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Procedures for Quick Estimation of Hydraulic Conductivity of Unsaturated Soils

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Procedures for Quick Estimation of Hydraulic Conductivity of Unsaturated Soils

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ABSTRACT

This paper describes a heuristic approach that combines limited number of experimental measurement with random finite element method (RFEM) to significantly accelerate the process in measuring the hydraulic conductivity of unsaturated soils. A microstructure-based RFEM model is established to describe the unsaturated soils with distribution of phases based on their respective volumetric contents. The intrinsic hydraulic properties of each phase (soil particle, water, and air) are applied based on the microscopic structures. The intrinsic permeability of each soil phase is firstly calibrated from soil measured under dry and saturated conditions, which is then used to predict the hydraulic conductivities at different extent of saturations. The results match closely with the experimental data. The pore size parameter was obtained from the variations of hydraulic conductivity with degree of saturation, from this the soil-water characteristic curve (SWCC) is predicted. The results show that the SWCC estimated matches very well with experimental data. Overall, this study provides a new modeling-based approach to predict the hydraulic conductivity function and SWCC of unsaturated soils based on measurement at complete dry or completely saturated conditions. An efficient way to measure these critical unsaturated soil properties will benefit introducing unsaturated soil mechanics into the engineering practice.

Key words: Finite-element modelling, Permeability, Seepage

INTRODUCTION

Hydraulic conductivity, or permeability, is a critical soil property describing the hydraulic behavior of unsaturated soils, and it is related to the water content and pore size of the soil (Das and Thyagaraj, 2016; Stipcevich *et al*. 2015; Jaafar and Likos, 2013; Arya *et al*. 1999). In the microstructural view, hydraulic conductivity depends on the structure of the soil skeleton, the connectedness of the pore space, and the intrinsic permeability of each phase of the unsaturated soil. The hydraulic conductivity of different type of soils (clay, silt, sand) varies with a wide range to the tune of several orders of magnitudes. It is challenging to determine the hydraulic conductivity due to the variation in the mineral composition and complex texture of the soil. The wide range in hydraulic conductivity and in its composition make it challenging to measure the hydraulic properties under unsaturated conditions.

Empirical method and experimental method have been proposed to determine the hydraulic conductivity of unsaturated soils. The empirical approach relates the hydraulic conductivity with soil index properties such as the water content (Liu *et al*. 2011), particle size distributions (Jaafar and Likos 2013), and soil texture. Table 1 lists a few representative types of empirical equations for the unsaturated hydraulic conductivity.

Table 1. A few representative empirical equations for the unsaturated hydraulic conductivity (k)

Equation †	Reference
$k = a\theta^b$	Gardner (1958)
$k = k_s\left(\theta / \theta_s\right)^n$	Campbell (1973)
$k = k_s exp\left[\alpha\left(\theta - \theta_s\right)\right]$	Davidson *et al*. (1969)
$k = k_s\left[\left(\theta - \theta_r\right) / \left(\theta_s - \theta_r\right)\right]^{3.5}$	Averjanov (1950)
$k = k_s S_e^{1/2}[1 - S_e(\mathrm{S}_e^{n/(1-\mathrm{n})} - 1)^{(n-1)/n}]^2$ and $S_e = \dfrac{S - S_r}{1 - S_r}$	Mualem (1976)

† k_s is the saturated hydraulic conductivity; θ is the volumetric water content; θ_s is the saturated volumetric water content; n is the pore size parameter; a, b and α are the fitting parameter; θ_r is the residual volumetric water content; S_e is the effective degree of saturation and S_r is the residual saturation.

The hydraulic conductivity of soils can also be measured by direct laboratory measurements. From these the empirical equations can be obtained between the hydraulic conductivity and volumetric water content (or degree of saturation), which is then used in the engineering practice. For saurated soils, Darcy's law is commonly used to determine the hydraulic conductivity from hydraulic experiments. The experimental approach in the laboratory include the transient-flow test, infiltration column tests and deformation test (McCartney *et al*. 2007; Lu *et al*. 2014); the experimental procedures in the field tests include small scale field tests that use observations of the water level in cavities installed in soil, and large scale field tests such as pump tests conducted in multiple wells. The experimental methods has the advantages of being reliable and accurate in determining the hydraulic conductivity, however, it is time-consuming and expensive to conduct the hydraulic experiments for unsaturated soils.

The hydraulic conductivity of unsaturated soil is typically obtained by multiplying that of the saturated soil with a hydraulic conductivity function (HCF). The theoretical basis for the hydraulic conductivity function (HCF) is provided by Fredlund *et al*. (1994). It has been proven that the HCF can be integrated from the soil-water characteristic curve (SWCC). The SWCC, or the soil-water retention curve, is a critical property describing the relationship between matric suction and moisture content. The experimental approach to determine the SWCC includes thermocouple psychrometry, pressure plate, suction plate, field tensiometer, pressure membrance and filter paper methods (Likos and Lu 2002; Likos and Lu 2003; Liu *et al*, 2012). Although these methods are commonly used in practice, they have the limitations of being labor intensive, time consuming, and not reliable in the high suction range (over 100,000 kPa). Empirical equations for the SWCC curve have been developed by Genuchten (1980), Maulem (1976),Fredlund and Xing (1994), Arya (1999) and Kosugi (1998), where the fitting parameters in the equations are determined through laboratory or field data.

This paper aims to provide a new method to provide quick estimation of the hydraulic conducitivty and soil water characteristics curve of unsaturated soils. This new method include phase-coded microstructure based REFM model and calibration experiments conducted at complete dry and saturated conditions. These calibration experiments allows to determine the transport properties of indiviudal phase and account for the effects of phase interactions. By use of the RFEM and calibrated parameters for individual phase, the bulk hydraulic conductivities at different degrees of saturation are predicted. Comparison of experimental data show that the results of hydraulic conductivity of unsaturated soils predicted by this new approach match very well with the experimental data. The approach is further extended to estimate the SWCC of unsaturated soils and demonstrates promising results.

THEORETICAL BACKGROUND

Random Finite Element Method

The hydraulic properties of soil are inherently variable due to its microstructure. The parameters affecting the bulk transport properties of soils (i.e., phases, pore structure, etc.) are randomly distributed throughout the soil due to the complex geological process. The effects of such random processes have been simulated by the random-finite element method (RFEM) (Fenton and Griffiths 2003; Griffiths *et al*. 2006; Griffiths *et al*. 2010), where the parameters including Young's modulus, Poisson's ratio, density, permeability, and a variety of other parameters demonstrate spatial randomness (Bharrucha-Reid 1968; Sobczyk 1985; Chamis 1987). The RFEM is an extension of the traditional FEM by adding randomness to the material properties. Griffiths and Fenton (1993) modeled soil as a spatially random medium and represented the soil's permeability as a stochastic field and used the RFEM to investigate the seepage beneath water retaining structures.

In the microstructural view, unsaturated soil is a three-phased material that includes soil particles, water, and air, arranged randomly due to complex geological processes (Andersland and Ladanyi, 2004). Microstructural based model has been developed for soil (Chang and Hicher, 2005; Ferber *et al*. 2006). Tracy et al. (2015) and Helliwell *et al*. (2013) obtained the structure of soil by using X-ray Computed Tomography (CT) images. Zhang et al. (2016) built the 3D digital images of the porous rock with porosity properties and mineral contents to study the elastic properties of the Longmaxi shale. The microstructure extracted from the high-resolution images provides a foundation to build models at multiple scales, i.e. micro-scale or pore-scale, of soil samples.

Principle for Hydraulic Conductivity Measurements

The theoretical basis for measuring the bulk hydraulic conductivity in the conventional geotechnical engineering settings is the Darcy's law,

$$q = -\frac{\kappa}{\mu}\nabla p \tag{1}$$

where ∇p is the pressure gradient, κ is the bulk permeability, μ is the dynamic viscosity of the pore fluid, and q is the flux per unit area.

From Equation (1), it is seen that the bulk hydraulic conductivity can be measured by use of different transport fluids. The most commonly used in the laboratory is water and for saturated conditions. Direct measurement of the hydraulic conductivity of soil in unsaturated conditions is much more complex, partly due to the difficulty to maintain the degree of unsaturation or suction.

To resolve this challenge, this paper proposes a microstructure-based RFEM model to describe the transport process in unsaturated soil and to predict its bulk transport properties including the SWCC. The soil models are phase coded with the distribution of different phases and properties based on the volumetric contents and type of the individual phases. The phase coded model is converted into a finite element model, where the hydraulic and mechanical properties of each soil phase (soil particle, water, and air) are applied based on its phase constituents. With this idea in mind, the transport of fluid through soil is assumed to occur within different phases following Darcy's law for each phase (Equation 2):

$$q_j = -\frac{\kappa_j}{\mu} \nabla p \tag{2}$$

where the terms bear similar physical interpretation as Equation (1), except that the subscript j denotes different phases of the poromaterial (i.e., air, water and soil solids) and represents the intrinsic hydraulic conductivity of each phase.

The total flux across per unit cross section area of the soil specimen is determined from Equation (3), which is then combined with Equation (1) to determine the bulk transport properties (i.e., hydraulic conductivity).

$$q = \sum a_k q_k \tag{3}$$

Where summation of k is over all the phase-coded pixels in the unit cross section area. a_k is the effective cross section area of each pixel, Σa_k equals to unit area (1). Flow rate at each pixel is determined by Equation (2). The percentage of each phase along the cross section area equals to its volume content in the bulk soil sample.

By use of the phase coded RFEM, the bulk hydraulic conductivity of the unsaturated soil is calculated from the finite element model which considers the phase distribution and hydraulic conductivities of individual phases.

CONSTRUCTION OF MICROSTRUCTURE-BASED RFEM MODEL

The microstructure-based RFEM models are firstly constructed from the bulk properties. Four types of soils (Denver claystone, BALT silt, Hopi silt and Ottawa sand) are included in this study, which properties obtained from the literature (Lu *et al*. 2014). Table 2 summarizes the important hydraulic parameters from the experimental data. Procedures in producing the microstructure based RFEM model are described in the following context.

