Downlink Sum-Rate Maximization for Rate Splitting Multiple Access (RSMA)

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Abstract*—In this paper, the sum-rate maximization problem is studied for wireless networks that use downlink rate splitting multiple access (RSMA). In the considered model, each base station (BS) divides the messages that must be transmitted to its users into a "private" part and a "common" part. Here, the common message is a message that all users want to receive and the private message is a message that is dedicated to only a specific user. The RSMA mechanism enables a BS to adjust the split of common and private messages so as to control the interference by decoding and treating interference as noise and, thus optimizing the data rate of users. To maximize the users' sum-rate, the network can determine the rate allocation for the common message to meet the rate demand, and adjust the transmit power for the private message to reduce the interference. This problem is formulated as an optimization problem whose goal is to maximize the sum-rate of all users. To solve this nonconvex maximization problem, the optimal power used for transmitting the private message to the users is first obtained in closed form for a given rate allocation and common message power. Based on the optimal private message transmission power, the optimal rate allocation is then derived under a fixed common message transmission power. Subsequently, a one-dimensional search algorithm is proposed to obtain the optimal solution of common message transmission power. Simulation results show that the RSMA can achieve up to 19.6% and 23.5% gains in terms of data rate compared to non-orthogonal multiple access (NOMA) and orthogonal frequency-division multiple access (OFDMA), respectively.

I. Introduction

Driven by the rapid development of advanced multimedia applications, next-generation wireless networks [1]–[4] must support high spectral efficiency and massive connectivity. By splitting users in the power domain, non-orthogonal multiple access (NOMA) can simultaneously serve multiple users at the same frequency or time resource [5]–[9]. Consequently, NOMA-based access scheme can achieve higher spectral efficiency than conventional orthogonal multiple access (OMA). However, using NOMA, the users must decode all of the interference as they receive the messages [10], which significantly increases the computational complexity needed for signal processing. To solve this problem, the idea of rate splitting multiple access (RSMA) was proposed in [11]–[17]. In RSMA, the message transmitted to the users is divided into a common message and a private message. The common message is a message that all users must receive and the private message is a message that only a specific intended user wishes to receive. To receive the common message, the users must decode the interference from other users. In contrast, to receive the private message, the users must only consider the interference from other users' private messages which can be treated as noise. Therefore, adjusting the split of common and private messages can control the computational complexity and the data rate achieved by RSMA.

Recently, a number of existing works such as in [12], [18]–[21] studied important problems related to RSMA. The work in [12] introduced the challenges and opportunities of using RSMA for multiple input multiple output (MIMO) based wireless networks. In [18], the authors proposed a distributed rate splitting method to maximize the data rates of the users. The authors in [19] evaluated the performance of RSMA, NOMA, and space-division multiple access (SDMA) and showed that RSMA achieves better performance than NOMA and SDMA. The authors in [20] investigated the use of linearly-precoded rate-splitting method for simultaneous wireless information and power transfer networks. In [21], the authors used RSMA to maximize the rate of all users in downlink multi-user multiple input single output (MISO) systems under imperfect channel state information at the transmitter. However, none of these existing works [12], [18]–[21] considers a successive interference cancelation (SIC) constraint for the private message transmission in RSMA, which is needed to guarantee the successful decoding of the common message.

The main contribution of this paper is an optimized rate allocation and power control scheme for RSMA in a downlink SISO system. Our key contributions include:

- We propose a wireless network that uses RSMA and in which one base station (BS) transmits message to multiple users using RSMA scheme. The sum-rate maximization problem is formulated via rate allocation and power control under both rate and SIC constraints.
- To solve this problem, we first derive a closed-form expression for the optimal transmit power of the private message. Then, we characterize the finite solution space for the optimal rate allocation. In order to obtain the optimal rate allocation and power control, we propose a one-dimensional search algorithm.
- Simulation results show that the optimized RSMA al-

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gorithm can achieve up to 15.6% and 21.5% gains in terms of data rate compared to NOMA and orthogonal frequency-division multiple access (OFDMA).

The system model and problem formulation are described in Section II. The optimal solution is presented in Section III. Simulation results are analyzed in Section IV. Conclusions are drawn in Section V.

II. System Model and Problem Formulation

A. System Model

Consider the downlink a single-cell wireless network that consists of one BS serving a set $\mathcal{O}$ of K users using RSMA [11]. In RSMA, the common message is decoded by all users, while the individual private message is only decoded by each user. At the receiver side, each user first decodes the common message and then decodes its private message using the previously decoded common message.

