The formation of preference in risky choice

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A key question in decision-making is how people integrate amounts and probabilities to form preferences between risky alternatives. Here we rely on the general principle of integration-to-boundary to develop several biologically plausible process models of risky-choice, which account for *both* choices and response-times. These models allowed us to contrast two influential competing theories: i) within-alternative evaluations, based on multiplicative interaction between amounts and probabilities, ii) within-attribute comparisons across alternatives. To constrain the preference formation process, we monitored eye-fixations during decisions between pairs of simple lotteries, designed to systematically span the decision-space. The behavioral results indicate that the participants' eye-scanning patterns were associated with risk-preferences and expected-value maximization. Crucially, model comparisons showed that within-alternative process models decisively outperformed within-attribute ones, in accounting for choices and response-times. These findings elucidate the psychological processes underlying preference formation when making risky-choices, and suggest that compensatory, within-alternative integration is an adaptive mechanism employed in human decision-making.

Author Summary

Decision-making under risk requires a selection between alternatives, such as lotteries, which offer a reward with a specified probability. Human decision between such alternatives is at the center of the normative decision theory, which assumes that decisions are rationally made by forming a value for each alternative and selecting the alternative with the highest value. To this day, there is still a considerable debate on how such values are computed. While the normative theory assumes that values of the alternatives reflect the statistically expected reward, more recent theories have argued that alternative-values are not computed and choices are only based on sequentially comparing the alternatives on amounts or on probabilities. Here, we carried out an experimental investigation of risky decision-making, in which participants chose between pairs of simple lottery alternatives that systematically span a range of probabilities and rewards, while we tracked their eye positions during their decision-making process. We found that the participants are sensitive to the expected-utility of the alternatives, as predicted by the normative decision theories, but they also exhibit risk-biases that correlate with the eye-scanning patterns. We then carry out computational modeling, comparing preference-formation models on their ability to account for both choices and their reaction-time. The results provide strong support for normative models which assume that the values of the alternative are computed via a multiplicative function of the amounts and probabilities. These results suggest that humans are closer to normative principles than previously assumed, and motivate further investigation into the neural mechanism that mediates these multiplicative computations.

1. Introduction

Decision-making under risk is ubiquitous in daily activities, such as deciding whether to take an umbrella when the weather forecast predicts 50% chance for rain or whether to purchase a lottery ticket with a winning probability of 1%. Such decisions are difficult because the outcomes of the alternatives are only known with some probability, and thus they are subject to risk tradeoffs. For example, when deciding between a lottery that offers $100 with a probability of 50% and an offer of $40 with certainty, one needs to balance between the appeal of the attractive amount ($100) and the risk of getting nothing (rather than gaining $40 for certain). Choices between such lotteries were the subject of intensive research in economics and experimental psychology that investigated how humans make risky decisions, starting from the normative Expected-Utility (*EU;* [1]), followed by random utility models [2] and culminating with Cumulative Prospect Theory (*CPT*; [3–6], see also Transfer of Attention eXchange [*TAX*], for a related type of model [7]). Yet despite the impressive success of *CPT* in accounting for risky choice data (e.g., the dependence of risk-aversion on the magnitude of the outcomes' probabilities [8]), the theory has been criticized for making assumptions that are inconsistent with capacity limitations of human online information processing, and for not explicating the process by which the preferences are formed [9,10].

Several process theories were developed to account for risky choice. First, heuristic models, such as Priority Heuristic (*PH*), suggest that preferences are not formed via a compensatory process of averaging over all outcomes (like in *EU* and *CPT*), but rather via a sequential process of comparing the alternatives over one specific attribute (probability or amount) at a time, in a specified order, and stopping at the first instance in which a termination criterion is satisfied [9]. Second, a number of models have relied on the sequential-sampling framework [11–14], which successfully accounted for choices in perceptual tasks, in order to develop a process model of risky choice. For example, in Decision Field Theory (*DFT*; [15]), as attention fluctuates between the alternatives, the preference dynamically evolves by integrating amounts, which are sampled with a frequency that is associated with their (subjective) probabilities [16]. In the Decision by Sampling model (*DbS*; [17–19]), like in *PH*, the sampling involves comparisons between the values of the alternatives on a specific attribute (i.e., amounts or probabilities, but not both). However, unlike *PH*, *DbS* does not assume a fixed order of attribute sampling, nor that the decision is settled at a single comparison, but rather a stochastic sampling, which continues until the accumulated difference of favorable comparisons reaches a decision boundary. Critically, as opposed to *EU* or *CPT*, in *DbS* the processing takes place within-attributes (i.e., comparison between amounts or between probabilities). Finally, in the Parallel Constraint Satisfaction model (*PCS*; [20]), a compensatory within-alternative process similar to *EU* (i.e., multiplication of amounts and probabilities) is carried out in a parallel and automatic manner; this process is mediated by a connectionist network of bottom-up and top-down connections. Although several qualitative predictions of the *PCS* model have been confirmed [20], this model has not been tested quantitatively in risky choice.

More recently, a number of studies have relied on eye-fixations during choice between alternatives, to gain insight into the preference formation process. For example, Krajbich, Rangel and colleagues have shown that an extension of the Drift Diffusion Model (*DDM*; [12,13]), the attentional *DDM* (*aDDM*), accounts well for observed preferences between consumer products, food items and 50-50 monetary gambles [21–24]. To do so, the *aDDM* assumes that the value of the sampled alternative is modulated by eye-fixations, so that the values of the non-fixated alternatives are attenuated compared with the fixated ones. In the domain of risky choice, a number of studies have contrasted within-alternative and within-attribute models, and reported partial support for both [20,24–28]. In particular, Glöckner and Herbold [20] analyzed risky choice while monitoring eye-movements, and provided evidence against the *PH* model and in favor of the *PCS* and *DFT* models (see also [29] for similar results). Finally, in a recent investigation of eye-movements during risky choice, Stewart, Hermens, & Matthews [30] concluded that, while eye-movements contribute to choice preference, this contribution is mostly independent of the values sampled. In other words, the more one looks at an alternative the more likely s/he is to choose it, independently of the magnitude of amount or probability.

The aim of the current study is to develop and contrast process models of risky choice, which are constrained by the eye movements of participants making decisions. In particular, we adopt an integration-to-boundary framework, which allows to predict *both* choices and their decision-time, and we extend the *aDDM* [21,22,31] approach to the domain of risky choice (see also [24] for a recent extension to 50-50 monetary gambles). In this regard, a central question is whether the preferences are formed by integrating global alternative-values, based on multiplicative interactions between amounts and probabilities (*within-alternative* processing), or by sampling and integrating attribute-comparisons (*within–attribute processing)*. Furthermore, using process models that include attentional modulation of fixated information, we wish to account for individual differences in risk preference. While previous work has highlighted the impact of task-complexity (e.g., number of alternatives and attributes) in determining the decision strategy that the participants adopt (e.g., [32]), here we focus on the simplest type of risky choice (between pairs of alternatives, each consisting of a probability p to win amount x, see Fig. 1A). Thus, our aim here is not to determine which of these two types of processes prevail in any choice scenario (we think that they both can take place, subject to task-conditions and individual differences). Rather, we wish to test if, at least for this simple case, the more "economically-normative" (within-alternative and multiplicative) strategies are within the capacity of participants resources. Towards this end, we carry out a systematic investigation of risky choice with simple two-outcome lotteries, while eye-fixations are monitored. To anticipate our results, we provide a clear demonstration that within-alternative and multiplicative evaluations are being used, subject to individual differences that correlate with choice normativity.

