lower limits correspond to two ends of $\mathbf{x}$. By nondimensionalizing s so that it ranges between −1 and 1, a polynomial that fits φ_i with an order r can be expressed by

$$\varphi_i^p\left(\widetilde{s}\right) = \sum_{q=0}^{r} a_q \widetilde{s}^q \tag{26}$$

where $\widetilde{s}$ denotes the nondimensionalized s, and a_q are coefficients of the polynomial. As pointed out in Ref. [16], an increase of r in the polynomial in Eq. (26) can improve the level of approximation of φ_i^p to φ_i. To determine a proper order of the polynomial fit, the modal assurance criterion (MAC) value between a mode shape of the damaged structure and that from a polynomial that fits the mode shape, which is defined by

$$\mathrm{MAC}\left(\varphi_i, \varphi_i^p\right) = \frac{\left(\varphi_i^{\mathrm{H}} \varphi_i^p\right)^2}{\left(\varphi_i^{\mathrm{H}} \varphi_i\right)\left(\varphi_i^{p\mathrm{H}} \varphi_i^p\right)} \times 100\% \tag{27}$$

where the superscript H denotes the conjugate transpose of a matrix or vector, is used. A proper order for the polynomial fit is two plus the minimum order with which $\mathrm{MAC}\left(\varphi_i, \varphi_i^p\right)$ is greater than 90% [22]. Two is added here in order to preserve smoothness of a curvature mode shape from the polynomial fit, since calculation of a curvature incurs second-order differentiation that reduces the order of a polynomial by two.

3. Numerical investigation

A finite element model of a damaged aluminum cantilever beam with a length $L = 0.8$ m, Young's modulus of 68.9 GPa, a mass density of 2700 kg/m^3 and a damping coefficient of Kelvin-Voigt damping model of 8×10^{-7} s is constructed using ABAQUS. The beam has a uniform square cross-section with a side length of 0.01 m. The damage is in the form of thickness reduction, which is located between $x = \frac{6}{16}L$ and $x = \frac{7}{16}L$, where x is the position of a point on the beam. The damaged portion of the beam has a height of 0.008 m and a length of 0.05 m. The beam has fixed and free ends at $x = 0$ and $x = L$, respectively. The first five natural frequencies and modal damping ratios of the beam are listed in Table 1(a) and (b), respectively. The first five mass-normalized mode shapes of the beam from its finite element model are shown in Fig. 1.

In this section, a single impulse is applied to the damaged cantilever beam. Assume that the beam has zero initial conditions; responses of the beam are calculated using the expansion theorem, where the number of included modes is five. A simulated CSLDV system is used to measure responses of the beam caused by the forces with a scan period $T = 2$ s and a sampling frequency of 16384 Hz. The number of measurement points on a free response shape is equal to the sampling frequency multiplied by the duration of a half-scan period. In this simulation, the number of measurement points on a free response shape is $16384 \times 1 = 16384$. The simulated CSLDV system is capable of measuring response in the form of displacement. Positions of the laser spot of the system on the beam in the first 8 s of a scan is shown in Fig. 2.

A single impulse with an intensity of 0.01 N is applied to the free end of the damaged beam $x = L$ at $t = 0$ s. Response of the beam at $x = 0.7$ m is measured in the form of displacement for 8 s, as shown in Fig. 3(a), and its auto-power spectrum is shown in Fig. 3(b). Natural frequencies of the beam can be identified in the spectrum as frequencies where prominent peaks are found, and identified natural frequencies are listed in Table 1(a). The largest error between the identified natural frequencies and those from the finite element model is 0.16%. Response of the beam is then measured using the simulated CSLDV system, and the measured response in the first 8 s are shown in Fig. 4(a); the associated spectrogram is shown in Fig. 4(b), where the window size is 2, 048. It can be observed that amplitudes of frequency components associated with the third through fifth modes

