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Beyond Truth-Telling: Preference Estimation with Centralized School Choice and College Admissions[†]

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We propose novel approaches to estimating student preferences with data from matching mechanisms, especially the Gale-Shapley deferred acceptance. Even if the mechanism is strategy-proof, assuming that students truthfully rank schools in applications may be restrictive. We show that when students are ranked strictly by some ex ante known priority index (e.g., test scores), stability is a plausible and weaker assumption, implying that every student is matched with her favorite school/college among those she qualifies for ex post. The methods are illustrated in simulations and applied to school choice in Paris. We discuss when each approach is more appropriate in real-life settings. (*JEL* D11, D12, D82, I23)

The past decade has seen the Gale-Shapley deferred acceptance (DA) becoming the leading centralized mechanism for the placement of students to public schools at every education level, and it is now used by many education systems around the world, including Amsterdam, Boston, Hungary, New York, Paris, and Taiwan.

One of the main reasons for the growing popularity of DA is its strategy-proofness (Abdulkadiroğlu and Sönmez 2003). When applying for admission, students are asked to submit rank-order lists (ROLs) of schools, and it is in their best interest to rank schools truthfully. Students and their parents are thus released from strategic

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considerations. Consequently, DA also provides policymakers "with more credible data about school choices, or parent 'demand' for particular schools," as argued by Thomas Payzant (former Boston Public Schools superintendent) (Abdulkadiroğlu et al. 2006, p. 25). Indeed, such rank-ordered data contain rich information on student preferences over schools, and are increasingly used in the empirical literature.

Due to the strategy-proofness of DA, one may be tempted to assume that the submitted ROLs reveal students' true preferences over schools. However, this truth-telling assumption can be restrictive in settings where students face only limited uncertainty about their admission outcomes. One such environment is the "strict-priority" setting in which schools rank students by some priority index, e.g., a test score, which is known to students when submitting their ROL. Consider a student who likes a highly selective school but has a low test score. The student may "skip the impossible" and choose not to apply to this school, as she rationally expects a zero admission probability based on available information such as past admission outcomes. This implies that not all students have strong incentives to rank all schools truthfully in their ROLs.[1]

Based on theoretical investigations of student incentive and behavior, we aim to provide empirical approaches to estimating student preferences in the strict-priority setting, which remains largely unexplored in the empirical literature on school choice and college admissions. Our proposed approaches can potentially be applied in many real-life systems, such as those in Table 1, including school choice in Finland, Paris, and Turkey (panel A) as well as college admissions in Chile, Norway, and Taiwan (panel B).

The paper's first contribution is to clarify the implications of the truth-telling assumption, which hypothesizes that students always report true preferences. Given the flourishing empirical literature on the setting in which schools rank students with post-application lotteries (Pathak and Shi 2014; Abdulkadiroğlu, Agarwal, and Pathak 2017), it is natural to extend those truth-telling-based approaches to the strict-priority setting. Unfortunately, strategy-proofness implies that truth-telling is a *weakly* dominant strategy, leaving open the issue of multiple equilibria because a student may obtain the same outcome by opting for non-truth-telling strategies—as shown in the "skipping the impossible" example above. Making truth-telling even less likely, many applications of DA restrict the length of submittable ROLs, which destroys strategy-proofness (Haeringer and Klijn 2009).

These arguments are formalized in a theoretical model. Deviating from the literature, we introduce an application cost that students incur when submitting ROLs, and the model therefore has the common real-life applications of DA as special cases. Conditional on both preferences and priorities being private information, we show that for truth-telling to be the unique equilibrium, two conditions are needed: no application cost and large uncertainty in admission outcomes. Neither is easily satisfied in the strict-priority setting. Even without limits on the length of submittable ROLs, students may find it costly to rank a long list of schools. As students

[1] In contrast, students can be more uncertain about their admission outcomes if (i) schools use lotteries to break ties ex post, or (ii) schools rank students by test scores that are ex ante unknown. In these cases, the aforementioned student may choose to apply to the highly selective school, since uncertainty in priority indices implies that admission probabilities are rarely zero ex ante.