Let the common message of all users be s_0 and the private message of each user k be s_k. The transmitted signal x of the BS is expressed as: $x = \overline{p_0}s_0 + \sum_{k=1}^{K} \overline{p_k}s_k$, where p_0 is the transmit power of the common message s_0 and p_k is the transmit power of the private message s_k.

The total received message at user k can be given by:

$$y_k = \sqrt{h_k}x + n_k = \sqrt{h_k p_0}s_0 + \sum_{j=1}^{K} \sqrt{h_k p_j}s_j + n_k, \quad (1)$$

where h_k represents the channel gain between user k and the BS and n_k is the additive white Gaussian noise with variance σ^2. The achievable rate of user k decoding common message s_0 can be expressed as:

$$c_k = B\log_2\left(1 + \frac{h_k p_0}{h_k \sum_{j=1}^{K} p_j + \sigma^2}\right), \quad (2)$$

where B is the bandwidth of the BS. Without loss of generality, the channel gains are sorted in ascending order, i.e., $h_1 \geq h_2 \geq \times\times\times \geq h_K$. To ensure that all users can successfully decode common message s_0, the rate of common message should be chosen as [19]:

$$\begin{aligned}\min_{k\in\mathcal{K}} c_k &= \min_{k\in\mathcal{K}} B\log_2\left(1 + \frac{p_0}{\sum_{j=1}^{K} p_j + \frac{\sigma^2}{h_k}}\right)\\ &= B\log_2\left(1 + \frac{p_0}{\sum_{j=1}^{K} p_j + \frac{\sigma^2}{\min_{k\in\mathcal{K}} h_k}}\right)\\ &\overset{(a)}{=} B\log_2\left(1 + \frac{p_0}{\sum_{j=1}^{K} p_j + \frac{\sigma^2}{h_1}}\right) = c_1, \quad (3)\end{aligned}$$

where equality (a) follows from the fact that $h_1 \geq \times\times\times \geq h_K$.

To successfully implement SIC operation at the receiver, the transmit power of each user must satisfy the following constraint [22]:

$$h_k p_0 \quad h_k \sum_{j=1}^{K} p_j \quad \sigma^2 \in \theta, \quad \mathcal{K}k \ / \ \mathcal{O}, \quad (4)$$

where θ is the minimum difference between the decoding signal power and the non-decoded inter-user interference signal power plus noise power [22]. This minimum difference is required to distinguish the common message to be decoded and the remaining non-decoded private message of all users (plus noise). Based on the channel condition $h_1 \geq h_2 \geq \times\times\times \geq h_K$, constraint (4) can be simplified as:

$$p_0 \quad \sum_{j=1}^{K} p_j \in \frac{\theta + \sigma^2}{h_1}. \quad (5)$$

Given the common message rate c_1 and the rate a_k allocated to user k, the constraint of each user k's data rate of receiving common message is given by: $\sum_{k=1}^{K} a_k \geq c_1$, which indicates that the total data rates of all users receiving common message must be less than the rate of common message c_1.

After having decoded the common message s_0, each user can decode its private message, the achievable rate of user k decoding its private message s_k is given by:

$$r_k = B\log_2\left(1 + \frac{h_k p_k}{h_k \sum_{j=1, j\neq k}^{K} p_j + \sigma^2}\right). \quad (6)$$

Given the common message rate a_k and achievable private message rate r_k, the total transmission rate of user k in RSMA is $r_k^{\text{tot}} = a_k + r_k$.

B. Problem Formulation

For our system model, the objective is to optimize the rate allocation and power control so as to maximize the sum-rate under a total power constraint and individual minimum rate requirements. Formally, the sum-rate maximization problem for RSMA can be formulated as:

$$\max_{\boldsymbol{a},\boldsymbol{p}} \quad \sum_{k=1}^{K}\left(a_k + B\log_2\left(1 + \frac{h_k p_k}{h_k \sum_{j=1, j\neq k}^{K} p_j + \sigma^2}\right)\right), \quad (7)$$

$$\text{s.t.} \quad \sum_{k=1}^{K} a_k \geq B\log_2\left(1 + \frac{h_1 p_0}{h_1 \sum_{j=1}^{K} p_j + \sigma^2}\right), \quad (7a)$$

$$a_k + B\log_2\left(1 + \frac{h_k p_k}{h_k \sum_{j=1, j\neq k}^{K} p_j + \sigma^2}\right) \in R_k, \quad \mathcal{K}k \ / \ \mathcal{O} \quad (7b)$$