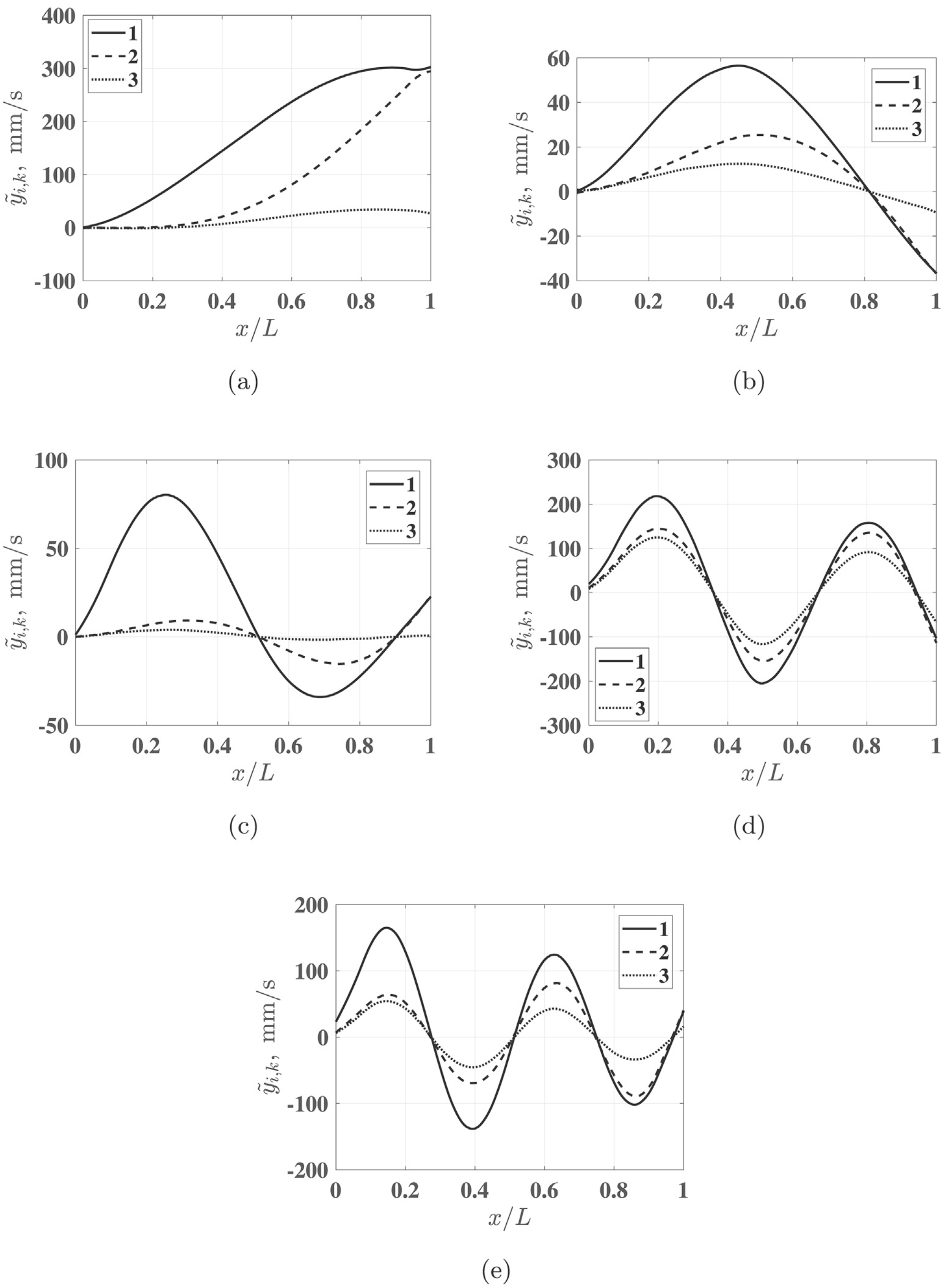

Fig. 12. Free response shapes of the damaged cantilever beam with its (a) first, (b) second, (c) third, (d) fourth and (e) fifth modes in the first three half-scan periods.