2. Results

The participants were tested on choices between simple lotteries of the type (x_1 with p_1 and otherwise 0, vs. x_2, with p_2 and otherwise 0; where $x_{1,2}$ are monetary amounts and $p_{1,2,}$ are the corresponding probabilities of winning). The choice problems (94 trials) were selected by systematically sampling a two-dimensional grid of probabilities and amounts (Fig. 1B). Dominated choice problems (in which both the amount and probability of one option were higher than in the other option) were excluded except for 10 catch-trials, which were used to assess task engagement. To discourage numerical calculations, the choice alternatives were presented in graphical format (Fig. 1A). The experiment was incentive compatible: it was explained to the participants that one of their choices will be randomly chosen and played out for real money at the end of the experiment (see *Methods and Suppl. Experimental instructions* for details on the stimuli and task instructions).

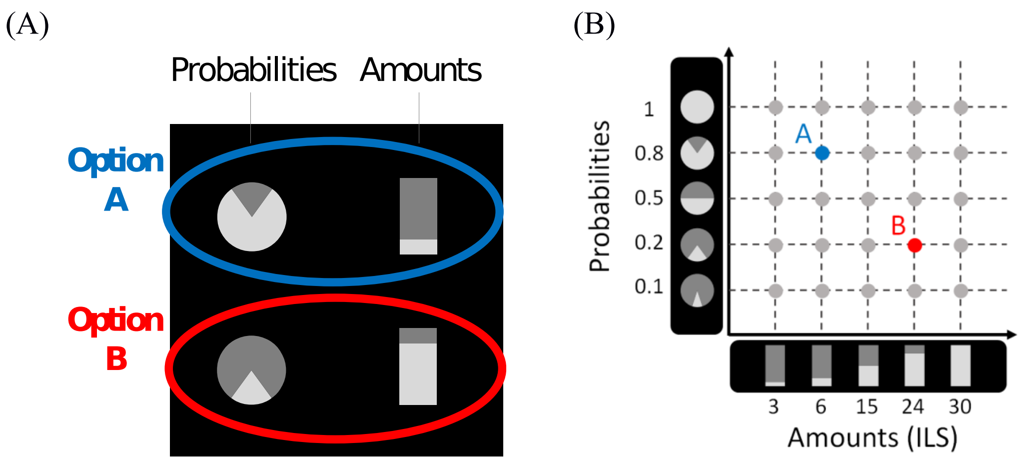

Figure 1. *Stimuli and study design. (A) Example of the stimuli used in the experiment. Amounts were represented by the lower (brighter) parts of divided bar graphs, and probabilities by the lower (brighter) sectors of pie charts. Note that the figure is not to scale, and the colored ellipses and labels are shown for illustration purposes only (they were not used in the actual experiment). (B) Choices were drawn from a 5x5 two-dimensional grid with amounts along one dimension, and probabilities along the other. The two stimuli from panel A are shown in this grid. Choice stimuli were presented without deadline until response.*

2.1 Choice behavior

We began by examining the basic psychometric properties of our choice-data. Analysis of the "catch-trials" showed that the participants chose the better option (higher in both amount and probability) in 97% of these trials (SD = 6%). Next, we conducted a mixed-effect logistic regression on the choice data, with the Expected-Value (EV) differences ($x_1{\cdot}p_1 - x_2{\cdot}p_2$) as a predictor, and with random intercepts and slopes at the participant level. The results indicated that, consistent with an “economically-normative” theory, the participants were sensitive to EV differences, and preferred lotteries with higher EVs over lotteries with lower ones (β = 0.40, p < .001; Fig. 2A). Additionally, using a Pearson correlation analysis, we showed that the reaction time (RT) of a decision decreased as the absolute *EV* difference between the lotteries increased (r = -0.8, p < .001; Fig. 2B). This finding is consistent with previous process models such as the *PCS* [20], the *aDDM* [21], and the *DFT* [16], indicating that the participants take longer to decide when the evidence (as measured by the *EV*-difference) is smaller.

Finally, we evaluated the risk-preferences of the participants. To this end, we focused on choice problems with similar *EV*s ($|\Delta EV| \leq 1$, $N_{choice\ problems} = 26$), and examined the proportion of trials in which high-payoff/low-probability lotteries (riskier options) were preferred over low-payoff/high-probability lotteries (safer options). Following the *CPT* regularity of differential risk-attitudes for low vs. medium/high probabilities (see *Suppl. Cumulative Prospect Theory (CPT) risk attitudes predictions*), we examined the risk-preferences separately for these two probability domains: i) low-probability cases, in which one of the lotteries has p < .25 (e.g., $24 with p = .1 vs. $6 with p = .5), and ii) high-probability cases, in which both lotteries have p ≥ .25 (e.g., $30 with p = .5 vs. $15 with p = 1); the .25 cutoff was selected to match *CPT* (see Fig. S1). A paired samples *t*-test indicated that, consistent with *CPT*, the participants showed higher levels of risk-aversion for medium/high probabilities as compared to low ones ($t(30) = 3.84$, p < .001). Follow-up one-sample *t*-tests (against .5) indicated that the participants showed risk-aversion for medium/high probabilities ($t(30) = 4.49$, p < .001); no risk-aversion, however, was obtained for low probabilities ($t(30) = -0.11$, p = .9).

2.2 Eye-fixations and individual differences

On average, the participants made 9.05 fixations (*SD* = 0.64) per trial, with a mean duration of 407ms (*SD* = 244 ms) per fixation. Also, on average across participants, there was no significant difference between the proportion of fixations towards amounts and probabilities ($t(30) = 0.78$, p = .44). There was, however, a remarkable difference between participants in this proportion, which was correlated with participants' risk preferences: the more a participant fixated on amounts, the more likely he or she was to choose the riskier alternatives (r = .48, p = .006; Fig. S2A). To understand this relationship we examined individual differences in fixating the higher of two amounts/probabilities, as this can explain risk-biases (looking more at higher amounts or at lower probabilities leads to risk-seeking according to the *aDDM* [21,22,24]). Importantly, we find that the more a participant tends to fixate on amounts the more s/he fixates on the larger of them (r = .47; p = .007; Fig. S2B), and similarly for probabilities (r = .46; p = .007; Fig. S2C). Finally, the frequency of fixations on the higher of the two amounts was positively correlated with risk-seeking (r = .58; p < .001; Fig. 2E), and the frequency of fixations on the higher of the two probabilities was negatively correlated with risk-seeking (r = .45; p = .01; Fig. S2D) see also [24,33].

We also examined the eye-trajectories in relation to their transitions between the four attributes (x_1, p_1, x_2, p_2). The transitions between decision attributes (amounts and probabilities) were classified into three categories [20,25,30]: i) Within-alternative transitions – transitions between attributes that belong to the same alternative. ii) Within-attribute transitions – transitions between different alternatives, within the same attribute. iii) “Diagonal” transitions – transitions between the amount of alternative A and the probability of alternative B and *vice versa*. Figures 2C-D show one example each for within-alternative and within-attribute trials, respectively. An Analysis of Variances (ANOVA) revealed significant differences of the transition probabilities between the three transitions types ($F(2,60) = 431.1$, p < .001). Post-hoc comparisons showed that the participants made more within-alternative than within-attribute transitions (p < .001), as well as more within-attribute than diagonal ones (p < .001). The proportion of within-alternative transitions (out of all transitions) was subject to individual differences and was correlated with the economically-normative choice performance (ΔEV), such that the higher the fraction of within-alternative transitions the higher was the proportion of the alternative with the higher EV to be chosen (r = .57, p < .001; Fig. 2F).