Table 1—Centralized School Choice and College Admissions Based on the Deferred Acceptance Mechanism with Strict Priority Indices: Examples

Education system	Assignment mechanism	Restrictions	Sources
Panel A. Secondary education			
Boston (exam schools)[a]	Student-proposing DA	Unrestricted	Abdulkadiroğlu, Angrist, and Pathak (2014)
Chicago (exam schools)[a]	DA (serial dictatorship)[c]	Up to 6 choices	Pathak and Sönmez (2013)
NYC (exam schools)[a]	DA (serial dictatorship)[c]	Unrestricted	Abdulkadiroğlu, Angrist, and Pathak (2014)
Finland	School-proposing DA	Up to 5 choices	Salonen (2014)
Ghana	DA (serial dictatorship)[c]	Up to 6 choices	Ajayi (2013)
Paris	School-proposing DA	Up to 8 choices	Hiller and Tercieux (2014)
Romania	DA (serial dictatorship)[c]	Unrestricted	Pop-Eleches and Urquiola (2013)
Singapore	DA (serial dictatorship)[c]	Up to 6 choices	Teo, Sethuraman, and Tan (2001)
Turkey	DA (serial dictatorship)[c]	Up to 12 choices	Akyol and Krishna (2017)
Panel B. Higher education			
Australia (Victoria)	College-proposing DA	Up to 12 choices	Artemov, Che, and He (2017)
Chile	Student-proposing DA	Up to 10 choices	Hastings, Neilson, and Zimmerman (2013)
Hungary	Student-proposing DA	Unrestricted[b]	Biró (2011)
Ireland	College-proposing DA	Up to 10 choices	Chen (2012)
Norway	College-proposing DA	Up to 15 choices	Kirkebøen, Leuven, and Mogstad (2016)
Spain	Student-proposing DA	Region-specific	Mora and Romero-Medina (2001)
Taiwan	College-proposing DA	Up to 100 choices	UAC (2014)
Tunisia	College-proposing DA	Up to 10 choices	Luflade (2018)
Turkey	College-proposing DA	Up to 24 choices	Saygin (2013)

Notes:

[a] For exam schools in Boston, selective enrollment high schools in Chicago, and specialized high schools in NYC, strict priority indices are used in the admission. In contrast, admissions to other schools often do not use strict priority indices.

[b] In Hungary, students may apply for any number of programs but they are charged a fee (of approximately ten euros) for every program after the third application.

[c] In all of the countries/cities listed in this table, students' priorities are based on various combinations of grades, entrance/exit exams, and other criteria (aptitude tests, interviews, etc.). When priority indices are not school-specific, i.e., schools/universities rank students in the same way, DA, whether student-proposing or school/college-proposing, is equivalent to serial dictatorship, under which students, in the order of their priority indices, are allowed to choose among the remaining schools or universities.

know their own priority indices, uncertainty about admission outcomes can also be limited.

Going beyond truth-telling, the paper's second contribution is to propose a set of novel empirical approaches that are theoretically founded. We consider a weaker assumption implied by truth-telling: stability, or justified-envy-freeness, of the matching (Abdulkadiroğlu and Sönmez 2003), meaning that every student is matched with her favorite feasible school. A school is feasible for a student if its ex post cutoff is below the student's priority index. These cutoffs are well-defined and often observable to the researcher: given the admission outcome, each school's cutoff is the lowest priority index of the students accepted there. Conditional on the cutoffs, stability therefore defines a discrete choice model with personalized choice sets, which is straightforward to analyze empirically.

We show that stability is a plausible assumption, as there exists an equilibrium outcome that is asymptotically stable under certain conditions. When school capacities and the number of students increase proportionally while the number of schools is fixed, the fraction of students not matched with their favorite feasible school tends to zero. Although stability, as an ex post optimality condition, is not guaranteed when students' information is incomplete, we provide numerical evidence suggesting that typical real-life markets are sufficiently large for stability to be almost exactly satisfied.

Based on the theoretical results, we propose a menu of approaches for preference estimation. We start by formalizing the truth-telling assumption under which one can apply rank-ordered models on submitted ROLs. In practice, students rarely rank all available schools, and, therefore, the truth-telling assumption often imposes the exogeneity of the length of a submitted ROL.[2]

Stability, but not asymptotic stability, leads to a discrete choice model with personalized choice sets, so the nonparametric identification in the discrete-choice literature can be applied (e.g., Matzkin 1993), under the assumption that priority indices and unobserved preference heterogeneity are independent conditional on observables. An advantage of this approach is that it enables estimation with data on admission outcomes only, although ignoring the information in ROLs entails some efficiency loss in the estimation.

We also provide a solution if neither truth-telling nor stability is satisfied: as long as students do not play dominated strategies, the submitted ROLs reveal true partial preference orders of schools (Haeringer and Klijn 2009),[3] allowing us to derive probability bounds for one school being preferred to another. The corresponding moment inequalities can be used for inference (for a survey, see Tamer 2010). When stability is satisfied and identifies student preferences, these inequalities provide over-identifying information that can improve estimation efficiency (Moon and Schorfheide 2009).