$$p_0 \quad \sum_{j=1}^{K} p_j \in \frac{\theta + \sigma^2}{h_1}, \quad (7c)$$

$$\sum_{k=0}^{K} p_k \geq P, \quad (7d)$$

$$a_k, p_0, p_k \in 0, \quad \mathcal{K}k \ / \ \mathcal{O}, \quad (7e)$$

where $\boldsymbol{a} = [a_1, a_2, \times\times\times, a_K]$, $\boldsymbol{p} = [p_0, p_1, p_2, \times\times\times, p_K]$, R_k is the minimum rate demand of user k, and P is the maximum transmit power of the BS. Constraint (7a) ensures that each user can decode the common message. The minimum rate constraints for all users are given in (7b). Constraint (7c) shows the successful SIC power requirement, and (7d) presents the maximum power constraint. Since the objective function is not concave, the sum-rate maximization problem (7) is nonconvex. Moreover, the rate and power vectors are coupled in the objective function and constraints, and hence, it is generally hard to solve problem (7). Despite the nonconvexity and coupling of variables in problem (7), the globally optimal solution to problem (7) can be effectively obtained in the following section.

III. Optimal Rate Allocation and Power Control

In this section, we first provide the optimal conditions of problem (7). Then, based on these optimal conditions, the optimal private message transmission power is obtained in closed form under a given rate allocation and common message transmission power. Substituting the optimal private message transmission power in (7), the optimal closed-form rate allocation is then derived under a fixed common message transmission power. Finally, a one-dimensional search algorithm is proposed to obtain the optimal solution of (7).

A. Optimal Conditions

Before solving problem (7), we provide some optimal conditions, which will be used to simplify problem (7).

Lemma 1: At the optimal solution $(\boldsymbol{a}^*, \boldsymbol{p}^*)$ of problem (7), both common message constraint (7b) and maximum power constraint (7d) hold with equality.

Since Lemma 1 can be easily proved by the contradiction method, the proof is omitted here.

Substituting $\sum_{j=1,j\neq k}^{K} p_j = P \quad p_0 \quad p_k$ from Lemma 1 to (7), we can then observe that problem (7) is equivalent to the following problem:

$$\max_{\boldsymbol{a},\boldsymbol{p}} \quad \sum_{k=1}^{K} a_k + \sum_{k=1}^{K} B\log_2\left(\frac{h_k(P \quad p_0)+\sigma^2}{h_k(P \quad p_0 \quad p_k)+\sigma^2}\right), \tag{8}$$

$$\text{s.t.} \quad \sum_{k=1}^{K} a_k = B\log_2\left(\frac{h_1 P+\sigma^2}{h_1(P \quad p_0)+\sigma^2}\right), \tag{8a}$$

$$a_k + B\log_2\left(\frac{h_k(P \quad p_0)+\sigma^2}{h_k(P \quad p_0 \quad p_k)+\sigma^2}\right) \in R_k, \mathcal{K}k \ / \ \mathcal{O}, \tag{8b}$$

$$\sum_{k=0}^{K} p_k = P, \tag{8c}$$

$$p_0 \in \frac{P}{2} + \frac{\theta+\sigma^2}{2h_1}, \tag{8d}$$

$$a_k, p_k \in 0, \quad \mathcal{K}k \ / \ \mathcal{O}. \tag{8e}$$

To solve problem (8), we can show that it is equivalent to another optimization problem, which admits a closed-form solution for the optimal private message transmission power.

Lemma 2: The optimal solution of problem (8) is equivalent to the following problem:

$$\max_{\boldsymbol{a},\boldsymbol{p}} \quad B\log_2\left(\frac{h_1 P+\sigma^2}{h_1(P \quad p_0)+\sigma^2}\right) + \sum_{k=1}^{K} B\log_2\left(\frac{h_k(P \quad p_0)+\sigma^2}{h_k(P \quad p_0 \quad p_k)+\sigma^2}\right), \tag{9}$$

$$\text{s.t.} \quad \sum_{k=1}^{K} a_k \geq B\log_2\left(\frac{h_1 P+\sigma^2}{h_1(P \quad p_0)+\sigma^2}\right), \tag{9a}$$

$$(8b), (8c) \tag{9b}$$

$$0 \geq a_k \geq R_k, p_k \in 0, \quad \mathcal{K}k \ / \ \mathcal{O}. \tag{9c}$$

Proof: See Appendix A. □

Note that the maximum rate limitation $0 \geq a_k \geq R_k$ is added in constraint (9c), which will be helpful in obtaining the optimal private message transmission power in closed form.