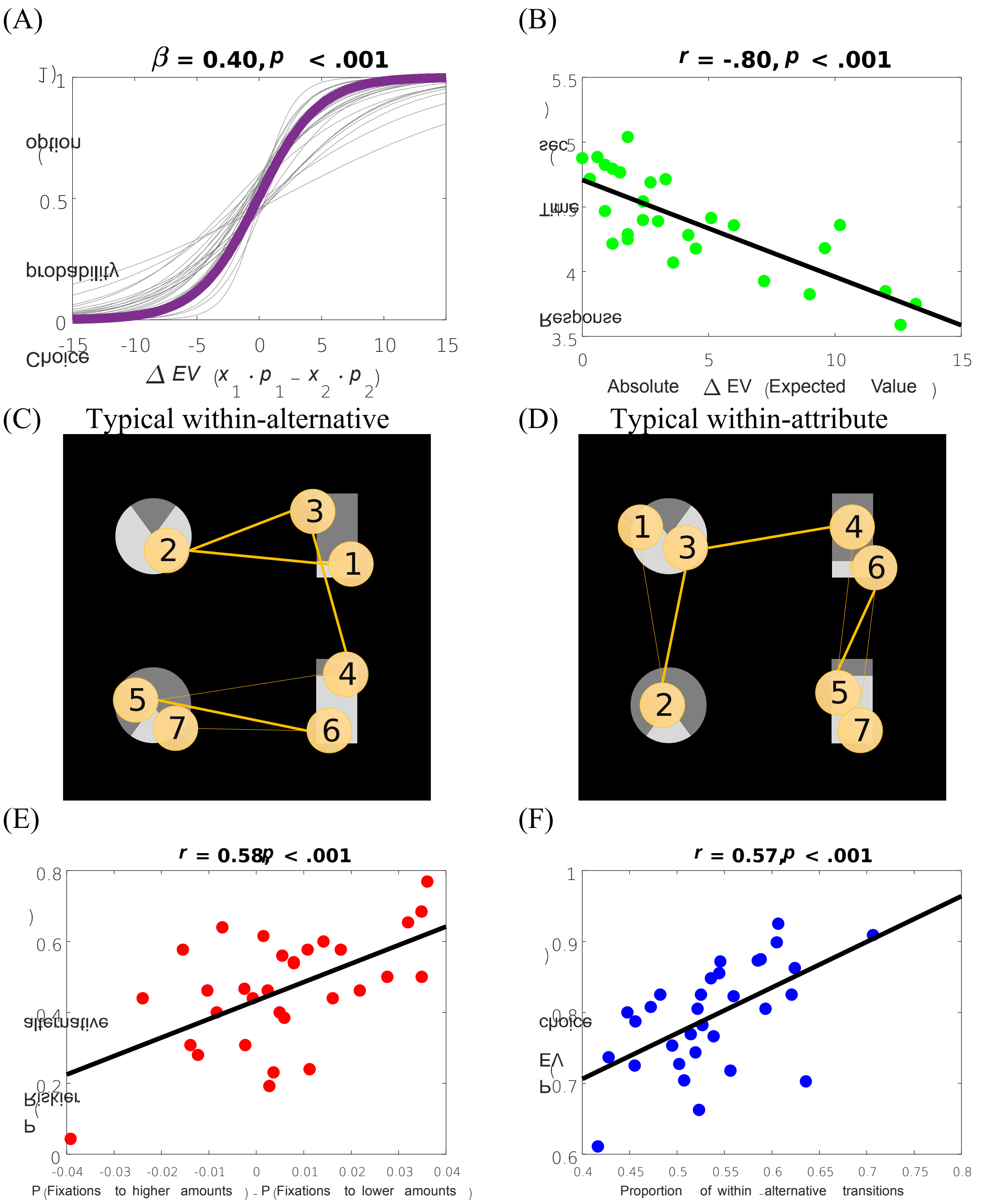

Figure 2. *Choice and eye-movements analysis. (A) The participants were sensitive to EV differences between the options. Solid purple line corresponds to the group fit; grey lines correspond to the fit of individual participants. (B) Response times were negatively correlated with the alternatives' EV differences. (C-D) Example eye-trajectory characterized by within-alternative transitions (in C) and by within-attribute transitions (in D). The numbers indicate the order of fixations. (E) The proportion of fixations to the higher amount correlated with risk-seeking preference. (F) The proportion of within-alternative transitions correlated with the proportion of higher EV choices.*

2.3 Predicting choices using eye-fixations

Recent research has demonstrated that attentional mechanisms play a key role in the development of preferences [24,34–38]. In particular, it was shown that the more an alternative is fixated, the more likely it is to be chosen [21,30,39].. We have confirmed this regularity in our data by carrying out a number of logistic regression models that predict choices based on the *EU* or *CPT* utility functions, and the relative number of fixations (or dwell-times) on each alternative (see *Suppl. Predicting choices using eye-fixations* for details).

(A)

Dwell Time Regression

$$P(\mathrm{A},\mathrm{B}) = \frac{1}{1+\ e^{-\beta(x_1^{\alpha}\cdot p_1\cdot t_1^{\tau} - x_2^{\alpha}\cdot p_2\cdot t_2^{\tau})}}$$

t_1 - Total looking time on option A

t_2 - Total looking time on option B

(B)

Number of Fixations Regression

$$P(A,B) = \frac{1}{1+\ e^{-\beta(x_1^{\alpha}\cdot p_1\cdot f_1^{\tau} - x_2^{\alpha}\cdot p_2\cdot f_2^{\tau})}}$$

f_1 - Number of fixations to option A

f_2 - Number of fixations to option B

(C)

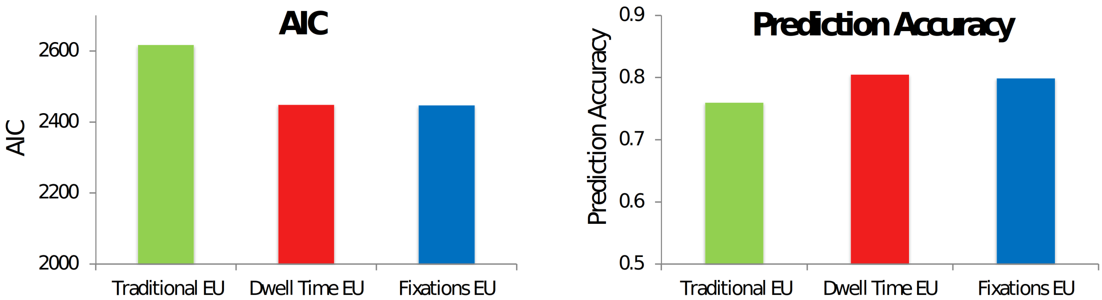

Figure 3. *Expected Utility based regression models. (A) $EU_{Dwell\text{-}time}$: the EU value of each of the alternatives is modulated by the total looking time on each alternative. (B) $EU_{Fixations}$: the EU value of each of the alternatives is modulated by the relative number of fixations toward it. (C) Prediction accuracy and AIC for the traditional, dwell time and number of fixations EU.*

As illustrated in Fig. 3A, we examined an *EU* × *time* model[1], in which the *EU* value of each alternative increases with its dwell time on the two alternatives (α is the risk-parameter of *EU*, τ is a saturation parameter, and β is a slope parameter). Additionally, we examined a similar regression model, in which dwell-times were replaced with the number of fixations each alternative is sampled (Fig 3B). Comparison of these models with the traditional *EU* (which does not take eye-movements into account) showed that using eye-movements significantly improved prediction accuracy and *AIC* comparted with the traditional *EU* (Fig. 3C). Note also, that the prediction accuracy

[1] We focus here on *EU* based models, however similar conclusions were obtained for *CPT* based models, see *Suppl. Predicting choices using eye-fixations.*

and *AIC* which were obtained using the number of fixations, equal (for *EU*) or surpasses (for *CPT*), the prediction accuracy and *AIC* obtained using the more traditional measure of dwell-time. In addition, the fitted values of saturation-parameter τ, were lower than 1, in both time based regressions (number of fixations and dwell time), indicating that, for example, looking twice as long at an alternative increases its value by a factor of less than 2. One way to understand this non-linear saturation is in relation to a leak of the accumulated values ([14,40,41]). In such leaky integration models, the accumulated evidence saturates at an asymptotic value, and remains constant even if more integration time is allowed. Accordingly, at each fixation one samples and accumulates a value, however, as the trial proceeds, the accumulated value leaks, resulting in a type of recency. Indeed, when we compute the percentage of match between the fixated alternative and the final choice as a function of fixation number (backwards from the end) we obtain a clear recency pattern (see Fig. S5 in *Suppl. Last fixations and choice*).

