Detection and Sensing Using Coupled Oscillatory Systems at the Synchronization Edge

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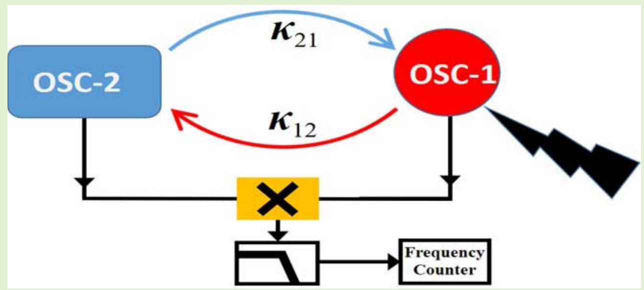
***Abstract*—We theoretically investigate the sensitivity of the beat frequency in a coupled oscillatory system comprising two originally synchronized similar or dissimilar oscillators up on weak perturbation of the frequency of one of the oscillators. Based on the well-known *Kuramoto model*, we analytically show that the variation of the beat frequency near the synchronization edge of two coupled oscillators induced by external perturbation of one of the oscillators, is much larger than the frequency shift of an isolated oscillator induced by the same level of perturbation. The theoretical predications are validated by experimental measurements of the response of two different coupled oscillatory systems to external perturbation: 1) Two bidirectionally coupled electronic Colpitts oscillators, when a photo induced current is applied to one of them. 2) An electronic oscillator unidirectionally coupled to an optoelectronic oscillator (OEO), when the temperature of the delay line of the OEO is changed.**



***Index Terms*—Kuramoto model, mutual coupling, oscillators, sensors, unidirectional coupling.**

I. Introduction

OSCILLATORS have been extensively used for various sensing applications. In particular for sensing and measuring physical parameters that can affect their oscillation frequency (e.g. mass, temperature, humidity, etc.). Typically the interaction of the measurand with the resonator or the feedback loop of the oscillator results in a change in the oscillation frequency; subsequently the magnitude of the measured frequency change can be used to extract the strength of the interaction with the measurand that is typically proportional to the magnitude of the measurand (e.g. mass, temperature, number of molecules,…). Frequency based sensing using a single oscillator has been the subject of extensive investigation. For example, electrical oscillators have been used for mass sensing [1], humidity sensing [2], load sensing [3], etc. Mechanical oscillators have been used for mass sensing [4], [5], charge detection [6], gas and pressure sensing [7], etc. Optomechanical oscillators have been used for mass sensing [8], [9]. Optoelectronic oscillators have been used for temperature sensing [10], [11], distance [12], load and strain [13] measurement, refractive index sensing [14] thermos-optical coefficient measurement [15].

To a lesser extent, synchronized coupled oscillators have been also considered for sensing applications. Juillard *et al.* [16], [17] showed that the phase difference between the two oscillators synchronized through mutual coupling is highly sensitive to the mismatch between the oscillators and can be used to detect the changes of certain physical parameters. It has been demonstrated that the amplitude change of the antisymmetric vibrational mode of two coupled cantilevers (micromechanical oscillators) is more sensitive than frequency change of single cantilever to the added mass on one of them [18]. Spletzer *et al.* [19] have shown that the amplitude of both symmetric and antisymmetric modes of two coupled cantilevers exhibits higher sensitivity than the frequency change of each one of them when upon adding a mass. Barbarossa and Celano [20] theoretically showed that a sensor based on a network of synchronized oscillators exhibits higher reliability than a sensor based on a single oscillator because the SNR of the sensor can be improved by the oscillator nodes. Beyond sensing, coupled oscillators have been also used in image sensors where nodal phase change in a network of 32×32 synchronized oscillators was used for imaging [21]. In almost

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all of these experimentally demonstrated sensing systems, the sensor includes two identical oscillators, which before coupling have the exact same oscillation frequency. Once they are coupled, either two distinctive modes with two different frequencies emerge or the two oscillators are synchronized. The amplitude change of one of these two emerged modes or the phase difference between the synchronized oscillators is then used as a sensing parameter to detect a change induced by perturbing a parameter in one of them.