B. Optimal Private Message Transmission Power

Given the simplified problem in (9), next, we find the optimal private message transmission power. With fixed $\boldsymbol{a}$ and p_0, problem (9) becomes

$$\max_{\bar{\boldsymbol{p}}} \quad \sum_{k=1}^{K} B\log_2\left(\frac{h_k(P \quad p_0)+\sigma^2}{h_k(P \quad p_0 \quad p_k)+\sigma^2}\right), \tag{10}$$

$$\text{s.t.} \quad \sum_{k=1}^{K} p_k = P \quad p_0, \tag{10a}$$

$$p_k \in p_k^{\min}, \quad \mathcal{K}k \ / \ \mathcal{O}, \tag{10b}$$

where $\bar{\boldsymbol{p}} = [p_1, p_2, \times\!\!\times\!\!\times, p_K]$ is a vector of power that is allocated to each user for receiving private message and

$$p_k^{\min} = \left(1 \quad 2^{\frac{a_k - R_k}{B}}\right)\left(P \quad p_0 + \frac{\sigma^2}{h_k}\right). \tag{11}$$

Due to constraint (9c), $p_k^{\min}$ is always non-negative.

Note that the fist term in objective function (9) is a constant with given common message power control p_0, thus the fist term in objective function (9) is omitted in problem (10). In (10b), $p_k^{\min}$ is used to meet the minimum rate constraint in (9b), and problem (10) is feasible if and only if $\sum_{k=1}^{K} p_k^{\min} \geq P \quad p_0$, which can be given as:

$$\sum_{k=1}^{K}\left(1 \quad 2^{\frac{a_k - R_k}{B}}\right)\left(P \quad p_0 + \frac{\sigma^2}{h_k}\right) \geq P \quad p_0. \tag{12}$$

Since objective function is convex, we can infer that the maximization problem (10) is nonconvex. To effectively solve problem (10), the following lemma is presented.

Lemma 3: For the optimal solution $\bar{\boldsymbol{p}}^*$ of problem (10), there exists one k such that $p_k^* = P \quad p_0 \quad \sum_{j=1,j\neq k}^{K} p_j^{\min}$ and $p_j^* = p_j^{\min}$, $\mathcal{K}j \ / \ \mathcal{O}, j \not\equiv k$.

Lemma 3 is reasonable due to the fact that the maximization of a convex function always lies in the corner points of the feasible solution. Since there are K corner points satisfying constraints (10a) and (10b), Lemma 3 is thus proved.

From Lemma 3, the structure of the optimal solution of problem (10) is revealed. Although problem (10) is nonconvex, the optimal solution can be obtained in closed form, which can be given by the following theorem.

Theorem 1: For nonconvex problem (10), the optimal power allocation $\bar{\boldsymbol{p}}^*$ is

$$p_k^* = P \quad p_0 \quad \sum_{j=1,j\neq k}^{K}\left(1 \quad 2^{\frac{a_j - R_j}{B}}\right)\left(P \quad p_0 + \frac{\sigma^2}{h_j}\right), \tag{13}$$

$$p_j^* = \left(1 \quad 2^{\frac{a_j - R_j}{B}}\right)\left(P \quad p_0 + \frac{\sigma^2}{h_j}\right), \quad \mathcal{K}j \ / \ \mathcal{O}, j \not\equiv k, \tag{14}$$

and the optimal sum-rate of private message is

$$B\log_2\left(\frac{P \quad p_0 + \frac{\sigma^2}{h_k}}{\sum_{j=1,j\neq k}^{K}\left(1 \quad 2^{\frac{a_j - R_j}{B}}\right)\left(P \quad p_0 + \frac{\sigma^2}{h_j}\right) + \frac{\sigma^2}{h_k}}\right) + \sum_{j=1,j\neq k}^{K}(R_j \quad a_j), \tag{15}$$

where

$$k = \arg\min_{j\in\mathcal{K}} 2^{\frac{a_j - R_j}{B}} \left(P - p_0 + \frac{\sigma^2}{h_j}\right). \tag{16}$$

Proof: See Appendix B. ■

Theorem 1 states that it is optimal for the BS to allocate more power to the user that can maximize the sum-rate while allocating the minimum transmit power that can meet the data rate requirement for all other users.

For the special case with $a_j = R_j$, $\forall j \in \mathcal{K}$, we can obtain $p_j^{\min} = 0$ and $k = \arg\min_{j\in\mathcal{K}} \frac{\sigma^2}{h_j} = K$ according to (16), i.e., all the power should be allocated to the user with the highest channel gain. This observation is trivial since allocating the maximum power to the user with the highest channel gain will always improve the rate.