decay faster than those with the first and second modes. Specifically, the frequency component associated with the fourth mode decays to almost zero before the second half-scan period ends and that associated with the fifth mode fast decays within the first half-scan period. Since the duration of a half-scan period is $\frac{T}{2} = 1$ s, non-zero-amplitude free response shapes of at least one half-scan period associated with the first through fourth modes can be obtained from the free response measured by the CSLDV system and that associated with the fifth mode cannot. Free response shapes obtained from the free response are shown in Fig. 5(a)–(e), which correspond to the first through fifth modes, respectively. It can be seen that amplitudes of the free response shapes associated with the fourth and fifth modes of the structure decay to almost zero faster than those associated with the first three modes, which verifies the observations on the spectrogram in Fig. 4(b).

Table 3
Scan frequencies, forms of excitation, and numbers of mode shape points of free response shapes associated with the first five modes of the beam.

Mode	1	2	3	4	5
Scan frequency (Hz)	0.1	1.0	1.0	10.0	10.0
Form of excitation	bending deformation	impact	impact	impact	impact
Number of mode shape points	1, 250, 000	125, 000	125, 000	12, 500	12, 500

Since there is only one non-zero-amplitude free response shape associated with the fourth mode and the free response shape associated with the fifth mode has decayed almost to zero before the end of the first half-scan period of the CSLDV system, modal damping ratios and mode shapes associated with the two modes cannot be estimated here. However, they can be estimated if a higher scan frequency is applied so that at least two non-zero-amplitude free response shapes associated with each of the two modes can be obtained from the response measured by the CSLDV system. By applying the MPE method in Sec. 2.3, modal damping ratios and mode shapes associated with the first three modes can be estimated with the obtained free response shapes here. Estimated damping ratios associated with the first three modes are listed in Table 1(b), and they compare well with those from the finite element model. Estimated mode shapes are shown in Fig. 6, and modal assurance criterion (MAC) values between mode shapes from the MPE method and finite element model are above 99.99%, which indicates that the mode shapes compare well with each other. Note that the mode shapes are normalized so that they have a maximum unit amplitude.

The estimated mode shapes are then used for structural damage identification. The three mode shapes of the damaged cantilever beam are fitted by polynomials with properly determined orders. The orders of the polynomial fits are 7, 8 and 11 for the first through third mode shapes, respectively. Curvature mode shapes corresponding to the first three mode shapes and those from the polynomial fits are shown in Fig. 7. It can be seen that local anomaly due to the damage can be well observed by comparing the curvature mode shapes of the beam with those from the polynomial fits. CDIs corresponding to the three mode shapes are shown in Fig. 8(a)–(c), and the auxiliary CDI corresponding to the three damage indices is shown in Fig. (d). The structural damage can be clearly and accurately identified in neighborhoods with consistently large values of the CDIs associated with different modes in Fig. 8(a)–(c) and the auxiliary CDI in Fig. 8(d) can well assist identification of the neighborhood.

4. Experimental investigation

An experimental investigation was performed in Ref. [16] to obtain free response shapes of a damaged cantilever beam and identify its damage. Experimental data and results from the investigation, including free response measurements of the beam under different excitation methods, which were measured by a CSLDV system with different scan frequencies, natural frequencies and free response shapes of the beam, were used here to perform the proposed MPE method and identify the damage. In this section, the configuration of the CSLDV system and experimental setup of the investigation are described, and MPE and damage identification results of the damaged cantilever beam are presented.