The sensing mechanism and coupled oscillatory system studied here, is based on two oscillators that when they are decoupled, have close but non-identical oscillation frequencies. These oscillators can be physically similar or dissimilar, but their oscillation frequencies are close enough so that after mutual or unidirectional coupling, they become synchronized. We demonstrate that when these two oscillators are coupled (mutually or unidirectionally) and their coupling is adjusted such that the coupled system is at the synchronization edge, the frequency difference between them can be used for enhanced sensing of an external perturbation affecting one of the oscillators (hereafter referred to as the "detector oscillator"). When a measurand perturbs a parameter of the detector oscillator, that changes its oscillation frequency, the two oscillators become desynchronized and their oscillation frequencies split. This frequency splitting can be converted to a measurable beat frequency that is proportional to the perturbation strength (i.e., the magnitude of the target measurand). We show that the variation of the beat frequency is much larger than the oscillation frequency shift of the isolated detector oscillator induced by the same type and magnitude of perturbation.

Previously the beat frequency generated by oscillation of two coupled mechanical modes of a single resonator (e.g., a single crystal) has been used for force and temperature sensing [22], [23]. In some other examples, the beat frequency between excited modes of two mechanical resonators fabricated on the same substrate has been used for temperature sensing [24]. In those systems, the coupling between the modes was naturally provided through the mechanical structure resulting in simultaneous perturbation of both modes by the measurand. As such, to make the beat frequency sensitive to a perturbation, the frequency of each mode had to be affected differently. Moreover, since in such configurations the coupling factor is determined by the structure, preparation of the system in a specific oscillatory state (e.g., synchronization edge) can be very challenging. Note that in other kinds of oscillators that support multimodal oscillations (e.g., optomechanical oscillators and optoelectronic oscillators [25], [26]), the response of each mode to a perturbation is an inherent property of the system and cannot be easily manipulated to provide significantly different response (to support a large beat frequency change up on exposure to a measurand).

In this paper, we first derive a general theory that explains the enhanced sensitivity provided by the coupled oscillatory system (compared to single oscillator sensors), then we demonstrate its validity by building and testing two oscillatory systems: 1) two non-identical mutually coupled electronic oscillators and 2) an optoelectronic oscillator unidirectionally coupled (injection locked) to an electronic oscillator.

II. General Theory

In this section, we theoretically analyze the performance of an oscillatory sensing system comprising two non-identical coupled oscillators that may be physically similar or dissimilar. The coupling strengths considered here are weak; in other words, the injected signal from one to the other oscillator is much smaller than the oscillation amplitude of the oscillator that receives the signal ($s_{inj}/s_{int} \ll 1$). With this assumption, the oscillation amplitude variation induced by coupling can be ignored, and the interaction between the two oscillators may be described by the well-known *Kuramoto* model [27]–[29]. As such the dynamic of the coupled oscillatory system can be captured by the coupled differential equations governing the phase of each oscillator:

$$\frac{d\theta_1}{dt} = \omega_{01} + \kappa_1 \sin(\theta_2 - \theta_1) \quad (1)$$

$$\frac{d\theta_2}{dt} = \omega_{02} + \kappa_2 \sin(\theta_1 - \theta_2). \quad (2)$$

where θ_1, θ_2 are the phases of the two oscillators, $\kappa_1, \kappa_2 \geq 0$ are the coupling strengths, and ω_{01}, ω_{02} are their isolated oscillation frequencies. ω_{01}, ω_{02} are close enough to support synchronization between the two oscillators (here we assume $\omega_{01} \geq \omega_{02}$). This simple phase model has been reported to be useful in predicting the behavior of a large variety of coupled oscillators [29], for example, it has been used in modelling biological oscillators [28], electrical oscillators [30], [31], chemical oscillators [32], [33], mechanical oscillators [34], [35] and optical oscillators [36], [37].