C. *Optimal Rate Allocation*

In the previous subsection, the optimal power allocation vector $\bar{\boldsymbol{p}}$ can be obtained as a function of the rate allocation vector $\boldsymbol{a}$ and common message power p_0. Thus, substituting the optimal power allocation vector $\bar{\boldsymbol{p}}$ given in (13) and (14) in Theorem 1, the original problem in (9) is simplified as:

$$\max_{\boldsymbol{a}, p_0} \quad B\log_2\left(\frac{h_1 P + \sigma^2}{h_1(P - p_0) + \sigma^2}\right) + \sum_{j=1, j\neq k}^{K} (R_j - a_j)$$
$$+ B\log_2\left(\frac{P - p_0 + \frac{\sigma^2}{h_k}}{\sum_{j=1,j\neq k}^{K}\left(1 - 2^{\frac{a_j - R_j}{B}}\right)\left(P - p_0 + \frac{\sigma^2}{h_j}\right) + \frac{\sigma^2}{h_k}}\right), \tag{17}$$

$$\text{s.t.} \quad \sum_{j=1}^{K} a_j \geq B\log_2\left(\frac{h_1 P + \sigma^2}{h_1(P - p_0) + \sigma^2}\right), \tag{17a}$$

$$k = \arg\min_{j\in\mathcal{K}} 2^{\frac{a_j - R_j}{B}}\left(P - p_0 + \frac{\sigma^2}{h_j}\right), \tag{17b}$$

$$\sum_{j=1}^{K}\left(1 - 2^{\frac{a_j - R_j}{B}}\right)\left(P - p_0 + \frac{\sigma^2}{h_j}\right) \geq P - p_0, \tag{17c}$$

$$p_0 \in \frac{P}{2} + \frac{\theta + \sigma^2}{2h_1}, \tag{17d}$$

$$0 \geq a_j \geq R_j, \quad \forall j \in \mathcal{K}, \tag{17e}$$

where constraint (17b) is added since more power should be allocated to user k to maximize the sum-rate while other users are allocated the minimum power to ensure the minimum rate constraint. Constraint (17c) follows from (12), which ensures that the private message power control problem is feasible.

Due to objective function and constraints (17a)-(17c), problem (17) is nonconvex and, hence, it is generally hard to directly optimize rate allocation $\boldsymbol{a}$ and private power p_0. To solve problem (17), we first fix the private message transmission power and derive the optimal rate allocation. Given common message transmission power p_0, problem (17) becomes

$$\max_{\boldsymbol{a}} \quad B\log_2\left(\frac{P - p_0 + \frac{\sigma^2}{h_k}}{\sum_{j=1,j\neq k}^{K}\left(1 - 2^{\frac{a_j - R_j}{B}}\right)\left(P - p_0 + \frac{\sigma^2}{h_j}\right) + \frac{\sigma^2}{h_k}}\right)$$
$$+ \sum_{j=1,j\neq k}^{K} (R_j - a_j), \tag{18}$$

$$\text{s.t.} \quad (17a) - (17c), (17e). \tag{18a}$$

Algorithm 1 Optimal Rate Allocation and Power Control

for $p_0 = \frac{P}{2} + \frac{\theta}{2} + \frac{\sigma^2}{2h_1} : \xi : P$ **do**
  Obtain the optimal rate allocation $\boldsymbol{a}$ of problem (18) according to Theorem 2.
  Calculate the optimal common message power allocation $\bar{\boldsymbol{p}}$ of (10) according to Theorem 1.
end for
Obtain the optimal p_0 with the maximum objective value (9a).

Despite the nonconvexity of (18), the optimal solution of problem (18) can be precisely formulated in the following theorem. Before presenting the theorem, rearrange users in the descending order $\pi_1, \pi_2, \cdots, \pi_K$ of $2^{\frac{-R_j}{B}}\left(P - p_0 + \frac{\sigma^2}{h_j}\right)$, i.e.,

$$2^{\frac{-R_{\pi_j}}{B}}\left(P - p_0 + \frac{\sigma^2}{h_{\pi_j}}\right) \in 2^{\frac{-R_{\pi_l}}{B}}\left(P - p_0 + \frac{\sigma^2}{h_{\pi_l}}\right), \quad \forall j < l. \tag{19}$$