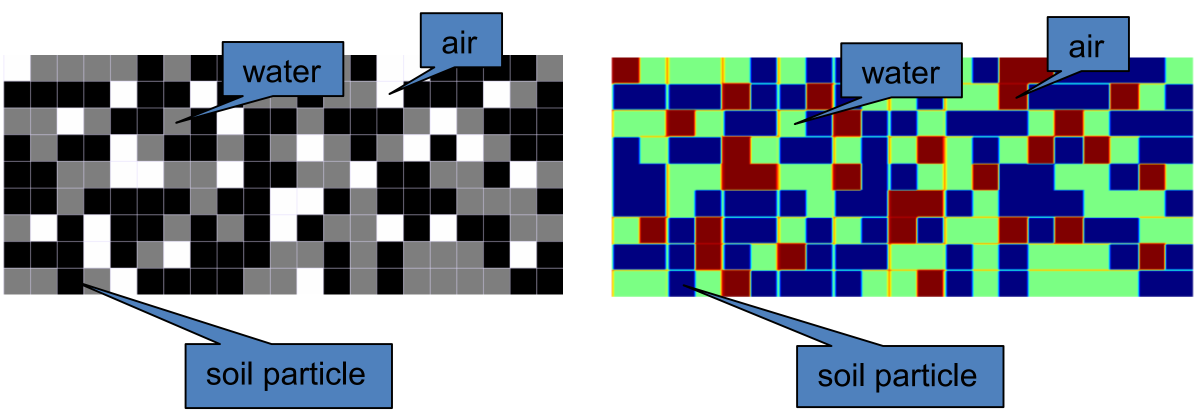
Table 2. Summary of major hydro-physical parameters determined from transient-flow experiments (Lu *et al*. 2014)

Soil	n	k_s (m/s)	Porosity
Denver claystone	1.40	8.58E-9	0.55
BALT silt	1.38	1E-6	0.47
Hopi silt	1.70	3E-8	0.48
Ottawa sand	6.00	2.56E-5	0.38

† n is the pore size parameter.

The following steps are undertaken to simulate the microstructure of these soils in unsaturated conditions, 1) determination of volume content of different phases. The volumetric content of each phase is calculated from the physical information including the dry density, water content, porosity, and specific gravity. 2) generation of matrix for soil specimen. A m×n matrix is generated by use of Matlab, where each cell in the matrix contains the corresponding phase coding. For the dimension of the matrix, m is set to be equal to the height of the soil specimens divided by the diameter of the soil particle, while n equals to the radius of the soil specimens divided by the diameter of the soil particle. Thus, m×n equals the total number of elements in the image of a 2-D soil model specimen. 3) Phase coding of digital soil specimen. Each element in the matrix is assigned with a value to represent a particular phase of the soil specimens. Three different numbers (0, 0.5, and 1) are assigned to represent different phases (soil particles, water and air) within the soil specimens. The phase is determined by a random number generator where the probability of occurrence of a particular phase is determined by the volumetric content of that phase. For fine grained soils, such as clay and silt, each cell is assumed to be independent and represent the smallest construction unit for hydraulic transport. For coarse grained particles, the soil particle size distribution is incorporated by use of the discrete element model.

The phase coded matrix can be visualized by a grey scale image. Figure 1a shows a zoom in view of the element. Each pixel of the image represents an element in the matrix, with the color of each pixel representing the types of that element. Three different colors (black, grey, and white) represent the soil particles, water, and air, respectively (Figure 1). The probability of occurrence of each phase (or color) is decided upon the volumetric content of that phase. The phase coded image produced by MATLAB is imported into a general finite element software COMSOL, which allows to set the material properties based on the value of the matrix.



a) b)

Figure 1. a) zoom-in gray scale image of model for unsaturated clay generated in Matlab (the white, grey and black pixel represents air, water and solid particle respectively; b) the corresponding phase coded image converted into Comsol (the red, green, and blue pixels represents air, water, soil particles respectively)

For coarse grained soil such as sand, the image is generated based on the particle size distribution of the sand, with three different colors (black, grey, and white) represent the soil particles, water, and air, respectively (Figure 2).

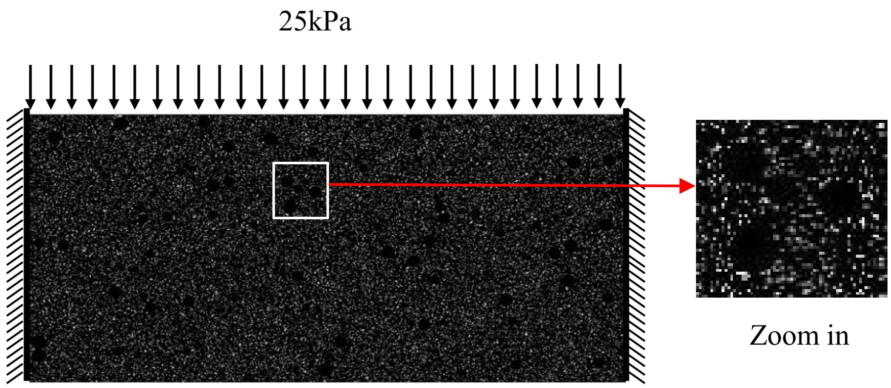


Figure 2. 2-D microstructure-based phase coded image of sand generated in Matlab based on particle size distribution (the white, grey and black pixel represents air, water and solid particle respectively).

The microstructure based phase coded image is imported to COMSOL and material properties are assigned based on the color coding of the image. Figures 3 shows examples of color coded digital specimen of 2-D silt at different volumetric water content in Matlab and after conversion into COMSOL, which then allows to set the properties of individual phase based on phase coding.

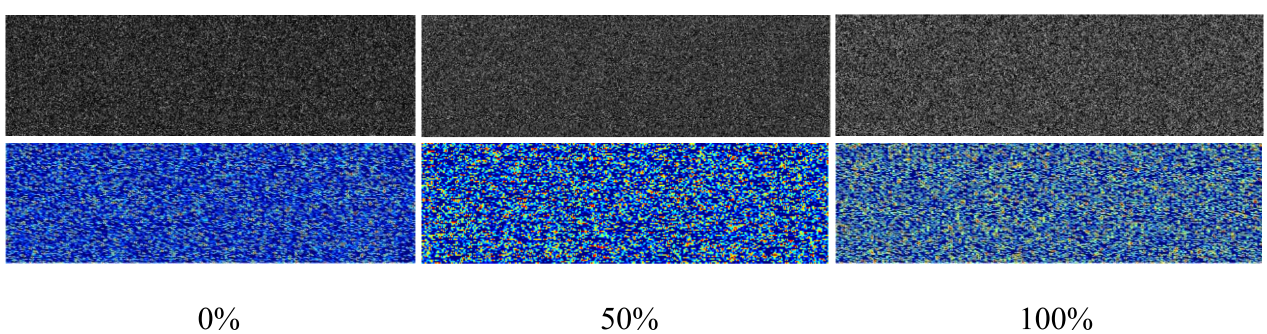


Figure 3. 2-D microstructure-based phase coded model for silt at different degree of saturation (the gray scale image is digital specimen produced by Matlab, the corresponding color image is the digital specimen after conversion into Comsol)

SIMULATION OF THE BULK HYDRAULIC CONDUCTIVITY BY RFEM

With the microstructure based random FEM model that is phase coded, the hydraulic parameters can be assigned to each pixel (phase) based on the phase coding. Here the hydraulic parameters refer to those for the individual phase, which is different from the bulk properties commonly referred or measured. Therefore, the term intrinsic hydraulic permeability is used in this paper to describe the hydraulic properties of each phase (i.e., solid particles, water, and air). These properties are different from the bulk hydraulic properties and should firstly be calibrated from controlled experiments.

Figure 4 and Figure 5 show the procedures to calibrate the intrinsic hydraulic properties of individual phase, which is then used to predict the hydraulic conductivity at different degrees of saturation.

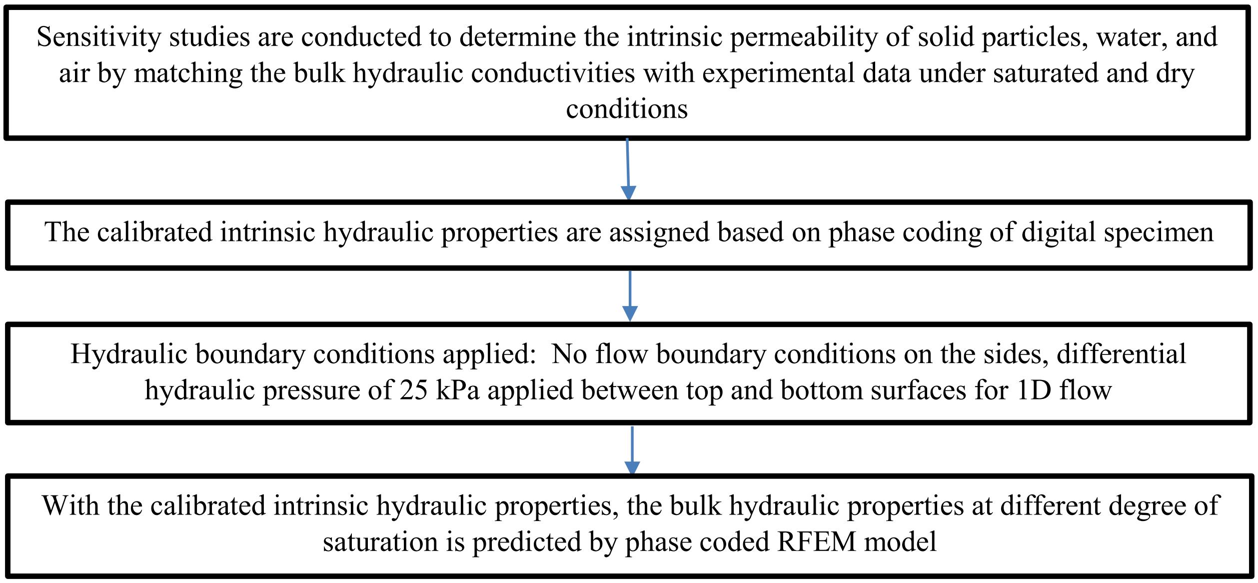


Figure 4. Flow chart for the prediction of the bulk hydraulic conductivity of soil at different degrees of saturation

To demonstrate the calibration and prediction procedures, experimental data by Lu *et al*. (2014), which include the hydraulic conductivity at different degrees of saturation (including completely dry or saturated conditions), are utilized in the analyses. The experimental data on soil samples under two phased conditions (i.e., complete dry or 100% degree of saturation) are used for calibration purpose. The phase coded images of totally dried soil specimens and totally saturated soil specimens (both are two-phased image) are generated in Matlab. Next, the images are converted into Comsol which allows to assign properties to individual phase based on phase coding. The saturated condition is used to calibrate the intrinsic hydraulic properties of water. The dry condition is used to calibrate the intrinsic hydraulic properties of air. The intrinsic properties of mineral are assumed to be magnitude smaller so that they are impermeable compared with other phases. The digital soil specimen is assumed to be subjected to hydraulic conductivity experiments with a hydraulic pressure difference of 25 kPa. No flow boundary conditions are applied on the sides with a hydraulic pressure of 25 kPa applied to the surface to produce one dimensional flow (Figure 2). Sensitivity studies are conducted to determine the intrinsic permeability values of solid particles, water, and air by matching the bulk hydraulic conductivities with experimental data (Figure 5). The intrinsic properties of different phases are determined based on this procedure. The calibration results for each phase of these four types of soils are listed in Table 3 and the parameters used for bulk hydraulic conductivity simulations are listed in Table 4.