Using (1) and (2) the temporal variation of the phase difference between the two coupled oscillators can be written as:

$$\frac{d(\theta_1 - \theta_2)}{dt} = (\omega_{01} - \omega_{02}) - (\kappa_1 + \kappa_2)\sin(\theta_1 - \theta_2). \quad (3)$$

If the two oscillators are synchronized, the temporal variation of their phase difference is zero, so (3) is simplified as:

$$(\omega_{01} - \omega_{02}) = (\kappa_1 + \kappa_2)\sin(\theta_1 - \theta_2). \quad (4)$$

Equation (4) shows that the necessary condition for synchronization is:

$$|(\omega_{01} - \omega_{02})| \leq (\kappa_1 + \kappa_2). \quad (5)$$

that is essentially the condition for (4) to have a real solution. In (5) equal sign corresponds to the frequency difference that for a given coupling can be considered the *synchronization edge*; meaning that a change in the original frequency difference or the coupling strength will desynchronize the two oscillators.

Under this condition the coupled system responds to an external perturbation (applied on one of the oscillators) with the highest level of sensitivity. Here we consider that oscillator #1 is the detector oscillator that is perturbed (a change induced in one or more parameters that determine its oscillation frequency). We assume that the magnitude of the perturbation is small enough such that the induced change in the oscillation frequency of the isolated oscillator (ω_{01}') is linearly proportional to the perturbation strength and can be written as

$$\omega_{01}' = \omega_{01} + \epsilon S. \quad (6)$$

where S is the strength of the perturbing signal and ϵ is the proportionality constant. Using (6), Equation (3) is modified as:

$$\frac{d(\theta_1 - \theta_2)}{dt} = \left(\omega_{01}{}' - \omega_{02}\right) - (\kappa_1 + \kappa_2)\sin(\theta_1 - \theta_2). \quad (7)$$

This equation is similar to Adler's equation that was developed in context of electronics oscillators [38].

We now introduce a phase variable $\varphi(\mathrm{t}) = \exp(j(\theta_1 - \theta_2))$ to capture the phase difference of the coupled oscillators. Using this phase variable, Equation (7) can be rewritten as:

$$\frac{d\varphi}{dt} = -\frac{\kappa_1 + \kappa_2}{2}\varphi^2 + j\left(\omega_{01}{}' - \omega_{02}\right)\varphi + \frac{\kappa_1 + \kappa_2}{2}. \quad (8)$$

following procedures similar to those presented in [31] and [39], the solution for $\varphi(\mathrm{t})$ may be expressed as:

$$\varphi(t) = \frac{\sigma_2 - C\sigma_1 e^{j\sqrt{(\omega_{01}{}' - \omega_{02})^2 - (\kappa_1 + \kappa_2)^2}t}}{1 - Ce^{j\sqrt{(\omega_{01}{}' - \omega_{02})^2 - (\kappa_1 + \kappa_2)^2}t}}. \quad (9)$$

where C is a constant, and σ_1, σ_2 are defined as:

$$\sigma_1 = j\left(\frac{\omega_{01}{}' - \omega_{02}}{\kappa_1 + \kappa_2} + \sqrt{\left(\frac{\omega_{01}{}' - \omega_{02}}{\kappa_1 + \kappa_2}\right)^2 - 1}\right). \quad (10)$$

$$\sigma_2 = j\left(\frac{\omega_{01}{}' - \omega_{02}}{\kappa_1 + \kappa_2} - \sqrt{\left(\frac{\omega_{01}{}' - \omega_{02}}{\kappa_1 + \kappa_2}\right)^2 - 1}\right). \quad (11)$$