Theorem 2: The optimal solution $\boldsymbol{a}^*$ of problem (18) is:
i) if $c_1 \geq \sum_{j=1}^{K} R_j$,

$$r_{\pi_j}^* = \begin{cases} R_{\pi_j}, & \text{if } j < l, \\ c_1 - \sum_{m=1}^{j-1} R_{\pi_m}, & \text{if } j = l, \\ 0, & \text{otherwise,} \end{cases} \tag{20}$$

where l satisfies $\sum_{j=1}^{l-1} R_j \geq c_1 \geq \sum_{j=1}^{l} R_j$.
ii) if $c_1 \in \sum_{j=1}^{K} R_j$,

$$r_{\pi_j}^* = R, \quad \forall j \in \mathcal{K}. \tag{21}$$

Proof: See Appendix C. ■

D. *Optimal Rate Allocation and Power Control*

To obtain the optimal rate allocation and power control of problem (9), we propose a novel solution, shown in Algorithm 1 where ξ is the minimum step size for searching p_0. In Algorithm 1, the optimal common message power p_0 is obtained by a one-dimensional search method, while the optimal rate allocation $\boldsymbol{a}$ and private message power $\bar{\boldsymbol{p}}$ are accordingly obtained in closed form given p_0.

In each step of Algorithm 1, the main complexity lies in solving (18) given p_0. The complexity of solving the problem in (18) is $\mathcal{O}(K)$ according to Theorem 3 and the complexity of Algorithm 1 is $\mathcal{O}(LK)$, where $L = \mathcal{O}\left(\frac{Ph_1 - \theta h_1 - \sigma^2}{2h_1\xi}\right)$ denotes the number of iterations for searching p_0.

IV. Simulation Results

In this section, we evaluate the performance of the proposed optimal rate allocation and power control algorithm. There are three users uniformly distributed in a square area of size 300 m $\times$ 300 m. The path loss model is $128.1 + 37.6\log_{10} d$ (d is in km) [22] and the standard deviation of shadow fading is 4 dB. In addition, the bandwidth of the BS is $B = 1$ MHz and the noise power is $\sigma^2 = -104$ dBm. We set maximum transmit power $P = 30$ dBm, and equal rate demand $R_1 = R_2 = \cdots = R_K = R = 1$ Mbits/s. The proposed

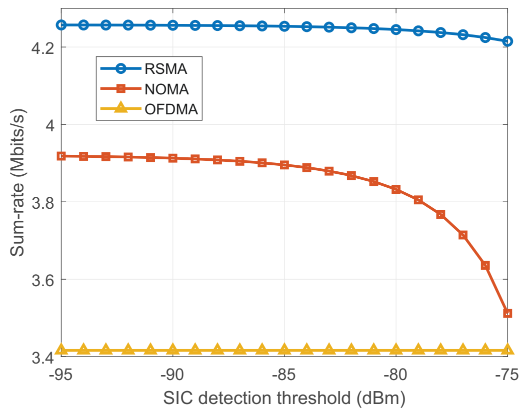

Fig. 1. Sum-rate versus SIC detection threshold.

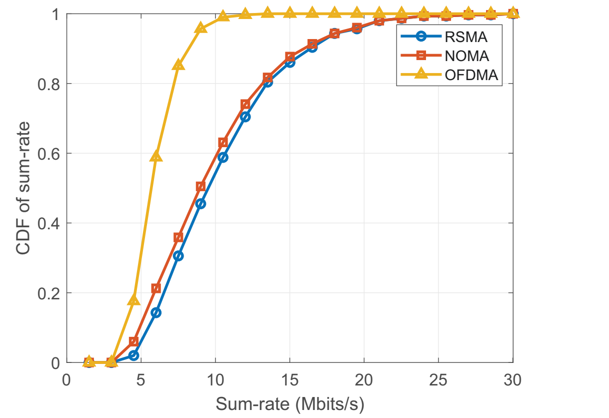

Fig. 2. CDF of the sum-rate resulting from RSMA, NOMA, and OFDMA with $\theta = -94$ dBm.

optimal rate allocation and power control algorithm for rate maximization of RSMA is labeled as 'RSMA'. We compare with the proposed algorithm with the optimal power control of NOMA for rate maximization [22], which is labeled as 'NOMA'. To compare conventional OMA, we use a OFDMA system as a baseline, which is labeled as 'OFDMA'.

