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Systemic Risk in Networks with a Central Node*

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Abstract. We examine the effects on a financial network of clearing all contracts though a central node (CN), thereby transforming the original network into a star-shaped one. The CN is capitalized with external equity and a guaranty fund. We introduce a structural systemic risk measure that captures the shortfall of end users. We show that it is possible to simultaneously improve the expected surplus of the banks and the CN as well as decrease the shortfall of end users. We determine the CN's equity and guaranty fund policies as a Nash bargaining solution. We illustrate our findings on simulated credit default swap networks compatible with aggregate market data.

Key words. star-shaped networks, central node, market design, financial network, contagion, systemic risk, credit default swap markets

AMS subject classifications. 91B30, 91G50, 90B15, 90B50, 90B10, 91B15, 60J10

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1. Introduction.

The reform of over-the-counter (OTC) derivatives markets lies at the core of the Dodd–Frank Wall Street Reform and Consumer Protection Act of 2010. Among the regulations is that the majority of OTC derivatives, of the order of dozens of trillions of US dollars in terms of notional, should be centrally cleared so as to ensure financial stability. The Basel Committee for Banking Supervision and European and UK regulators have enacted similar proposals. Introducing a central node (CN) modifies the intermediation structure of the market: any financial obligation among banks is now intermediated by the CN, while part of the banks' liquidity is transferred to the CN.

Centralized clearing is complex and multifaceted, and a variety of viewpoints must be weighed. The CN must be *designed* so as to decrease the risk imposed on the outside economy, i.e., systemic risk, and at the same time be Pareto optimal; otherwise a unanimous agreement cannot be reached. The question we ask is whether a unanimous agreement can be achieved: can one can find parameters of a CN design that are Pareto optimal and that can be achieved as a solution of a bargaining game?

We use a network representation of the OTC market, with financial institutions (banks) interlinked by liabilities. We focus on scenarios where the survival of the CN is threatened,

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and we refer to these as extreme scenarios. We can think of the interbank liabilities as large variation margin calls following a catastrophic event on the market. Yet another question (not yet answered in the literature) is how a CN would be replenished or continue following an extreme event. This is beyond our model (the recovery mechanism is the last point in the default waterfall—see https://www.theice.com/clear-europe/risk-management).

To account for the risk imposed on the outside economy, banks have end users. We capture systemic risk by using a structural measure defined as the expected loss imposed on the end users. A key ingredient in the model is the early liquidation losses: banks receive endowments of nonpledgeable assets, such as long-term investments which are liquidated at a loss as banks pay the realized liabilities. Negative network externalities due to default contagion further increase the liquidation losses and vice versa. As the CN changes the network structure to a star-shaped network and members' default management resources are transferred to the CN in the form of guaranty fund contributions, the CN changes the equilibrium payments and liquidations across the network.

The CN in our paper is not meant to represent an actual clearinghouse. Indeed, current clearinghouses segregate the members' contributions into well-determined layers: variation margin, initial margin, and default fund. Under an extreme event, the critical layers are the initial margin and the default fund.[1] Our guaranty fund is closest to *a default fund in the absence of the initial margin.* Bank shares in the guaranty fund are pooled and absorb the losses imposed by members. It is understood that members would first wipe out their own guaranty fund contribution before the remaining losses would cascade into the other members' pooled shares of the guaranty fund. Because the initial margin is not present in our model, morally, a real-world CCP's initial margin plus its default fund should be larger than the guaranty fund of our model.

We show that the CN does not necessarily reduce systemic risk and we derive rigorous conditions for a systemic risk reduction. We give guidance on how to select the guaranty fund contributions and the CN external capital. We use as a solution concept the Nash bargaining solution; see, e.g., Roth (1979).[2] We show that all parties can (via bargaining) reach a binding agreement to form a CN. By the axioms of the Nash bargaining solution, such an agreement is Pareto optimal. We verify that the main US clearinghouse for credit derivatives has default resources that are larger than those implied by our model.

One important insight emerging from our model is that under the Nash solution, the seniority of the CN's own equity will not change the expected surplus across all parties. The banks and the CN adapt and their utilities in the Nash solution stay the same. While the utilities are the same, the Nash solution capital levels differ. When the CN's equity is junior with respect to the guaranty fund, also known as "skin in the game," the level of CN equity at the Nash solution is almost a third as for the case where CN equity is senior. When CN capital is junior the equilibrium shifts and it is the banks that make higher contributions to decrease systemic risk.

This shows that the prevailing debate should not be about "skin in the game" but rather about the adequate levels of capitalization under a junior versus a senior CN equity setup.

[1] For examples of a real-world waterfall procedure we refer the reader to https://www.theice.com/clear-europe/risk-management or https://www.cmegroup.com/clearing/risk-management/financial-safeguards.html.

[2] Because we are under a cooperative game setup, banks form binding commitments (the model already features binding liabilities).

The overall resources in the Nash bargaining solution could actually increase when the CN's capital is senior.

Relation to prior literature. There is an emerging literature on centralized clearing. Duffie and Zhu (2011) focus on netting effects and point out the trade-off between multilateral netting achieved through a CN and bilateral netting across asset classes. Amini, Filipović, and Minca (2016a) investigate partial netting of a subset of liabilities and account for network knock-on effects as well as asset liquidation effects. Glasserman, Moallemi, and Yuan (2016) study margin provision in the case of one dealer and competing clearinghouses. Capponi, Cheng, and Rajan (2014) obtain endogenous build-up of asset concentration in centrally cleared markets. Cont and Minca (2016) propose algorithms for generating multilayered OTC networks and assess the impact of centralized clearing by simulation. Armenti and Crépey (2017) introduce a quantitative model for determining the clearing (including margin) costs for default-free clearinghouses.

There cannot be a single model that deals with all different operational regimes of a CCP. Models on variation margins rely on CVA approaches and stochastic processes and they assume that the CCP is default free. Our paper assumes the opposite: we abstract away from normal operations, the CCP can default and cause losses. For normal operations, models determine variation margins and this is very minutiose. We are giving a "bird's eye view" of what happens under Armageddon liabilities ("extreme events"). In a real-world CCP example (see https://www.theice.com/clear-europe/risk-management), the events that concern the initial margin and default funds are referred to as "extreme."

Our paper is part of the larger literature on contagion in financial networks and in particular on payment equilibrium models; see, e.g., Eisenberg and Noe (2001), Cifuentes, Ferrucci, and Shin (2005), Rogers and Veraart (2013), Glasserman and Young (2015). Similarly to the more recent works of this strand, the liquidation losses are a critical driver of our results. Our model suggests that fees and collateral are not substitutes, consistent with Capponi and Cheng (2015). They propose a model with a default-free CN and no contagion effects. In our case fees and collateral are not substitutes because larger guaranty fund contributions serve to satisfy the regulator-imposed constraint to decrease systemic risk, while the fees serve to make the setting attractive for the CN.

Our work is related to the literature on network structure and contagion. We show a type of phase transitions in the *ex post* (statewise) effects on end users. If the liabilities to end users are fully "reinsured," i.e., the banks fully offset liabilities to end users by using contracts with other banks (akin to primary insurers and reinsurers), then the CN decreases the shortfall to end users. However, in heterogeneous settings where some of the banks are overreinsured and some are underreinsured, the loss to end users increases with the CN. In a particular example of a stylized financial network reduced to an intermediation chain, we show that as the length of the intermediation chain becomes large, the CN always decreases the shortfall on end users. In contrast, for medium length intermediation chains, the shortfall on end users increases. Our main results are about the *ex ante* situation, where the effects are in expectation over the networks realized under a set of extreme scenarios.

We can apply our results to derivatives classes for which risk can be propagated to end users, and for which bank-end user trades are not centrally cleared. Dodd–Frank (and similar frameworks in Europe) generally seek to mandate all transactions to be centrally cleared, but this is not the case for nonfinancial end users such as corporations that are hedging. The end users we are concerned with are the real economy firms who have hedging needs.

We are focusing on (CDS) markets because we consider extreme events: "the issue with credit default swap is that because a default is a discrete event, it can lead to large jumps in the value of these contracts" (Stulz (2010)).

Outline. The reminder of the paper is organized as follows. In section 2, we introduce the OTC financial network and a generic interbank liability clearing equilibrium without centralized clearing. In section 3, we add a CN to the financial network and explicitly solve for the corresponding interbank liability clearing equilibrium. In section 4, we study the *ex post* impact of the CN on banks, CN, and end users. We compare liquidation losses and shortfall losses imposed on the end users. In section 5, we introduce the systemic risk measure and give *ex ante* conditions for the CN to decrease systemic risk from an ex ante perspective. In section 6 we present the Nash bargaining solution: the utilities of all parties involved make use of all previous results. In section 7, we analyze numerically the impact of CNs. Section 8 concludes. Appendix A contains some additional sensitivity results and all proofs.

2. Financial network. We consider m interlinked financial institutions (banks) $i = 1, \dots, m$. There are three dates $t = 0, 1, 2$. Values at date $t = 0$ are deterministic and values at date $t = 1, 2$ are random variables on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with Ω a set of *extreme* scenarios that realize at time 1. Liabilities $L_{ij} = L_{ij}(\omega)$ represent claims that become due at date 1 given an extreme scenario $\omega \in \Omega$. For example, following the default of a large reference entity, the payments due to CDS can amount to billions of dollars (after the default of Lehman Brothers, recovery rates were set in auction at only 8% and AIG faced large liabilities upon the default event, and eventually needed a government bailout that topped \$180 billion). It is this kind of (single) catastrophic event that we consider, and the meaning of the dates $0, 1, 2$ is before the event, the realization of the event, and after the event.

At time 0 bank i holds $\gamma_i \geq 0$ units of cash, with zero return. At time 1 banks receive a random endowment $Q_i \geq 0$ of a nonpledgeable asset. The asset can be liquidated at its fundamental value Q_i only at $t = 2$ (e.g., the long-term investment matures at $t = 2$). If the asset is liquidated at $t = 1$ only a fraction of its fundamental value can be recovered, since early liquidation is costly. Instead of fixed early liquidation value we can use an exogenous inverse demand function as in (Cifuentes, Ferrucci, and Shin 2005) and (Amini, Filipović, and Minca 2016b). The fixed early liquidation value for illiquid assets simplifies the exposition a lot and is without loss of generality with respect to the case of exogenous inverse demand function, as in an earlier version of the present paper. We denote by P_i the liquidation value of the asset at $t = 1$ and we assume that $P_i < Q_i$ if $Q_i > 0$ (naturally, $P_i = 0$ if $Q_i = 0$). We assume that the asset can be partially liquidated.

This assumption that the asset is nonpledgeable (i.e., banks cannot borrow against it) is reasonable in our setting because the scenarios we consider represent extreme events. There is no funding liquidity in this situation, unless a lender of last resort provides it, which we do not consider here. Both assumptions are common in the literature on systemic risk; see, e.g., Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015) and Rogers and Veraart (2013).

In addition to interbank liabilities, banks have liabilities toward the end users. If there are any liabilities from end users to the banks, then these liabilities are included in the random endowment. We note that there is a time mismatch between assets and the liabilities to the end users. This is realistic: "hedging with differing maturities is commonplace"; see Chen et al. (2011).

Nominal interbank liabilities. The nominal interbank liabilities, realized at time 1, are represented by a nonnegative random matrix (L_{ij}), where $L_{ij} \geq 0$ (with $L_{ii} = 0$) denotes the cash amount that bank i owes bank j at $t = 1$. The liability of bank i to the end users is denoted by $D_i \geq 0$. The total nominal interbank liabilities of bank i sum up to

$$L_i = \sum_{j=1}^{m} L_{ij}.$$

Bank i in turn claims a total nominal cash amount of $\sum_{j=1}^{m} L_{ji}$ from the other banks.

The nominal balance sheet of bank i at $t = 1$ is given by

- Assets: $\gamma_i + \sum_{j=1}^{m} L_{ji} + Q_i$;
- Liabilities: $L_i + D_i +$ nominal net worth.

The nominal cash balance is $\gamma_i + \sum_{j=1}^{m} L_{ji} - L_i - D_i$.

The liability matrix represents contingent claims, i.e., financial derivatives such as swaps, which are the relevant claims in OTC markets. *The network of contingent claims is random: both the direction and the size of liabilities among banks are random and depend on the scenario* $\omega \in \Omega$.

Bank constraints. We let for each bank

$$\Lambda_i = \sum_{j=1}^{m} L_{ji} - L_i, \tag{1}$$

the net receivables from the interbank network. We assume that the interbank dealer network provides hedging services to the end users. We also assume that end user trades are not centrally cleared. We assume that dealer banks offset exposures to end users ("customers") by entering opposite contracts with other banks. This translates into the condition

$$D_i > 0 \implies \Lambda_i > 0. \tag{2}$$

The example of CDS clarifies this assumption because of the similarity to the insurance business: D_i is the amount of primary "insurance" that dealer i sells to end users and Λ_i^+ is the net amount of "reinsurance" that dealer i buys from the rest of the network. The assumption states that a bank exposed to end users will offset this risk at least partially by reinsuring. If all banks are liquid and reinsured, then the end users have no risk. Of course, if either banks are not "reinsured" or the "reinsurance" does not work, then there are knock-on effects on end users and there is an economic loss for the latter. We will examine the impact of central clearing on this economic loss of end users.

In the other sense, dealer banks that receive random endowments act as net "reinsurers" of the network (note that "reinsurers" do take risks even as they have a random endowment because of the maturity mismatch)

$$Q_i > 0 \implies \Lambda_i < 0. \tag{3}$$

Network rationale. While a model of network formation is beyond our scope, we provide a rationale for the network we consider, under the example of CDS. Assume that that banks learn at time 1 whether under the realized scenario they receive an endowment or they are liable to the end user. At time 1 the bank learns its role, “primary insurer” or “reinsurer,” but this role is of course not known at time 0. The network rationale is to make sure that endowments are matched to the end users. Since all these are random, so is the network. In aggregate, we would require that $\sum_i P_i = \sum_i D_i$ statewise or, at least in expectation $\sum_i \mathbb{E}[P_i] = \sum_i \mathbb{E}[D_i]$. These conditions recall dealer models in equity markets (where the flow of buyers equates in expectation the flow of sellers). A dealer in equity market models can manage inventory risk by offsetting her positions intertemporally as buyers and sellers arrive intermittently. In the case of an OTC market one dealer cannot manage inventory risk over time because the endowments and liabilities to end users arrive randomly at different nodes, and the flow of these arrivals is much smaller that the flow of buyers and sellers in equity markets. OTC dealers manage inventory risk spatially by entering offsetting contracts with other dealers. As a whole, the network of dealer banks fulfills its role of matching buyers and sellers. There is liquidation risk when this network of contracts is realized.

Finally, we discuss the implication of our assumption that the probability distribution of liabilities does not change when introducing the CN. Again, take the CDS example. If the reference entity is outside the network, then the value of the CDS is not affected by the CN. In turn, if the reference entity is in the network, then its default risk is affected by the CN and therefore the CDS liabilities change too.

Interbank liability clearing equilibrium. We assume that the payables to the end users are senior with respect to the interbank payables. We let the cash, net of outside payables, be

$$\Gamma_i := \gamma_i - D_i.$$

If bank i's cash balance is negative, $\Gamma_i + \sum_{j=1}^m L_{ji} < L_i$, then bank i has a liquidity shortfall and is forced to sell its illiquid asset (in part or in full) at price $P_i < Q_i$. If the revenue from the illiquid does not cover the negative cash balance, $\Gamma_i + \sum_{j=1}^m L_{ji} + P_i < L_i$, then bank i defaults. Bank j will in turn receive a fraction of the cash value of bank i's total assets.

Definition 1. *An interbank liability clearing equilibrium consists of a random matrix of clearing payments (L^*_{ij}) that satisfies $0 \leq L^*_{ij} \leq L_{ij}$ and the clearing condition*

$$L^*_i = L_i \wedge \Big(\Gamma_i + \sum_{j=1}^m L^*_{ji} + P_i\Big)^+, \quad i = 1, \dots, m,$$

where we denote by $L^*_i = \sum_{j=1}^m L^*_{ij}$ the total clearing payments of bank i.

Note that the clearing condition makes use of the fact that the bank liquidates as much as needed (and up to its entire endownment) in order to pay its liabilities. Once the clearing equilibrium is determined, and using that no bank liquidates voluntarily more than what it needs to cover its cash shortfall, we can determine the amount of asset liquidations in equilibrium, as shown below in (5).

An interbank liability clearing mechanism is defined in the following example, for which an equilibrium always exists.