Equation (9) indicates that $\varphi(\mathrm{t})$ is a harmonically oscillating parameter with an oscillation frequency equal to:

$$\omega_{\mathrm{B}} = \omega_{01\mathrm{c}} - \omega_{02\mathrm{c}} = \sqrt{(\omega_{01}{}' - \omega_{02})^2 - (\kappa_1 + \kappa_2)^2}. \quad (12)$$

where $\omega_{01\mathrm{c}}$ and $\omega_{02\mathrm{c}}$ are the oscillation frequencies of the two coupled oscillators after perturbing the detector oscillator. ω_{B} is essentially the beat frequency that can be extracted from the oscillatory systems by subtracting the output frequency of the two coupled oscillators (in practice, ω_{B} can be generated using a frequency mixer followed by a low pass filter).

If the coupled system is tuned to oscillate at the *synchronization edge* (i.e. $\kappa_1 + \kappa_2 = \omega_{01} - \omega_{02}$), Equation (12) can be written as:

$$\begin{aligned}\omega_{\mathrm{B}} &= \sqrt{(\omega_{01} + \epsilon S - \omega_{02})^2 - (\kappa_1 + \kappa_2)^2} \\ &= \sqrt{(\epsilon S)^2 + 2(\omega_{01} - \omega_{02})\epsilon S}. \end{aligned} \quad (13)$$

A comparison between (6) (the oscillation frequency shift for the isolated detector oscillator) and (13) (the beat frequency for the coupled system) shows ω_{B} is much larger than ϵS, especially when the perturbing signal is very weak (i.e. $\epsilon S \ll \omega_{01} - \omega_{02}$). One can define an enhancement factor as the ratio between the beat frequency ω_{B} and the frequency shift $\omega_{01}' - \omega_{01}$ as:

$$\eta = \frac{\sqrt{(\epsilon S)^2 + 2(\omega_{01} - \omega_{02})\epsilon S}}{\epsilon S} = \sqrt{1 + 2\frac{\omega_{01} - \omega_{02}}{\epsilon S}}. \quad (14)$$

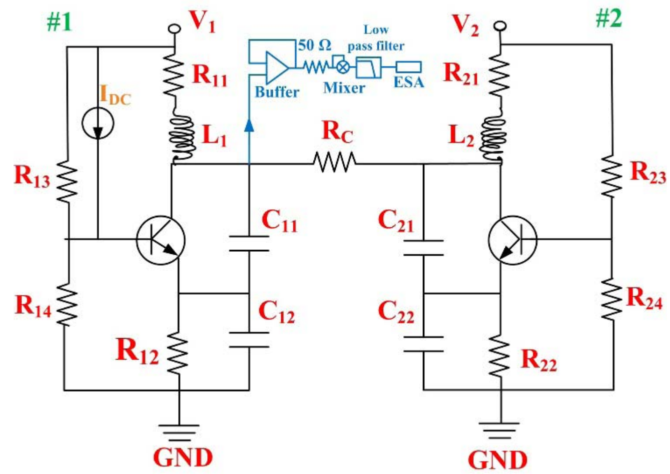

Fig. 1. Two resistively coupled Colpitts oscillators used to detect a DC photocurrent (I_{DC}) applied to the base port of the first oscillator (serving as the detector oscillator). The blue circuit is used to generate the beat frequency and its output is measured using an electric spectrum analyzer (ESA). Here, $V_1 = 3$ V, $L_1 = 16\ \mu$H, $R_{11} = 33\Omega$, $R_{12} = 68\Omega$, $R_{13} = 83\Omega$, $R_{14} = 327\Omega$, $C_{11} = 27$ nF, $C_{12} = 33$ nF, $V_2 = 3$ V, $L_2 = 22\mu$H, $R_{21} = 33\Omega$, $R_{22} = 69\Omega$, $R_{23} = 75\Omega$, $R_{24} = 325\Omega$, $C_{21} = 31$ nF, $C_{22} = 21$ nF, and $R_c = 3500\ \Omega$.