Fig. 1 shows the sum-rate versus SIC detection threshold θ. For both RSMA and NOMA, we find that the sum-rate decreases as the SIC detection threshold increases. This is due to the fact that, as the SIC detection threshold increases, the BS must allocate more power to the common message in RSMA and the user with worse channel gain in NOMA. For OFDMA, naturally, the sum-rate remains the same when the SIC detection threshold increases. The proposed RSMA algorithm outperforms the NOMA in terms of sum-rate, particularly for cases with a high SIC detection threshold. Besides, RSMA can achieve up to 19.6% and 23.5% gains in terms of data rate compared to NOMA and OFDMA, respectively. Moreover, the sum-rate decreases faster for NOMA than RSMA as the SIC detection threshold increases, which implies that RSMA is more suitable for high SIC detection threshold.

Fig. 2 shows the cumulative distribution function (CDF) of the sum-rate. From Fig. 2, the CDFs for RSMA and NOMA all improve significantly vs. OFDMA especially for high sum-rate region, which shows that both RSMA and NOMA are suitable for high sum-rate transmission. Moreover, we can find that RSMA outperforms NOMA at regions with moderate data rates, i.e., 5-15 Mbits/s. This is because RSMA can adjust the split between common and private messages so as to control the interference decoding and thus optimize the sum-rate.

V. Conclusions

In this paper, we have investigated the allocation of data rate of common message transmission and the transmit power used for common and private message transmission in a SISO RSMA system. We have formulated the problem as a sum-rate maximization problem. To solve this problem, we have derived the optimal transmit power of private message in closed form. Then, we have characterized the finite solution space for the optimal rate allocation. Finally, we have proposed a one-dimensional search algorithm to find the optimal rate allocation and power control solutions. Simulation results show that RSMA achieves higher sum-rate than NOMA and OFDMA.

Appendix A
Proof of Lemma 2

If the pair $(\boldsymbol{a}, \boldsymbol{p})$ is feasible in problem (9), then the pair $(\boldsymbol{a}, \boldsymbol{p})$ is also feasible in problem (8) with the same objective value. It follows from the fact that the optimal value of (9) is less than or equal to the optimal value of (8).

Conversely, if the pair $(\boldsymbol{a}, \boldsymbol{p})$ is feasible in (8), we can construct a new pair $(\boldsymbol{a}', \boldsymbol{p})$, where

$$a'_k = \min\}a_k, R_k \;,\; \mathcal{K}k \;/\; \mathcal{O}. \tag{A.1}$$

It can be shown that solution $(\boldsymbol{a}', \boldsymbol{p})$ is feasible in problem (9). Moreover, the objective value of problem (8) is the same as problem (9). Thus, we conclude that the optimal value of (9) is greater than or equal to the optimal value of (8). Hence, problem (9) is equivalent to problem (8).

Appendix B
Proof of Theorem 1

Substituting the optimal power $p_k^* = P \quad p_0 \quad \sum_{j=1,j\neq k}^{K} p_j^{\min}$ and $p_j^* = p_j^{\min}$, $\mathcal{K}j \;/\; \mathcal{O}, j \not\forall k$ to problem (10) based on Lemma 3, we can obtain the objective value (10a) as

$$\sum_{j=1,j\neq k}^{K} B\log_2\left(\frac{h_j(P \quad p_0)+\sigma^2}{h_j(P \quad p_0 \quad p_j^{\min})+\sigma^2}\right) + B\log_2\left(\frac{h_k(P \quad p_0)+\sigma^2}{h_k \sum_{j=1,j\neq k}^{K} p_j^{\min}+\sigma^2}\right). \tag{B.1}$$

To maximize (B.1), the optimal k should be chosen as

$$\begin{aligned} k &= \arg\max_{m\in\mathcal{K}} \sum_{j=1}^{K} B\log_2\left(\frac{h_j(P \quad p_0)+\sigma^2}{h_j(P \quad p_0 \quad p_j^{\min})+\sigma^2}\right) \\ &\qquad B\log_2\left(\frac{h_m(P \quad p_0)+\sigma^2}{h_m(P \quad p_0 \quad p_m^{\min})+\sigma^2}\right) \\ &\qquad + B\log_2\left(\frac{h_m(P \quad p_0)+\sigma^2}{h_m \sum_{j=1,j\neq m}^{K} p_j^{\min}+\sigma^2}\right) \\ &= \arg\max_{m\in\mathcal{K}} \frac{h_m(P \quad p_0 \quad p_m^{\min})+\sigma^2}{h_m \sum_{j=1,j\neq m}^{K} p_j^{\min}+\sigma^2} \quad 1 \\ &= \arg\min_{m\in\mathcal{K}} \frac{\sigma^2}{h_m} \quad p_m^{\min}. \end{aligned} \tag{B.2}$$

Substituting (11) to (B.2), we have

$$k = \arg\min_{j\in\mathcal{K}} \frac{\sigma^2}{h_j} \quad \left(1 \quad 2^{\frac{a_j - R_j}{B}}\right)\left(P \quad p_0 + \frac{\sigma^2}{h_j}\right)$$
$$= \arg\min_{j\in\mathcal{K}} 2^{\frac{a_j - R_j}{B}} \left(P \quad p_0 + \frac{\sigma^2}{h_j}\right). \tag{B.3}$$

This completes the proof of Theorem 1.