Example 1. In the setup of Eisenberg and Noe (2001) one assumes a proportional sharing rule, where in case of default of bank i any counterparty bank j receives the proportion

$$\Pi_{ij} = \begin{cases} \frac{L_{ij}}{L_i} & \text{if } L_i > 0, \\ 0 & \text{otherwise,} \end{cases}$$

of the cash value of bank i's total assets. The clearing vector of payments $\mathbf{L}^* = (L_1^*, \dots, L_m^*)$ can be determined statewise as a fixed point, $\Phi(\mathbf{L}^*) = (\mathbf{L}^*)$, of the map Φ on $[0, (L_1, \dots, L_m)]$ given by

$$\Phi(\ell)_i = L_i \wedge \left(\Gamma_i + \sum_{j=1}^m \ell_j \Pi_{ji} + P_i\right)^+, \quad i = 1, \dots, m. \tag{4}$$

It can be shown as in Eisenberg and Noe (2001) that the mapping Φ has a largest fixed point (the set of fixed points forms a lattice). The matrix of clearing payments is then given by $L_{ij}^* = L_i^* \Pi_{ij}$. *An extension of the* Eisenberg and Noe (2001) *setup is given in* Amini, Filipović, and Minca (2016b), *who consider multiple seniority classes and an inverse demand function for the illiquid asset.*

We henceforth assume that (L_{ij}^*) is a matrix of clearing interbank liability payments. The following results hold irrespectively of whether this equilibrium is based on the proportional sharing rule of Eisenberg and Noe (2001) or not. The liquidated fraction of the asset of bank i in the clearing equilibrium is given by

$$Z_i = \frac{\left(\Gamma_i + \sum_{j=1}^m L_{ji}^* - L_i\right)^-}{P_i} \wedge 1 \text{ for } i \text{ with } P_i > 0. \tag{5}$$

Note that if bank i is in default, then its asset is liquidated in full, $Z_i = 1$. The actual payment to end users is given by

$$D_i^* = D_i \wedge \left(\gamma_i + \sum_{j=1}^m L_{ji}^* + P_i\right)^+.$$

Note that this is compatible with Definition 1 since a fixed point $L_i^*, i = 1, \dots, m$, would not change if Γ_i were defined as $\gamma_i - D_i^*$ for all $i = 1, \dots, m$. In particular, note that when $D_i^* < D_i$ we have $L_i^* = 0$, consistent with the seniority assumption for the liability to end users.

Terminal net worth. The value of bank i's assets at $t = 2$ becomes

$$A_i = \Gamma_i + Z_i P_i + (1 - Z_i) Q_i + \sum_{j=1}^m L_{ji}^*.$$

The net worth of bank i at $t = 2$ is defined by

$$C_i = A_i - L_i. \tag{6}$$

Subtracting the nominal—rather than the clearing—value of its liabilities from the value of its assets accounts for the shortfall in case of default of bank i. Indeed, bank i is in default if and only if $C_i < 0$. In this case, $Z_i = 1, L_i^* = (\Gamma_i + \sum_{j=1}^m L_{ji}^* + P_i)^+$ and we have

$$C_i^- = L_i - A_i = \begin{cases} L_i - L_i^* & \text{if } D_i^* = D_i, \\ L_i + D_i - D_i^* & \text{if } D_i^* < D_i \quad (L_i^* = 0). \end{cases}$$

The shortfall imposed on the financial network and on end users by bank i is thus given by

$$C_i^- = \underbrace{L_i - L_i^*}_{\text{shortfall on other banks}} + \underbrace{D_i - D_i^*}_{\text{shortfall on end users}} \tag{7}$$

and we denote that the shortfall imposed by $D_i^- = D_i - D_i^* = C_i^- - (L_i - L_i^*)$. From the perspective of the bank's utility, the relevant quantity we will consider is C_i^+. The aggregate utility of the end users is represented by their aggregate receivables $\sum_{i=1}^m D_i^*$.

Table 1 summarizes the notations for the financial network without CN.

Aggregate surplus identity. We now establish a fundamental relation between aggregate surplus and aggregate liquidation losses.

Lemma 1. *The aggregate surplus satisfies*

$$\sum_{i=1}^m C_i^+ + \sum_{i=1}^m D_i^* = \sum_{i=1}^m \gamma_i + \sum_{i=1}^m Q_i - \sum_{i=1}^m Z_i(Q_i - P_i). \tag{8}$$

Hence, the aggregate surplus depends on the interbank liabilities only through the implied liquidation losses. Forced liquidation of the assets lowers the aggregate surplus. Absent any illiquid asset, cash gets only redistributed and there are no dead weight losses.

Table 1

Overview of model notation for the financial network without CN.

γ_i	units of liquid asset (cash) hold by bank i at $t = 0$
Q_i	fundamental value of asset hold by bank i at $t = 2$
P_i	liquidation value of asset hold by bank i at $t = 1$
D_i	liability of bank i to the end users at $t = 1$
L_{ij}	liability (cash amount) that bank i owes bank j at $t = 1$
$L_i = \sum_{j=1}^m L_{ij}$	total nominal interbank liabilities of bank i to all other banks
$\Lambda_i = \sum_{j=1}^m L_{ji} - L_i$	net receivables of bank i from the interbank network
$\Gamma_i = \gamma_i - D_i$	cash hold by bank i, net of outside payables
$\Pi_{ij} = L_{ij}/L_i$	relative (proportional) nominal liability of bank i to bank j
L_{ij}^*	clearing liability payment of bank i to bank j in equilibrium
$L_i^* = \sum_{j=1}^m L_{ij}^*$	total clearing payments of bank i in equilibrium
D_i^*	actual payment of bank i to end users in equilibrium
Z_i	liquidated fraction of the asset of bank i in equilibrium
A_i	value of bank i's assets in equilibrium at $t = 2$
$C_i = A_i - L_i$	net worth of bank i in equilibrium at $t = 2$
$C_i^- = \max\{-C_i, 0\}$	shortfall imposed by bank i in equilibrium

3. Central counterparty clearing. We extend the preceding setting by adding a CN to the financial network. We formally label it as entity $i = 0$. We assume that all interbank liabilities among banks (but not toward end users) are cleared through the CN, so that the interbank network becomes a star-shaped network. The CN is capitalized in cash with equity γ_0 and a guaranty fund, $\sum_{i=1}^m g_i$, which is funded by up-front cash-contribution $g_i \le \gamma_i$ from every bank i.

Nominal interbank liabilities. The net exposure of bank i to the CN is given by its net receivables, $\Lambda_i = \sum_{j=1}^m L_{ji} - L_i$ in (1). In our model, we allow for netting the nominal liability of bank i to the CN against the up-front guaranty payment g_i:

$$\widehat{L}_{i0} = \Big(\Lambda_i + g_i\Big)^-. \tag{9}$$

Note that $\widehat{L}_{i0}$ is positive if and only if Λ_i^- exceeds g_i. Netting bank i's liabilities against its up-front payment reduces the forced liquidation need of its asset. This in turn has a positive effect on the aggregate surplus, as seen in Lemma 1. In a Nash bargaining game of the banks and the CN it will thus be possible to reach an unanimous agreement to form a CN in which all parties are better off.

The CN charges a proportional fee $f \in [0, 1]$ to its interbank liabilities in exchange for putting its own equity γ_0 at risk. For tractability, this charge is *ex post*. The nominal liability of the CN to bank i is given by

$$\widehat{L}_{0i} = (1 - f)\Lambda_i^+, \tag{10}$$

and the total nominal liability of the CN equals

$$\widehat{L}_0 = \sum_{i=1}^m \widehat{L}_{0i} = (1 - f)\sum_{i=1}^m \Lambda_i^+. \tag{11}$$

Nominal guaranty fund. We define the nominal share of bank i in the guaranty fund as

$$G_i = \Big(\Lambda_i + g_i\Big)^+ - \Lambda_i^+ = \begin{cases} g_i & \text{if } \Lambda_i > 0, \\ g_i + \Lambda_i & \text{if } -g_i < \Lambda_i \le 0, \\ 0 & \text{otherwise,} \end{cases}$$

which is contingent on the realization of Λ_i. As a result we have

$$G_i - \widehat{L}_{i0} = g_i - \Lambda_i^-, \tag{12}$$

and $G_i \times \widehat{L}_{i0} = 0$. Figure 1 shows G_i and $\widehat{L}_{i0}$ as functions of Λ_i.

We denote by $G_{\mathrm{tot}} = \sum_{i=1}^m G_i$ the total nominal value of the guaranty fund. The nominal balance sheet of the CN at $t = 1$ becomes

- Assets: $\gamma_0 + \sum_{i=1}^m g_i + \sum_{i=1}^m \widehat{L}_{i0}$;
- Liabilities: $\widehat{L}_0 + G_{\mathrm{tot}} +$ nominal net worth $(\gamma_0 + \sum_{i=1}^m f\Lambda_i^+)$.

The guaranty fund is loss-absorbing equity of the CN. It can absorb losses before or after the CN's external capital. We refer to these cases as "senior CN" and respectively "junior CN," also known as "skin in the game." Banks' remaining shares in the guaranty fund are repaid at $t = 2$.

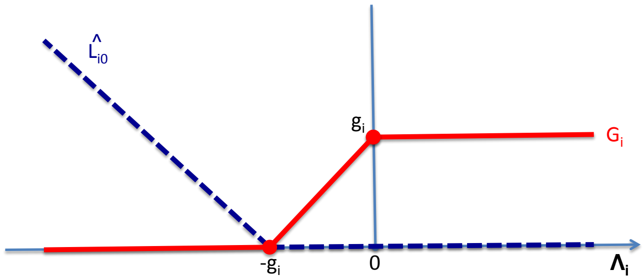

Figure 1. *Nominal share in the guaranty fund, G_i, and nominal liability to the CN, $\widehat{L}_{i0}$, of bank i as functions of its net exposure to the CN, Λ_i, in terms of its guaranty fund contribution, g_i.*

The guaranty fund share is mutualized but the member retains the right to net his own liability with this share under an extreme event. Clearly, in the context of our two-period model, the meaning of netting with a guaranty fund must be made precise.

Our model does not represent the daily margining with a CN but one "armageddon" realization of liabilities. We are not suggesting that members net their daily liabilities against the posted guaranty fund, but we allow this in the extreme scenario. In practice, if a member nets its guaranty fund contribution with its liability, then a new guaranty fund must be set up after the extreme event. Old members who cannot replenish the guaranty fund can be excluded. The CN recovery (including guaranty fund replenishment) following the extreme event is beyond our model.

Interbank liability clearing equilibrium with CN. Bank i has either a positive liability to the CN, $\widehat{L}_{i0} > 0$, or a positive receivable from the CN, $\widehat{L}_{0i} > 0$, but never both, $\widehat{L}_{i0} \times \widehat{L}_{0i} = 0$. This fact allows us to solve explicitly for the clearing payments at $t = 1$. As in the case without the CN, the payables to the end users are senior, and we let $\Gamma_i = \gamma_i - D_i$. The nominal cash balance of bank i at $t = 1$ is $\Gamma_i - g_i + \widehat{L}_{0i} - \widehat{L}_{i0}$.

Let i with $\Lambda_i < 0$. Then $\widehat{L}_{i0} = 0$ and moreover $D_i = 0$ from (2). The cash balance is given in this case by $\gamma_i - g_i - \widehat{L}_{i0}$ and bank i is forced to liquidate the fraction

$$\widehat{Z}_i = \frac{\left(\gamma_i - g_i - \widehat{L}_{i0}\right)^-}{P_i} \wedge 1 \tag{13}$$

of the asset. The clearing payment of bank i to the CN is

$$\widehat{L}_i^* = \widehat{L}_{i0} \wedge \left(\gamma_i - g_i + P_i\right).$$

The value of the CN's total assets becomes

$$\widehat{A}_0 = \gamma_0 + \sum_{i=1}^{m} g_i + \sum_{i=1}^{m} \widehat{L}_i^*. \tag{14}$$

The clearing interbank liability payment of the CN to bank i is defined according to the proportionality rule, $\widehat{L}_0^* \times \widehat{\Pi}_{0i}$, with total clearing interbank liability payment

$$\widehat{L}_0^* = \widehat{A}_0 \wedge \widehat{L}_0$$

and relative liability weights

$$\widehat{\Pi}_{0i} = \begin{cases} \frac{\Lambda_i^+}{\sum_{j=1}^m \Lambda_j^+} & \text{if } \widehat{L}_0 > 0, \\ 0 & \text{otherwise.} \end{cases}$$

Liquidation of the guaranty fund. We now determine the clearing payments from the guaranty fund to the banks at $t = 2$. With "senior CN," the guaranty fund is the first layer to absorb shortfall losses imposed by defaulted banks, prior to the nominal net worth of the CN. The guaranty fund's surplus in the clearing equilibrium is given by

$$G^*_{\text{tot}} = G_{\text{tot}} \wedge \left(\widehat{A}_0 - \widehat{L}_0 - \gamma_0 - \sum_{i=1}^m f \Lambda_i^+ \right)^+, \tag{15}$$

where we used that the nominal net worth of the CN amounts to $\gamma_0 + \sum_{i=1}^m f \Lambda_i^+$.

Remark 1. With "junior CN," the guaranty fund's surplus in the clearing equilibrium is given by

$$G^*_{\text{tot}} = G_{\text{tot}} \wedge \left(\widehat{A}_0 - \widehat{L}_0 \right)^+. \tag{16}$$

The clearing payment from the guaranty fund to bank i is defined by the proportionality rule,

$$G_i^* = \frac{G_i}{G_{\text{tot}}} \times G^*_{\text{tot}} \quad (= 0 \text{ if } G_{\text{tot}} = 0).$$

This means that banks' shares absorb losses in the guaranty fund proportionally.

If $\Lambda_i > 0$, then the bank has net payables to the end users, hedged with net receivables from the financial network. The cash balance of bank i is given by $\Gamma_i - g_i + \widehat{L}_0^* \times \widehat{\Pi}_{0i} + G_i^*$, and the actual payable to the end users is

$$D_i^* = D_i \wedge \left(\gamma_i - g_i + \widehat{L}_0^* \times \widehat{\Pi}_{0i} + G_i^* \right).$$

Terminal net worth. The net worth of the CN at $t = 2$ is defined by

$$\widehat{C}_0 = \widehat{A}_0 - \widehat{L}_0 - G^*_{\text{tot}}. \tag{17}$$

The shortfall of the CN becomes

$$\widehat{C}_0^- = (\widehat{A}_0 - \widehat{L}_0)^- = \widehat{L}_0 - \widehat{L}_0^*. \tag{18}$$

The value of bank i's assets, including its share in the guaranty fund, at $t = 2$ becomes

$$\widehat{A}_i = \Gamma_i + \widehat{Z}_i P_i + (1 - \widehat{Z}_i) Q_i + \widehat{L}_0^* \times \widehat{\Pi}_{0i} + G_i^* - g_i. \tag{19}$$

As before, the net worth of bank i is obtained by subtracting the nominal value of its liabilities from the value of its assets,

$$\widehat{C}_i = \widehat{A}_i - \widehat{L}_{i0}. \tag{20}$$

If $\Lambda_i < 0$ the shortfall of bank i equals

$$\widehat{C}_i^- = \widehat{L}_{i0} - \widehat{L}_i^*. \tag{21}$$

In this case, bank i is in default, $\widehat{C}_i < 0$, if and only if $\widehat{L}_{i0} > \widehat{L}_i^*$, where in this case $\widehat{L}_i^* = \gamma_i - g_i + P_i$. This again implies that $\widehat{A}_i = \gamma_i - g_i + P_i = \widehat{L}_i^*$, which proves (21).

If $\Lambda_i > 0$, then the shortfall of bank i equals

$$\widehat{C}_i^- = D_i - \widehat{D}_i^*. \tag{22}$$

Indeed, in this case $\widehat{C}_i < 0$ if and only if $\widehat{A}_i < 0$. This in turn is equivalent to $\gamma_i - g_i + \widehat{L}_{0i}^* + G_i^* < D_i$, which implies $D_i^* = \gamma_i - g_i + \widehat{L}_{0i}^* + G_i^*$. Then

$$\widehat{C}_i^- = \widehat{A}_i^- = D_i - (\gamma_i - g_i + \widehat{L}_{0i}^* + G_i^*) = D_i - D_i^*.$$

By combining (21) and (22) we obtain

$$\widehat{C}_i^- = (\widehat{L}_{i0} - \widehat{L}_i^*) + (D_i - \widehat{D}_i^*). \tag{23}$$

Table 2 summarizes the notations for the financial network with CN.