III. Experiment

In order to test the proposed sensing scheme and validate the corresponding theory, we fabricated two different kinds of oscillators to detect two different measurands. The first experiment uses two mutually coupled Colpitts electronic oscillators to detect a current change in one of them (more specifically optically induced current change or a photocurrent). The second experiment uses an optoelectronic oscillator (OEO) injection locked to an electronic oscillator to detect the temperature change that affects the OEO's optical time delay. For both measurements we compare the frequency shift of the isolated detector oscillator (that in the first case can be one of the two Colpitts and in the second case is the OEO) with the frequency difference between the two coupled oscillators (beat frequency), for a given change in the measurand (photocurrent and temperature). We also calculate the frequency shift for the isolated detector oscillator, beat frequency for the coupled system and the sensitivity enhancement using (6), (13), and (14) and show that the calculated results are in good agreement with the experimental results.

A. Using Electrical Colpitts Oscillators to Detect DC Current

Fig. 1 shows the first coupled oscillatory system that consists of two resistively coupled Colpitts oscillators. Each oscillator uses an NPN bipolar transistor configured as a common emitter amplifier, and an LC tank as the feedback. The oscillation frequency of the first oscillator (#1) is $f_{\mathrm{C1}} = 398.20$ kHz and its oscillation linewidth is 11.94 Hz. The oscillation frequency of the second oscillator (#2) is $f_{\mathrm{C2}} = 395.00$ kHz and its linewidth is 12.60 Hz. This system is used to compare the sensitivity of the oscillation frequency of a single oscillator and beat frequency of a coupled system to a current change applied to the base port of the first oscillator

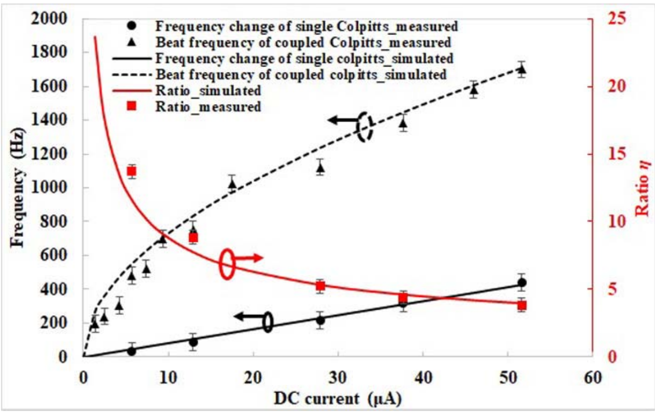

Fig. 2. Response of a single isolated Colpitts oscillator and a coupled Colpitts oscillatory system to induced current change in one of the oscillators. Black dots and black triangles are the measured values of Δf_{C1} and f_B plotted against applied photocurrent (I_{DC}) respectively. The squares are the measured values for $\eta = f_B/(\Delta f_{C1})$. The lines are the calculated values for f_B (dashed black), Δf_{C1} (solid black) and η (solid red) using Equations (6), (13) and (14) for $\epsilon = 2\pi \times 8.3184$ Hz/μA (extracted from the measured values of Δf_{C1}) and $\omega_{01} - \omega_{02} = 2\pi \times 3200$ Hz.

(serving as the detector oscillator). Firstly, we use oscillator #1 (as a single isolated oscillator) to detect the DC photocurrent generated by a photodiode. The magnitude of the DC current is controlled by changing the intensity of the incident light. The black dots in Fig. 2 are the measured data points for the frequency change (Δf_{C1}) plotted against the applied photocurrent (I_{DC}). The oscillation frequency is monitored through the collector port of the transistor using a buffer circuit (to make sure the measurement does not affect the oscillation). Next, oscillator #1 (the detector oscillator) is coupled to the oscillator #2. The two oscillators are bidirectionally coupled through a resistor R_C that can be selected according to desired coupling strength.