2.4 Towards a process model of risky choice based on eye-movements

The central aim of this study is to develop and contrast two classes of process models that differ in the way attentional (or eye) transitions affect the integration of amounts and probabilities. Both types of models assume that: a) fixated objects receive enhanced attention, b) attention modulates the weight of value integration [21], and c) recently sampled values are weighted more than earlier ones [14,40,41]. The models differ, however, on how the values are integrated into preferences. Note that we do not aim to test specific models but rather distinguish between broad classes of models based on certain principles, in particular, between *within-attribute* vs. *within-alternative* models [20,25,32,42]. While the former is used in models such as *PH* and *DBS*, the latter is used in models such as *EU*, *CPT* and *PCS*. We also examined a more hybrid model, which still relies on multiplicative within-alternative computations, but also allows some extent of competition between the attributes.

Within-attribute integration models. Models from this class assume that when decision-makers attend to one attribute (e.g., amount or probability), they accumulate the value-difference (or categorical difference) of the two alternatives on that attribute, according to:

$$Y_A(t+1)=(1-\lambda)\cdot Y_A(t)+D_A(t)$$

$$Y_B(t+1)=(1-\lambda)\cdot Y_B(t)+D_B(t)$$

where Y_i, $i\in\{A,B\}$ is the accumulated preference for alternative i, λ is an integration-leak factor that emphasizes recent values, and D_i is the value (or categorical) difference between the attributes, which depends on eye-fixation and model variant (see *Suppl. Within-attribute integration models* for a detailed description of the models). This mechanism was implemented in two model variants. In the first one, preferences were generated by accumulating the normalized differences (min-max normalization, over the whole set of decision problems) of the attended attribute values. For example, if the participant had to choose between A:($20, 0.2) and B:($10, 0.5), then the difference between the normalized amount values (of $20 and $10, respectively), is accumulated whenever the representations of amounts are fixated. Likewise, the difference between the normalized probability values (of 0.2 and 0.5, respectively) is accumulated whenever the representations of probabilities are fixated. The second model assumes integration of categorical differences; this follows the *DBS* assumption that people have access to ordinal comparisons rather than values [18]. Therefore, in the above example, the accumulator associated with alternative A increases by one unit at each fixation of an amount (since $20 is more than $10), and the accumulator of alternative B increases by one unit at each fixation of a probability (since 0.5 is larger than 0.2). This means that the mechanism accumulates binary counts of comparison between the same attribute in different alternatives [17,19]. To enhance these models' performance we allowed an additional parameter: attentional modulation, which enhances the weight of sampled attributes ([21,22]; see *Suppl. within-attribute integration models).* Note that since the values of both attributes are used in the comparison, these models assume either the existence of some degree of peripheral vision, or reliance on memory. Since memory cannot play a role during the first fixation of an attribute (and since peripheral vision is less sensitive to the low contrast of our stimuli in any fixation, including the first), we assumed that default values (mid-range of the amounts and probabilities values used in the experiment) are used for the yet un-scanned attributes. The default values were replaced with the actual attributes' values at the first fixation to each attribute. This treatment was implemented in all versions the process models.

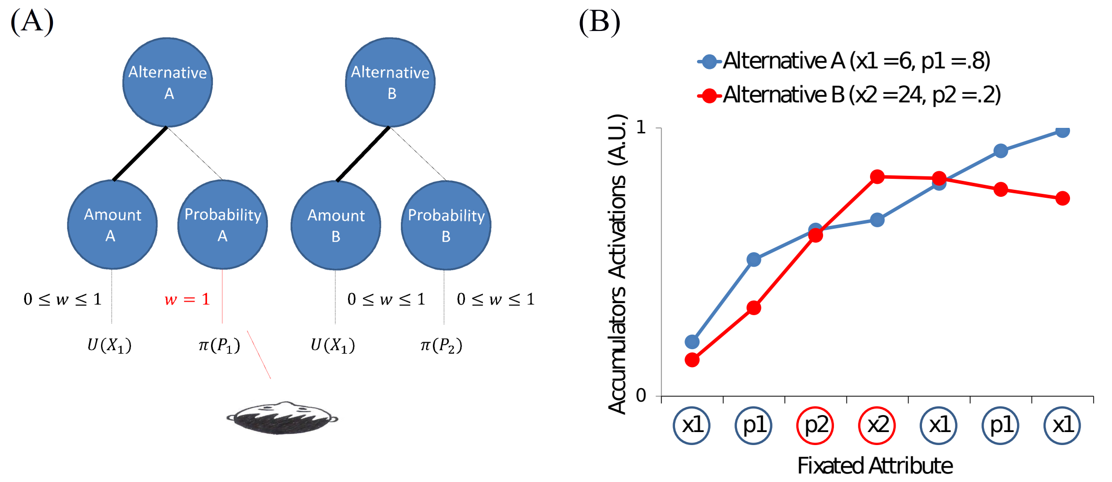

Figure 4. *Illustration of the two-layer leaky accumulator model and its dynamics. (A) The first layer consists of four leaky-accumulators associated with the different attributes, and the second consists of two leaky accumulators associated with the alternatives' values. The units in the first layer are updated with the attentionally modulated subjective values of each attribute (the red arrow indicates the input from the attended attribute, whereas the black arrows indicate the attenuated inputs from the unattended attributes). The units in the second layer are fed with the first layer units' activations, and accumulate their product. (B) Simulated run of the two-layer leaky accumulators model using the average best fitted parameters (see Table S2), in a choice between: A(\$6, .8) and B(\$24, .2); A-wins. Blue circles (x-axis) correspond to fixation toward A, and red circles correspond to fixation toward B. Values on the y-axis correspond to the activations of the second-layer accumulators.*

Within-alternative integration models. The second class of models assumes that the values that are integrated are associated with the alternatives and are multiplicatively formed from the attributes (as in expected utility). This mechanism was also implemented in two models. The first model has single-layer architecture and involves two accumulator units, one for each alternative (A or B). On each fixation, the accumulators are updated with the integrated subjective utilities of the fixated alternative (which is based on multiplication of the subjective-amounts and subjective-probabilities; see *Suppl. Within-alternative selection/One-layer leaky accumulators*), according to:

$$Y_A(t+1)=(1-\lambda)\cdot Y_A(t)+SU_A(t)$$

$$Y_B(t+1)=(1-\lambda)\cdot Y_B(t)+SU_B(t)$$

where Y_i, $i \in \{A,B\}$ is the accumulated preference of alternative-i, λ is an integration-leak, and SU_i is the subjective expected utility of alternative-i (similar to *CPT*), subject to attentional modulation that depends on eye-fixation. As in the *aDMM* model [21], this model assumes that the inputs are modulated by gaze direction, i.e., higher weight is assigned to the fixated alternative than to the non-fixated one. Note that in this model the update does not depend on whether the current fixation is on amount or probability, but only which alternative is fixated, with the non-fixated alternative being attenuated. For example, when one looks at either the amount or the probability of alternative A, the corresponding accumulator is updated with the integrated subjective utility of that alternative, while the other accumulator is updated with an attenuated value of the subjective utility of alternative B.