Table 2

Overview of model notation for the financial network with CN.

γ_0	units of liquid asset (cash) hold by CN at $t = 0$
g_i	guaranty fund contribution by bank i at $t = 0$
$f \in [0, 1]$	volume based fee charged by the CN on banks' receivables
$\Lambda_i = \sum_{j=1}^m L_{ji} - L_i$	net exposure of bank i to the CN
$\widehat{L}_{i0} = \Big(\Lambda_i + g_i\Big)^-$	nominal liability of bank i to the CN
$\widehat{L}_{0i} = (1 - f)\Lambda_i^+$	nominal liability of the CN to bank i
$\widehat{L}_0 = \sum_{i=1}^m \widehat{L}_{0i}$	total nominal liability of the CN
$G_i = \Big(\Lambda_i + g_i\Big)^+ - \Lambda_i^+$	nominal share of bank i in the guaranty fund
$G_{\text{tot}} = \sum_{i=1}^m G_i$	total nominal value of the guaranty fund
$\widehat{L}_i^*$	clearing liability payment of bank i to the CN in equilibrium
$\hat{Z}_i$	liquidated fraction of the asset of bank i in equilibrium
$\widehat{A}_0$	value of the CN's total assets in equilibrium
$\widehat{L}_0^* = \widehat{A}_0 \wedge \widehat{L}_0$	total clearing interbank liability payment of the CN in equilibrium
$\widehat{\Pi}_{0i} = \widehat{L}_{0i}/\widehat{L}_0$	relative (proportional) liability of the CN to bank i
$G_{\text{tot}}^* = G_{\text{tot}} \wedge \Big(\widehat{A}_0 - \widehat{L}_0\Big)^+$	guaranty fund's surplus in the clearing equilibrium
$G_i^* = G_i/G_{\text{tot}} \times G_{\text{tot}}^*$	clearing payment from the guaranty fund to bank i
$\widehat{D}_i^*$	actual payable to the end users by bank i in equilibrium
$\hat{A}_i$	value of bank i's assets in equilibrium
$\hat{C}_i = \hat{A}_i - \hat{L}_{i0}$	net worth of bank i in equilibrium
$\widehat{C}_0 = \widehat{A}_0 - \widehat{L}_0 - G_{\text{tot}}^*$	net worth of the CN in equilibrium

Aggregate surplus identity with CN. We now establish the aggregate surplus identity with CN, which is the exact analogue of the aggregate surplus identity for the uncleared network in Lemma 1.

Lemma 2. *The aggregate surplus with CN satisfies*

$$\sum_{i=0}^{m} \widehat{C}_i^+ + \sum_{i=1}^{m} \widehat{D}_i^* = \sum_{i=0}^{m} \gamma_i + \sum_{i=1}^{m} Q_i - \sum_{i=1}^{m} \widehat{Z}_i (Q_i - P_i). \tag{24}$$

Hence, the aggregate surplus depends on the clearing mechanism only through the implied liquidation losses. The implications are the same as for the uncleared network given following Lemma 1.

4. Ex post effects of centralized clearing. We analyze the main statewise effects of centralized clearing on the net worth of all network participants. A critical determinant of the overall net worth is the amount of asset liquidations. These are determined by the network structure, which becomes a star-shaped network with a CN. From the banks' perspective, their net worth depends on the guaranty fund contributions (their own contributions and the contributions of the other members) and on the CN external capital. Of course, the fee charged by the CN also affects the bank's net worth, but the effect is secondary. We denote by $\boldsymbol{g} = (g_1, \ldots, g_m)$ the vector of guaranty fund contributions. We refer to (γ_0, g, f) as "CN policy."

We first state the following invariance result for banks with net payables to the interbank network (and consequently who have a net payable to the CN).

Lemma 3. *For any bank i with net payables to the interbank network, i.e., $\Lambda_i < 0$, the shortfall on interbank liabilities $\widehat{C}_i^- = (\widehat{L}_{i0} - \widehat{L}_i^*)$ and the liquidated fraction of the asset $\widehat{Z}_i$ do not depend on the guaranty fund $\boldsymbol{g}$, on the CN external capital γ_0, or on the fee f.*

The intuition behind this result is that ex post fee collection from receivables does not affect the default risk of banks with net liabilities toward the financial network. The amount of liquidations of these banks is independent of the amount of guaranty fund contributions (because their liabilities are net of the guaranty fund contribution under the extreme event). In contrast, banks that are net receivers from the CN are also liable to the end users. Their shortfall on those liabilities does depend on the policy and we will give conditions that the shortfall is smaller than in the case without CN.

We next provide some basic identities which lead to simple formulas for the net worth across the network as a function of the banks' aggregated shortfall, $\sum_{i,\ \Lambda_i<0} \widehat{C}_i^-$. Note that a bank cannot at the same time have a positive nominal share in the guaranty fund and impose a positive shortfall loss on the network, i.e., $\widehat{L}_i^* < \widehat{L}_{i0}$ implies $G_i = 0$: the first defense in the CN against losses is the guaranty fund of the member itself. In the lemma, we are providing the identities for the case "senior CN" and respectively "junior CN."

Lemma 4. *We have that*

$$\widehat{A}_0 - \widehat{L}_0 = \gamma_0 + f \sum_{i=1}^{m} \Lambda_i^+ + G_{\text{tot}} - \sum_{i,\ \Lambda_i<0} \widehat{C}_i^-.$$

The surplus of the guaranty fund in the case of "senior" CN (resp., "junior" CN) satisfies

$$G^*_{\text{tot}} = \Big(G_{\text{tot}} - \sum_{i,\ \Lambda_i<0} \widehat{C}_i^-\Big)^+ \qquad \Big(\textit{resp.},\ G_{\text{tot}} \wedge \big(\widehat{A}_0 - \widehat{L}_0\big)^+\Big). \tag{25}$$

The net worth of the CN equals

$$\widehat{C}_0 = \gamma_0 + \sum_{i=1}^m f\Lambda_i^+ - \Big(G_{\text{tot}} - \sum_{i,\ \Lambda_i<0} \widehat{C}_i^-\Big)^- \Big(\ \textit{resp.},\ \Big(\gamma_0 + f\sum_{i=1}^m \Lambda_i^+ - \sum_{i,\ \Lambda_i<0} \widehat{C}_i^-\Big)^+ - \big(\widehat{A}_0 - \widehat{L}_0\big)^-\Big). \tag{26}$$

It is nondecreasing in $(\boldsymbol{g}, f)$*, for* $i = 1, \dots, m$*,*

$$\frac{\partial \widehat{C}_0}{\partial f} \geq 0 \quad \textit{and} \quad \frac{\partial \widehat{C}_0}{\partial g_i} \geq 0. \tag{27}$$

The net worth of bank i *equals*

$$\widehat{C}_i = \Gamma_i + Q_i + \Lambda_i - \widehat{\Pi}_{0i}\widehat{C}_0^- - \widehat{Z}_i(Q_i - P_i) - f\Lambda_i^+ - \frac{G_i}{G_{\text{tot}}}\Big(G_{\text{tot}} - G^*_{\text{tot}}\Big), \tag{28}$$

the nominal net worth minus a share in the shortfall of the CN, minus the liquidation loss, minus the fees, minus a loss in the share in the guaranty fund.

Lemma 4 is essentially a check of the waterfall procedure in the CN, which takes place when there are members who deplete their own guaranty fund contributions and impose shortfall losses on the network. In the case of "senior CN," the shortfall of one bank (after depletion of its own guaranty fund contribution) is absorbed by the nominal shares of the other banks and then by the external capital of the CN. In the case of "junior CN" the order is reversed. Identity (25) states that the shares of the banks have limited liability.[3] For both senior and junior CN, we check that if $\widehat{C}_0 < 0$, then the guaranty fund is wiped out, $G^* < 0$. Moreover, we check that $\widehat{C}_0^-$ is the same for senior/junior CN:

$$\widehat{C}_0^- = (\widehat{A}_0 - \widehat{L}_0)^- = \Big(\gamma_0 + \sum_{i=1}^m f\Lambda_i^+ + G_{\text{tot}} - \sum_{i,\ \Lambda_i<0} \widehat{C}_i^-\Big)^-. \tag{29}$$

In turn, CN seniority affects the surplus and the banks' shares in the guaranty fund. Its sensitivities with respect to the fee and guaranty fund policy shown in (27) confirm economic intuition. More sensitivity results for banks' individual and aggregate net worth, surplus, and shortfall are given in Appendix A.

We now state our result on the statewise effects of centralized clearing on the net worth of the banks. To make the comparison of the financial network with and without CN, we treat the CN in the latter case as dummy bank $i = 0$ without any liabilities and with constant net worth equal to its equity, $C_0 = \gamma_0$.

[3] More recently, there are CNs where membership does not come with limited liability in the guaranty fund. The CN can make capital calls so as to recapitalize the guaranty fund if the initial shares are depleted. We do not consider this case here.

Theorem 1.

(i) *For a bank i with $\Lambda_i \leq 0$, introducing the CN reduces liquidation losses, $\widehat{Z}_i \leq Z_i$, and shortfall losses, $\widehat{C}_i^- \leq C_i^-$.*

(ii) *The impact of the CN on the net worth of bank i can be decomposed in three components,*

$$\widehat{C}_i - C_i = T_1(i) + T_2(i) + T_3(i), \tag{30}$$

corresponding to

- *counterparty default,*

$$T_1(i) = -\widehat{\Pi}_{0i}\widehat{C}_0^- + \sum_{j=1}^{m}(L_{ji} - L_{ji}^*);$$

- *change in liquidation loss,*

$$T_2(i) = (Z_i - \widehat{Z}_i)(Q_i - P_i) \geq 0;$$

- *fees and loss in the share in the guaranty fund,*

$$T_3(i) = -f\Lambda_i^+ - \frac{G_i}{G_{\text{tot}}}\left(G_{\text{tot}} - G_{\text{tot}}^*\right) \leq 0.$$

(iii) *For the bank i with $\Lambda_i > 0$, if $T_1(i) + T_3(i) \geq 0$, then the CN reduces the shortfall on end users: $\widehat{C}_i^- \leq C_i^-$. If $T_1(i) + T_3(i) \leq 0$, then the CN increases the shortfall on end users $\widehat{C}_i^- \geq C_i^-$.*

From Theorem 1 we see that the netting effects due to the CN reduce the liquidation losses for banks with net liabilities to the financial network, and also the shortfall that these banks impose on the rest of the financial network. In other words, the “reinsurers” have less shortfall. The natural consequence of this is that if the “primary insurers” are fully reinsured, then the CN will always have a positive effect on the aggregate shortfalls. We will demonstrate this formally below. However, as the empirical analysis (Chen et al. (2011)) of the CDS market shows, the offset or “reinsurance” is not perfect. Moreover, other banks may take larger positions than their own exposure to the end users (“overreinsurance”). In this case, the shortfalls of the underreinsured may increase with a CN and this effect increases with the guaranty fund share at risk.

The CN impact on the individual net worth of bank i results from weighing three components that do not all go in the same direction, as expressed by the decomposition (30). The CN always reduces liquidation losses, $T_2 \geq 0$ for banks with net liabilities toward the CN. On the other hand, as captured by $T_3 \leq 0$, the CN always puts the guaranty fund contribution at risk and charges a volume-based fee that comes along with membership of the CN. The latter effect becomes more negative with increasing guaranty fund contribution and especially in the setting “senior CN.” The CN may have a positive impact on the reduction of counterparty risk, as captured by the term T_1, but this is not guaranteed and depends critically on the capitalization of the CN. For banks with net receivables from the CN, it is not clear in general that CN decreases the shortfall to the end users. The last point of the theorem gives the necessary and sufficient condition for a decrease of shortfall on end users of these banks. The sum of the three components, $T_1 + T_2 + T_3$, is not always positive and a positive impact of the CN on bank i's surplus cannot be guaranteed statewise. In the next section, we aggregate the statewise effects using a systemic risk measure.

We now consider the loss imposed on the end users. This loss is imposed by banks which have $D_i > 0$ (and thus $\Lambda_i > 0$). We consider two extremes: one in which the realized network is fully symmetric and fully "reinsured": D_i constant over i and $\Lambda_i = D_i$. In this case, the CN always decreases the aggregate shortfall to end users and consequently increases the aggregate surplus of end users.

Proposition 1. *Suppose that all primary insurers are fully reinsured, $D_i = \Lambda_i$ for all i with $\Lambda_i > 0$, and that $L_{ij} = 0$ for all i,j with $\Lambda_i\Lambda_j > 0$ (there are no linkages between two primary insurers or two reinsurers). Moreover we assume that the primary insurers are symmetric: D_i constant over i with $D_i > 0$ and that $\gamma - g - f\sup_i \Lambda_i > 0$, with $\sup_i \Lambda_i$ the supremum of the random variable Λ_i. Then, the aggregate shortfall on external users decreases with the CN:*

$$\sum_{i,\ \Lambda_i>0} \widehat{C}_i^- \le \sum_{i,\ \Lambda_i>0} C_i^- .$$

The previous proposition assumes a network with full reinsurance. In reality, the primary primary insurers may be either over- or underinsured. In this case the CN can increase the shortfall of end users, as the following example shows.

Assume a network in which all "primary insurers," i.e., i with $D_i > 0$, either do not "reinsure" ($L_{ji} = 0$ for all j) or are overreinsured and have the same "reinsurer," say, node m: $L_{mi} > D_i$. We assume that under the given realization $\Gamma_i = 0$ for all i with $D_i > 0$: the cash holdings suffice to pay the liabilities to the end users in the case without the CN. In the case with CN, this may be no longer the case if the guaranty fund contributions of the banks that are not reinsured are effectively transferred to the banks which are overreinsured.

Let i with $D_i > 0$ and $\Lambda_i = 0$ (bank i is not reinsured). Then

$$\widehat{C}_i^- = \left(\Gamma_i - \frac{G_i}{G_{\text{tot}}}\left(G_{\text{tot}} - G^*_{\text{tot}}\right)\right)^- = \frac{G_i}{G_{\text{tot}}}\left(G_{\text{tot}} - G^*_{\text{tot}}\right),$$

while $C_i^- = \Gamma_i^- = 0$.

Let i with $D_i > 0$ and $\Lambda_i > D_i$ (bank i is overreinsured). Then

$$\widehat{C}_i^- = \left(\Gamma_i + \Lambda_i - \widehat{\Pi}_{0i}\widehat{C}_0^- - f\Lambda_i^+ - \frac{G_i}{G_{\text{tot}}}\left(G_{\text{tot}} - G^*_{\text{tot}}\right)\right)^-$$

and $C_i^- = (\Gamma_i + L^*_m \Pi_{mi})^- = 0$. We clearly have

$$\sum_{i,\Lambda_i=D_i} \widehat{C}_i^- + \sum_{i,\Lambda_i>D_i} \widehat{C}_i^- \ge \sum_{i,\Lambda_i=D_i} C_i^- + \sum_{i,\Lambda_i>D_i} C_i^- .$$

The inequality can be strict. To see that, let $\Lambda_i \to \infty$ for an overreinsured i. In this limit case the reinsurer m defaults, and the guaranty fund suffers losses (the effect being stronger with senior CN equity). For any i with $D_i > 0$ and $\Lambda_i = 0$, the shortfall equates the depleted guaranty fund share and therefore the shortfall of these banks on end users is strictly positive. On the other hand, overreinsured banks will have capital surplus (partly coming from the transfer of the guaranty fund shares of the banks that are not reinsured under the realized scenario). In sum, banks that are not reinsured may independently have sufficient cash to pay their end users. With the CN, part of their cash is transferred to the surplus of the overreinsured bank and they are led to impose losses on their end users.

5. Does centralized clearing reduce systemic risk?

We have seen that the CN reduces banks' liquidation and shortfall on other banks, and it improves the aggregate surplus. This comes at the cost of potentially increased shortfall on the end users in the case the realized network is asymmetric. This cost is higher if the CN capital is senior. We compare these shortfall losses imposed on the end users, with and without CN.

There are a variety of well-known systemic risk indicators, such as CoVAR (Adrian and Brunnermeier (2011)) or SES (Acharya et al. (2010)), which are based on measuring losses in terms of market equity. In contrast to these approaches, the systemic risk measure we employ here uses a structural model of loss propagation in the network of liabilities. Chen, Iyengar, and Moallemi (2013) extend Brunnermeier and Cheridito (2014) and the classic axiomatic risk measure theory to an axiomatic theory of systemic risk measures. More recent works on systemic risk measures include Biagini et al. (2019), Feinstein, Rudloff, and Weber (2015), Weber and Weske (2017), and Kusnetsov and Veraart (2019). These works have shown that a systemic risk measure can be expressed using a single firm risk measure and an aggregation function in a variety of settings and have explored the properties of such measures.