Table 3. Calibrated results of intrinsic hydraulic conductivities

Constant	Value	Units	Description
κ_s	10^{-19}	m^2	Intrinsic permeability of solid particle for Denver claystone
κ_w	8.08×10^{-16}	m^2	Intrinsic permeability of water for Denver claystone
κ_a	8.92×10^{-19}	m^2	Intrinsic permeability of air for Denver claystone
κ_s	10^{-17}	m^2	Intrinsic permeability of solid particle for BALT silt
κ_w	3.08×10^{-13}	m^2	Intrinsic permeability of water for BALT silt
κ_a	1.31×10^{-19}	m^2	Intrinsic permeability of air for BALT silt
κ_s	10^{-18}	m^2	Intrinsic permeability of solid particle for Hopi silt
κ_w	8.08×10^{-15}	m^2	Intrinsic permeability of water for Hopi silt
κ_a	1.31×10^{-19}	m^2	Intrinsic permeability of air for Hopi silt
κ_s	10^{-16}	m^2	Intrinsic permeability of solid particle for Ottawa sand
κ_w	3.08×10^{-10}	m^2	Intrinsic permeability of water for Ottawa sand
κ_a	8.92×10^{-16}	m^2	Intrinsic permeability of air for Ottawa sand

Table 4. Parameters used for hydraulic conductivity simulations

Constant	Value	Units	Description
D	100	mm	Diameter of soil specimens
H	21	mm	Height of soil specimens
E_s	12.7	GPa	Young's modulus of solid particle
E_w	3.9×10^{-5}	Pa	Young's modulus of water
E_a	0	Pa	Young's modulus of air
ρ_s	2600	kg/m^3	Density of solid particle
ρ_w	1000	kg/m^3	Density of water
ρ_a	1.29	kg/m^3	Density of air
μ_s	0.3	1	Poisson's ratio of solid particle
μ_w	0.5	1	Poisson's ratio of water
μ_a	0	1	Poisson's ratio of air
μ	8.9×10^{-4}	Pa·s	Dynamic viscosity of water
χ_f	4.6×10^{-10}	1/Pa	Compressibility of water
d_c	2	μm	Diameter of clay particle
d_s	0.01	mm	Diameter of silt particle
D_{10}	0.181	mm	The particle size where 10% of the particles in the sand are smaller
D_{60}	0.347	1	The particle size where 60% of the particles in the sand are smaller

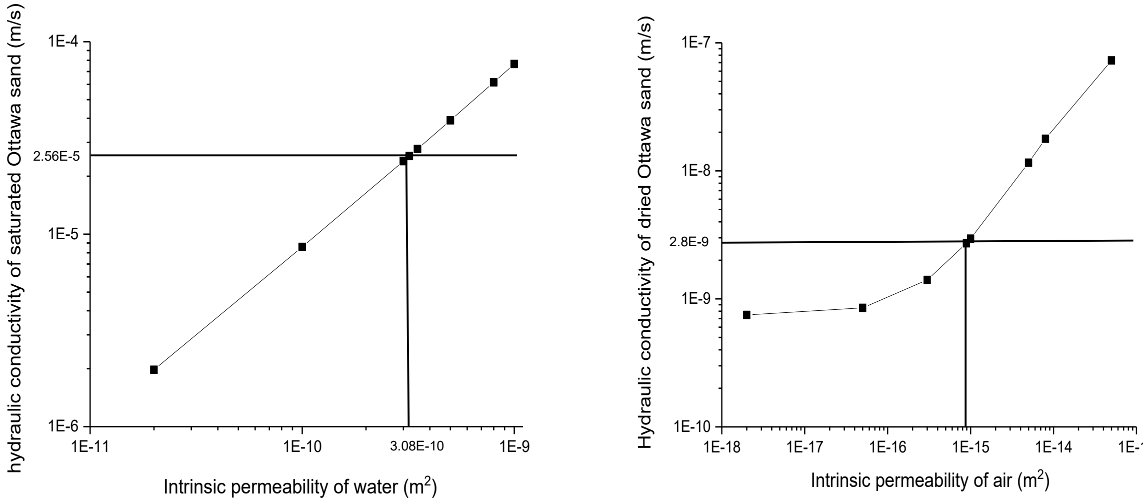
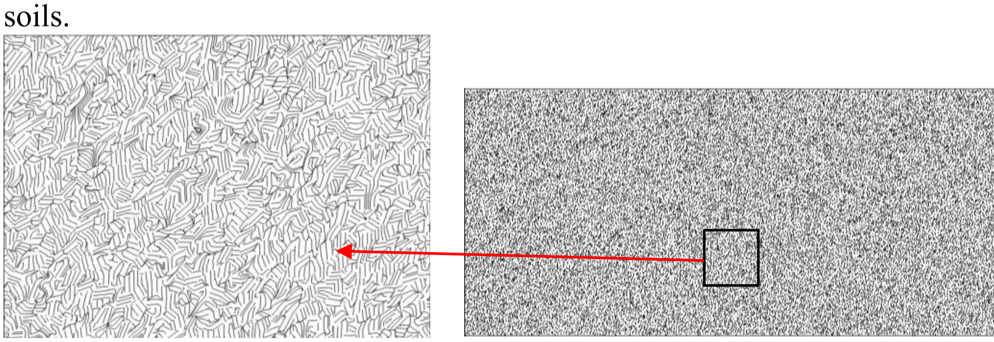
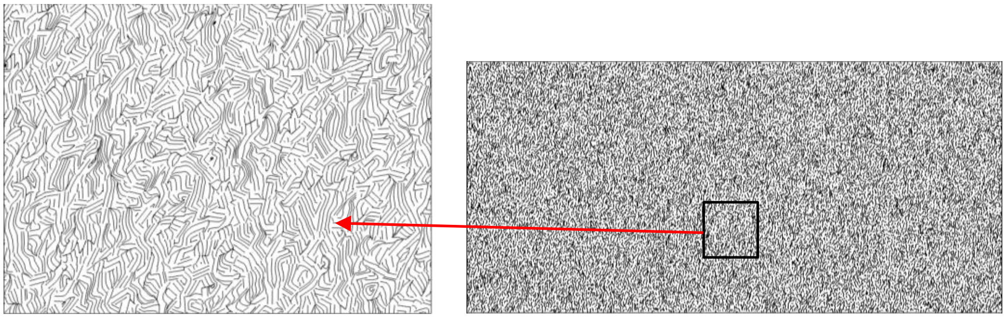


Figure 5. Sensitivity analysis to determine the intrinsic permeability of water and air for Ottawa sand

Figure 6 shows the flow lines across the digital soil specimen. Interesting observations are made. As shown in Figure 6, the flowlines in the clay specimen are mostly zigzagged because the directions of the water flow are altered by the solid particles and the numbers of clay particles are magnitudes higher than that of silt and sand. In addition, clay contains more tortuous pores that abruptly end, so clay has the least permeability. For the silt specimen, the directions of the water flow are less zigzagged due to larger intrinsic hydraulic conductivity of water as well as there are possibly fewer solid particles in silt. Besides that, the pore size in silt is larger than that in clay, so the dissipative force is larger and therefore the bulk permeability is larger in silt than that in clay. The moisture flows in the sand specimen are most vertically distributed because sand is a material with very open pores that pass completely and directly through the sand. Besides that, the numbers of sand particles are magnitudes smaller than that of clay and silt, which caused decreased resistance to the moisture flow. The last but not the least, the pore size in which the water is flowing is the largest in sand, so the sand has the largest permeability among these three different kinds of soils.

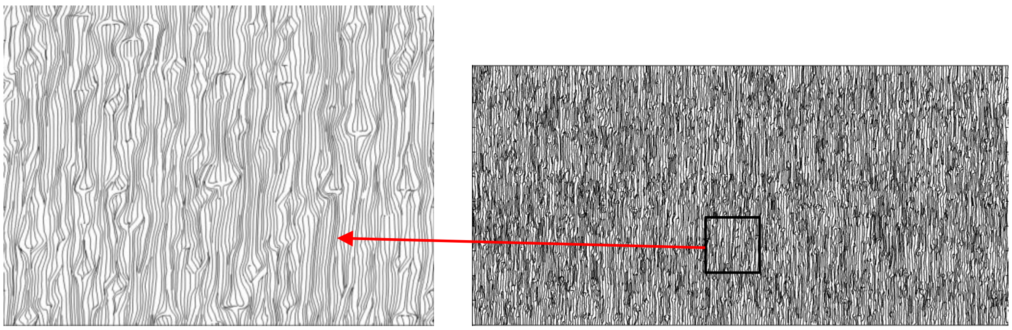

m=10500;n=25000

Clay



m=2100; n=5000

Silt



m=210; n=500

Sand

Figure 6. Flowline simulated in the hydraulic conduction experiment in three different type of soils (the left figures are the enlarged images of the flowlines)

To further illustrate the impact of the microstructure on the fluid transport process, the contour plots of the hydraulic head distribution with and without considering the microstructure of the soil specimen are shown in Figure 7. For a uniform specimen without microstructure, the hydraulic head contours are horizontally distributed. For specimen considering the microstructure, the contours of hydraulic head are more complicated and does not show a horizontal line, which is evidently affected by the randomly distributed microstructure of the soil. In reality, the hydraulic head in soil is not uniformly distributed and varies from point to point obtained from the borehole data on site. The phase coded microstructure-based model is built based on the structure of the soil specimen from the experiment. The simulation results clearly demonstrate the microstructure are introduced in the new model to make the prediction closer to the reality.