The second within-alternative model contains two-layers of leaky-accumulators in cascade (Fig 4A); as we will show, this model allows to apply attentional modulations to specific attributes and not only to the whole alternative. The first layer of the model consists of four leaky-accumulators associated with the four different attributes (x_1, p_1; x_2, p_2). Unlike in the previous (single layer) version, these units are updated with the attentionally modulated subjective values of each *attribute*. For example, when a participant looks at the amount of alternative A (x_1), the accumulator of that attribute is updated with the subjective value associated with it (i.e., x_1^{α}, where α is a free parameter), while the other accumulators (of p_1, x_2 and p_2) are updated with attenuated subjective values of these attributes. The second layer of the model consists of two leaky-accumulators corresponding to the integrated preference of the two alternatives. At each fixation, each second layer (alternative) accumulator is fed with the activations of the first layer units associated with it, by accumulating the product of their values (see Fig 4B for illustration of the model dynamics and *Suppl. Within-alternative selection/Two-layer leaky accumulators* for details). In one version of the two-layer model, we also introduced mutual inhibition between the amount units (i.e., competition between x_1 *and* x_2) and the probability units (i.e., competition between p_1 *and* p_2). One can think of such a model as implementing a hybrid between within-attribute and within-alternative processes: while the alternatives units still receive multiplicative input from both their attributes units, the mutual inhibition (depending on its strength) can polarize the difference in activation, subject to attentional modulation based on the current fixation. We note that the level of activation-leakage (in both types of models) is a free-parameter, so that the case in which $\lambda = 0$, corresponds to the more standard Drift-Diffusion models, and is thus explicitly tested as part of our model fit procedures.

The models were fitted to the data in two steps. In the first, we fitted the models to choice data, based on the values of the alternatives and the eye-fixations made for that decision, using maximum likelihood estimations to obtain the best model parameters (see *Suppl. Model Fitting*). In the second step, we used the models with their fitted parameters from step 1, to make predictions for decision-time, under an integration-to-boundary framework (in which we included a new set of parameters that correspond to the response boundary). At this stage we also compared the models on their ability to predict *both* choices and decision-times.

In addition to the process models, we also fitted a number of *benchmark* non-integration to boundary models. Specifically we examined traditional compensatory models, such as *EV*, *EU* and *CPT*, as well as non-compensatory heuristic models such as *Maximax* [43], *Least-Likely* [44] and *PH* ([9]; see *Suppl. Heuristics* for detailed description of these models).

To evaluate the models' capabilities to fit the data, we used several selection criteria: prediction-accuracy, *AIC* and cross-validation measures (see *Suppl. Model Selection*). For the heuristic models, whose choices are deterministic, we only examined their accuracy measures [8]. Because it can be argued that the prediction-accuracy of models with fixed (or no) parameter values (such as the heuristic models) cannot be compared to the prediction-accuracy of models with fitted parameter values [45], we compare them using the Cross-Validation/Accuracy measure.

2.4.1 Step-1: choice data

The most complex of the models (in terms of number of parameters) is the within-alternative process model, which has four free parameters. The first two, α and γ correspond to the *CPT* parameters [3] for risk aversion and probability weighting, respectively, θ corresponds to the *aDDM* attentional modulation parameter, and λ is the activation-leak. As we show in *Suppl. Parameters Recovery*, we carried out a recovery exercise, showing that our fitting procedure is able to provide a good recovery for all those parameters over a wide range of values that correspond to those found in the actual data. This non-trivial result is helped by the fact that our 94 choice problems systematically span the choice space. As shown in Table-2, the *within-alternative* process models with attention modulation and leak gave the best fit and showed the highest cross-validation prediction accuracy. They outperformed both the within-attribute process models, as well as the traditional, non-integration to boundary models (compensatory and non-compensatory heuristics). These results speak against the hypothesis that the participants accumulate only the differences of the attended attributes. We also found that the within-attribute models with perfect (rather than leaky) integration (Normalized and Binary differences), resulted in much worse *AIC*, prediction accuracy, and cross-validation (therefore in Table-2 we report only the within-attribute models which include leak as a free parameter). We also note that the within-alternative choice models required a significant degree of information leak (λ_{group}= 0.58). As we show in the *Suppl. Additional model variations*, we explicitly tested four versions of within-alternative models that included an attentional modulation but no activation-leak, all of which resulted in much poorer prediction-accuracy and *AIC* fit values (these models reached a prediction accuracy that while exceeding that of the simple *EU*, did not exceed that of the *CPT* without eye-fixations; see *Suppl. Additional model variations*)[2]. By contrast, the within-alternative process models (with leak) outperformed (on prediction accuracy, *AIC* and cross-validation) the regression models that include either *EU* or *CPT* together with the number of fixations (see *Suppl. Predicting choices using eye-fixations*). This suggests that considering dynamic processes, such as attentional shifts and leak of activation improves prediction accuracy and fit measures beyond what is achievable by using

[2] The hybrid model resulted in fits that did not exceed (*AIC* and prediction accuracy) those of the within-alternative model (see *Suppl. Additional model variations*) and with a moderate mutual inhibition value (.13), which does not trigger a full all-or-none dynamics. As this model has two extra parameters (the mutual inhibition values between the *x* and the *p* units), we kept two of the other parameters (leak and attentional modulation) to the optimal values of the model without mutual inhibition. Due to its complexity, we leave a full investigation of this model to future research. We wish to point out, however, that inhibition at the level of attributes is not motivated by Connectionist principles ([72]), which suggested mutual inhibition between units that correspond to different alternatives (for example, this could apply to inhibition between alternative A and B at the 2nd alternative layer, see Fig. 4A).

only the number of fixations. Note that the within-alternative two-layer leaky accumulators model outperforms the single layer accumulator model. This result suggests that the perception of the attributes is dynamic and is subject to modulation by attentional processes.

Table-2: Model comparison

Model	***AIC***	***Prediction Accuracy***	***Cross-Validation (-2●LogLikelihood)***	***Cross-Validation (Accuracy)***
Traditional Models				
EV	2789	75.1%	567	75.1%
EU	2617	76.5%	527	76.0%
CPT	2364	81.2%	484	79.6%
Heuristics				
MaxiMax	-	44.8%	-	44.8%
Least-Likely	-	55.2%	-	55.2%
Priority Heuristic	-	58.3%	-	58.3%
Fixation based regression models				
$EU_{Fixations}$	2447	79.9%	506	78.7%
$CPT_{Fixations}$	2173	83.9%	453	81.7%
Within-attribute Integration				
Normalized differences	2716	76.8%	551	75.4%
Categorical differences	2724	76.4%	576	74.4%
Within-alternative Integration				
one-layer leaky accumulators	1980	86.1%	445	83.8%
two-layer leaky accumulators	**1877**	**87.2%**	**436**	**84.1%**

AIC values are rounded to the nearest integers. Bold entry indicates the best fitting models. Note that AIC differences exceeding 10 are considered very strong evidence in favor of the model with the lower numerical values.

Finally, we carried out a comparison of the predictive accuracy of our best performance model – the two-layer leaky accumulators - with that of the traditional *EU* and *CPT* models across all decisions as a function of *EV*-differences. The comparison demonstrates that the difference in prediction accuracy is especially large for difficult choices (low *EV*-differences, 1-3 Quantiles; Fig. 5), suggesting that attentional modulations are particularly significant in difficult decisions [46].