4.1. CSLDV system and experimental setup

The CSLDV system used in the experimental investigation is shown in Fig. 9. The system consists of a Cambridge 6240H scanner, a Polytec OFV-353 single-point laser vibrometer and a dSPACE MicroLabBox controller board that controls a pair of orthogonal scan mirrors of the scanner termed as X and Y mirrors. Triangular and constant input signals were given to the X and Y mirrors, respectively, so that the laser spot could continuously sweep along the length of the damaged beam. The schematic diagram of the experimental setup and dimensions of the damaged beam are shown in Fig. 10(a). The undamaged portion of the beam had a thickness of 6.35 mm. There was a region of machined thickness reduction of 1.27 mm on one side of the beam along its length, as shown in Fig. 10(a) and (b). The thickness reduction is about 20% of the thickness of the undamaged portion; its location and length are shown in Fig. 10(a). A bench vice was used to clamp the left end of the beam to simulate a fixed boundary. A straight scan path was assigned on the intact side of the beam, as shown in Fig. 10(a) and (c). A strip of retroreflective tape was attached on the intact side to enhance laser reflection that directly determined the signal-to-noise ratio of measurement by the CSLDV system. The scan path was non-dimensionalized with respect to its length so that it ranged from 0 to 1, where 0 and 1 represented left and right ends of the scan path, respectively. The damage was located between 0.46 and 0.51 on the scan path. The experimental setup is shown in Fig. 10(a), (c) and (d), where the CSLDV system was used to measure free response of the beam along the scan path. The sampling frequency of the system was 250, 000 Hz.

4.2. MPE and structural damage identification results

The free response of a point of the damaged cantilever beam caused by an impact was measured to estimate its natural frequencies. Based on the auto-power spectrum of the measured free response in Fig. 11, the first five identified natural frequencies of the beam were estimated, as listed in Table 2. Free response shapes of the beam under two forms of excitations measured by the CSLDV system with different scan frequencies were used to estimate its modal damping ratios and mode shapes. One form of excitation is an initial non-zero bending deformation of the beam along its length under a transverse concentrated force at its