For the system in Fig. 1 when R_C is 3500 Ω, the coupled oscillatory system will oscillate at the *synchronization edge* (as defined by Equation (5)). Here the ratio between the amplitude of the injected current through R_C to each oscillator and amplitude of the intrinsic current flowing in the oscillator (I_{inj}/I_{int}) is 1.3×10^{-4}. Once a system oscillating at the *synchronization edge* is prepared, the photocurrent is induced only in the base port of oscillator #1 by illuminating the photodiode connected between V_1 and the base port of the transistor. In this case the readout circuit (blue circuit in Fig. 1) includes a mixer and a low-pass filter (in addition to the buffer) that together they generate an output proportional to Sin(ω_Bt). The frequency of this signal (f_B) is monitored using an electric spectrum analyzer.

The triangles in Fig. 2 are the measured values of f_B plotted against applied photocurrent (I_{DC}). The squares in the same figure are the calculated ratio (η) between measured beat frequency (f_B) and the measured frequency shift (Δf_{C1}). The solid lines are the calculated values of f_B, Δf_{C1} and η using (6), (13) and (14) for $\epsilon = 2\pi \times 8.3184$ Hz/μA (extracted from the measurement) and measured value of $\omega_{01} - \omega_{02} = 2\pi \times 3200$ Hz. It is worth mentioning that when the photocurrent is too small (less than 6 μA) the resulting frequency shift (Δf_{C1}) in the single Colpitts oscillator is not detectable since its magnitude is in the same order or smaller than the oscillation linewidth. However, the magnitude of f_B of the coupled system is large enough to be resolved. As such the limit of detection (LoD) for the coupled system is significantly larger compared to the single oscillator system (~500 times larger based on the linewidth of the Colpitts oscillator).

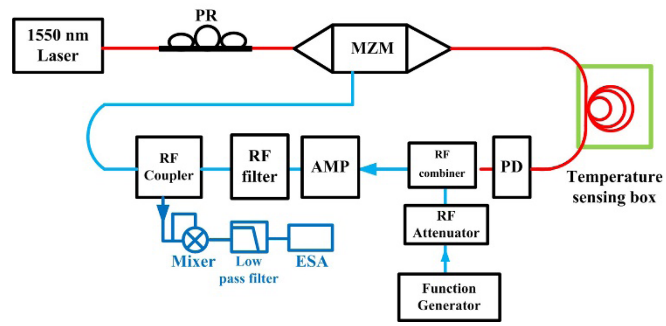

Fig. 3. A coupled heterogeneous oscillatory system consisting of an OEO injection locked (unidirectionally coupled) to an electronic oscillator, the fiber-optic delay loop is enclosed in a temperature-controlled chamber. The blue circuit is used to generate the beat frequency and its output is measured using an electric spectrum analyzer (ESA).

B. Using OEO to Detect Temperature Change

The second coupled oscillatory system studied here is a heterogeneous system consisting of an optoelectronic oscillator (OEO) [40] injection locked to an electronic oscillator. Fig. 3 shows the schematic diagram of the coupled oscillatory system. The electronic oscillator is a commercially available signal generator (HP, 8648B). The OEO that serves as detector oscillator, is fabricated using a simple single loop architecture with an optical delay line consisting of 1 km of single mode fiber. The RF filter in the OEO feedback loop has been selected to force OEO to oscillate at 10.5650 MHz. The measured linewidth of the resulting oscillation is 16.09 Hz. Here the measurand is temperature and the affected OEO parameter is the optical delay. As such the fiber optic delay has been enclosed in a chamber so that the temperature of the entire loop can be controlled with an electrical heater placed inside the chamber. A Commercial psychrometer (EXTECH, RH350) with temperature sensing resolution of 0.1°C, is used to characterize the temperature.