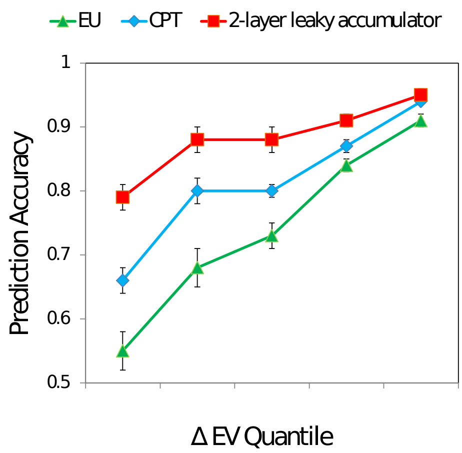

Figure 5. *Predictive accuracy of the EU, CPT and the two-layer within-alternative integration models as a function of ΔEV quantile. The two-layer model outperformed all other models especially in difficult decisions (low EV-differences). Bars denote S.E., clustered by subjects.*

2.4.2 Step-2: Accounting for both choice and decision-time.

We contrasted the *within-alternative* and the *within-attribute* models, in accounting simultaneously for choices and decision-times. To this end, we adopt an integration-to-boundary framework, which assumes that preferences are accumulated until they cross a decision criterion [47,48]; this introduced a few more parameters (for the boundary) into the model (see *Suppl. Model Fitting*). The models are now set to estimate the probability of a subject's choice conditioned on its decision time and fixations. This probability is accumulated for all choice trials of the participant to a total likelihood, which is used to optimize the boundary parameters. Two families of decision boundaries were tested, for each of the models: i) the standard fixed (time-invariant) boundary, which introduces a single new boundary parameter, and ii) a collapsing (time-variant) boundary model, which introduces three new parameters (see *Suppl. Optimization procedure: choices and decision-times*, for further details regarding the implementations of these two types of models). The collapsing boundary model has been the focus of recent investigations in decision neuroscience [41, 42], and appears to be favored in experimental tasks that span over longer time intervals (more than 2-3 sec [49,50]).

(A) (B)

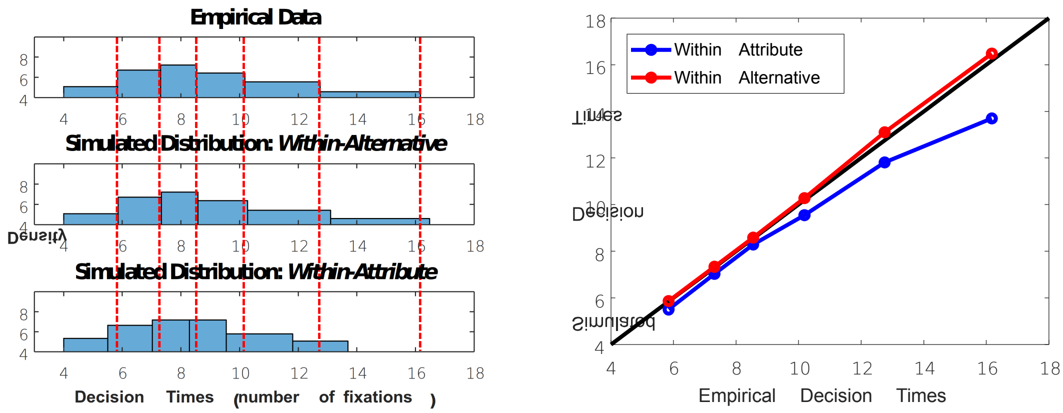

Figure 6. Accounting for choices and decision times. (A) Group quantile (Vincentizing; [12]*) decision-time density distribution (in number of fixations to decision). Dashed red lines indicate the quantiles of the actual data distribution ([.1, .3, .5, .7, .9, .99]). (B) Group Quantile-Quantile plot comparing the actual decision time and the simulated decision time of the within-alternative (red) and within-attribute models (blue). Note that the within-alternative model captures better the tail of the distribution.*

The results show that, with both decision boundary families, the *two-layer leaky accumulator* model outperformed all the other models. Among the two types of boundary families, the best fits by far were obtained under the collapsing boundary model (*AIC* and cross-validation), despite the cost of the two extra parameters (see *Suppl. Process model comparison* for all models). For this reason, we only report below the results for this type of boundary. We find that the *within-alternative/two-layer leaky accumulator* model (*AIC* = 14,492) decisively outperformed the *within-attribute/normalized differences* model (*AIC* = 15,815; Δ*AIC* = 823), in accounting for decision-times (conditioned on the actual fixation patterns). Finally, we used these models to predict the distribution of decision times (measured in number of fixations), for novel but statistically matched patterns of fixations. To this end, for each trial we simulated a fixation sequence that is based on a statistical model of the participant's fixations towards the four attributes as a function of their values [21,30]. The results indicate that for the two-layer leaky accumulator model, the predicted and actual decision-time distributions show a good match, however for the normalized differences model, the tail of the predicted decision-time distribution deviates from that of the actual decision-time distribution (Fig. 6 A-B).

2.5 *Accounting for individual differences in risk-bias and in economically normative choice*

Our best within-alternative integration model accounts also for the empirical correlation we reported between the proportion of fixations a participant makes to the higher of the two amounts and his or her risk-preference bias (Fig. 2E; see also [24]). To show this, we simulated choices for each participant, based on his or her fitted model-parameters and the participant's actual fixation sequence. The correlation between the model's risk-preference prediction and the proportion of fixations to the higher amount (r = .58, p < .001), was exactly equal to the empirical correlation obtained in the data (Fig. 2E). Next, we sought to demonstrate that this relation is associated with the fixation pattern and not merely with differences in model parameters. To this end, we simulated choices for each participant, by using his or her actual fixation sequences, however, this time we used model parameters that correspond to the group mean (rather than the individually fitted parameters). This resulted in a significant correlation (r = .52; p = .002; Fig. 7A) between the risk-preference and the proportion of fixating on the higher amount. This correlation between risk-biases and fixation-pattern relies upon the model's attentional component, which gives higher weights to the attributes on which the participant fixates. For example, assume that a participant is asked to choose between A:($20, 0.5) and B:($10, 1). If s/he fixates more the amount of alternative A than the amount of alternative B, higher weights would be given to the former, and thus the riskier alternative (A) would be preferred by the model over the safer one (B).

(A) (B)

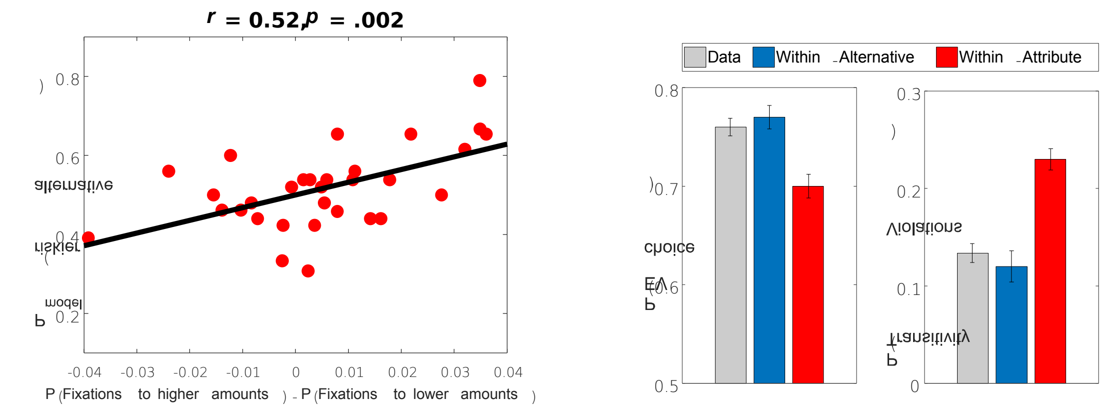

Figure 7. Model predictions and individual differences. (A) The proportion of fixations

toward the higher amount was correlated with the risk-seeking preference predicted by the two-layer within-alternative integration model with parameters fixed at the group-mean, but with actual fixations. (B) The within-alternative model is closer to the data and predicts a higher fraction of economical-normative EV-choices (left panel) and a lower fraction of (irrational) transitivity violations (right panel), compared with the within-attribute model. Error bars represent the standard error of the mean.