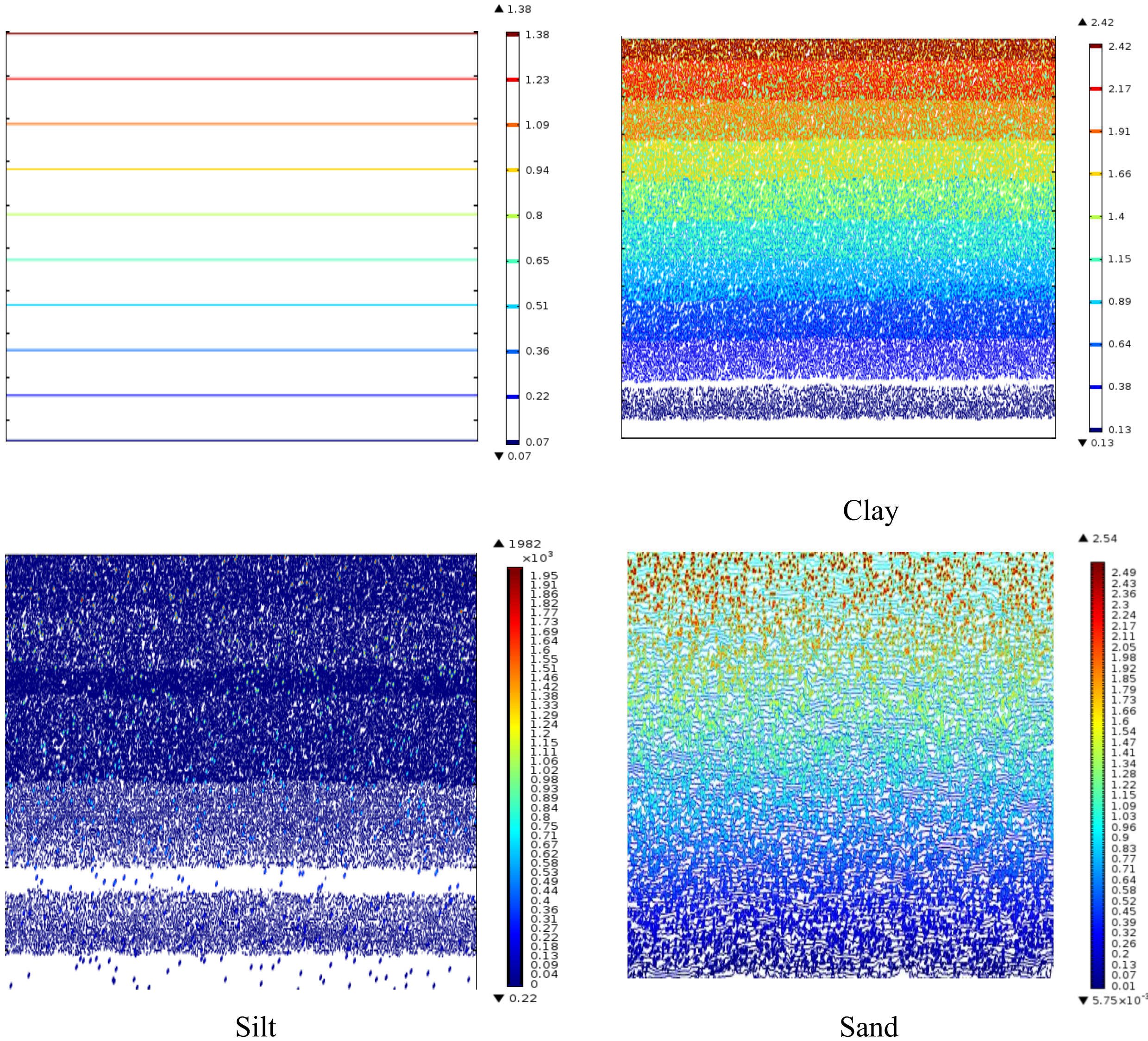


Figure 7. Contour plots of the hydraulic head distribution in different types of soil specimens (the microstructure is not considered in the upper-left figure)

With the calibrated intrinsic hydraulic properties for individual phases, the bulk hydraulic properties at different extent of saturation can be predicted via the microstructure based model by use of the simulated hydraulic conductivity test on the digital specimens. The results of the hydraulic conductivities at different degrees of saturations are compared with those from direct experimental measurements.

Mualem (1976) proposed the following model to describe the variation of hydraulic conductivity K with the effective degree of saturation S_e:

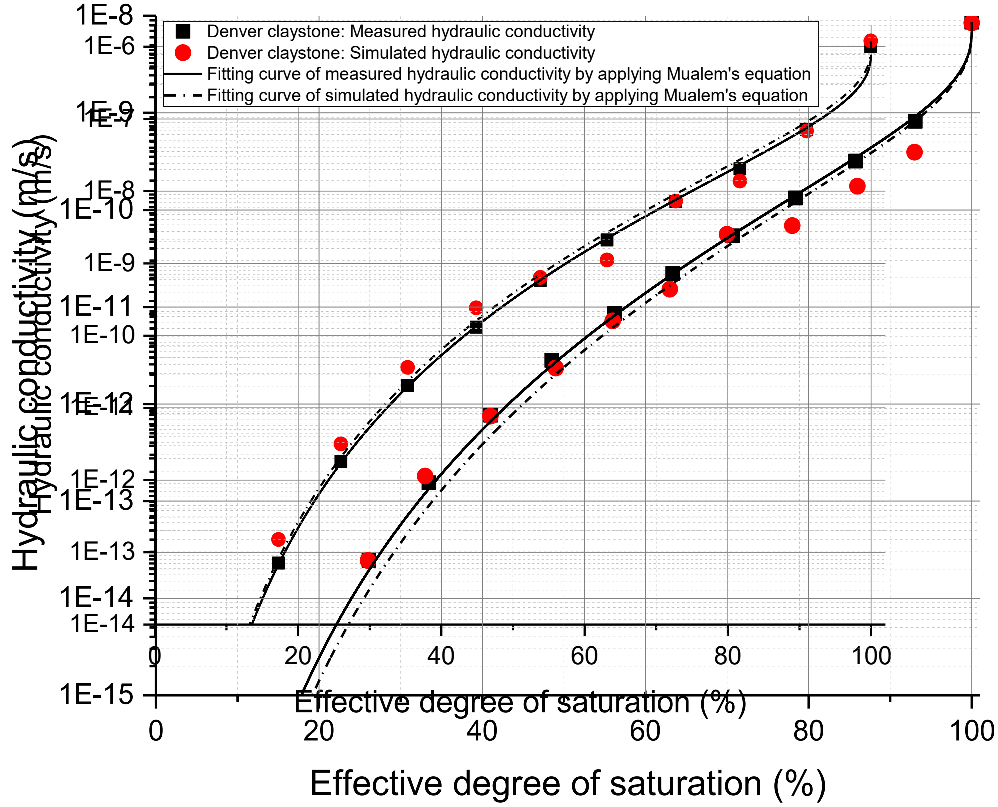
$$K = K_s S_e^{1/2}[1 - S_e(S_e^{n/(1-n)} - 1)^{(n-1)/n}]^2 \tag{4}$$

where n represents pore size parameter, it is determined by the distribution of the pore size, or the structure of the soil; K_s represents saturated hydraulic conductivity; and the effective degree of saturation can be defined as

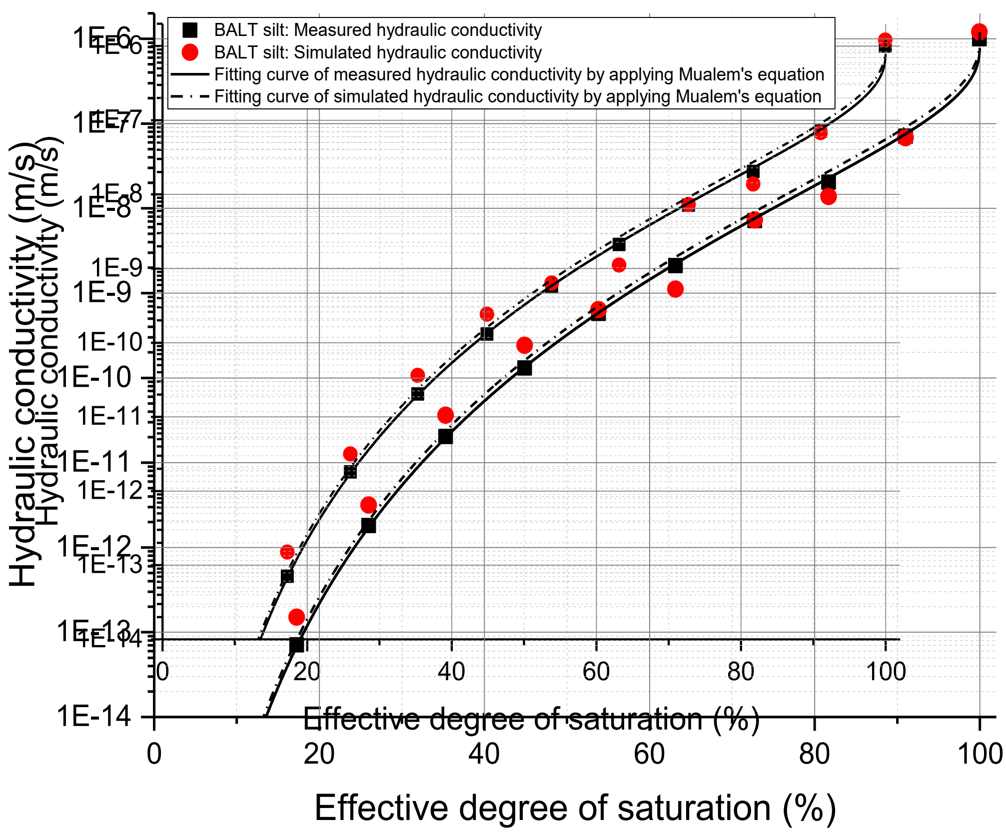
$$S_e = \frac{S - S_r}{1 - S_r} \quad (5)$$

where S_r represents residual saturation.

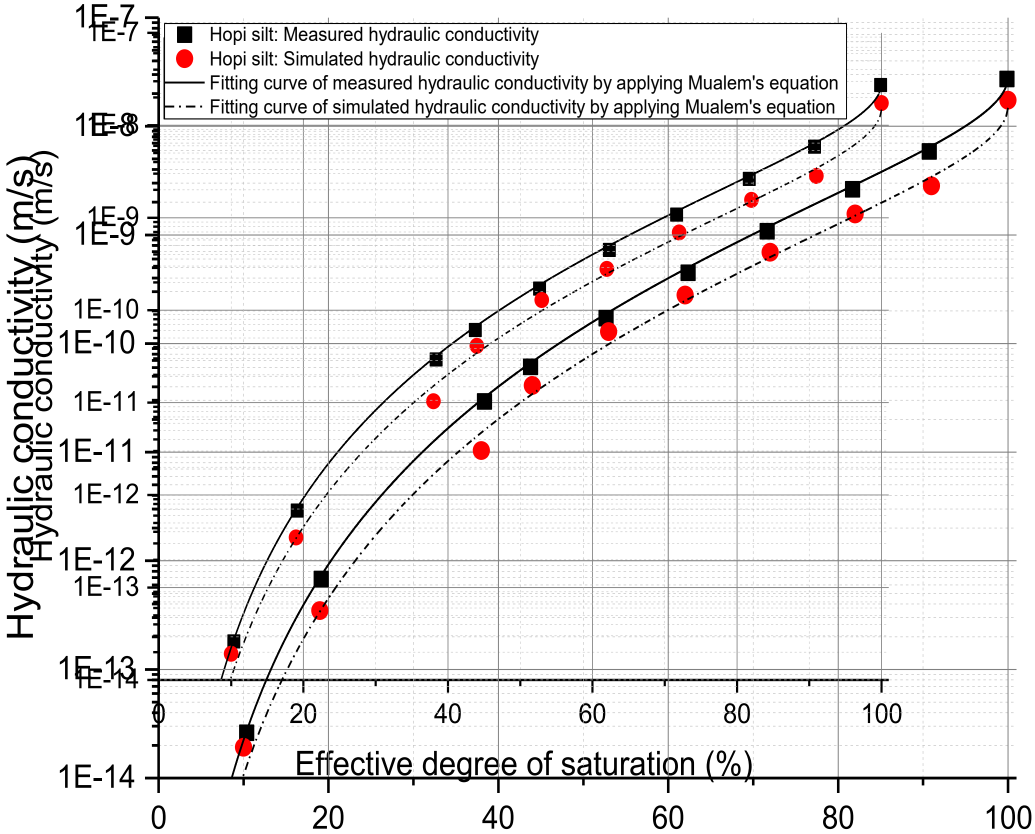
This model is also used to fit the predicted data. The results compared favorably with experimental results for clay, silt, and sand. As shown in Figure 8, hydraulic conductivity of soils generally decreased abruptly (as much as 7 orders of magnitude) from its saturated value, K_s, with decreasing degrees of saturation. The simulated hydraulic conductivities agree well with the measured hydraulic conductivities of Denver claystone, BALT silt, Hopi silt and Ottawa sand. By fitting Mualem's equation to the simulated hydraulic conductivity, the pore size parameters n can be back calculated as 1.38, 1.37, 1.69, and 6.10 for Denver claystone, BALT silt, Hopi silt and Ottawa sand, respectively. The simulated pore size parameters (n) are close to the measured ones (in Table 2, n=1.40, 1.38, 1.70, and 6.00 for Denver claystone, BALT silt, Hopi silt and Ottawa sand, respectively).