First, we measure the oscillation frequency change (Δf_{OEO}) of the isolated OEO as a function of the temperature of the optical delay. In this experiment the oscillating RF power inside the OEO is monitored using a direction RF coupler that couples 19 dB of the RF power circulating in OEO's feedback loop out. The readout circuit for the single oscillator characterization only includes a buffer that isolates the ESA from the feedback loop. The black dots in Fig. 4 are the measured oscillation frequency of the free running OEO as a function of the temperature of fiber optic delay.

Next, the output RF power of a tunable electronic oscillator is fed to OEO's feedback loop using a 3-port RF combiner as shown in Fig. 3. The strength of the coupling is adjusted by tuning the oscillation frequency of the electronic oscillator and controlling the magnitude of the coupled (injected) RF power

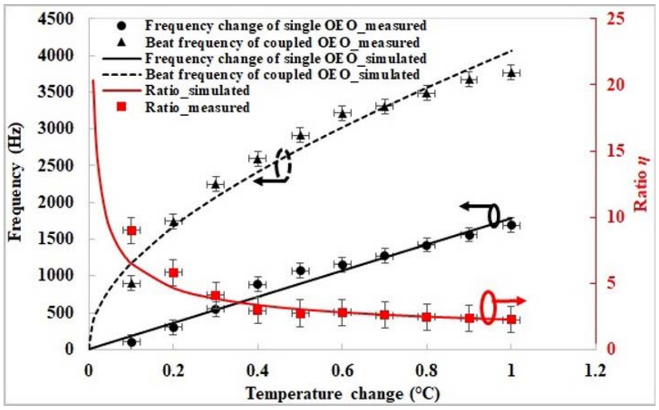

Fig. 4. Single and coupled OEO is used to detect the temperature change in the chamber containing the fiber-optic delay, Black dots and black triangles are the measured values of Δf_{OEO} and f_{B} plotted against the temperature, respectively. The squares are the measured values for $\eta = f_{\mathrm{B}}/(\Delta f_{OEO})$. The lines are the calculated values for f_{B} (dashed black), Δf_{OEO} (solid black) and η (solid red) using Equations (6), (13) and (14). Here $\epsilon = 2\pi \times 1794.20$ Hz/ °C, and $\omega_{01} - \omega_{02} = 2\pi \times 3700$ Hz.

using a tunable RF attenuator. When the frequency of the electronic oscillator is 10.5687 MHz and the ratio between injected voltage amplitude and the intrinsic oscillating voltage amplitude of the OEO (V_{inj}/V_{int}) is 0.02, the coupled system oscillates at the *synchronization edge*. Similar to the previous experiment, a mixer and a low-pass filter are used after the buffer to generate an output proportional to Sin(ω_{B}t) without affecting the oscillation of the system (see the blue circuit in Fig. 3). The resulting beat frequency is monitored using an ESA.

The black triangles in Fig. 4 are the measured values of the beat frequency (f_{B}) at different temperatures. The red squares are the magnitude of the enhancement ratio (η) calculated based on measured frequencies (f_{B}, Δf_{OEO}). The solid lines are the calculated values of f_{B}, Δf_{OEO} and η using (6), (13) and (14) for $\epsilon = 2\pi \times 1794.20$ Hz/°C (extracted from the measurement) and measured value of $\omega_{01} - \omega_{02} = 2\pi \times 3700$ Hz. As expected, the temperature sensitivity of the beat frequency is much larger than that of the single OEO oscillation frequency change. In particular between 0 and 0.1°C, η can be as large as 20. Note that the smallest data point measured is limited by the resolution of our temperature sensor and it is not related to the limit of detection of the system.