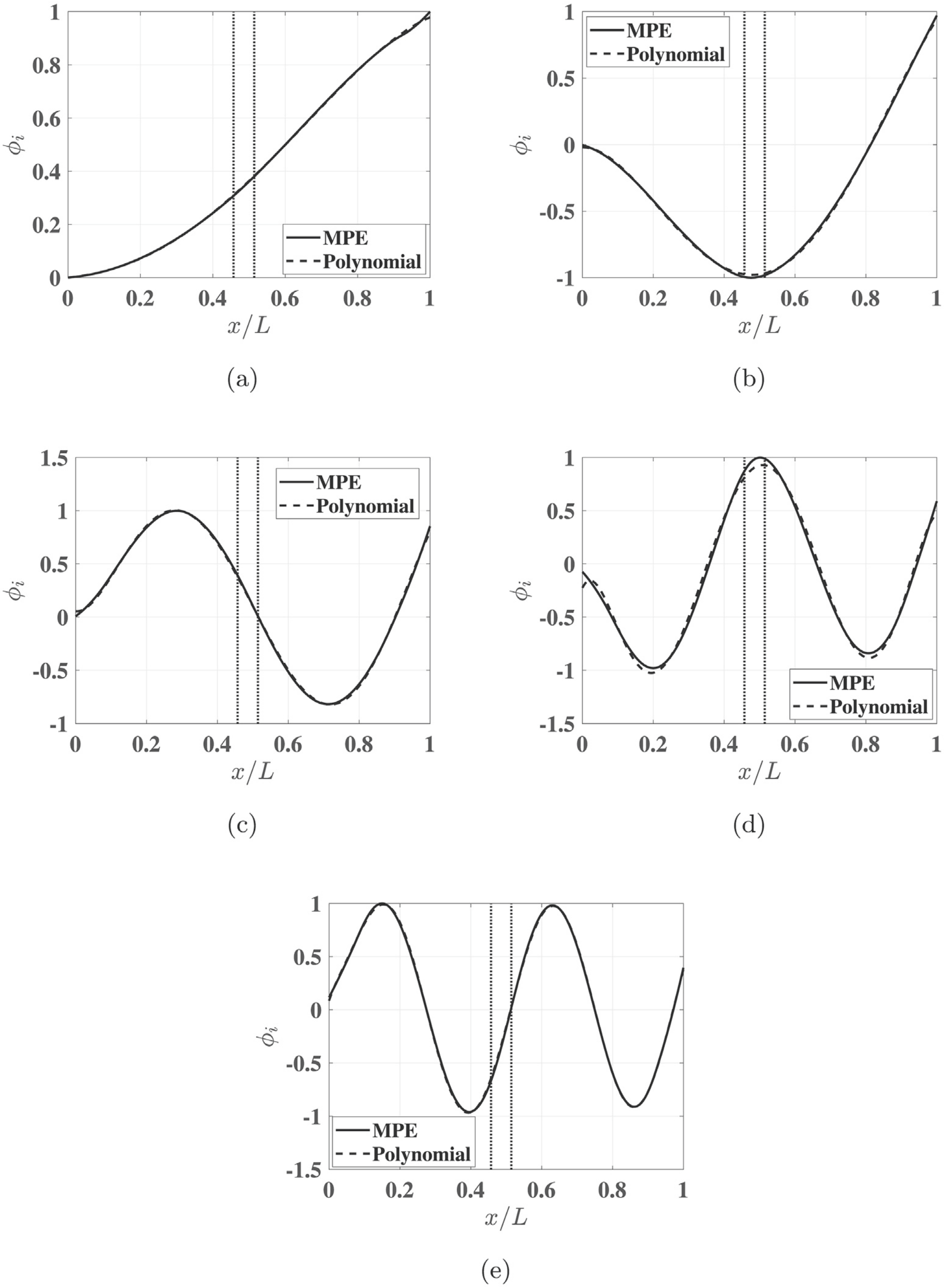

Fig. 13. Estimated mode shapes of the damaged cantilever beam associated with its (a) first, (b) second, (c) third, (d) fourth and (e) fifth modes and those from polynomial fits with properly determined orders. Locations of damage ends are indicated by dotted lines.

free end, with the initial velocity of the beam being zero. The other form is an impact on the beam with zero initial conditions with the impact point on the damaged side of the beam. Free response shapes associated with the first five modes of the beam are shown in Fig. 12 and their corresponding forms of excitations and scan frequencies are listed in Table 3. Note that different scan frequencies were used for different modes. The reason is that the higher the scan frequency the lower the signal-to-noise ratio of estimated mode shapes due to speckle noise, which is a type of measurement noise in CSLDV measurements. Since curvature mode shapes are sensitive to measurement noise in mode shapes, a low scan frequency is desired. However, the proposed MPE method requires that free-response shapes of a mode of at least two half-scan periods be measured, and the scan frequency needs to be high enough to satisfy this requirement.