Denver claystone: Measured hydraulic conductivity
Denver claystone: Simulated hydraulic conductivity
Fitting curve of measured hydraulic conductivity by applying Mualem's equation
Fitting curve of simulated hydraulic conductivity by applying Mualem's equation
Hydraulic conductivity (m/s)
Effective degree of saturation (%)

1


BALT silt: Measured hydraulic conductivity
BALT silt: Simulated hydraulic conductivity
Fitting curve of measured hydraulic conductivity by applying Mualem's equation
Fitting curve of simulated hydraulic conductivity by applying Mualem's equation
Hydraulic conductivity (m/s)
Effective degree of saturation (%)

2



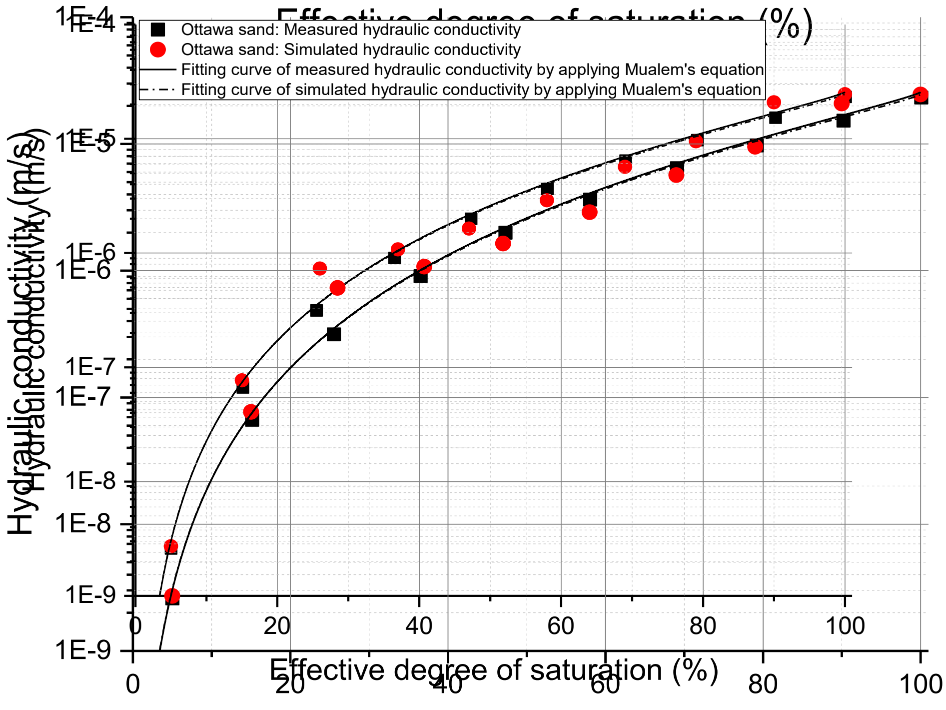
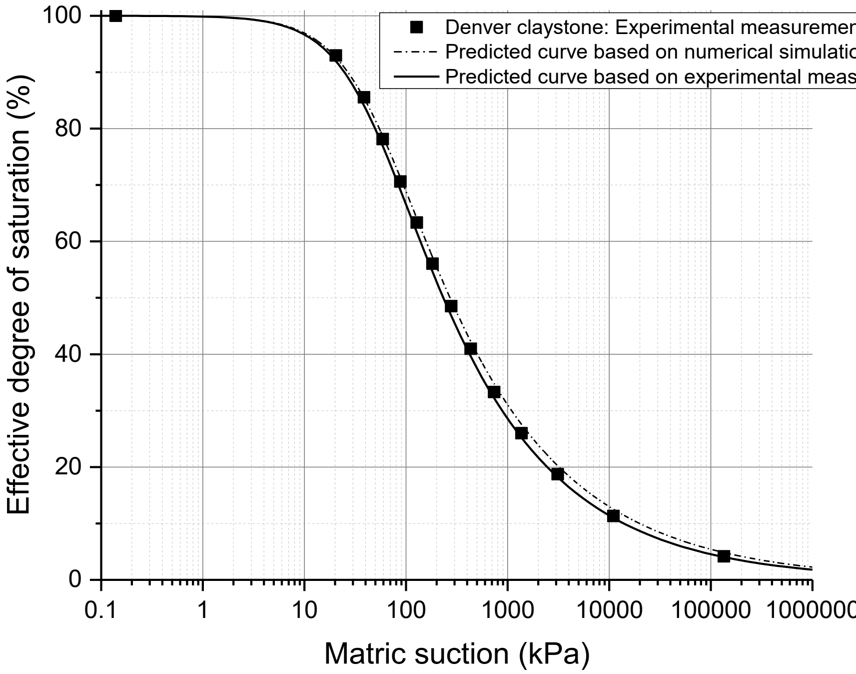


Figure 8. Comparison of the simulated hydraulic conductivity with the measured data from the experiments for the four soils and Mualem's equation

PREDICTION OF SWCC

From Figure 8, the fitted parameter n is calculated for different type of soils. Based on the soil water retention equation proposed by Genuchten (1980): $S_e = (\alpha\psi^n + 1)^{\frac{1-n}{n}}$, the SWCC curve of different soil specimens can be produced based on curve fitting of the simulation results. In the equation, Ψ is the matric suction and α is the inverse of the air-entry suction. Abdelkabir et al. (2014) proposed an equation to predict the air-entry values: $1/\alpha = 19.57 * (h_{c0})^{(2.8E-03/D_{60})}$, in which, h_{co} (cm) is the equivalent capillary rise and D_{60} is the grain diameter at 60% passing.

For Denver claystone, BALT silt, Hopi silt and Ottawa sand, α are 0.02 kPa^{-1}, 0.07 kPa^{-1}, α=0.03 kPa^{-1} and α=0.23 kPa^{-1}, respectively (Lu et al. 2014).


Denver claystone: Experimental measurement
Predicted curve based on numerical simulation
Predicted curve based on experimental measurement
Effective degree of saturation (%)
Matric suction (kPa)
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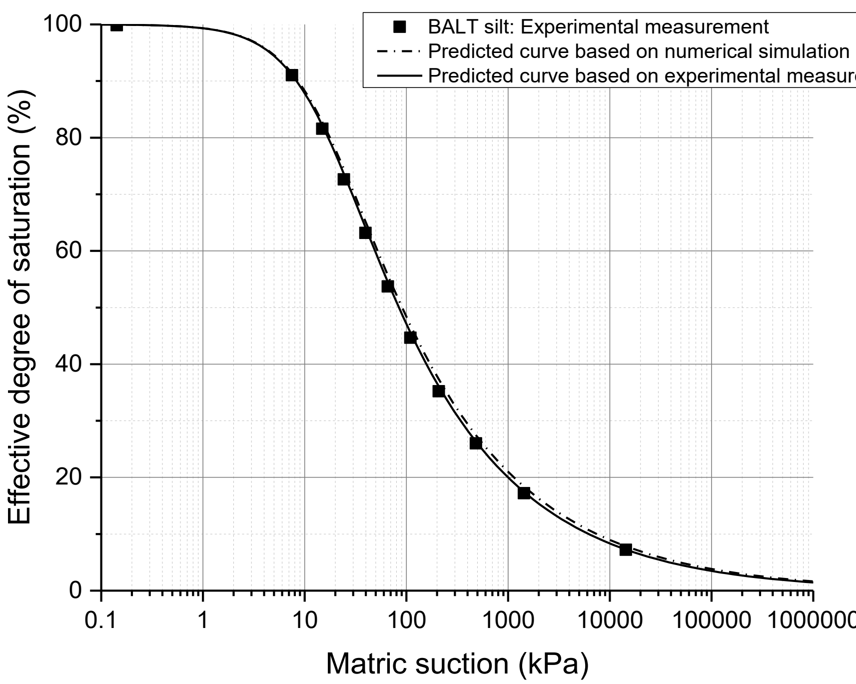
40
60
80
100
0.1
1
10
100
1000
10000
100000
1000000

1


BALT silt: Experimental measurement
Predicted curve based on numerical simulation
Predicted curve based on experimental measurement
Effective degree of saturation (%)
Matric suction (kPa)
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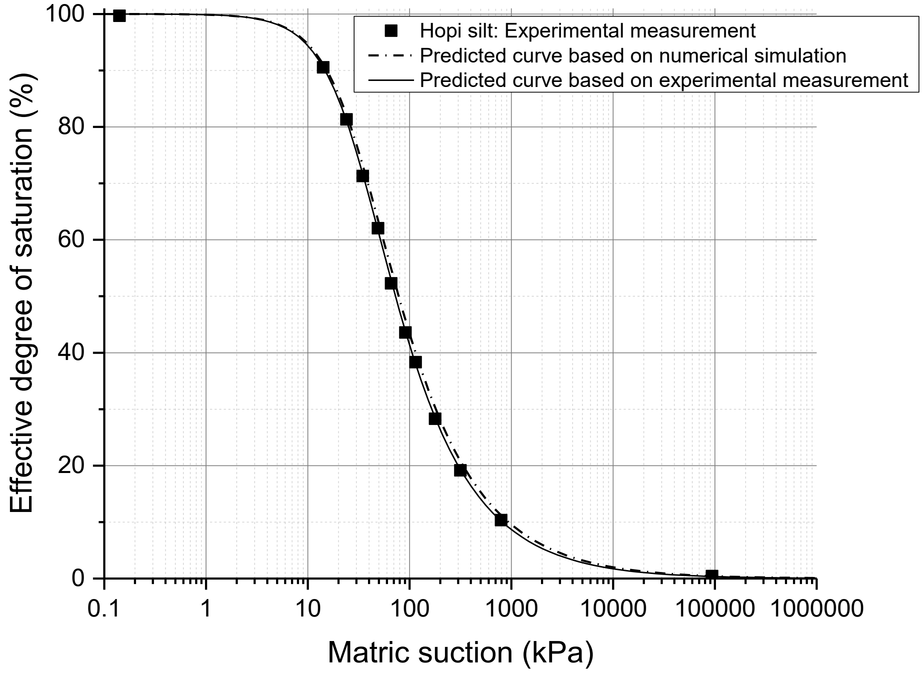