In contrast to these approaches, the systemic risk measure we employ here uses a structural model of loss propagation through the financial network. This is a new example of an aggregation function, of independent interest to the literature on systemic risk measures.

We aggregate the economic loss of the end users, $\sum_{i=1}^m (D_i - \widehat{D}_i^*)$, which can be partially offset (for example, via tax) by a fraction $\alpha \geq 0$ of the surplus of the banks,

$$\mathcal{A}_\alpha = \sum_{i=1}^m (D_i - D_i^*) - \alpha \sum_{i=0}^m C_i^+, \tag{31}$$

and respectively

$$\widehat{\mathcal{A}}_\alpha = \sum_{i=1}^m (D_i - \widehat{D}_i^*) - \alpha \sum_{i=0}^m \widehat{C}_i^+$$

for the case with the CN. The regulator measures systemic risk using

$$\mathcal{R} = \mathbb{E}\Big[\mathcal{A}_\alpha\Big], \tag{32}$$

which can be interpreted as an expected shortfall of the end users. Our results are robust to aggregation functions (equivalent to the "externality function" in Brunnermeier and Cheridito (2014) and related to the expectile in Bellini et al. (2014)) of the type $\mathcal{A}_\alpha = \alpha \sum_{i=0}^m C_i^- - (1-\alpha) \sum_{i=0}^m C_i^+$ but are less interesting from a societal viewpoint because they consider the losses of the banks and not of the end users.

We now compare the systemic risk in the financial network with and without CN. The following theorem shows that a CN does not always reduce systemic risk. More importantly, it gives a simple and tight condition for a reduction of systemic risk in terms of the expected shortfall on end users and a threshold which can be computed using only the uncleared interbank network.

Theorem 2. *We have that*

$$\widehat{\mathcal{A}}_\alpha - \mathcal{A}_\alpha = -\alpha \sum_{i=1}^{m} (Z_i - \widehat{Z}_i)(Q_i - P_i) + (1-\alpha) \sum_{i=1}^{m} (D_i^* - \widehat{D}_i^*). \tag{33}$$

Moreover, $\sum_{i=1}^m (Z_i - \widehat{Z}_i)(Q_i - P_i) \geq 0$ *and does not depend on* γ_0 *and the fee and guaranty fund policy* $(f, \boldsymbol{g})$*. The CN reduces the systemic risk in the financial network if and only if*

$$\mathbb{E}\Big[\sum_{i=1}^{m} (D_i^* - \widehat{D}_i^*)\Big] < \mathbb{E}\Big[\alpha \sum_{i=1}^{m} (Z_i - \widehat{Z}_i)(Q_i - P_i)/(1-\alpha)\Big]. \tag{34}$$

The shortfall risk on end users on the left-hand side of (34) is decreasing in its equity γ_0 and has a nonmonotonous dependence on (g, f), while the threshold on the right-hand side depends only on the parameters of the initial, uncleared interbank network. Condition (34) thus provides a regulatory criterion for a reduction of systemic risk in terms of the policy $(\gamma_0, f, \boldsymbol{g})$ of the CN.

On the right-hand side, $\mathbb{E}[\alpha \sum_{i=1}^m (Z_i - \widehat{Z}_i)(Q_i - P_i)/(1-\alpha)]$ quantifies the mitigation effects of the CN on liquidation costs. The shortfall imposed by bank i without the CN, Z_i, can be determined by multiple iterations of the "fictitious default" algorithm of Eisenberg and Noe (2001) that converges increasingly to the equilibrium liquidation losses. The liquidated quantity by bank i with the CN, $\widehat{Z}_i$, corresponds to the first iteration of the algorithm. As such, the term $Z_i - \widehat{Z}_i$ corresponds to second and higher rounds of liquidations in the network without CN.

In the following example, we apply Theorem 2 to a stylized network in which two banks have opposite net positions and there exists a chain of intermediaries between these banks fully hedged, i.e., with zero net positions. CDS markets (and generally OTC markets) constitute examples of markets with this topology; see, e.g., Stulz (2010) and Minca (2011). We analyze the impact of introducing a clearing facility in the network.

Example 2 (intermediation chain). Consider a set of m banks in which banks $2, \ldots, m-1$ are intermediaries for a claim between banks 1 and m. More specifically, we let $\Lambda > 0$ a bounded random variable, and the interbank claims are given as

$$L_{i,i+1} = \Lambda \text{ for all } i = 1, \ldots, m-1.$$

We let $D_m = \Lambda$ and $D_i = 0$ for all $i = 1, \ldots, m-1$ and we further assume that $f \sup \Lambda + g \leq \gamma$.

For the systemic risk measure we set $\alpha \in (0,1)$ such that

$$\mathcal{R} = \mathbb{E}\Big[\sum_{i=1}^{m} (D_i - D_i^*) - \alpha \sum_{i=1}^{m} C_i^+\Big]. \tag{35}$$

We also assume that the guaranty fund contributions are identical across banks, $g_i = g$, and that $\gamma_i = \gamma_1$ and $P_i = P_1$ for all $i = 1, \ldots, m$. Introducing a CN will replace the intermediation chain with one intermediary, which is the CN. We have that

$$\Lambda_i = \begin{cases} 0, & i = 2, \ldots, m-1, \\ \Lambda, & i = m, \\ -\Lambda, & i = 1. \end{cases} \tag{36}$$

Consequently, the shares in the guaranty fund are given by

$$G_i = \begin{cases} g, & i = 2, \ldots, m, \\ (\Lambda - g)^- - \Lambda^-, & i = 1. \end{cases} \tag{37}$$

We obtain

$$\widehat{C}_0^- = \Big(\gamma_0 + f\Lambda + g(m-1) - \Big(-\Lambda + P_1 + \gamma_1\Big)^-\Big)^-$$

and

$$\begin{aligned} D_m - \widehat{D}_m^* = \widehat{C}_m^- &= \Big(\gamma_m - f\Lambda - \widehat{C}_0^- - \frac{G_m}{G_{\text{tot}}}\Big(G_{\text{tot}} - G_{\text{tot}}^*\Big)\Big)^- = \mathbb{1}_{\widehat{C}_0<0}\Big(\gamma_m - f\Lambda - \widehat{C}_0^- - g\Big)^- \\ &= \mathbb{1}_{\widehat{C}_0<0}\Big(\gamma_m - f\Lambda + \gamma_0 + f\Lambda + g(m-1) - \Big(-\Lambda + P_1 + \gamma_1\Big)^- - g\Big)^- \\ &= \mathbb{1}_{\widehat{C}_0<0}\Big(\gamma_m + \gamma_0 + g(m-2) - \Lambda + P_1 + \gamma_1\Big)^-, \end{aligned}$$

where in the second line we used the assumption $f \sup \Lambda + g \leq \gamma$, in which case there is a shortfall on the end users only if the CN defaults. For the case without the CN we have

$$D_m - D_m^* = C_m^- = \Big(\gamma_m - \Lambda\Big)^- = \Big(\gamma_m + (m-1)(\gamma_1 + P_1) - \Lambda\Big)^-.$$

The total liquidated amount in the case with the CN is

$$\widehat{Z}_1 = \frac{(\gamma - \Lambda)^-}{P} \wedge 1,$$

whereas the total liquidated amount in the case without the CN is

$$\sum_{i=1}^{m-1} \frac{(i\gamma + (i-1)P - \Lambda)^-}{P} \wedge 1.$$

The difference in systemic risk is $\mathbb{E}[\widehat{\mathcal{A}}_\alpha - \mathcal{A}_\alpha]$, where

$$\begin{aligned} \widehat{\mathcal{A}}_\alpha - \mathcal{A}_\alpha = (1-\alpha)&\mathbb{1}_{\widehat{C}_0<0}(\gamma_m + \gamma_0 + g(m-2) - \Lambda + P_1 + \gamma_1)^- \\ &- (1-\alpha)(\gamma_m + (m-1)(\gamma_1 + P_1) - \Lambda)^- \\ &- \alpha\left(\sum_{i=1}^{m-1} \frac{(i\gamma + (i-1)P - \Lambda)^-}{P} \wedge 1 - \frac{(\gamma - \Lambda)^-}{P} \wedge 1\right)(Q_i - P_i). \end{aligned} \tag{38}$$

We note that the last term corresponds to the difference in liquidation losses and is always negative:

$$-\Big(\sum_{i=1}^{m-1}\frac{(i\gamma+(i-1)P-\Lambda)^-}{P}\wedge 1-\frac{(\gamma-\Lambda)^-}{P}\wedge 1\Big)(Q_i-P_i)<0.$$

In the uncleared network there are liquidations at each intermediary bank. The first two terms in (38) represent the difference in the shortfall to the end users with and without CN. Because each intermediary is liable, the total pool of assets that absorbs the potential loss induced by node 1 is $m\gamma+(m-1)P$ in the case without CN. In the case with the CN, the total pool of assets is $\gamma_1+P_1+\gamma_m+(m-2)g+\gamma_0=2\gamma+P+(m-2)g+\gamma_0$: the intermediaries only contribute up to the amount of g to the pool of loss absorbing assets.

The difference in the loss absorbing pool with CN and without CN is $\gamma_0-(m-2)(\gamma+P-g)$. As long as $\gamma_0>(m-2)(\gamma+P-g)$, the shortfall on end users is smaller with the CN and we have $\widehat{D}^*_m>D^*_m$. For these case, we have $\widehat{\mathcal{A}}_\alpha-\mathcal{A}_\alpha<0$ and the systemic risk decreases with the CN.

However, the loss absorbing pool can decrease with a CN. To see that, we analyze the $\widehat{\mathcal{A}}_\alpha-\mathcal{A}_\alpha$ under varying length of the intermediation chain. If $m=2$, then the last term in (38) is zero as there is no difference in liquidation losses. We have that the shortfall on end users is smaller with the CN because $\gamma_0>(m-2)(\gamma+P-g)=0$ and the difference in the first two terms is negative, so $\widehat{\mathcal{A}}_\alpha<\mathcal{A}_\alpha$ for $m=2$. For the limit case $m\to\infty$, we have that the first two terms in (38) tend to zero (since Λ is bounded), so $\widehat{\mathcal{A}}_\alpha<\mathcal{A}_\alpha$.

We conclude by virtue of Theorem 2 that *for large and small intermediation chains the CN reduces systemic risk.* However, for medium size intermediation chains, the pool of loss absorbing assets of the entire chain is larger than the pool of the CN (but still not enough for the shortfall on end users to be zero)—$\gamma_0<(m-2)(\gamma+P-g)$—but m still small enough such that $D^*_m<D_m$. In this case, the shortfall increases with the CN.

6. Nash bargaining solution.

We now consider the cooperative[4] game among the CN and the banks. We place ourselves under an axiomatic model of bargaining, namely the Nash bargaining solution. For a textbook introduction to the axiomatic model, following Nash, we refer the reader to Roth (1979). The banks and the CN are risk neutral. Their utility is given by their expected surplus. We assume that the cash resources in the system are fixed, and the players enter a bargaining game in which they decide on the membership, the external CN capital, and guaranty fund contributions $(\gamma_0,\boldsymbol{g})$. Under a pure bargaining problem, the group of players is faced with a set of possible outcomes. Any such outcome will be reached if there is unanimous agreement. In this section we fix the fee f and focus on the CN capital as this is of higher importance in our context of an extreme event.

The utility vector for an agreement outcome is $\widehat{\mathbf{U}}=(\mathbb{E}[\widehat{C}^+_i(\gamma_0,\boldsymbol{g})])_{i=0,\ldots,m}$. There is also a disagreement outcome, namely that the network remains uncleared. The utility vector for the disagreement outcome is $\mathbf{U}=(\mathbb{E}[C^+_i])_{i=0,\ldots,m}$. The game is played under constraints imposed by the regulator that systemic risk, as measured by (32), decreases by at least a given threshold level $\ell\geq 0$: $\mathcal{R}-\widehat{\mathcal{R}}\geq\ell$.

[4] Because the model already has binding commitments in the form of liabilities, it is natural to consider the cooperative setting, with binding agreements.

Any bargaining game is described by the set of feasible utility payoffs, i.e., those outcomes that can be achieved by unanimous agreement and in which all players receive higher utility than in the disagreement outcome. In our case, the set of feasible utility payoffs is given by

$$\mathcal{F} := \Big\{ \widehat{\mathbf{U}} \mid \gamma_0 \geq 0 \text{ for all } i \in 0,\dots,m: \; g_i \in [0,\gamma_i], \mathbb{E}[\widehat{C}_i^+(\gamma_0,\boldsymbol{g})] \geq \mathbb{E}[C_i^+], \mathcal{R} - \widehat{\mathcal{R}}(\gamma_0,\boldsymbol{g}) \geq \ell \Big\}.$$

For the game to be well defined, the set $\mathcal{F}$ must be nonempty. Only in this case may the bargaining problem offer a potential reward to the players for reaching an agreement. If $\mathcal{F}$ is empty, then disagreement is the only rational outcome of the bargaining game. Nash proposed that a solution to the bargaining game should verify four axioms: independence of equivalent utility representations, symmetry (which states that symmetry of players implies symmetry of the solution), independence of irrelevant outcomes, and Pareto optimality; see Roth (1979).

If $\mathcal{F}$ is nonempty, then any rational outcome of the bargaining game is among the Pareto optimal points of $\mathcal{F}$. Under the four axioms there exists a unique solution to the bargaining game, called the Nash solution.

We assume that the network is symmetric, in the sense that

$$C_i \overset{d}{=} C_j \text{ and } \widehat{C}_i \overset{d}{=} \widehat{C}_j \text{ for all } i,j = 1,\dots,m. \tag{39}$$

A sufficient condition for this is that $g_i = g$, $\gamma_i = \gamma$, and that the random vector

$$\big(Q_i, P_i, \{L_{ij}\}_{j=1,\dots,m}, \{L_{ji}\}_{j=1,\dots,m}\big)_{i=1,\dots,m}$$

is exchangeable, i.e., has the same distribution under any permutation of indices $i = 1,\dots,m$. We also assume that all interbank liabilities L_{ij} form integrable random variables with a continuous distribution (this is a technical property used in the proof of Theorem 3). We also assume the case of full reinsurance, which implies that in absence of defaults of the intermediary banks, the banks with liabilities to the end users do not default on those liabilities.

By a symmetry axiom of the Nash solution, it follows that any solution of the bargaining game is symmetric for the banks. This ensures that either all banks will agree to join the CN or none of them will. This leads to analytical tractability, while it does not trivialize the problem. There is a conflict of interest among the banks and the CN, as evidenced by (27) and the sensitivity results stated in Lemma 5 in the appendix. More importantly, banks are symmetric from an ex ante perspective but not ex post.

The condition $\mathbb{E}[\widehat{C}_i^+(\gamma_0,\boldsymbol{g})] \geq \mathbb{E}[C_i^+]$ for all $i \in 0,\dots,m$ required for feasibility is written

$$\gamma_0 \leq \mathbb{E}[\widehat{C}_0^+] \leq \gamma_0 + \sum_{i=1}^m \mathbb{E}[(Z_i - \widehat{Z}_i)(Q_i - P_i)] + \sum_{i=1}^m \mathbb{E}[D_i^* - \widehat{D}_i^*], \tag{40}$$

where in the second inequality we used the surplus identities with and without CN.

Underlying our results is the remarkably simple condition for a reduction of systemic risk given in Theorem 2. In the general case when banks are heterogeneous, the pure bargaining solution is also nonsymmetric. The following theorem states that there exists a level of systemic risk reduction required by the regulator, for which the feasible payoff set is non-empty.

We take $f > 0$. If the CN capital is senior, we assume that the set

$$\Big\{g \in [0,\gamma] \mid \mathbb{E}\Big[\sum_{i=1}^{m} f\Lambda_i^+ - \Big(G_{\text{tot}} - \sum_{i,\ \Lambda_i<0} \widehat{C}_i^-\Big)^-\Big] \geq 0\Big\}$$

is nonempty. We thus exclude from the outset the cases in which the CN will have an expected net loss for all possible guaranty fund contributions. If the CN capital is junior, the corresponding condition is stronger, namely the set $\{g \in [0,\gamma] \mid \mathbb{E}[\sum_{i=1}^m f\Lambda_i^+ - \sum_{i,\ \Lambda_i<0} \widehat{C}_i^-] \geq 0\}$ is nonempty. Second, we assume (for both the junior and the senior case) that

$$\sum_{i=1}^{m} \mathbb{E}[D_i - D_i^*] \leq \sum_{i=1}^{m} \mathbb{E}[(Z_i - \widehat{Z}_i)(Q_i - P_i)] - \sum_{i=1}^{m} f\Lambda_i^+, \tag{41}$$

which means that in the initial network the losses induced on the end users are lower than the liquidation losses. This is a reasonable assumption since the majority of liabilities are interbank liabilities and not to the end users. Therefore, in the uncleared network liquidation losses are amplified in multiple rounds. The loss to the end users is much smaller than the overall liquidation losses at the level of the dealer banks. The fee gains of the CN $\sum_{i=1}^m f\Lambda_i^+$ are typically negligible with respect to the liquidation losses in the uncleared network and appear in our assumption for technical reasons.