Finally, we address an important question: which preference-formation mechanism (*within-alternative* or *within-attribute*) results in better normative performance, and thus can be regarded as more adaptive? To answer this, we simulated the two types of models based on the participants' best fitted parameters and actual fixation sequences, and we examined two measures of normative choice predicted by each model: i) the fraction of *EV*-choices (for simplification we discuss normativity in terms of *EV*, but the same would hold in terms of *EU*), and ii) the fraction of transitivity violations – a direct measure of choice irrationality ([51,52]; see *Suppl. Transitivity Violations*). As seen in Fig. 7B, the normative performance is higher for the within-alternative model than for the within-attribute model, for both measures: *EV*-choices: $t(30) = 6.27$, $p < .001$ and transitivity violations: $t(30) = 5.15$, $p < .001$. This is expected because our within-alternative model, like *CPT*, assumes a multiplication between subjectively transformed amounts and probabilities, which also maintains choice-consistency. Although the economically-normative model requires a multiplication of objective values whereas our model requires a multiplication of subjective values, this discrepancy is relatively minor compared with non-multiplicative strategies (i.e., within-attribute integration or heuristics). Moreover, we have found that the more within-alternative transitions a person makes, the higher is his or her fraction of economically-normative *EV* choices (Fig. 2F; see also [28]). This correlation can be naturally understood, since the participants rely on a within-alternative multiplicative mechanism, and this operation is likely to be more precise following an actual transition between amounts and probabilities (i.e., a fixation on one attribute of an alternative followed immediately by a fixation to the other attribute of the same alternative), than following a non-direct transition (where one of the to-be-multiplied attributes is based on memory or defaults). Consistent with this, we found a correlation across participants between the prediction accuracy of the *within-*

alternative model and the proportion of within-alternative transitions (r = 0.39, p = .03).

3. Discussion

The main aim of our study was to elucidate mechanisms by which different attributes (amounts and probabilities) are integrated to generate an overall subjective value of choice alternative. To this end, we focused on choices between simple lotteries and developed process models of risky choice, which are constrained by eye-fixations and we assumed a fixation-based attentional modulation. In addition, we introduced activation-leak and examined two types of decision-boundaries, in order to account for decision times. Within these models we specifically contrasted *within-alternative* multiplicative models and *within-attribute* type models, and carried out a systematic parametric investigation of choices between simple lotteries (x_1 with p_1 vs. x_2 with p_2), while tracking participants' eye-fixations.

First, we replicated previous findings indicating that participants show preference for lotteries that reflect the economic-normative theory: choice probability of the alternative with the higher *EV* increases (and choice-RT decreases) with *EV*-difference between the lotteries. Nevertheless, participants also exhibited risk biases that are probability-dependent, being risk-averse at high/medium probabilities, but not at low probabilities. Second, we found that, on average, the eye-scan patterns were dominated by within-alternative as compared to within-attribute or diagonal transitions (Fig. 2C-D, respectively), and that individual differences on this eye-scan pattern correlate with *EV*-choice (see also [28], for a similar result). Third, we used eye-fixations to constrain a number of process models that accumulate preference across fixations, using an *aDDM* approach with two attributes [21,53]. Here we contrasted two types of integration-to-boundary process models: i) within-attribute models, and ii) within-alternative models. As shown in Table-2, the latter resulted in the best predictive accuracy and measures of fit. Importantly, the two-layer model also accounted well for decision times (see Fig. 6) and for individual differences in risk biases. Finally, the worst performance in our task was obtained for the non-compensatory heuristic models. For example, the best of the heuristics (the *PH*) resulted in a worse fit than even the simple *EV* model (see cross-validation measure in Table-2).

The conclusions favoring the within-alternative multiplicative models may need to be the task conditions we used here. First, we used simple lotteries with single non-zero outcomes (*x* with *p*, 0 with *1-p*). It is possible that the amount of non-compensatory, within-attribute processing would increase when more complex choices are used ([25,32]). While we cannot rule out this possibility, recent research in the domain of probabilistic inferences ([54–56]), and risky choice ([20,57]), indicate that when decision processes are monitored via eye-tracking (which does not slow down the decision process) rather than via mouse pressing techniques (e.g., [10]), participants are able to use compensatory strategies for relatively high complexity levels (see also [58] for a multi-attribute choice task). With regards to our study, we need to also qualify the results to our use of analog (graphic rather than symbolic) presentation of the data[3], and the fact that our alternatives where presented top/bottom (and thus the amounts and probabilities left/right, Fig. 1A)[4]. Here we wish to support the following conclusion: humans possess the ability to deploy an 'economic' (multiplicative across-dimension) type computation, supporting the idea that humans are closer to normative principles than previously thought (see also [56,59]). Future research will be needed to further quantify to what extent the use of this mechanism (or strategy) depends on the task complexity and type of stimuli.

There are several important properties of our winning process model that we want to highlight. First, it assumes two layers of leaky accumulators, one for the estimation of subjective amounts and of subjective probabilities, and the one for the evaluation of the integrated subjective values (the combination of subjective amounts and probabilities). Second, it assumes that the units in the second layer are updated via a

[3] We used here analog representations because we wanted to prevent our participants (who are students that may be familiar with *EV*-principles), and are required to do 94 choice problems, from adopting an explicit *EV* calculation strategy. We believe that such a strategy is less likely with analog information and thus our results favoring an implicit multiplicative mechanism are even more remarkable.

[4] It is possible to argue that our experimental layout (horizontal) favors within attribute processing (left to right, or right to left for our Hebrew speaking participants). Note, however, that the alternative layout (setting amounts bars horizontally), would trigger a strong bias favoring within-attribute processing (in particular, comparing the aligned bars). Nevertheless, we report in the *Suppl. Pilot study* data from a pilot Experiment (*N*=13) using this layout (i.e., vertical), which shows that even under such within-attribute favorable conditions, we still find dominance for within alternative transitions.

multiplication of the activation of the corresponding, first layer units (Fig. 4A). This is similar to how the *CPT* model generates subjective utilities. In fact, we find a high correlation between the utility function's curvature parameter (α) of the classical *CPT* and the corresponding parameter of our process model (r = .91, p < .001), with higher α -values for the classical *CPT* (see *Suppl. Relationship between the models' utility parameters*). This suggests that the classical *CPT*-parameters reflect a combination of several processes, such as attention allocation and subjective-value transformation [60]. Note also, that the model assumes an activation-leak, a feature that allows it to account for recency effects in the data (see *Suppl. Last fixations and choice*), and prevents a double-integration that would occur in the two-layer model in its absence. Third, in addition to predicting choices, the model also predicts decision times, describing the preference formation dynamics under the integration to boundary framework with inputs that correspond to a multiplicative transformation of subjective amounts and probabilities. In particular, we found support for a collapsing boundary, consistent with choice studies that span longer intervals [49,50]).

Other process models of risky choice, such as *DFT* [15,16] also assume an implicit multiplicative interaction between amounts and probabilities. In *DFT*, however, this is not due to the multiplication of amounts and probabilities but rather to the sampling frequency of the amounts, which changes with the corresponding probabilities. This implies that observers look (or attend) more to a given amount if the corresponding probability is higher. In our data, while we find that the relative number of fixations to an amount increases with its probability, this increase was quite minor (about 1%), and therefore cannot explain the multiplicative interaction [20]. However, it is possible that eye-fixations under-estimate the differential of covert attention modulation.

Future research is also needed to better understand the neural mechanisms underlying these computations [61–63]. While the computation of subjective amounts and probabilities can be understood to involve simple psychophysical transformations over amounts (unbounded scale; [64]) and probabilities (bounded scales; [65]), the nature of the multiplicative interaction between neural activations requires future investigations. Note that a multiplicative interaction is also assumed in the *PCS* risk model [20]. To do so, *PCS* had to assume different neural substrates for amounts (neural activations) and for probabilities (synaptic weights). The latter assumption, however, may be difficult to justify for one-shot decisions, which allow little opportunity for learning synaptic weights. We thus suggest that the multiplicative interactions involve neural activations. While less standard than linear interactions [66], a number of neural mechanisms have been proposed to mediate multiplication of neural activity in neural systems [67,68]. Future research is also needed to extend the scope of this investigation from simple lotteries to more complex ones (with multiple outcomes) and from binary to multiple choices.