Appendix C
Proof of Theorem 2

We first show that $a^*_{\pi_m} \in a^*_{\pi_n}$ for all $m < n$ for the optimal solution $\boldsymbol{a}^*$ of problem (18). This can be proved by the contradiction method. If there exists $m < n$ such that $a^*_{\pi_m} < a^*_{\pi_n}$, we can construct a new solution $\boldsymbol{a}'$ with $a'_{\pi_m} = a^*_{\pi_n}$, $a'_{\pi_n} = a^*_{\pi_m}$, $a'_j = a^*_j$ for $j \not\forall m, n$. Then, we have:

$$\sum_{j=1}^{K}\left(1 \quad 2^{\frac{a'_{\pi_j} - R_{\pi_j}}{B}}\right)\left(P \quad p_0 + \frac{\sigma^2}{h_{\pi_j}}\right)$$
$$= \sum_{j=1}^{K}\left(1 \quad 2^{\frac{a^*_{\pi_j} - R_{\pi_j}}{B}}\right)\left(P \quad p_0 + \frac{\sigma^2}{h_{\pi_j}}\right)$$
$$+ \left(2^{\frac{a^*_m}{B}} \quad 2^{\frac{a^*_n}{B}}\right)\left(2^{\frac{-R_{\pi_m}}{B}}\left(P \quad p_0 + \frac{\sigma^2}{h_{\pi_m}}\right)\right.$$
$$\left. 2^{\frac{-R_{\pi_n}}{B}}\left(P \quad p_0 + \frac{\sigma^2}{h_{\pi_n}}\right)\right)$$
$$\geq \sum_{j=1}^{K}\left(1 \quad 2^{\frac{a^*_{\pi_j} - R_{\pi_j}}{B}}\right)\left(P \quad p_0 + \frac{\sigma^2}{h_{\pi_j}}\right), \tag{C.1}$$

where the inequality follows from (19). Based on (C.1), we can claim that the new solution $\boldsymbol{a}'$ is feasible with better or equal objective value than solution $\boldsymbol{a}^*$.

Then, we show that the objective function (18) monotonically increases with a_j for $j \not\forall k$. To show this, the first derivative of the objective function (18) with respect to a_j can be given by:

$$\frac{\partial f(\boldsymbol{a})}{\partial a_j} = \quad 1 + \frac{2^{\frac{a_j - R_j}{B}}\left(P \quad p_0 + \frac{\sigma^2}{h_j}\right)}{\sum_{j=1, j\neq k}^{K}\left(1 \quad 2^{\frac{a_j - R_j}{B}}\right)\left(P \quad p_0 + \frac{\sigma^2}{h_j}\right) + \frac{\sigma^2}{h_k}}$$
$$\in \quad 1 + \frac{2^{\frac{a_k - R_k}{B}}\left(P \quad p_0 + \frac{\sigma^2}{h_k}\right)}{\sum_{j=1, j\neq k}^{K}\left(1 \quad 2^{\frac{a_j - R_j}{B}}\right)\left(P \quad p_0 + \frac{\sigma^2}{h_j}\right) + \frac{\sigma^2}{h_k}}$$
$$= \frac{P \quad p_0 \quad \sum_{j=1}^{K}\left(1 \quad 2^{\frac{a_j - R_j}{B}}\right)\left(P \quad p_0 + \frac{\sigma^2}{h_j}\right) \quad \frac{\sigma^2}{h_k}}{\sum_{j=1, j\neq k}^{K}\left(1 \quad 2^{\frac{a_j - R_j}{B}}\right)\left(P \quad p_0 + \frac{\sigma^2}{h_j}\right) + \frac{\sigma^2}{h_k}} \in 0, \tag{C.2}$$

where $f(\boldsymbol{a})$ denotes the objective function (18). The first inequality in (C.2) follows from constraint (18b), and the second inequality in (C.2) follows from constraint (18c).

Based on (C.2), the objective function (18) increases with each rate a_j, while (C.1) shows that it is optimal to allocate rate to the user that has the lowest channel gain. As a result, the optimal rate allocation can be given in (20) and (21). This completes the proof of Theorem 2.

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