Theorem 3. *Assume that the systemic risk measure is given by* (35) *for some weight parameter* α. *Then, for any level of systemic risk reduction imposed by the regulator* $\ell \in [0, \mathbb{E}[\alpha \sum_{i=1}^m \mathbb{E}[(Z_i - \widehat{Z}_i)(Q_i - P_i)] + (1-\alpha)\mathbb{E}[\sum_{i=1}^m (D_i - D_i^*)]]$, *the set of feasible utility payoffs* $\mathcal{F}$ *is nonempty.*

Solution to the bargaining problem. Following Nash's theorem (see Roth (1979, Theorem 1)), since the set of feasible payoff outcomes $\mathcal{A}$ is nonempty, there is a unique solution to the Nash bargaining problem, where uniqueness is understood in terms of utility payoffs. It is given by

$$\max_{\gamma_0;\boldsymbol{g}} \Big(\mathbb{E}[\widehat{C}_0^+] - \gamma_0\Big)\Big(\gamma_0 - \mathbb{E}[\widehat{C}_0^+] + \sum_{i=1}^{m} \mathbb{E}[(Z_i - \widehat{Z}_i)(Q_i - P_i)] + \sum_{i=1}^{m} \mathbb{E}[D_i^* - \widehat{D}_i^*]\Big) \tag{42}$$

$$\text{subject to } (\gamma_0, \boldsymbol{g}) \in \mathcal{F}. \tag{43}$$

By the Pareto optimality axiom of the Nash bargaining solution, we must have that (γ_0, g) lies on an aggregate utility indifference curve, where the aggregate utility is understood for the banks and the CN. The aggregate utility (or more conveniently, the difference in aggregate utility to the case without the CN) is given by $\Delta + \sum_{i=1}^m \mathbb{E}[D_i^* - \widehat{D}_i^*]$ with $\Delta := \sum_{i=1}^m \mathbb{E}[(Z_i - \widehat{Z}_i)(Q_i - P_i)]$ which does not depend on (γ_0, g).

The condition $\mathcal{R} - \widehat{\mathcal{R}}(\gamma_0, \boldsymbol{g}) \geq \ell$ is written as

$$\sum_{i=1}^{m} \mathbb{E}[D_i^* - \widehat{D}_i^*] \leq \frac{\alpha\Delta - \ell}{(1-\alpha)}. \tag{44}$$

Letting x the variable total utility minus Δ, the solution to the bargaining problem is written as

$$\max_{\gamma_0, x} \Big(\mathbb{E}[\widehat{C}_0^+] - \gamma_0\Big)\Big(\gamma_0 - \mathbb{E}[\widehat{C}_0^+] + \Delta + x\Big)$$
$$\text{subject to } x \le \frac{\alpha\Delta - \ell}{(1-\alpha)}.$$

The constraint is saturated at the optimal solution, and the solution to the bargaining problem (g^*, γ_0^*) (which exists and is unique by virtue of Nash's theorem) is the solution of the following system of equations:

$$\sum_{i=1}^{m} \mathbb{E}[D_i^* - \widehat{D}_i^*(\gamma_0, g)] = \frac{\alpha\Delta - \ell}{(1-\alpha)},$$
$$\mathbb{E}[\widehat{C}_0^+(\gamma_0, g)] - \gamma_0 = \frac{\Delta - \ell}{2(1-\alpha)}. \tag{45}$$

In the particular case of "junior CN," the solution is trivial because the CN utility $\mathbb{E}[\widehat{C}_0^+] - \gamma_0$ does not depend on g and is a decreasing function of γ_0. Because $\mathbb{E}[\widehat{C}_0^+]$ is lower in the case of "junior CN" and due to the monotonicity of $\gamma_0 \to \mathbb{E}[\widehat{C}_0^+] - \gamma_0$, we have that the solution γ_0^* is also lower for this case.

In other words, the setup "junior" or "senior" CN does not have an impact on the CN utility in the Nash solution. This is because the right-hand side of (45), $\frac{\Delta-\ell}{2(1-\alpha)}$, does not depend on seniority of the CN capital. The banks and the CN adapt to the configuration, and the utilities in the Nash solution are the same. In what follows, we consider the case of senior CN capital.

"First-best" benchmark. We conclude this section by comparing the outcome under our CN design with the first-best benchmark in which all liabilities are netted out in the economy. A CN as we consider here achieves the same aggregate surplus as in the first-best benchmark.

The CN's layers of capitalization do not change the aggregate surplus, but they change how this surplus is distributed to the banks, the CN, and the end users. In terms of this distribution, the first-best benchmark coincides with a CN with zero equity and guaranty fund requirements. However, such CN is not reducing systemic risk and has the largest spillover to the end users; see Figure 5 (note that this Figure plots the inverse of systemic risk). The first-best benchmark is highly unfair to the end users. If we start with an uncleared network in which the end users are senior to the interbank liabilities, then netting out across the network will make the end users more junior. To see this, consider the following example. Bank 1 owes 10 to bank 2; bank 2 owes 10 to bank 3 and 10 to the end users. Assume bank 2 has 0 in its own cash. If the network is not netted out, then 2 pays 10 to the end users which are senior and defaults on its liability to bank 3. If the network is netted out, then 1 pays directly to 3, 2 has no receivables or payables from the network and defaults on its liability to the end users. Because of netting out, the end users have become more junior! This example illustrates why only a CN with adequate policy can achieve both the best aggregate surplus and Pareto improvements. Our results are primarily existence results for such policies. Given the existence results, the Nash bargaining solution selects the levels g^* and γ^*.

7. Simulation study.

We analyze numerically the impact of centralized clearing of OTC derivatives in the particular case of CDS, under the assumption of a symmetric network as in the previous section. We calibrate to a gross market value of all contracts $W = 100$bn (all figures are in US dollars) and a gross notional $N = 1.8$tn.[5] The top dealers concentrate more than 85% of the CDS market (see, e.g., Cont and Minca (2016)), so it is reasonable to approximate the whole market by distributing these market values over the top dealers only. Given the volatility of the CDS asset class and the large positions concentrated on the dealers, we capture extreme risks of large liabilities.

We let $m = 14$ banks. We assume that the market is symmetric and set the market share of bank i as $\frac{1}{m}$.[6] Moreover, we assume $g_i = g$ and $\gamma_i = .5$bn for all $i = 1, \dots, m$. We take fundamental and liquidation values $Q_i = .2$bn and $P_i = 70\% Q_i$ constant and equal over all banks $i = 1, \dots, m$.

We let $V_{ij} \geq 0$ denote the random value at $t = 1$ of all contracts of bank i with bank j where bank i owes to bank j. We may think of V_{ij} as the sum of values of CDS contracts where bank j bought protection from bank i on some reference entities, and where the premiums were fully paid at some time earlier than $t = 0$. According to our symmetric market assumption, the notional underlying V_{ij} amounts to $N/(m(m-1))$. The gross market value is given by $W = \sum_{i \neq j} \mathbb{E}[V_{ij}]$.

We specify V_{ij} as a half-normal random variable, $V_{ij} = |X_{ij}|$, where X_{ij} are normal random variables with mean zero and standard deviation σ. The pair (X_{ij}, X_{ji}) have correlation ρ and are independent of other such pairs. The OCC (2019) reports significant bilateral netting benefits, close to 90%, i.e., the sum of interbank liabilities after bilateral netting is approximately 10% of the sum of the liabilities before bilateral netting. This means that there is high correlation between X_{ij} and X_{ji}. In the following we set the correlation parameter $\rho = 90\%$.

We obtain

$$W = \frac{2}{\sqrt{2\pi}} m(m-1)\sigma,$$

which serves as calibration of σ. We assume that the interbank liabilities clear as in Eisenberg and Noe (2001). Hence in the absence of a CN there results a unique matrix of clearing payments (L^*_{ij}) as outlined in Example 1. In what follows we consider the netted liabilities of bank i to j (the amount that bank i owes to bank j) are obtained from the netting of V_{ij} and V_{ji},

$$L_{ij} = (V_{ij} - V_{ji})^+.$$

Our framework allows for the case of nonnetted liabilities, but here we consider an initial network with bilateral netting agreements. Because we isolate a single asset class, when we introduce the CN the multilateral netting includes the bilateral netting. If we allowed for several asset classes, then there would be a trade-off between bilateral netting across asset

[5]This represents the sum (in absolute terms) of the positive market value of all market participants. In (BIS, 2010, Table 19), the ratio between gross market value and notional is 1:18 for CDS.

[6]The actual market shares in terms of notional amounts outstanding and gross positive fair values for top US dealers are given in OCC (2019).

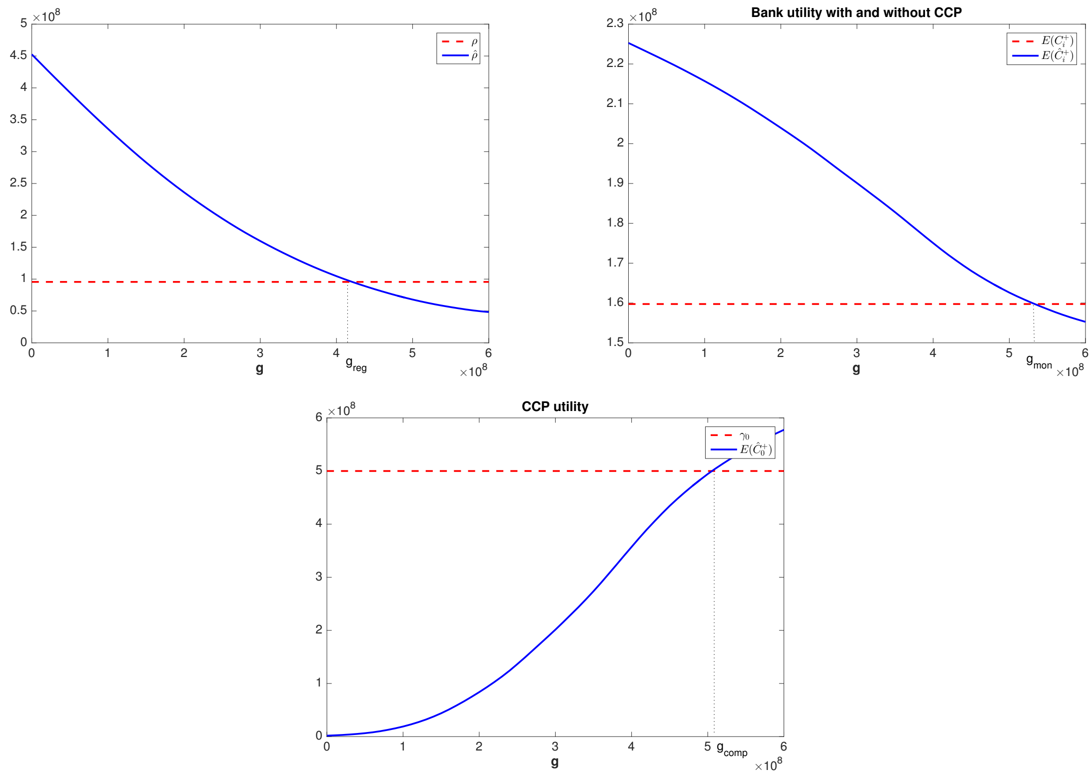

Figure 2. *Systemic risk and banks' and CN utility with CN (solid lines) and without CN (dashed lines) as functions of guaranty fund contribution g. g_{reg} is the minimal regulatory acceptable, g_{mon} is the maximal banks' utility improving, and g_{comp} is the minimal CN utility improving guaranty fund contribution. Number of banks is $m = 14$. CN equity is $\gamma_0 = .5bn$. Fee is $f = 2\%$.*

classes and multilateral netting with the CN (Duffie and Zhu (2011)), and the implication of this in our model is left for future work.

Thinking of L_{ij} as a variation margin under an extreme event, this liability is considered after bilateral netting. The random vector $\left(\{L_{ij}\}_{j=1,\dots,m}, \{L_{ji}\}_{j=1,\dots,m}\right)_{i=1,\dots,m}$ is exchangeable and therefore (39) holds.

The systemic risk measure is $\mathcal{R}(\boldsymbol{C}) = \mathbb{E}[\sum_{i=1}^m (D_i - \widehat{D}_i^*)]$, meaning that the regulator is exclusively concerned by the loss of the end users. The regulatory threshold level for acceptability is set to $\ell = 0$. We fix the number of banks to $m = 14$. The CN charges a fee of $f = 2\%$,[7] and the external equity varies $\gamma_0 \in [0, .5bn]$.

Figure 2 shows systemic risk and banks' and CN utility with and without CN for varying guaranty fund contribution g. The external capital of the CN is fixed at $\gamma_0 = 5bn$. Systemic

[7]Note that the fee is expressed here as a percentage of the netted liabilities. Given that net liabilities represent approximately 10% of the market value before netting (OCC (2019)), the fee is of the order $0.2\% = 20bp$ in terms of market value. In practice, clearinghouses charge fees per traded volume.

risk and banks' utility is decreasing in g and CN utility is increasing in g. The level $g = g_{\text{reg}}$ is the minimal guaranty fund contribution for which the systemic risk with CN is at most as large as without CN, $\mathcal{R}(\widehat{\boldsymbol{C}}) \le \mathcal{R}(\boldsymbol{C})$. Any $g \ge g_{\text{reg}}$ corresponds to a level acceptable by a regulator. The level $g = g_{\text{mon}}$ is the maximal guaranty fund contribution for which banks' utility with CN is at least as large as without CN, $\mathbb{E}[\widehat{C}_i^+] \ge \mathbb{E}[C_i^+]$. It corresponds to the "monopolistic" situation where all bargaining power is with the CN. The level $g = g_{\text{comp}}$ is the minimal guaranty fund contribution for which CN utility is at least as large as its equity, $\mathbb{E}[\widehat{C}_0^+] \ge \gamma_0$. It corresponds to the "competitive" situation where all bargaining power is with the banks. As predicted by Theorem 3, the set of Pareto improving policies $[g_{\text{reg}}, \infty) \cap [g_{\text{comp}}, g_{\text{mon}}]$ is nonempty for γ_0 sufficiently large. Of course, Nash's theorem gives the solution to the bargaining problem, i.e., both the guaranty fund level g^* and the CN capital γ_0^*, and we will have that $g^* \in [g_{\text{reg}}, \infty) \cap [g_{\text{comp}}, g_{\text{mon}}]$. At the bargaining solution, the banks and the CN share equally the surplus gain from clearing minus the transfer for this surplus to the end users (which is imposed via the regulator's constraint)

We further investigate numerically the components of the difference in expected net worth of a bank with and without CN, using the expectation of the different components T_1, T_2, T_3 in the decomposition (30). Figure 3 plots the dependence of $\mathbb{E}[T_1]$, $\mathbb{E}[T_2]$ and respectively $\mathbb{E}[T_3]$ on the guaranty fund contribution g. We see that the CN reduces counterparty risk in expectation, $\mathbb{E}[T_1] > 0$, and the term $\mathbb{E}[T_1]$ increases with g. This comes at the cost of an expected loss due to the fee and the risk of a possible use of the share in the guaranty fund to absorb shortfall losses of defaulting banks, $\mathbb{E}[-T_3]$, which increases with g. On top of these two opposite effects, the CN has a mitigating effect on forced liquidations, which is constant in g and given by the expected reduction in liquidations $\mathbb{E}[T_2]$. Both, $\mathbb{E}[T_2]$ and $\mathbb{E}[-T_3]$ are positive as predicted by Theorem 1.