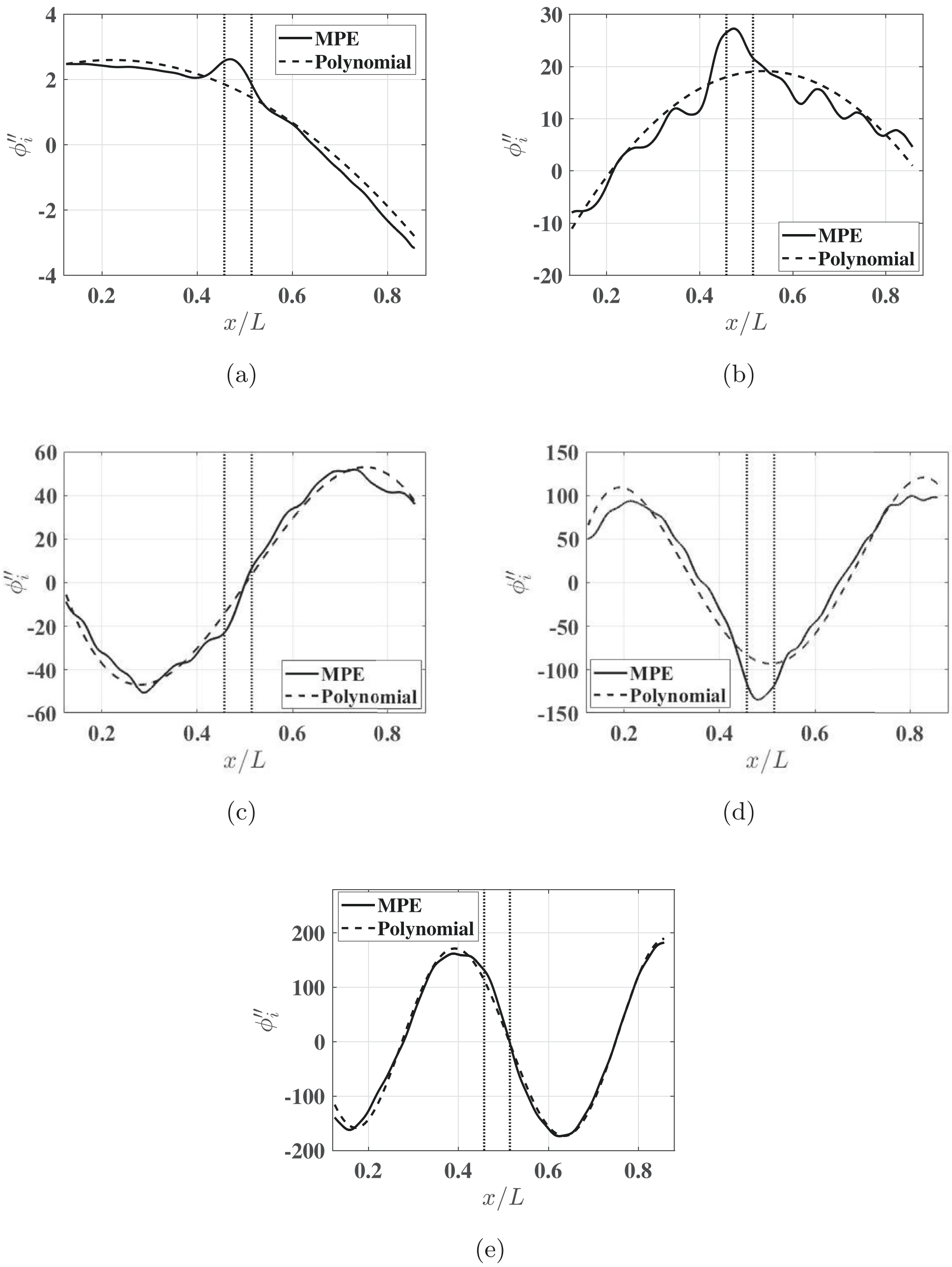

Fig. 14. Curvature mode shapes of the damaged cantilever beam with its (a) first, (b) second, (c) third, (d) fourth and (e) fifth modes and those from polynomial fits with properly determined orders. Locations of damage ends are indicated by dotted lines.

With use of the MPE method, modal damping ratios and mode shapes associated with the first five modes of the damaged cantilever beam were estimated using the measured free response shapes in their first three half-scan periods. The estimated modal damping ratios are listed in Table 2 and the estimated mode shapes of the five modes are shown in Fig. 13. Mode shapes corresponding to the associated undamaged cantilever beam are obtained from polynomials that fit the mode shapes of the damaged beam with properly determined orders, as shown in Fig. 13. The orders for the first through fifth modes are determined to be 4, 4, 5, 7 and 9, respectively. Curvature mode shapes corresponding to the first five mode shapes and those from the polynomial fits are shown in Fig. 14. CDIs associated with the first through fifth modes of the damaged beam are shown in Fig. 15(a)–(e), respectively. The structural damage can be clearly identified in neighborhoods with consistently large values of the CDIs. However, relatively high values can be observed in CDIs associated with the third and fifth modes outside the damage region, possibly due to speckle noise that was caused by continuously sweeping the laser spot of the CSLDV system. False