4. Methods

4.1 Experiment

Participants. 35 Tel-Aviv University undergraduate students (24 females; ages range from 19 to 26, $Median_{age} = 23$) were recruited to the experiment. All of them reported having normal or corrected-to-normal vision. Four of the participants were not able to carry out the eye tracker calibration task, and thus did not take part in the main experiment, leaving 31 participants. The participants received course credit in exchange for participating, as well as a bonus fee ranging from 0 to 30 Israeli Shekels (ILS), which was contingent upon their choices. The experiment was approved by the ethics committee at Tel-Aviv University.

Apparatus. Eye-movements were recorded using a Tobii TX300 desk-mounted eye-tracker (23" monitor with 1920 x 1080 pixels resolution, sampling rate: 300Hz, spatial accuracy: 0.5°), attached to an Intel i7 personal computer. Displays were presented using Psychtoolbox for MATLAB [69]. Viewing distance was approximately 60 cm. Responses were collected via the computer keyboard. A chin rest was not used.

Stimuli. Each choice consisted of two simple lotteries in the form of p_1 chance to get x_1 ILS (otherwise nothing) vs. p_2 chance to get x_2 ILS (otherwise nothing). An example of the display is presented in Fig. 1A. Amounts were represented by the lower parts of divided bar graphs, and probabilities were represented by the lower sectors of pie charts. These attributes appeared at the vertices of an imaginary square subtending 14.5° (15.25 cm), and located in the center of a black screen. The height of each bar graph subtended 2.07° (2.17 cm) and its width subtended 0.67° (0.69 cm); the radius of each pie chart subtended 0.67° (0.69 cm). Thus, the bar graphs and pie charts had exactly the same surface. The amounts and probabilities of each alternative were displayed horizontally (one lottery was placed over the other); but see footnote 4 and the results of a pilot study reported in *Suppl. Pilot study.*

Choices. Choice problems were constructed in the following way: we generated a 2-dimensional grid with amounts (3, 6, 15, 24 and 30 ILS) along one dimension, and probabilities (0.1, 0.2, 0.5, 0.8 and 1) along the other dimension. The resulting grid contained 25 lotteries (Fig. 1B), each of which was paired with all other possible lotteries. Stochastically dominated choices (in which both the amount and probability of one alternative were higher than those of the other) were excluded, except for 10 choice problems which served as "catch-trials". Overall, the experiment consisted of 104 separate choices: 94 non-dominated trials and 10 "catch-trials" (all the choice problems are given in *Table S1*).

Procedure. The participants signed an informed consent form prior to the experiment. Then, a calibration of the eye-tracker took place. In case the calibration was successful, the experiment started, otherwise recalibration was performed. At the beginning of the experiment, instructions were given to the participants (see *Suppl. Experimental instructions*). The experimenter emphasized that choices should be made in accordance with subjective preferences and that there is no "correct" choice. The experiment consisted of two blocks of 52 choice trials each. A short break was allowed between blocks, and a recalibration procedure was performed before the second block. Each trial began with a fixation display which consisted of a red 0.2° × 0.2° fixation cross (+) that remained on screen until a continuous fixation of 500 ms duration was made. Then, the two lotteries were presented until response. Choice was made using the up and down arrow keys. Participants were told that after completing the experiment, one of the choices will be randomly chosen and payed out. The whole experiment took approximately 30 min per participant. Choice order as well as the horizontal position (left/right) of the amounts and probabilities were randomized for each participant; the vertical position of each lottery (up/down) was randomized between subjects.

Eye-movements. Fixations were classified as being directed to a certain attribute, if they were within 100 pixels of the center of that attribute and lasted at least 50 ms. Two consecutive fixations to the same attribute were joined and considered as one fixation. Trials longer than 10 sec or shorter than 500 ms (4% of all trials), as well as trials in which the participants did not look at all of the attributes (4% of all trials), were excluded from further analysis.

4.2 Models of Risky-choice

Here we briefly describe the key features of the models applied (for a full description see *Suppl. Models of risky choice).* In all of the models (except for the Heuristics models), the probability of choosing each alternative is calculated using an exponential version of Luce's choice rule [70,71]:

$$P(x_1, p_1; x_2, p_2) = \frac{1}{1+e^{-\beta(U_1-U_2)}}$$

where U_1 and U_2 are the utilities of the alternatives, and β is a free parameter indicating the sensitivity of the model to their difference.

Traditional Models

Expected Value (EV)

- Participants choose the alternative with the higher Expected-Value: $EV = x \cdot p$.

Expected Utility (EU)

- Participants choose the alternative with the higher Expected-Utility: $EU = u(x) \cdot p \; ; u(x) = x^{\alpha}$.

Cumulative Prospect Theory (CPT)

- Participants choose the alternative with the higher Subjective-Utility: $SU = u(x) \cdot \pi(p); \pi(p) = \frac{p^{\gamma}}{(p^{\gamma}+(1-p)^{\gamma})^{\frac{1}{\gamma}}}$.

Fixation based regression models

$EU_{Fixations}$

- $U_{alternative} = u(x) \cdot p \cdot n^{\tau}$, where n is the number of fixations to the alternative, and τ is a saturation parameter.

$CPT_{Fixations}$

- Defined analogously to *Multiplicative $EU_{Fixations}$*, but with p replaced with $\pi(p)$ according to *CPT*.

Heuristics
MaxiMax • Choose the alternative with the highest maximum amount.
Least-Likely • Choose the alternative with the lowest probability of its worst outcome**.**
Priority Heuristic • Compare sequentially the following attributes: minimum amounts, probability of minimum amounts, maximum amount. • Stop when difference between the attributes reaches a termination criterion.
Within-attribute Integration
Normalized differences • Normalize the amounts and probabilities using min-max normalization: $y' = \frac{y - min(y)}{max(y) - min(y)}$ • On each fixation, the normalized differences of the attended attribute are accumulated. Unattended attribute are underweighted. • Example: if one fixates on x_1, accumulator A increases by: $x_1' - \theta \cdot x_2'$, and accumulator B by: $\theta \cdot x_2' - x_1'$. $\theta \in [0,1]$ represents attentional modulation.
Categorical differences • This model was implemented as the *normalized differences* model, except that instead of accumulating normalized differences, the model accumulates counts based on categorical comparisons. • Example: if one fixate on x_1 and $x_1 > \theta \cdot x_2$, accumulator A increases by one unit, and B remains the same. $\theta \in [0,1]$ represents attentional modulation.
Within-alternative Integration
One-layer leaky accumulators • On each fixation, this model accumulates the *SU* of the two alternatives, defined as in *CPT*. • At fixation toward alternative A, the input of alternative B is attenuated (and vice versa). • Example: if one fixate on x_1, accumulator A increases by: SU_A, whereas the accumulator B increases by: $\theta \cdot SU_B$. • The activations of the alternatives' accumulators are subject to leak.

Two-layer leaky accumulators

- Two layers of leaky accumulators, the first estimates subjective amounts and probabilities (defined as in *CPT*), and the second estimates integrated *SU*.
- On each fixation, the first-layer units are updated with the subjective amounts and probabilities, with the inputs of the unattended attributes attenuated.
- Example, if one fixates on x_1 the inputs of p_1, x_2 and p_2 are $\theta \cdot \pi(p_1), \theta \cdot u(x_2)$ and $\theta \cdot \pi(p_2)$, respectively. The second-layer units are fed with the activations of the first layer units, and accumulate the product of their values.
- The units of both layers (first and second) are subject to leak.

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Authors' contributions:

D.L., E.N., M.G., M.U. and V.T. developed the study concept. M.G., M.U. and O.S. designed the experiment. M.G. and O.S. performed research and analyzed the data. M.G. and M.U. developed the models. M.G. fitted the models. M.G. and M.U. wrote the paper. All authors provided critical feedback and final approval for publication.

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