As a variation of our experiment, we now reduce the number of banks while keeping total notional N and total gross fair market value W. This increases banks' average exposure. The cash $\gamma = .84bn$ so as to keep the total cash invariant. Figure 4 points to important

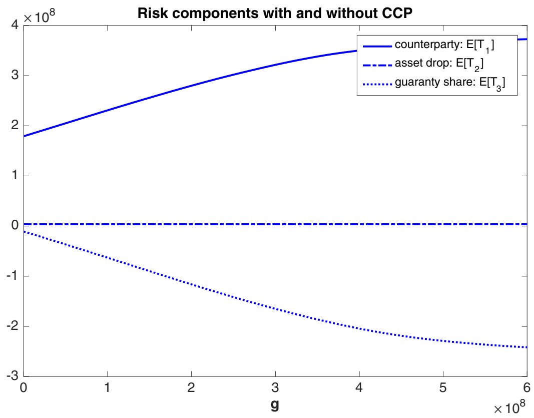

Figure 3. *Expected differences in stand-alone risk components with and without CN given in decomposition (30) as functions of guaranty fund contribution g. Number of banks is $m = 14$. CN equity is $\gamma_0 = .5bn$. Fee is $f = 2\%$.*

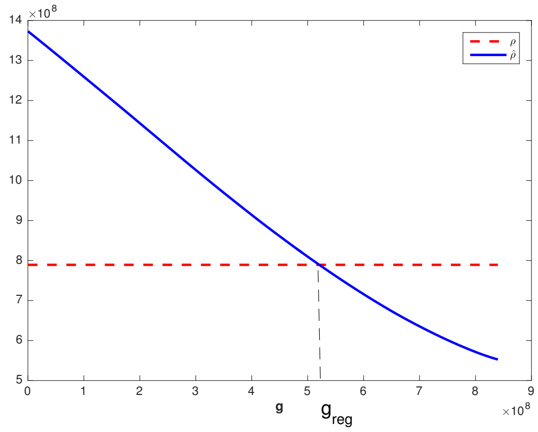

Figure 4. *Systemic risk with and without CN as a function of guaranty fund contribution g. g_{reg} is the minimal regulatory acceptable. Number of banks is $m = 10$. CN equity is $\gamma_0 = .5bn$. Fee is $f = 2\%$.*

nonlinearities. Namely for a decrease of the number of banks from $m = 14$ to $m = 10$, all else equal, the value g for which systemic risk is reduced is $g = g_{reg} = .5bn$, where in the case of 14 banks it was close to $.4bn$. Concentrating the liabilities on fewer banks translates to higher levels of CN capitalization to achieve systemic risk reduction.

We now vary both the guaranty fund contribution and the CN equity γ_0. Again, we fix the fee to $f = 2\%$ and consider $m = 14$ banks. Figure 5 shows that the systemic risk measure is much more sensitive with respect to the guaranty fund contributions than with respect to the CN equity.

For the design parameters which make the guaranty fund acceptable by a regulator, the risk that CN equity will be impacted is very small. Consequently, the systemic risk measure will be less sensitive to the CN equity.

The CN utility increases in g, while increasing γ_0 too much will disincentivize it from joining the network. The bank's utility increases in γ_0, while increasing g too much will disincentivize it from joining the CN. However, as we have seen, there is a wide range of parameters in which all parties are better off under centralized clearing of the network, and the regulator will also see the systemic risk decrease. Within that range, the bargaining game between the CN and the banks determines the parameters g^* and γ_0^* as shown in Figure 6. We obtain $\gamma_0^* = .26bn$ and $g = .49bn$.

Junior CN capital ("skin in the game"). As the theory predicts, the setup "junior" or "senior" CN does not have an impact on the CN and bank's utility in the Nash solution. This is because the surplus $\frac{\Delta-\ell}{(1-\alpha)}$ (which is divided between the CN and the banks) does not depend on seniority of the CN capital. The banks and the CN adapt to the configuration, while the utilities in the Nash solution stay the same. Of course, with a "junior" CN capital, the value of the CN equity which is solution to

$$\left(\mathbb{E}[\widehat{C}_0^+(\gamma_0)] - \gamma_0\right) = \frac{\Delta - \ell}{2(1-\alpha)}$$

in (45) is much lower than in the case of a "senior" CN.

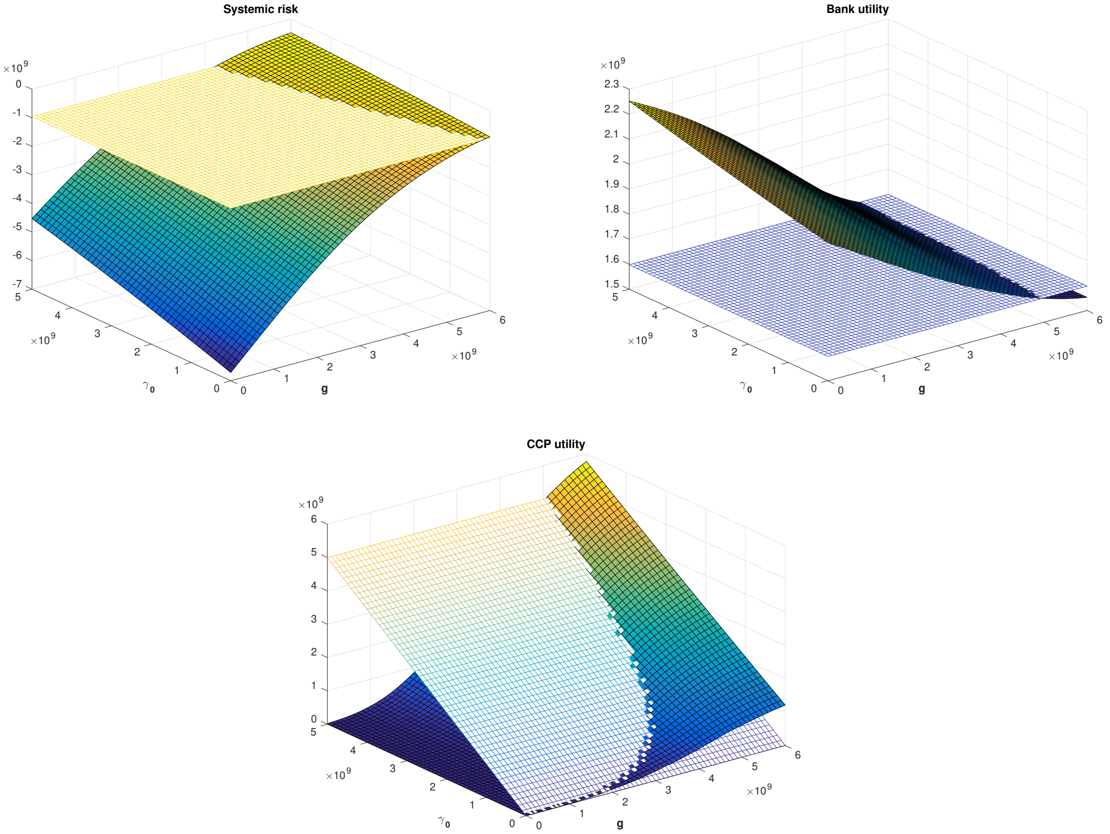

Figure 5. *Systemic risk and banks' and CN utility as functions of CN equity γ_0 and guaranty fund contribution g. Planes represent the case without a CN. Policies (g, γ_0) corresponding to values above the planes are regulatory acceptable and incentive compatible for banks and CN, respectively (note that the Z axis is reversed in the systemic risk figure). Number of banks is $m = 14$. Fee is $f = 2\%$.*

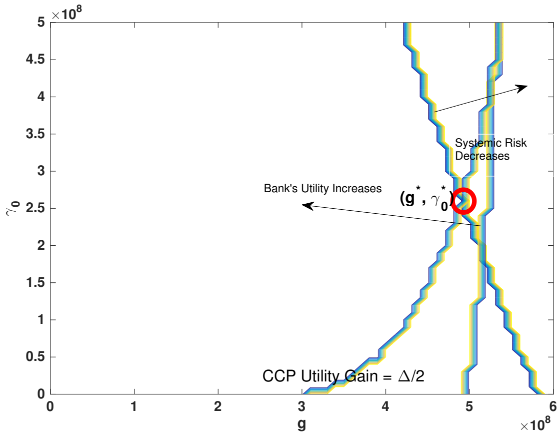

Figure 6. *(Nash bargaining solution.) (γ_0^*, g^*) is the unique solution of the equations $\sum_{i=1}^m \mathbb{E}[D_i^* - \widehat{D}_i^*(\gamma_0, g)] = 0$ (the systemic risk zero line) and $\mathbb{E}[\widehat{C}_0^+(\gamma_0, g)] - \gamma_0 = \frac{\Delta}{2}$ (the CN utility gain $= \frac{\Delta}{2}$); see (45) for $\ell = 0$ and $\alpha = 0$. We note that for (γ_0^*, g^*) we are in the domain in which the banks' utility is above the noncleared case. We obtain $\gamma_0^* = .26bn$ and $g = .49bn$.*

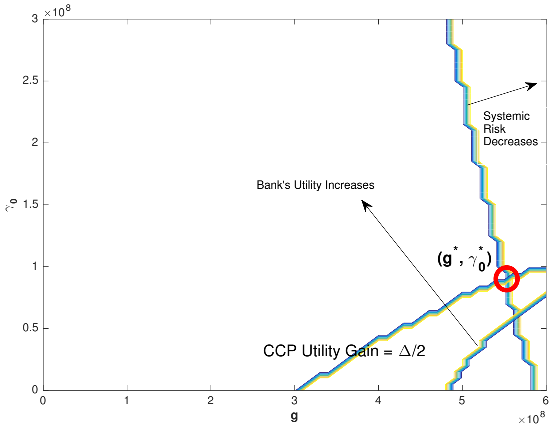

Figure 7. *The CN equity is junior ("skin in the game"). (γ_0^*, g^*) is the unique solution of the equations $\sum_{i=1}^m \mathbb{E}[D_i^* - \widehat{D}_i^*(\gamma_0, g)] = 0$ (the systemic risk zero line) and $\mathbb{E}[\widehat{C}_0^+] - \gamma_0 = \frac{\Delta}{2}$ (the CN utility gain $= \frac{\Delta}{2}$). For all g and γ_0 the utility of the banks is higher than in the non-CN case. We obtain $\gamma_0^* = .09bn$ and $g = .55bn$.*

We now compute the parameters g^* and γ_0^* in Figure 7. With the CN junior, we obtain $\gamma_0^* = .09$ bn and $g^* = .55$bn. We stress that the utilities of the banks and the CN are the same at the respective solution (g^*, γ_0^*) and these respective solutions satisfy $\sum_{i=1}^m \mathbb{E}[D_i^* - \widehat{D}_i^*(\gamma_0, g)] = \frac{\alpha\Delta - \ell}{(1-\alpha)}$. The utilities of all parties involved are the same at the respective Nash bargaining solutions; the difference in resources that are needed in the case of a CN with junior capital versus the case with a senior capital is $(.55 - .49) \times 14 - (.26 - .09) = .67bn$. When the CN has junior capital, also known as "skin in the game," the CN equity (at the Nash solution) is almost a third of the case in which the CN capital is senior. To satisfy the systemic risk decrease constraint, the banks need to make higher guaranty fund contributions. The overall resources in the Nash bargaining solution actually increase when the CN's capital is junior. Our simulation suggests that a CCP clearing 1.8 tn in CDS in notional should hold at least 7 bn in default resources. This is largely satisfied by a real-world CCP that clears comparable amounts of CDS and holds over 30 bn in total default resources.

8. Conclusion and future directions.

We propose a stochastic framework with endogenous systemic risk measurement that takes into account the dependence structure among banks induced by a network of interbank liabilities. Interbank liability clearing can force early liquidation of banks' assets at a fraction of their fundamental value. Our theoretical analysis sheds light on several aspects related to centralized clearing. Introduction of a CN reduces banks' liquidation and thus improves aggregate surplus. The impact on banks' individual surplus is indefinite and depends on the trade-off between CN shortfall versus reduced bank counterparty risk from an ex ante perspective. We find that a CN does not always reduce systemic risk. We provide sufficient conditions in terms of the CN's equity and guaranty fund policy for a reduction of systemic risk. We perform a surplus analysis in the context of a symmetric financial network with homogeneous banks for which we characterize the Pareto improving CN equity and guaranty fund policies. A simulation study calibrated to market data reveals those policies for which centralized clearing improves banks' utility while reducing systemic risk. Several directions emerge from the current study. Our results are stated under the symmetry assumption, as well as under a specific form of utility, expected surplus. The insight that clearing members are compensated for the risk of guaranty fund loss by lower expected liquidations may carry over to different forms of banks' utilities. The other crucial element is the initial margin, which represents additional default funds that are segregated for each member. The part of the resources that are pooled guaranty funds versus segregated initial margins would result from strategic decisions. A full analysis of this matter as well as the relaxation of the symmetry assumption warrants a careful study. A direction of particular importance is systemic risk measurement and clearing facility design in a dynamic network setting. A Nash equilibrium solution concept would be interesting in a multiperiod version of the game, where a binding agreement would be only for one period, while in the second period some banks may exit the CN. We plan to investigate this in future extensions, along with the question of replenishment: the CN can also ask banks for additional guaranty fund contributions. Also, we expect that game theoretic arguments would lead to design optimal rules for guaranty fund contributions in the general case with heterogeneous banks and multiple CNs.

Appendix A.

A.1. Sensitivity analysis. In this appendix, following up on (27), we provide the statewise sensitivities of banks' individual and aggregate net worth, surplus, and shortfall with respect to the fee and guaranty fund policy $(f, \boldsymbol{g})$, respectively. The result confirms economic intuition and it clearly marks the opposing sensitivities of the parities involved: on one hand the CN and the banks, and on the other hand the banks among each other.

Lemma 5.

(i) *Bank's individual net worth is nonincreasing in f and its own guaranty fund contribution, while it is nondecreasing in the other banks' guaranty fund contributions, for $j = 1, \dots, m$,*

$$\frac{\partial \widehat{C}_i}{\partial f} \leq 0, \quad \text{and} \quad \frac{\partial \widehat{C}_i}{\partial g_j} \begin{cases} \geq 0 & \text{if } j \neq i, \\ \leq 0 & \text{if } j = i. \end{cases}$$

(ii) *The aggregate net worth of the financial network including the CN is nondecreasing in the fee and guaranty fund policy $(f, \boldsymbol{g})$, for $i = 1, \dots, m$,*

$$\frac{\partial \sum_{k=0}^m \widehat{C}_k}{\partial f} = -\frac{\partial \widehat{C}_0^-}{\partial f} \geq 0, \quad \text{and} \quad \frac{\partial \sum_{k=0}^m \widehat{C}_k}{\partial g_i} = -\frac{\partial \widehat{C}_0^-}{\partial g_i} \geq 0.$$

(iii) *The aggregate net worth of the banks is nonincreasing in the fee and guaranty fund policy $(f, \boldsymbol{g})$, for $i = 1, \dots, m$,*

$$\frac{\partial \sum_{k=1}^m \widehat{C}_k}{\partial f} \leq 0, \quad \text{and} \quad \frac{\partial \sum_{k=1}^m \widehat{C}_k}{\partial g_i} \leq 0.$$

A.2. Proofs. This appendix contains the proofs of all lemmas and theorems.

Proof of Lemma 1. We have

$$\begin{aligned}\sum_{i=1}^m C_i &= \sum_{i=1}^m \Gamma_i + \sum_{i=1}^m Q_i - \sum_{i=1}^m Z_i(Q_i - P_i) + \sum_{i=1}^m \sum_{j=1}^m L_{ji}^* - \sum_{i=1}^m L_i \\ &= \sum_{i=1}^m \gamma_i + \sum_{i=1}^m Q_i - \sum_{i=1}^m Z_i(Q_i - P_i) - \sum_{i=1}^m (L_i - L_i^*) - \sum_{i=1}^m D_i \\ &= \sum_{i=1}^m \gamma_i + \sum_{i=1}^m Q_i - \sum_{i=1}^m Z_i(Q_i - P_i) - \sum_{i=1}^m C_i^- - \sum_{i=1}^m D_i^*,\end{aligned}$$

which proves the aggregate surplus identity (8).