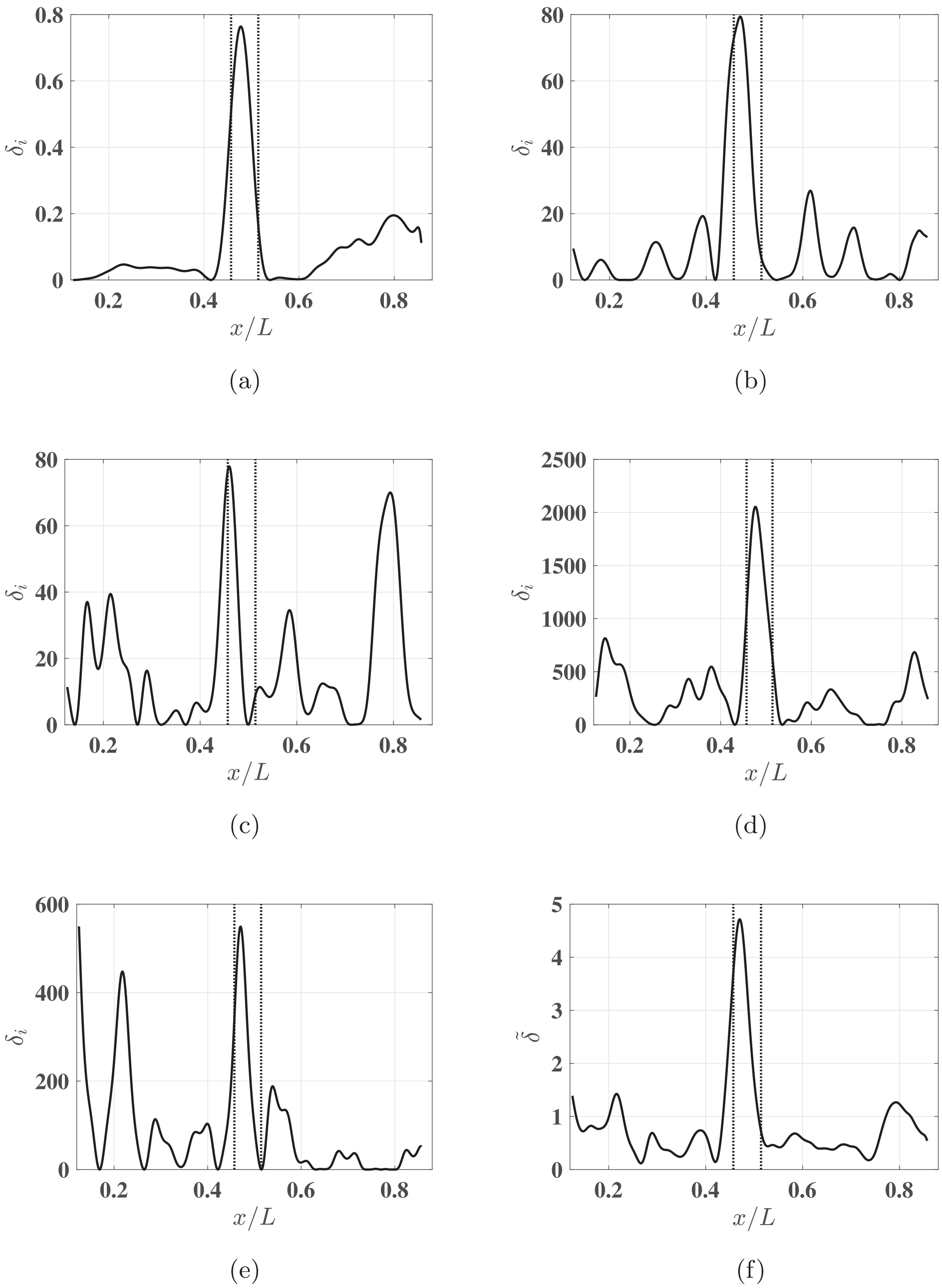

Fig. 15. CDIs of the damaged cantilever beam associated with its (a) first, (b) second, (c) third, (d) fourth and (e) fifth modes; (f) the auxiliary CDI associated with the CDIs in (a) through (e). Locations of damage ends are indicated by dotted lines.

damage identification results could have been obtained if CDIs of only one of the two modes were available. Based on the CDIs corresponding to the five mode shapes, the auxiliary CDI is shown in Fig. 15(f), which can assist accurate identification of the damage in the region with high auxiliary CDI values, and the false damage identification results were excluded.