IV. Discussion

The sensing method described above relies on the fact that the perturbation of the sensor oscillator increases the difference between the intrinsic oscillation frequencies of the two oscillators. So assuming the two oscillators are synchronized and the coupling coefficients are selected so that the system is at the synchronization edge, two scenarios are possible: 1) the intrinsic oscillation frequency of the sensor oscillator is larger than the other oscillator; in this case the perturbation should increase the frequency of the sensor oscillator. 2) the intrinsic oscillation frequency of the sensor oscillator is smaller than the other oscillator; in this case the perturbation should decrease the frequency of the sensor oscillator. Given that the oscillation frequency of any oscillator is selectable by design, once the response of the sensor oscillator to a target measurand is known, the frequency of any other oscillators can be selected to provide maximum sensitivity to a change in the target measurand in a given direction. Clearly, this requirement imposes a limit on the direction or sign of the measurand change; in other words, if the system is designed to detect an increase in certain measurand with highest sensitivity (initially synchronized at synchronization edge), then it will be insensitive to the decrease of that measurand. Alternatively, the system may be tuned such that initially it is not right at the synchronization edge so that the beat frequency is present even in the absence of a measurand change; in this case the system may detect both an increase and decrease of the measurand but with a lower sensitivity.

Since the limit of detection (LoD) of the proposed oscillatory sensor system is ultimately limited by the smallest measurable beat frequency, the signal-to-noise ratio (SNR) of the beat signal may impose a bound on the limit of detection beyond the limit defined by the resolving power of the frequency measurement system. Our preliminary theoretical analysis and experimental observations indicate that the amplitude of the beat signal decreases as the perturbation (and therefore the beat frequency) becomes smaller. Given that generally (at least in electronic circuits) the level of noise increases at lower frequencies (e.g., due to $1/f$ noise mechanisms), we expect the SNR to degrade as the magnitude of the perturbation decreases. A comprehensive study and analysis of the amplitude variations of the beat frequency and the noise in such system is beyond the scope of this paper but it should be considered as an important limitation in particular for applications where a low LoD is required.

Generally, the temperature dependence of the oscillation frequency can be different for the two coupled oscillators in the sensing system. As such if temperature of the oscillators varies during the measurement, the measured value of the beat frequency cannot be used to accurately monitor a change in the measurand (due to residual shift resulted from temperature variations). This problem can be mitigated by stabilizing the temperature of the oscillators (by active control) and thermally isolating them from the sensing element of the sensor oscillator that is used to detect the measurand (e.g., the fiber delay in the OEO).

Another possible limitation of the proposed method is the fact that the beat frequency is much smaller than the oscillation frequencies of the individual oscillator. As such, depending on the frequency measurement system used (e.g., a frequency counter, RF spectrum analyzer, and the like), measuring small changes in a small beat frequency may require longer integration time compared to the time required for monitoring the frequency of isolated oscillators. This factor should be taken into account when evaluating the enhanced sensitivity provided by the coupled system.

V. Conclusion

The enhanced sensitivity and the fact that the oscillatory system and the sensing mechanism used in this work support dissimilar oscillators with slightly different frequencies, suggest that this work may become an effective technique in many applications. The tolerance for small frequency difference between two oscillators is an advantage over previously demonstrated systems, since fabrication of oscillators with identical oscillation frequencies (a prerequisite of most previously demonstrated systems), is a challenging task. The heterogeneous systems may provide the added benefit of simultaneous detection of small perturbations of physically distinctive measurands (e.g., temperature, optical power, current, magnetic field, etc.).

The proposed approach is particularly suitable for detection of extremely small perturbations where the frequency shift of a single oscillator may be screened by noise, but the high sensitivity of the beat frequency change in the proposed coupled oscillatory systems is large enough to be measured (resulting in significantly lower limit of detection). While the variation of the beat frequency in a coupled oscillatory system is inherently nonlinear, for detecting small changes around a background value or starting from a zero perturbation, the sensitivity can be approximated by linear slope. Moreover, in many applications only detection (as opposed to measurement) of a small change of a parameter is the objective, in which case the nonlinearity of the response becomes irrelevant. An example of such applications is detection of small quantity of hazardous molecules (in particular in gaseous state) and triggering an alarm when the detected signal exceeds a pre-set threshold.

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