Proof of Lemma 2. We have

$$\begin{aligned}\sum_{i=0}^m \widehat{C}_i = \widehat{A}_0 - \widehat{L}_0 - G_{tot}^* + \sum_{i=1}^m \widehat{A}_i - \sum_{i=1}^m \widehat{L}_{i0}& \\ = \gamma_0 + \sum_{i=1}^m g_i + \sum_{i=1}^m \widehat{L}_i^* - \widehat{L}_0 - G_{tot}^* + \sum_{i=1}^m \Gamma_i + \sum_{i=1}^m Q_i& \\ - \sum_{i=1}^m \widehat{Z}_i(Q_i - P_i) - \sum_{i=1}^m g_i + \sum_{i=1}^m \widehat{L}_0^* \times \widehat{\Pi}_{0i} + \sum_{i=1}^m G_i^* - \sum_{i=1}^m \widehat{L}_{i0}& \\ = \gamma_0 + \sum_{i=1}^m \Gamma_i + \sum_{i=1}^m Q_i - \sum_{i=1}^m \widehat{Z}_i(Q_i - P_i) - (\widehat{L}_0 - \widehat{L}_0^*) - \sum_{i=1}^m (\widehat{L}_{i0} - \widehat{L}_i^*),&\end{aligned}$$

where in the last equality we used the fact that $\sum_{i=1}^m \widehat{\Pi}_{0i} = 1$. We obtain that

$$\begin{aligned}\sum_{i=0}^m \widehat{C}_i + \sum_{i=1}^m \widehat{D}_i^* &= \sum_{i=0}^m \gamma_i + \sum_{i=1}^m Q_i - \sum_{i=1}^m \widehat{Z}_i(Q_i - P_i) - \widehat{C}_0^- - \sum_{i=1}^m (\widehat{L}_{i0} - \widehat{L}_i^*) - \sum_{i=1}^m (D_i - \widehat{D}_i^*) \\ &= \sum_{i=0}^m \gamma_i + \sum_{i=1}^m Q_i - \sum_{i=1}^m \widehat{Z}_i(Q_i - P_i) - \widehat{C}_0^- - \sum_{i=1}^m \widehat{C}_i^-,\end{aligned}$$

where in the second line we used (24). We obtain the aggregate surplus identity with CN.

Proof of Lemma 3. We let i be a bank with net liabilities to the CN: $\Lambda_i < 0$. It is sufficient to prove that capital and liquidity shortfall of bank i satisfy

$$\widehat{C}_i^- = \widehat{L}_{i0} - \widehat{L}_i^* = (\Lambda_i + P_i + \gamma_i)^- \tag{46}$$

and

$$(\gamma_i - g_i - \widehat{L}_{i0})^- = (\Lambda_i + \gamma_i)^-, \tag{47}$$

respectively, and these identities will be useful throughout other proofs as well.

By (21) we have (since $g_i \le \gamma_i$ and $P_i \ge 0$) for bank i's shortfall

$$\begin{aligned}\widehat{C}_i^- &= \widehat{L}_{i0} - \widehat{L}_i^* = \widehat{L}_{i0} - \widehat{L}_{i0} \wedge \Big(P_i + \gamma_i - g_i\Big) = \Big(\widehat{L}_{i0} - \Big(P_i + \gamma_i - g_i\Big)\Big)^+ \\ &= \Big(\Big(\Lambda_i + g_i\Big)^- - \Big(P_i + \gamma_i - g_i\Big)\Big)^+ = \Big(-\Lambda_i - g_i - \Big(P_i + \gamma_i - g_i\Big)\Big)^+ \\ &= \Big(\Lambda_i + P_i + \gamma_i\Big)^-,\end{aligned}$$

which proves (46). Note that the right-hand side does not depend on the fee and guaranty fund policy $(f, \boldsymbol{g})$.

By (9), we have $\widehat{L}_{i0} = (\Lambda_i + g_i)^-$. Clearly, for any $i = 1, \dots, m$ and $j \neq i$, the quantity liquidated by bank i, $(\gamma_i - g_i - \widehat{L}_{i0})^-$ does not depend on g_j. Let us now show that the quantity liquidated by bank i does not depend on g_i. We have (since $g_i \le \gamma_i$)

$$\begin{aligned}(\gamma_i - g_i - \widehat{L}_{i0})^- &= (\widehat{L}_{i0} - \gamma_i + g_i)\mathbb{1}_{\widehat{L}_{i0} > \gamma_i - g_i} = (\widehat{L}_{i0} - \gamma_i + g_i)\mathbb{1}_{\Lambda_i < -\gamma_i} = (-\Lambda_i - \gamma_i)\mathbb{1}_{\Lambda_i < -\gamma_i} \\ &= (\Lambda_i + \gamma_i)^-,\end{aligned}$$

which proves (47). From (13), it follows that the fraction of liquidated assets $\widehat{Z}_i$, and thus the aggregate surplus (24), do not depend on the fee and guaranty fund policy $(f, \boldsymbol{g})$.

Proof of Lemma 4. Recall from (14) that $\widehat{A}_0 = \gamma_0 + \sum_{i=1}^m g_i + \sum_{i=1}^m \widehat{L}_i^*$, which combined with (46) yields

$$\begin{aligned}\widehat{A}_0 &= \gamma_0 + \sum_{i=1}^m g_i + \sum_{i=1}^m \widehat{L}_{i0} - \sum_{i=1}^m \Big(\Lambda_i + P_i + \gamma_i\Big)^- \\ &= \gamma_0 + \sum_{i=1}^m G_i + \sum_{i=1}^m \Lambda_i^- - \sum_{i=1}^m \Big(\Lambda_i + P_i + \gamma_i\Big)^- \\ &= \gamma_0 + \sum_{i=1}^m \Lambda_i^+ + \sum_{i=1}^m G_i - \sum_{i=1}^m \Big(\Lambda_i + P_i + \gamma_i\Big)^-,\end{aligned}$$

where in the second equality we have used (12) and in the last equality we have used that $\sum_{i=1}^m \Lambda_i^- = \sum_{i=1}^m \Lambda_i^+$. We define

$$\mathcal{T} = G_{\text{tot}} - \sum_{i,\ \Lambda_i < 0} \widehat{C}_i^- = \sum_{i=1}^m \Big(G_i - \Big(\Lambda_i + P_i + \gamma_i\Big)^-\Big). \tag{48}$$

From (11) and (48) we infer $\widehat{A}_0 - \widehat{L}_0 = \gamma_0 + f\sum_{i=1}^m \Lambda_i^+ + \mathcal{T}$.

For the case of "senior CN" we note that $(\widehat{A}_0 - \widehat{L}_0 - \gamma_0 - f\sum_{i=1}^m \Lambda_i^+)^+ = \mathcal{T}^+$ and from (15) we obtain that $G_{\text{tot}}^* = G_{\text{tot}} \wedge \mathcal{T}^+ = \mathcal{T}^+$, since $\mathcal{T} \le G_{\text{tot}}$. For the case of "junior CN" we obtain from (16) that $G_{\text{tot}}^* = G_{\text{tot}} \wedge (\gamma_0 + f\sum_{i=1}^m \Lambda_i^+ + \mathcal{T})^+$. This proves (25).

From (17), it now follows that

$$\widehat{C}_0 = \gamma_0 + f\sum_{i=1}^m \Lambda_i^+ + \mathcal{T} - \mathcal{T}^+ = \gamma_0 + f\sum_{i=1}^m \Lambda_i^+ - \mathcal{T}^-, \tag{49}$$

and respectively

$$\widehat{C}_0 = \gamma_0 + f\sum_{i=1}^m \Lambda_i^+ + \mathcal{T} - G_{\mathrm{tot}} \wedge \left(\gamma_0 + f\sum_{i=1}^m \Lambda_i^+ + \mathcal{T}\right)^+$$

$$(50) \qquad = \left(\gamma_0 + f\sum_{i=1}^m \Lambda_i^+ - \sum_{i,\ \Lambda_i<0} \widehat{C}_i^-\right)^+ - (\widehat{A}_0 - \widehat{L}_0)^-,$$

which proves (26).

From (48) we have

$$\frac{\partial \mathcal{T}}{\partial g_i} = \frac{\partial G_{tot}}{\partial g_i} = \mathbb{1}_{\Lambda_i+g_i>0}; \quad \frac{\partial \mathcal{T}}{\partial f} = 0.$$

From (26) we have $\frac{\partial \widehat{C}_0}{\partial g_i} = -\frac{\partial \mathcal{T}^-}{\partial g_i} = \mathbb{1}_{\mathcal{T}<0}\mathbb{1}_{\Lambda_i+g_i\geq 0}$ and respectively $\mathbb{1}_{\widehat{C}_0<0}\mathbb{1}_{\Lambda_i+g_i>0}$ for the "junior CN" case. Thus

$$\frac{\partial \widehat{C}_0^+}{\partial g_i} = \mathbb{1}_{\widehat{C}_0>0}\mathbb{1}_{\mathcal{T}<0}\mathbb{1}_{\Lambda_i+g_i\geq 0} \geq 0 \text{ (resp., 0)} \quad \text{and} \quad \frac{\partial \widehat{C}_0^-}{\partial g_i} = -\mathbb{1}_{\widehat{C}_0<0}\mathbb{1}_{\Lambda_i+g_i\geq 0} \leq 0,$$

where we used that $\mathbb{1}_{\widehat{C}_0<0} = \mathbb{1}_{\widehat{C}_0<0}\mathbb{1}_{\mathcal{T}<0}$. We also obtain from (26) that

$$\frac{\partial \widehat{C}_0}{\partial f} = \sum_{i=1}^m \Lambda_i^+ \geq 0.$$

This proves (27).

From (12), (19), and (20) we obtain

$$\begin{aligned}\widehat{C}_i &= \widehat{A}_i - \widehat{L}_{i0} = \Gamma_i + Q_i - \widehat{Z}_i(Q_i - P_i) + \widehat{L}_0^* \times \widehat{\Pi}_{0i} - (G_i - G_i^*) - \Lambda_i^- \\ &= \Gamma_i + Q_i - \widehat{Z}_i(Q_i - P_i) + (\widehat{L}_0 - \widehat{C}_0^-) \times \widehat{\Pi}_{0i} - \frac{G_i}{G_{\mathrm{tot}}}\left(G_{\mathrm{tot}} - G_{\mathrm{tot}}^*\right) - \Lambda_i^-,\end{aligned}$$

where in the third line we used (18) and (25). Combining this with (10) proves (28).

Proof of Theorem 1.

(i) Let i be a bank with net liability to the CN: $\Lambda_i < 0$. By (47), we have that the liquidity shortfall of bank i is given by

$$(\gamma_i - g_i - \widehat{L}_{i0})^- = (\Lambda_i + \gamma_i)^- = \left(\gamma_i + \sum_{j=1}^m L_{ji} - L_i\right)^-,$$

where in the second equality we have used the explicit definition of Λ in (1). Combining this with (13) we obtain (for i with P_i ¿ 0)

$$(51) \qquad \widehat{Z}_i = \frac{(\gamma_i + \sum_{j=1}^m L_{ji} - L_i)^-}{P_i} \wedge 1.$$

On the other hand, in the case without CN, the clearing vector is obtained as a fixed point of the mapping Φ given in (4). The number of units of asset liquidated by bank i is

$$Z_i = \frac{(\gamma_i + \sum_{j=1}^m L_{ji}^* - L_i)^-}{P_i} \wedge 1,$$

where we use that $D_i = 0$ (and thus $\Gamma_i = \gamma_i$) whenever bank i has $\Lambda_i < 0$. Note that for all $j = 1, \ldots, m$, from the definition of the mapping Φ, $L_{ji}^* \leq L_{ji}$. We now conclude that the CN reduces liquidation losses,

$$\widehat{Z}_i \leq Z_i \quad \text{for all } i \text{ with } \Lambda_i < 0. \tag{52}$$

To prove that the CN reduces bank shortfall losses, recall that by (7),

$$C_i^- = L_i - L_i^* = \Big(L_i - \sum_{j=1}^m L_{ji}^* - P_i - \gamma_i\Big)^+ \text{ for all } i \text{ with } \Lambda_i < 0.$$

In view of $L_{ji} \geq L_{ji}^*$ and (46) we conclude

$$C_i^- \geq \Big(L_i - \sum_{j=1}^m L_{ji} - P_i - \gamma_i\Big)^+ = \Big(-\Lambda_i - P_i - \gamma_i\Big)^+ = \widehat{C}_i^- \text{ for all } i \text{ with } \Lambda_i < 0.$$

(ii) The decomposition (30) follows by subtracting (6) from (28).

(iii) Suppose that i is such that $\Lambda_i > 0$. In this case, from (3) we have that $Q_i = 0$ so we automatically have that $T_2(i) = 0$. From (28) we have moreover that $\widehat{C}_i^- = \Big(\Gamma_i + \Lambda_i - \widehat{\Pi}_{0i}\widehat{C}_0^- - f\Lambda_i^+ - \frac{G_i}{G_{\text{tot}}}\Big(G_{\text{tot}} - G_{\text{tot}}^*\Big)\Big)^- = \Big(\Gamma_i + \Lambda_i - \widehat{\Pi}_{0i}\widehat{C}_0^- + T_3(i)\Big)^-$. From (6) we have $C_i^- = (\Gamma_i + \Lambda_i - \sum_{j=1}^m (L_{ji} - L_{ji}^*))^-$. Then $T_1(i) + T_3(i) > 0$ is written as $-\widehat{\Pi}_{0i}\widehat{C}_0^- + T_3(i) > -\sum_{j=1}^m (L_{ji} - L_{ji}^*)$ and implies that $\widehat{C}_i^- < C_i^-$.

Proof of Proposition 1. We use that $\Gamma_i + \Lambda_i = \gamma_i$ under the full "reinsurance" assumption. We have $\sum_{i,\ \Lambda_i>0} \widehat{C}_i^- = (\gamma_i - \widehat{\Pi}_{0i}\widehat{C}_0^- - f\Lambda_i^+ - \frac{G_i}{G_{\text{tot}}}(G_{\text{tot}} - G_{\text{tot}}^*))^-$.

We have that $\sum_{i,\ \Lambda_i>0} \widehat{C}_i^- = \sum_{i,\ \Lambda_i>0} (\gamma_i - \widehat{\Pi}_{0i}\widehat{C}_0^- - f\Lambda_i^+ - g_i)^-$, where we used that $G_i = g_i$ when $\Lambda_i > 0$ and $G_{\text{tot}}^* = 0$ when $C_0 < 0$.

We consider the symmetric case: D_i constant over i with $D_i > 0$ implies that $\widehat{\Pi}_{0i} = \frac{1}{\#\{i, D_i>0\}}$. In the symmetric case we also have $g_i = g$, $\gamma_i = \gamma$. Under the assumption that $\gamma - g - f\sup_i \Lambda_i > 0$, all terms $\widehat{C}_i^-$ (for $\Lambda_i < 0$) are equal. There can be a shortfall $\widehat{C}_i < 0$ only if the CN fails ($\widehat{C}_0 < 0$),

$$\begin{aligned}\sum_{i,\ \Lambda_i>0} \widehat{C}_i^- &= \sum_{i,\ \Lambda_i>0} \Big(\gamma_i - \widehat{\Pi}_{0i}\widehat{C}_0^- - f\Lambda_i^+ - g_i\Big)^- \\ &= \Big(-\widehat{C}_0^- - \sum_{i,\ \Lambda_i>0} f\Lambda_i^+ + \sum_{i,\ \Lambda_i>0} \gamma_i - \sum_{i,\ \Lambda_i>0} g_i\Big)^-.\end{aligned}$$

The only case when $(-\widehat{C}_0^- - \sum_{i,\ \Lambda_i>0} f\Lambda_i^+ + \sum_{i,\ \Lambda_i>0} \gamma_i - \sum_{i,\ \Lambda_i>0} g_i) < 0$ is when $C_0 < 0$. In this case we use (29) to further obtain

$$\begin{aligned}
\sum_{i,\ \Lambda_i>0} \widehat{C}_i^- &= \Big(\gamma_0 + f\sum_{i=1}^m \Lambda_i^+ + G_{\text{tot}} - \sum_{i,\Lambda_i<0} \widehat{C}_i^- - \sum_{i,\ \Lambda_i>0} f\Lambda_i^+ - \sum_{i,\ \Lambda_i>0} \gamma_i + \sum_{i,\ \Lambda_i>0} g_i\Big)^- \\
&= \Big(\gamma_0 + G_{\text{tot}} - \sum_{i,\Lambda_i<0} \widehat{C}_i^- + \sum_{i,\ \Lambda_i>0} \gamma_i - \sum_{i,\ \Lambda_i>0} g_i\Big)^- \\
&= \Big(\gamma_0 + \sum_{i,\ \Lambda_i<0} G_i - \sum_{i,\Lambda_i<0} \widehat{C}_i^- + \sum_{i,\ \Lambda_i>0} \gamma_i\Big)^- \\
&\le \Big(\gamma_0 - \sum_{i,\Lambda_i<0} \widehat{C}_i^- + \sum_{i,\ \Lambda_i>0} \gamma_i\Big)^- \le \Big(- \sum_{i,\Lambda_i<0} \widehat{C}_i^- + \sum_{i,\ \Lambda_i>0} \gamma_i\Big)^-,
\end{aligned}$$

In the third line we used that $G_i = g_i$ if $\Lambda_i > 0$. By Theorem 1(i) we have

$$\sum_{i,\ \Lambda_i>0} \widehat{C}_i^- \le \Big(- \sum_{i,\Lambda_i<0} C_i^- + \sum_{i,\ \Lambda_i>0} \gamma_i\Big)^-. \tag{53}$$

On the other hand, under the assumption of "full reinsurance" we have

$$\begin{aligned}
\sum_{i,\ \Lambda_i>0} C_i^- = \sum_{i,\ \Lambda_i>0} \Big(\gamma_i - \sum_{j=1}^m (L_{ji} - L_{ji}^*)\Big)^- &\ge \Big(\sum_{i,\ \Lambda_i>0} \gamma_i - \sum_{i,\ \Lambda_i>0}\sum_{j=1}^m (L_{ji} - L_{ji}^*)\Big)^- \\
&= \Big(\sum_{i,\ \Lambda_i>0} \gamma_i - \sum_{j,\Lambda_j<0} C_j^-\Big)^-,
\end{aligned} \tag{54}$$

where in the last line we used the assumption that there is a link only between "reinsurers" and primary "insurers": in this case $\sum_{i,\ \Lambda_i>0}\sum_{j=1}^m (L_{ji} - L_{ji}^*) = \sum_{i,\ \Lambda_i>0}\sum_{j,\ \Lambda_i<0} C_j^- \Pi_{ji}$ and $\sum_{i,\ \Lambda_i>0} \Pi_{ji} = 1$ for all j with $\Lambda_j < 0$. Inequalities (53) and (54) give that $\sum_{i,\ \Lambda_i>0} \widehat{C}_i^- \le \sum_{i,\ \Lambda_i>0} C_i^-$, i.e., the CN reduces the shortfall to the end users.