5. Conclusions

Derivation of free response shapes of a linear, time-invariant, viscously damped structure is shown. The only current application of free response shapes is structural damage identification and they cannot be used for model validation and updating. A new MPE method using free response measured by a CSLDV system is proposed to estimate modal parameters, including natural frequencies, modal damping ratios, and mode shapes based on the concept of free response shapes. A critical assumption of the method is that all damped natural frequencies of the structure are well separated. Natural frequencies can be estimated by measuring free response of fixed points on the structure. A free response shape associated with an elastic mode corresponds to the mode shape associated with the same mode. The amplitude of the free response shape exponentially decays with a decay rate that is directly related to the modal damping ratio and natural frequency of the mode. When the decay is compensated with an accurately estimated decay rate, the mode shape of the mode can be automatically obtained as a result. In the proposed MPE method, the modal damping ratio and mode shape associated with an elastic mode can be simultaneously estimated by solving an optimization problem. Free response shapes of one elastic mode in at least two half-scan periods are needed to estimate its modal damping ratio and mode shape. A half-scan period starts when the laser spot of the CSLDV system arrives at one end of a scan path and ends when the laser spot arrives at the other end of the scan path. Using relatively low scanning frequencies can yield mode shapes of high qualities. A baseline-free method is applied to identify structural damage using mode shapes estimated by the MPE method. In the numerical investigation, the proposed MPE method is applied to estimate modal parameters of a viscously damped damaged beam using its free response measured by a simulated CSLDV system. Estimated modal parameters compare well with their theoretical ones and the damage can be accurately identified using the auxiliary curvature damage index. In the experimental investigation, modal parameters of a damaged cantilever beam were estimated using the proposed MPE method and its damage could be accurately identified using the auxiliary curvature damage index. The proposed MPE method can be extended to structures under random excitation.

CRediT authorship contribution statement

Y.F. Xu: Methodology, Funding acquisition, Software, Writing - original draft. **Da-Ming Chen:** Software, Validation. **W.D. Zhu:** Conceptualization, Methodology, Funding acquisition, Writing - review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Short-time Fourier transform

The short-time Fourier transform of $\widetilde{z}$, denoted by $\widetilde{V}_w(t, f)$, can be expressed by

$$\widetilde{V}_w(t,\omega) = \int_{-\infty}^{\infty} \widetilde{z}(\tau)\, g_s^*(\tau - t)\, \mathrm{e}^{-\mathrm{j}\omega\tau} \mathrm{d}\tau \tag{A.1}$$

where g_s is a window function with a scale s, the superscript * denotes complex conjugation, and $\mathrm{j} = \sqrt{-1}$. The scale s determines the width of g_s in the time domain, which should be smaller than that of a half-scan period. When $\widetilde{V}_w$ at the i-th natural frequency of the structure becomes almost zero at an instant $t_{i,0}$, the amplitude of $\widetilde{y}_i$ is considered to be zero. Note that in Eq. (A.1), $\widetilde{V}_w(t,\omega)$ is visualized by use of a spectrogram whose intensity denotes the power spectral density associated with $\widetilde{V}_w(t,\omega)$; g_s is a Hamming function that can be expressed by

$$g_s(t) = \begin{cases} 0.54 - 0.46\cos\left(\dfrac{2\pi t}{s}\right), & 0 \le t \le s \\ 0, & \text{otherwise} \end{cases} \tag{A.2}$$

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