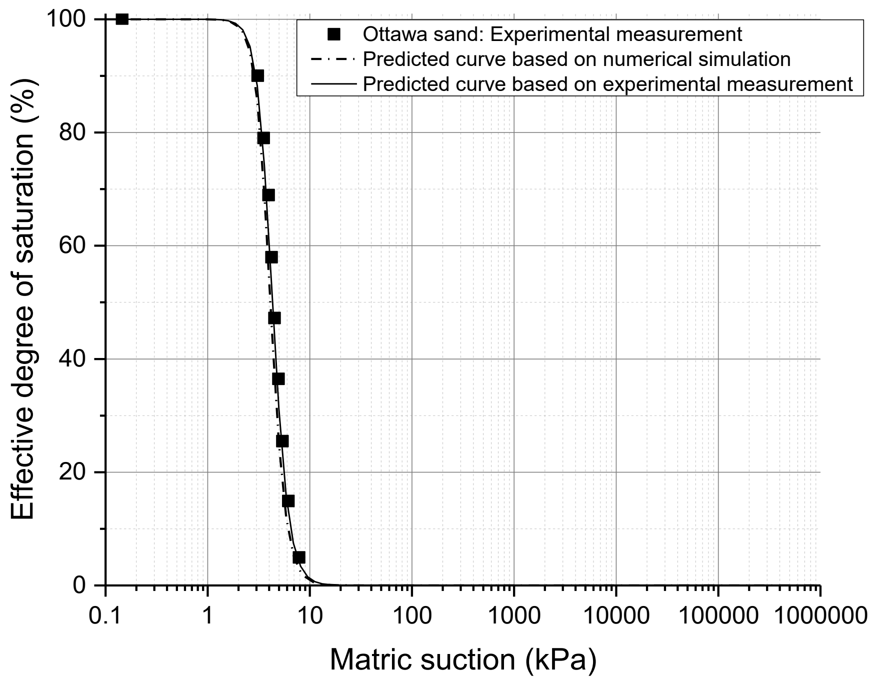

Figure 9. Comparison of the predicted SWCC curve with the measured data for the four soils

In Figure 9, the SWCC curves predicted by the numerical simulation agree well with the experimental data of different type of soils. The simulated SWCC curves compare favorably with the measured ones. This indicates that the proposed procedures will provide an inexpensive and efficient way to predict the SWCC curve. (i.e., the SWCC curve can be predicted by knowing the dried and saturated hydraulic conductivity of certain type of soil as well as the air-entry suction of that soil). It is noted that the SWCC curves are predicted under the drying process and phenomena such as hysteresis are not considered. This is an aspect that requires further investigation.

CONCLUSION

This research develops a method to estimate the hydraulic conductivity and soil water characteristic curve of unsaturated soils by use of microstructure-based random finite element model with limited number of calibration experiments that are conducted under conventional settings. The randomly generated soil structures are linked to the particle size and the volumetric content of each phase in unsaturated soils. Therefore, important factors affecting the hydraulic and transport properties such as the microstructure and phase distribution in the soils are considered. This combined modeling and experimental approach simulate the bulk transport behaviors based on the behaviors of individual phase, which allows the efforts to be focused on understanding the behaviors of indiviudal phase. Besides, the calibration experiments for the intrinsic hydraulic properties of individual phase, which are conducted under complete saturated or dry conditiomns, are easily implemented with conventional experimental procedures. Experimental data of unsaturated soils of different types are used to demonstrate the proposed approach. The results showed that the hydraulic conductivity under unsaturated conditions are prediced with reasonable accurate. In addition, the method is further extended to predice the SWCC at different extent of saturation, whose results are consistent with experimental data. The developed microstructure based modeling approach combines the advantages of discrete element model in simulating the bulk behaviors of particulate system and the computational efficiency of finite element model. It provides a new and reliable simulation-based tool to predict unsaturated soil behaviors, which is anticipated significantly save the time and cost in introducing unsaturated soil mechanics into the engineering practice.

ACKNOWLEDGMENTS

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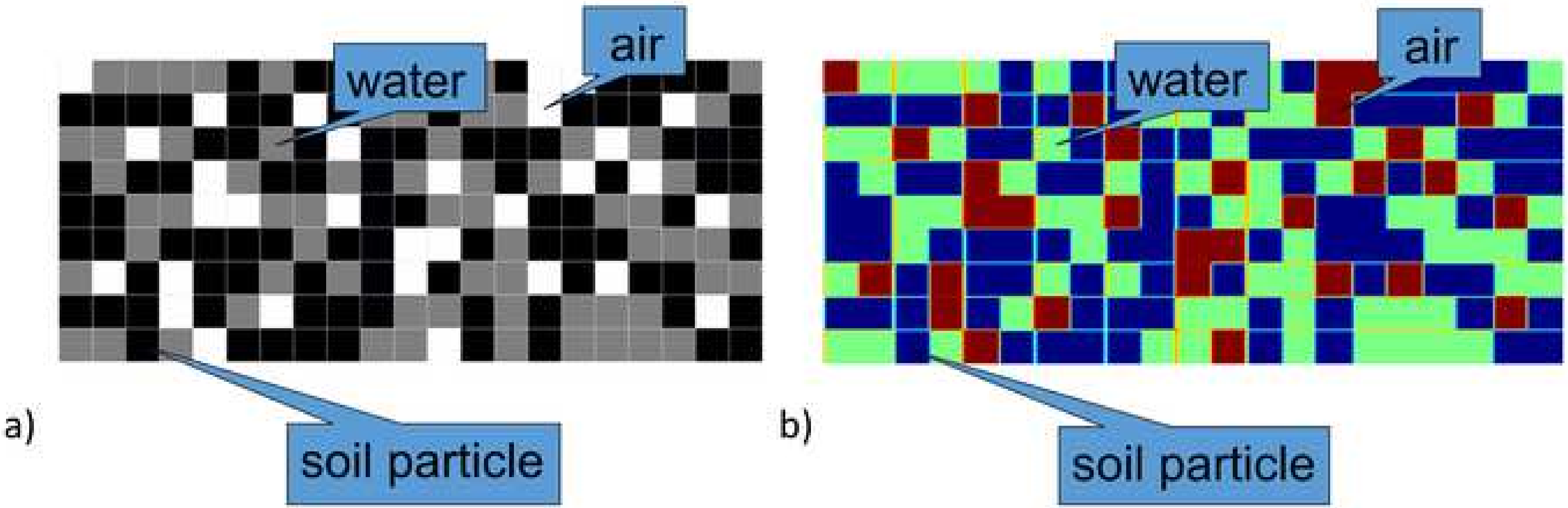