Proof of Lemma 5.

(i) First consider the case of "senior CN." From (25) and (28), we infer (for $j \neq i$)

$$\begin{aligned}
\frac{\partial \widehat{C}_i}{\partial g_j} &= -\widehat{\Pi}_{0i} \frac{\partial \widehat{C}_0^-}{\partial g_j} + G_i \frac{\partial (\mathcal{T}^+/G_{\text{tot}})}{\partial g_j} \\
&= \Big(\widehat{\Pi}_{0i} \mathbb{1}_{\widehat{C}_0<0} + \frac{G_i}{G_{\text{tot}}}\Big(1 - \frac{\mathcal{T}}{G_{\text{tot}}}\Big)\mathbb{1}_{\mathcal{T}>0}\Big)\mathbb{1}_{g_j+\Lambda_j>0} \ \ge 0.
\end{aligned}$$

We also have

$$\begin{aligned}
\frac{\partial \widehat{C}_i}{\partial g_i} &= -\widehat{\Pi}_{0i} \frac{\partial \widehat{C}_0^-}{\partial g_i} - \frac{\partial G_i}{\partial g_i}\Big(1 - \frac{\mathcal{T}^+}{G_{\text{tot}}}\Big) + G_i \frac{\partial (\mathcal{T}^+/G_{\text{tot}})}{\partial g_i} \\
&= \Big(\widehat{\Pi}_{0i} \mathbb{1}_{\widehat{C}_0<0} - \Big(1 - \frac{\mathcal{T}^+}{G_{\text{tot}}}\Big) + \frac{G_i}{G_{\text{tot}}}\Big(1 - \frac{\mathcal{T}}{G_{\text{tot}}}\Big)\mathbb{1}_{\mathcal{T}>0}\Big)\mathbb{1}_{g_i+\Lambda_i>0} \\
&= \Big(\widehat{\Pi}_{0i} \mathbb{1}_{\widehat{C}_0<0} - 1 + \Big(\frac{\mathcal{T}}{G_{\text{tot}}} + \frac{G_i}{G_{\text{tot}}}\Big(1 - \frac{\mathcal{T}}{G_{\text{tot}}}\Big)\Big)\mathbb{1}_{\mathcal{T}>0}\Big)\mathbb{1}_{g_i+\Lambda_i>0} \\
&\le \Big(\widehat{\Pi}_{0i} \mathbb{1}_{\widehat{C}_0<0} - 1 + \mathbb{1}_{\mathcal{T}>0}\Big)\mathbb{1}_{g_i+\Lambda_i>0} \le 0
\end{aligned}$$

where in fourth line we used that $\frac{G_i}{G_{\text{tot}}} \le 1$.

From (28) and (27) we obtain

$$\frac{\partial \widehat{C}_i}{\partial f} = -\Lambda_i^+ - \widehat{\Pi}_{0i}\frac{\partial \widehat{C}_0^-}{\partial f} = -\Lambda_i^+ + \frac{\Lambda_i^+}{\sum_{j=1}^m \Lambda_j^+}\sum_{j=1}^m \Lambda_j^+ \mathbb{1}_{\widehat{C}_0<0} = -\Lambda_i^+\mathbb{1}_{\widehat{C}_0\geq 0} \leq 0.$$

Consider now the case "junior CN."
From (25) and (28), we infer (for $j \neq i$)

$$\begin{aligned}\frac{\partial \widehat{C}_i}{\partial g_j} &= -\widehat{\Pi}_{0i}\frac{\partial \widehat{C}_0^-}{\partial g_j} + G_i\frac{\partial(1\wedge(\widehat{A}_0-\widehat{L}_0)^+/G_{\text{tot}})}{\partial g_j}\\ &= \Big(\widehat{\Pi}_{0i}\mathbb{1}_{\widehat{C}_0<0} + \frac{G_i}{G_{\text{tot}}}\Big(1-\frac{\widehat{A}_0-\widehat{L}_0}{G_{\text{tot}}}\Big)\mathbb{1}_{0<\widehat{A}_0-\widehat{L}_0<G_{\text{tot}}}\Big)\mathbb{1}_{g_j+\Lambda_j>0} \ \geq 0,\end{aligned}$$

Moreover,

$$\begin{aligned}\frac{\partial \widehat{C}_i}{\partial g_i} &= -\widehat{\Pi}_{0i}\frac{\partial \widehat{C}_0^-}{\partial g_i} - \frac{\partial G_i}{\partial g_i}\Big(1-1\wedge(\widehat{A}_0-\widehat{L}_0)^+/G_{\text{tot}}\Big) + G_i\frac{\partial(1\wedge(\widehat{A}_0-\widehat{L}_0)^+/G_{\text{tot}})}{\partial g_j}\\ &= \Big(\widehat{\Pi}_{0i}\mathbb{1}_{\widehat{C}_0<0} - \Big(1-1\wedge(\widehat{A}_0-\widehat{L}_0)^+/G_{\text{tot}}\Big)\\ &\quad + \frac{G_i}{G_{\text{tot}}}\Big(1-\frac{\widehat{A}_0-\widehat{L}_0}{G_{\text{tot}}}\Big)\mathbb{1}_{0<\widehat{A}_0-\widehat{L}_0<G_{\text{tot}}}\Big)\mathbb{1}_{g_i+\Lambda_i>0}\\ &= \Big(\widehat{\Pi}_{0i}\mathbb{1}_{\widehat{C}_0<0}\mathbb{1}_{\widehat{A}_0-\widehat{L}_0>G_{\text{tot}}} + \Big(\frac{G_i}{G_{\text{tot}}}-1\Big)\Big(1-\frac{\widehat{A}_0-\widehat{L}_0}{G_{\text{tot}}}\Big)\mathbb{1}_{0<\widehat{A}_0-\widehat{L}_0<G_{\text{tot}}}\Big)\mathbb{1}_{g_i+\Lambda_i>0}\\ &= -\Big(1-\frac{G_i}{G_{\text{tot}}}\Big)\Big(1-\frac{\widehat{A}_0-\widehat{L}_0}{G_{\text{tot}}}\Big)\mathbb{1}_{0<\widehat{A}_0-\widehat{L}_0<G_{\text{tot}}}\mathbb{1}_{g_i+\Lambda_i>0} \leq 0\end{aligned}$$

where in the fourth line we used that $\mathbb{1}_{\widehat{C}_0<0}\mathbb{1}_{\widehat{A}_0-\widehat{L}_0>G_{\text{tot}}} = 0$ and $\frac{G_i}{G_{\text{tot}}} \leq 1$.
From (28) and (27) we obtain similarly as in the case with "senior CN"

$$\frac{\partial \widehat{C}_i}{\partial f} = -\Lambda_i^+ - \widehat{\Pi}_{0i}\frac{\partial \widehat{C}_0^-}{\partial f} = -\Lambda_i^+ + \frac{\Lambda_i^+}{\sum_{j=1}^m \Lambda_j^+}\sum_{j=1}^m \Lambda_j^+ \mathbb{1}_{\widehat{C}_0<0} = -\Lambda_i^+\mathbb{1}_{\widehat{C}_0\geq 0} \leq 0.$$

(ii) From the aggregate surplus identity (24) we have

$$\sum_{k=0}^m \widehat{C}_k = \sum_{k=0}^m \gamma_k + \sum_{i=1}^m Q_i - \sum_{i=1}^m \widehat{Z}_i(Q_i-P_i) - \widehat{C}_0^- - \sum_{i=1,\ \Lambda_i<0}^m \widehat{C}_i^- - \sum_{i,\ \Lambda_i>0} D_i.$$

It is immediate to see, using Lemma 2, that the dependence of this sum on $(f, \boldsymbol{g})$ comes only from $\widehat{C}_0^-$. Thus, by (27), this sum is nondecreasing in $(f, \boldsymbol{g})$, as stated.

(iii) We have

$$\frac{\partial \sum_{k=1}^m \widehat{C}_k}{\partial g_i} = \frac{\partial \sum_{k=0}^m \widehat{C}_k}{\partial g_i} - \frac{\partial \widehat{C}_0}{\partial g_i} = -\frac{\partial \widehat{C}_0^-}{\partial g_i} - \frac{\partial \widehat{C}_0}{\partial g_i} = -\frac{\partial \widehat{C}_0^+}{\partial g_i} \leq 0,$$

and, similarly,

$$\frac{\partial \sum_{k=1}^m \widehat{C}_k}{\partial f} = -\frac{\partial \widehat{C}_0^-}{\partial f} - \frac{\partial \widehat{C}_0}{\partial f} = -\frac{\partial \widehat{C}_0^+}{\partial f} \leq 0.$$

Proof of Theorem 2. We have that

$$\begin{aligned}\widehat{\mathcal{A}}_\alpha - \mathcal{A}_\alpha &= -\alpha\sum_{i=0}^m(\widehat{C}_i^+ - C_i^+) - \sum_{i=1}^m(\widehat{D}_i^* - D_i)\\ &= -\alpha\sum_{i=1}^m(Z_i - \widehat{Z}_i)(Q_i - P_i) + (1-\alpha)\sum_{i=1}^m(D_i - \widehat{D}_i^*),\end{aligned}$$

where in the second line we used (8). The first term is always negative by Theorem 1, and contributes to a reduction of systemic risk. The second term, as we have seen, can be positive or negative depending on the realized network: in networks with large asymmetries ("overinsured" and "underinsured") the shortfall on end users increases with a CN, while in symmetric networks the shortfall decreases. As a consequence of Lemma 3, $\sum_{i=1}^m(Z_i - \widehat{Z}_i)(Q_i - P_i)$ does not depend on γ_0 and the fee and guaranty fund policy $(f, \boldsymbol{g})$ and can be computed using only the parameters of the initial, uncleared interbank network.

Now using (32), we obtain

$$\begin{aligned}\mathcal{R}(\widehat{\boldsymbol{C}}) - \mathcal{R}(\boldsymbol{C}) &= \mathbb{E}\Big(\mathcal{A}_\alpha(\widehat{\boldsymbol{C}})\Big) - \mathbb{E}\Big(\mathcal{A}_\alpha(\boldsymbol{C})\Big) = \mathbb{E}\Big(\mathcal{A}_\alpha(\widehat{\boldsymbol{C}}) - \mathcal{A}_\alpha(\boldsymbol{C})\Big)\\ &= (1-\alpha)\mathbb{E}\Big(D_i - \widehat{D}_i^*\Big) + \mathbb{E}\Big(-\alpha\sum_{i=1}^m(Z_i - \widehat{Z}_i)(Q_i - P_i)\Big).\end{aligned}$$

Proof of Theorem 3. By Theorem 2 we have $\mathcal{R}(\boldsymbol{C}) - \mathcal{R}(\widehat{\boldsymbol{C}}) = \mathbb{E}[\alpha\sum_{i=1}^m(Z_i - \widehat{Z}_i)(Q_i - P_i) - (1-\alpha)\sum_{i=1}^m(D_i^* - \widehat{D}_i^*)]$, so that the CN reduces the systemic risk by at least ℓ, i.e., $\mathcal{R}(\boldsymbol{C}) - \mathcal{R}(\widehat{\boldsymbol{C}}) \geq \ell$, if and only if

$$\alpha\sum_{i=1}^m\mathbb{E}[(Z_i - \widehat{Z}_i)(Q_i - P_i)] + (1-\alpha)\sum_{i=1}^m\mathbb{E}[\widehat{D}_i^* - D_i^*] \geq \ell. \tag{55}$$

We first prove that there exists γ_0^* such that (55) holds for any $\gamma_0 \geq \gamma_0^*$, $f \in [0,1]$, and $g \in [0,\gamma]$. By Lemma 4 we have

$$\widehat{C}_0^- = \Big(\gamma_0 + \sum_{i=1}^m\Big(f\Lambda_i^+ + G_i - \widehat{C}_i^-\Big)\Big)^-,$$

which statewise and monotonically tends to 0 as $\gamma_0 \to \infty$. Under the assumption of full reinsurance, we have that $\widehat{D}_i^* = D_i$ for all i with $\Lambda_i > 0$, and thus $\lim_{\gamma_0\to\infty}\mathbb{E}[\sum_{i=1}^m(\widehat{D}_i^* - D_i^*)] \to \mathbb{E}[\sum_{i=1}^m(D_i - D_i^*)] > 0$ and the convergence is uniform over all $g \in [0,\gamma]$ and $f \in [0,1]$. The limit is attained due to the integrability assumptions on L.

Recall from Theorem 2 that $\widehat{Z}_i$ does not depend on γ_0, and we assume that $\ell \leq \alpha\sum_{i=1}^m \mathbb{E}[(Z_i - \widehat{Z}_i)(Q_i - P_i)] + (1-\alpha)\mathbb{E}[\sum_{i=1}^m(D_i - D_i^*)]$. By monotone convergence, there exists γ_0^* such that (55) holds for any $\gamma_0 \geq \gamma_0^*$, $f \in [0,1]$ and $g \in [0,\gamma]$.

For the case "senior CN," we show that there exists γ_0' such that (40) is verified for $\gamma_0 > \gamma_0'$ uniformly over $\mathcal{G} := \{g \mid \mathbb{E}[\sum_{i=1}^m f\Lambda_i^+ - (G_{\mathrm{tot}} - \sum_{i,\ \Lambda_i<0}\widehat{C}_i^-)^-] > 0\}$. Using the convexity we have that $\mathbb{E}[\widehat{C}_0^+] = \mathbb{E}[(\gamma_0 + f\Lambda_i^+ - (G_{\mathrm{tot}} - \sum_{i,\ \Lambda_i<0}\widehat{C}_i^-)^-)^+] \geq \gamma_0$ for all γ_0 and $g \in \mathcal{G}$. For the second inequality, we have that $\lim_{\gamma_0\to\infty}\mathbb{E}[\widehat{C}_0^+]-\gamma_0=\mathbb{E}[\sum_{i=1}^m(f\Lambda_i^+-(G_{\text{tot}}-\sum_{i,\ \Lambda_i<0}\widehat{C}_i^-)^-)]$, whereas $\sum_{i=1}^m\mathbb{E}[(Z_i-\widehat{Z}_i)(Q_i-P_i)]+\sum_{i=1}^m\mathbb{E}[D_i^*-\widehat{D}_i^*]\to\sum_{i=1}^m\mathbb{E}[(Z_i-\widehat{Z}_i)(Q_i-P_i)]+\sum_{i=1}^m\mathbb{E}[D_i^*-D_i]$. The second inequality of (40) holds in the limit under assumption (41). For the case "junior CN" the first inequality of (40) is verified under condition $\mathbb{E}[\sum_{i=1}^m f\Lambda_i^+-\sum_{i,\ \Lambda_i<0}\widehat{C}_i^-]>0$. The second inequality holds in the limit under assumption (41).

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