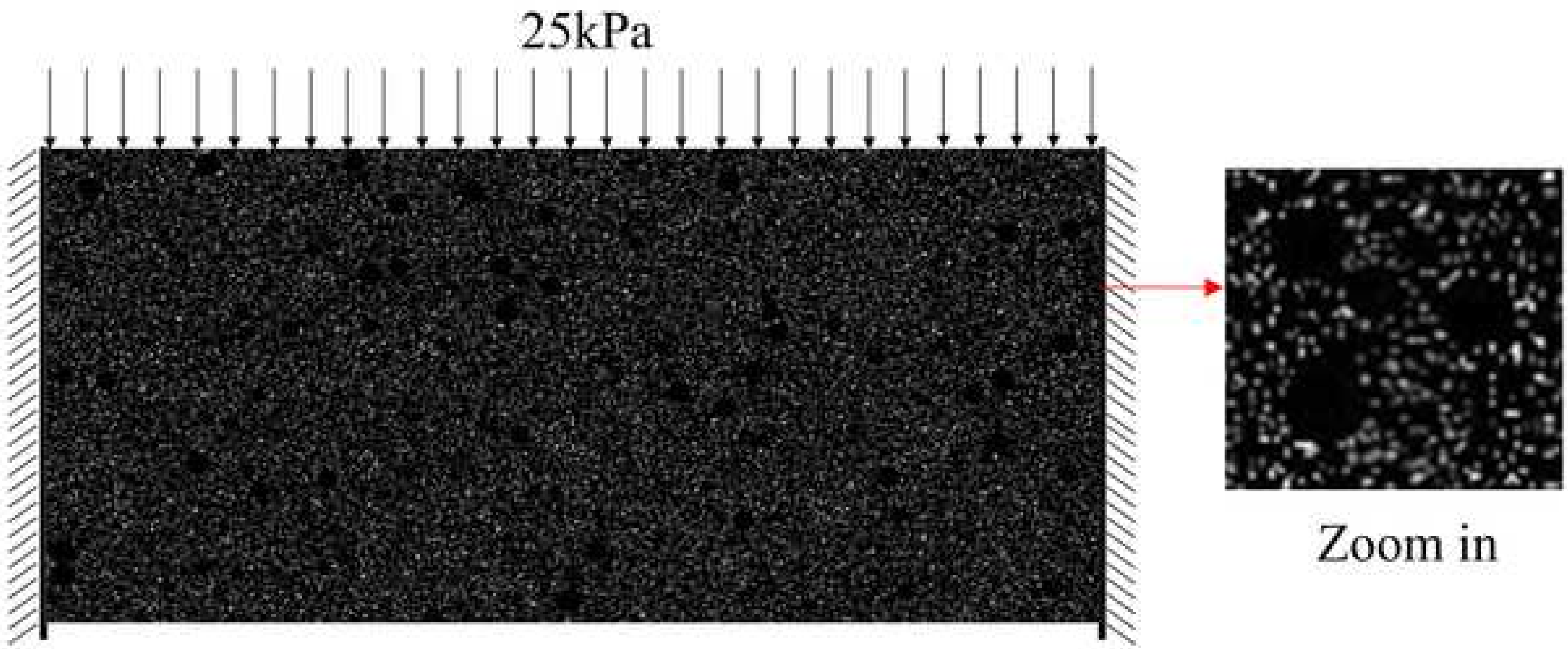

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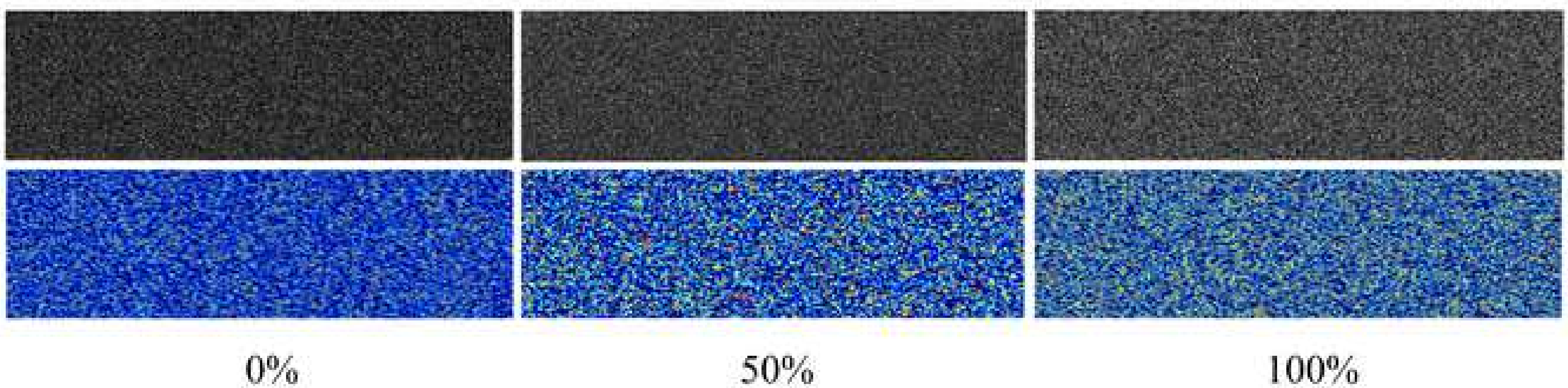
Zoom in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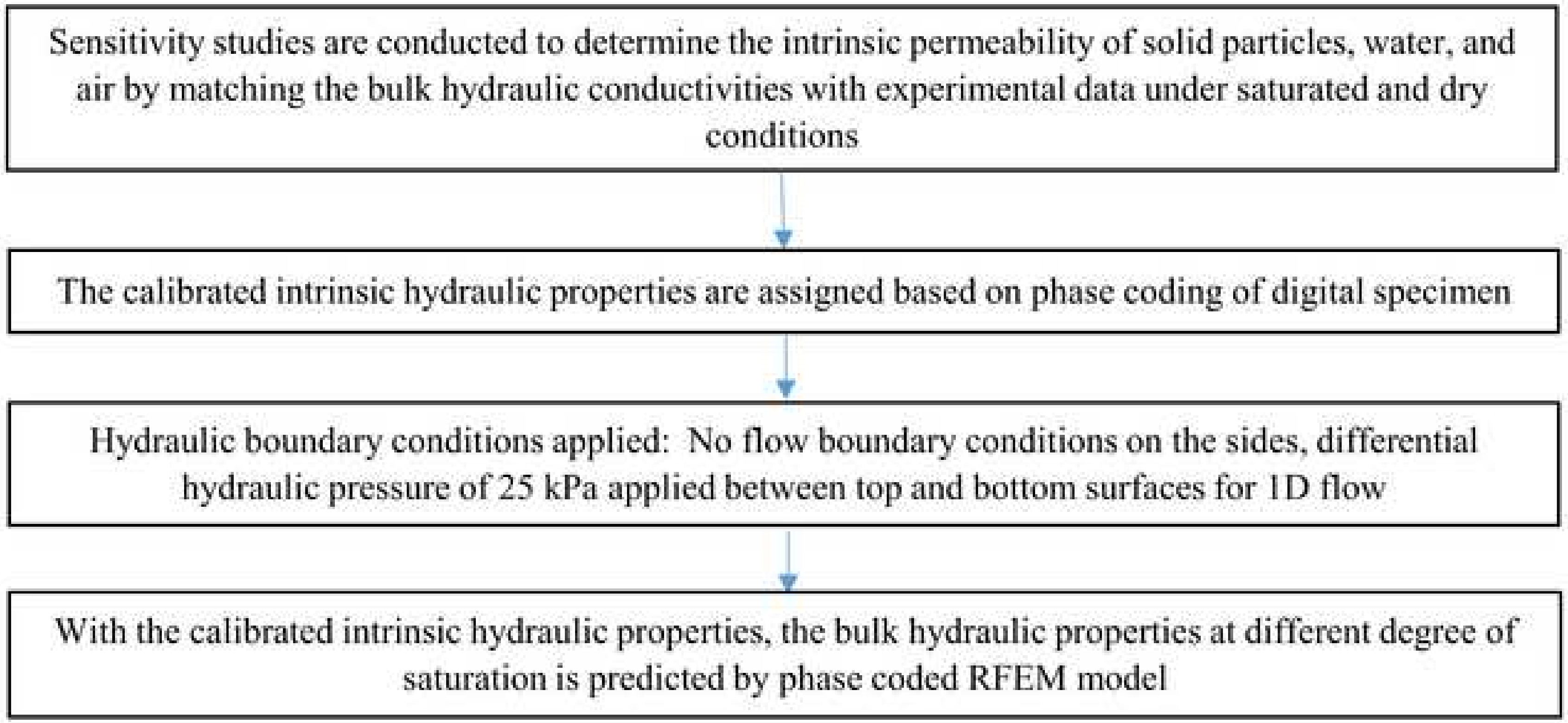
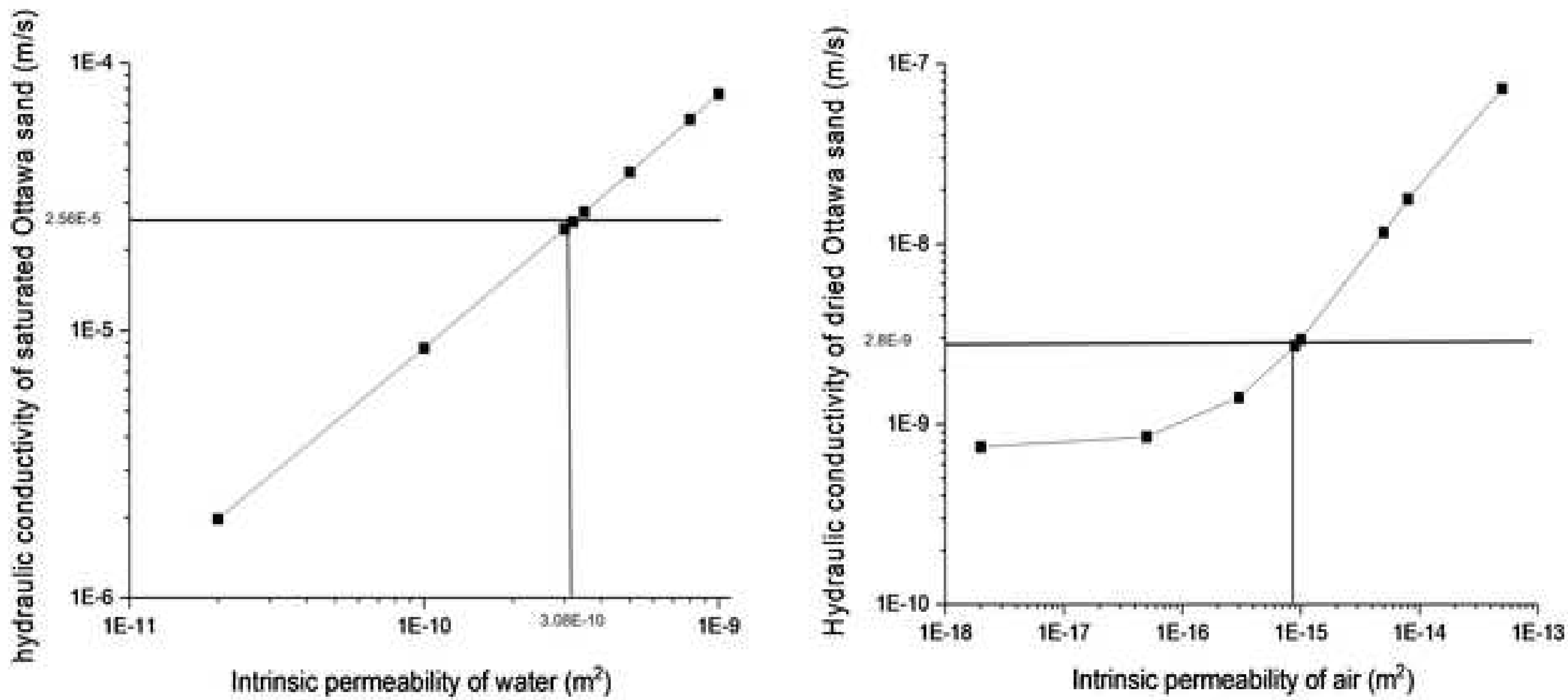

Sensitivity studies are conducted to determine the intrinsic permeability of solid particles, water, and air by matching the bulk hydraulic conductivities with experimental data under saturated and dry conditions
The calibrated intrinsic hydraulic properties are assigned based on phase coding of digital specimen
Hydraulic boundary conditions applied: No flow boundary conditions on the sides, differential hydraulic pressure of 25 kPa applied between top and bottom surfaces for 1D flow
With the calibrated intrinsic hydraulic properties, the bulk hydraulic properties at different degree of saturation is predicted by phase coded RFEM model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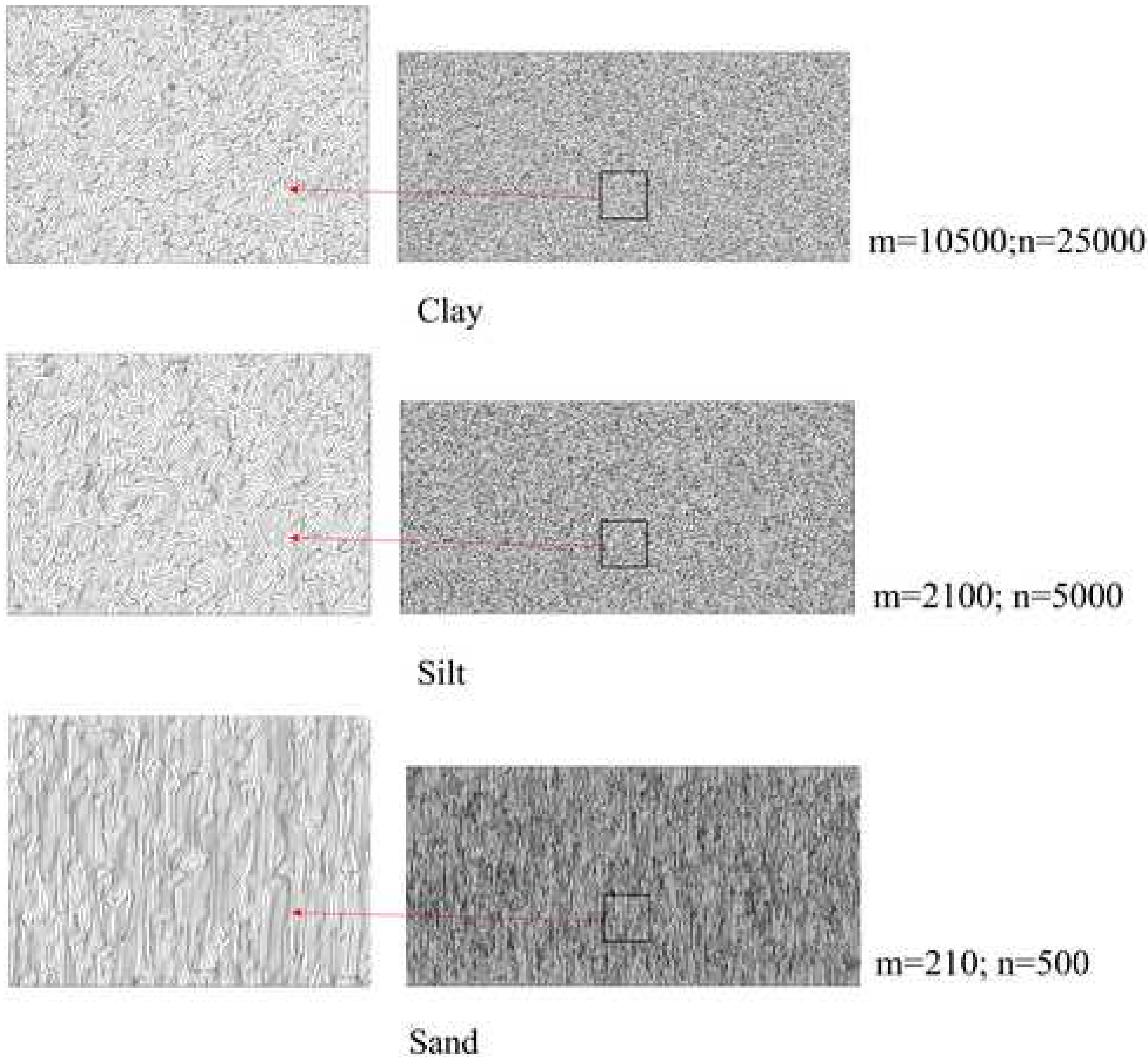
1E-5
1E-6
1E-11
1E-10
3.08E-10
1E-9
Intrinsic permeability of water (m^2)
Hydraulic conductivity of dried Ottawa sand (m/s)
1E-7
1E-8
2.8E-9
1E-9
1E-10
1E-18
1E-17
1E-16
1E-15
1E-14
1E-13
Intrinsic permeability of air (m^2)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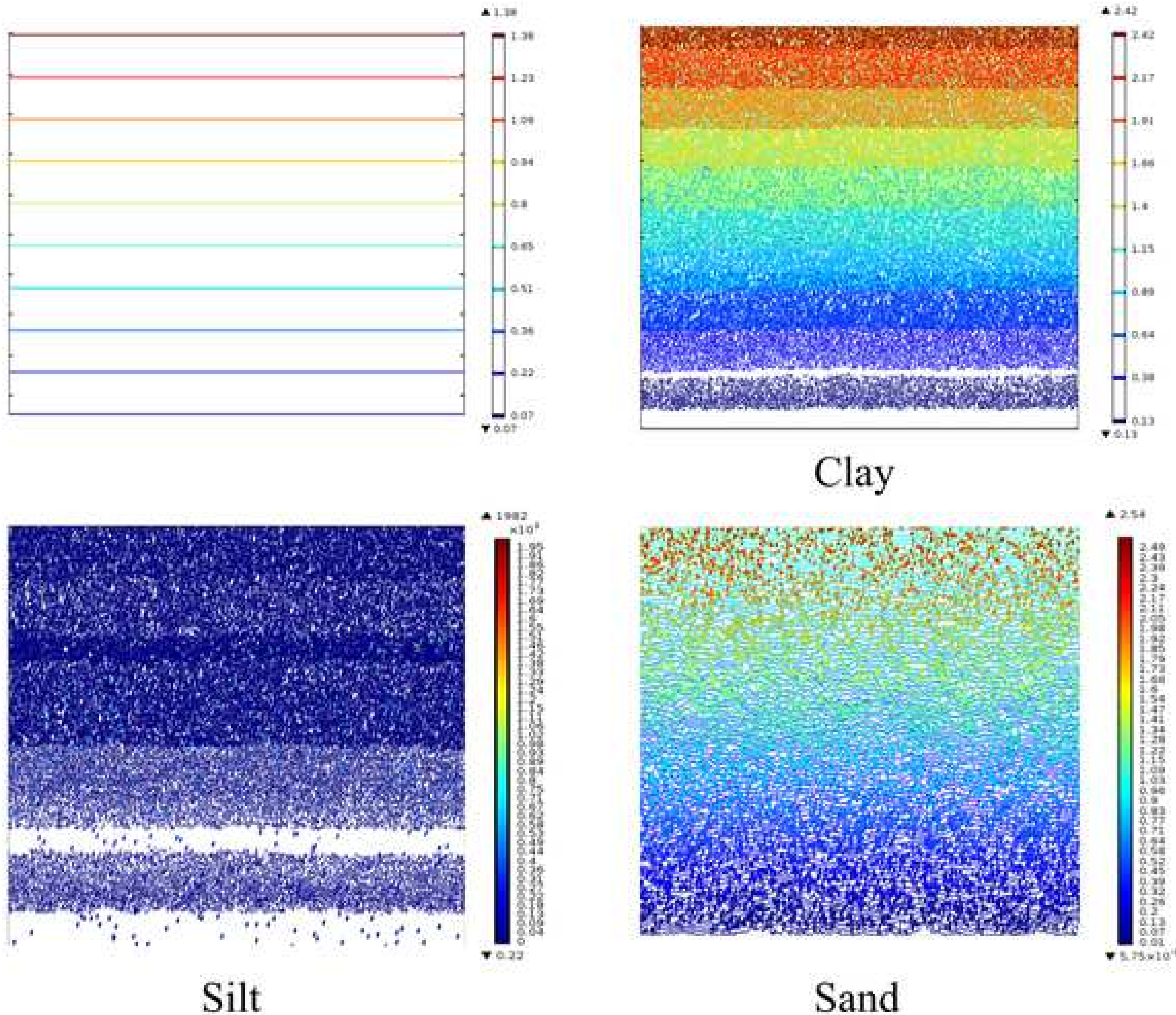
Clay
m=2100; n=5000
Silt
m=210; n=500
Sand


Clay
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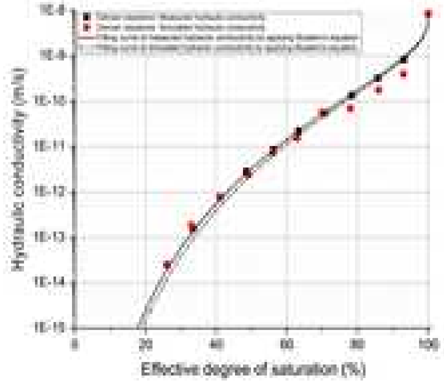
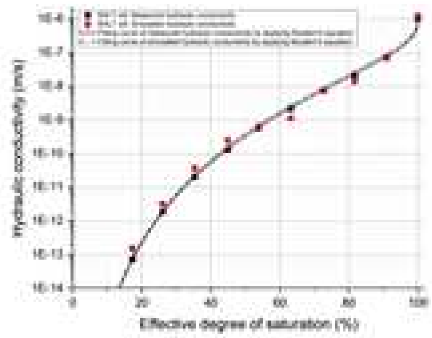
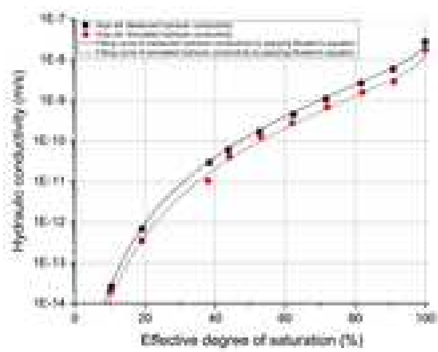


Denver claystone



BALT silt



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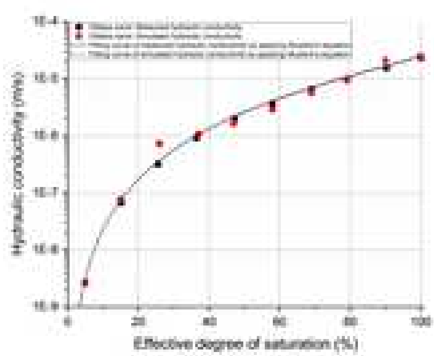
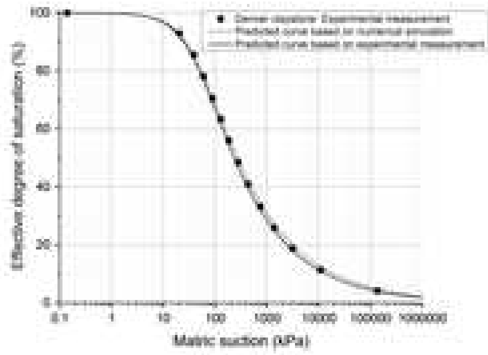


Ottawa sand



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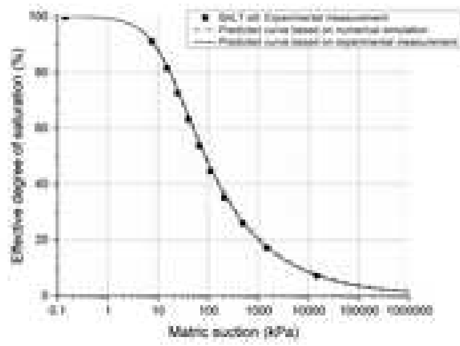

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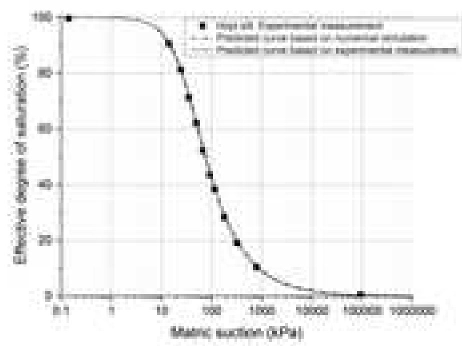

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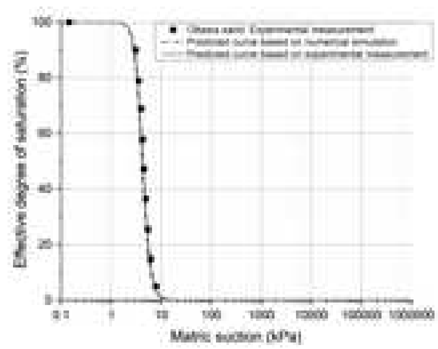

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