To guide the choice between these identifying assumptions, we consider several statistical tests, provided that the model is correctly specified and identified. Truth-telling, leading to more restrictions than stability, can be tested against stability using a Hausman-type test (Hausman 1978) or a test of over-identifying restrictions (Hansen 1982). Similarly, stability can be tested against undominated strategies: if the outcome is unstable, the stability restrictions are incompatible with the moment inequalities implied by undominated strategies, allowing us to use tests such as Bugni, Canay, and Shi (2015).

Our third contribution is to evaluate the performance of each approach based on simulated and real-life data. Having illustrated the main theoretical results with Monte Carlo simulations, we apply the empirical approaches to school choice data from Paris. There are 1,590 middle school students applying for admissions to 11 academic-track high schools in Paris' southern district through a version of DA. Schools rank applicants by their academic grades but give priority to low-income students. The findings are more consistent with stability than truth-telling. Our proposed statistical tests reject truth-telling in favor of stability but fail to reject stability against undominated strategies. The tests, however, do not provide definitive proof against truth-telling, since they are conditional on the model's parametric assumptions. Additionally, we provide reduced-form evidence on students' ranking behavior suggesting that some students may have omitted the most selective schools from their ROLs because of low admission probabilities. Moreover, the truth-telling-based

[2] Hence, we distinguish *strict* from *weak* truth-telling. The former assumes that every student ranks all schools truthfully, while the latter requires students to rank their most-preferred schools truthfully and allows them to omit the least-preferred schools.

[3] An ROL is a true partial preference order if the listed schools are ranked according to true preferences.

estimator is outperformed by the stability-based one when it comes to predicting admission outcomes and student preferences.

To highlight the differences between the proposed approaches and their underlying behavioral assumptions, we summarize the theoretical results and describe the nesting structure of the assumptions in Section V. We also emphasize the key features of school choice and college admissions in practice that can help researchers to choose the most appropriate empirical approach to preference estimation.

Other Related Literature.—Since the seminal work of Abdulkadiroğlu and Sönmez (2003), the theoretical study of student behavior and matching properties under DA has been extensive, and large-market asymptotics are a common analytical tool (see the survey by Kojima 2015). Closely related to our study is Azevedo and Leshno (2016), who show the asymptotics of stable matchings. Our paper extends theirs to outcomes of Bayesian Nash equilibrium, whereas they assume that students are always truth-telling.

There is a burgeoning literature on preference estimation using centralized school choice data. One strand of this literature uses data from settings in which researchers argue that truth-telling behavior by students is plausible. For example, Hastings, Kane, and Staiger (2008) use data from Charlotte-Mecklenburg public school district, and Abdulkadiroğlu, Agarwal, and Pathak (2017) study school choice data from New York City, which is a "lottery" setting.[4] Both papers estimate student preferences under the assumption that students truthfully report their preferences. In the same spirit, assuming students report their true preferences in surveys, Budish and Cantillon (2012) and De Haan et al. (2018) use reported student ordinal preferences to conduct analysis without estimating preferences.

The second strand of the empirical literature explicitly considers students' strategic behavior when estimating student preferences, especially if the mechanism is not strategy-proof, e.g., the (Boston) immediate-acceptance mechanism (Calsamiglia, Fu, and Güell 2014; He 2015; Hwang 2016; Kapor, Neilson, and Zimmerman 2016; Agarwal and Somaini 2018). In those settings, observed ROLs are sometimes considered as solutions to the maximization of students' expected utility. Avoiding some difficulties of this strategy-based approach, we instead propose methods that rely on equilibrium outcome of the school choice game.[5]

As to the strict-priority setting, there are only a handful of empirical studies (Ajayi 2013, Burgess et al. 2014, Akyol and Krishna 2017). Most of them use ad-hoc solutions to the potential problem of students' non-truth-telling behavior.[6] Akyol and Krishna (2017) is an exception. Observing the outcome and the cutoffs

[4] The authors perform robustness checks, e.g., only considering students' top three submitted choices.

[5] With assumptions on students' beliefs, the strategy-based approach formulates a discrete choice problem defined on the set of possible ROLs. It faces some challenges. (i) Degenerate admission probabilities can occur, leading to multiple equilibria (He 2015). (ii) Application costs, especially those related to cognitive load, are often unobservable, necessitating additional assumptions in the maximization of expected utility. (iii) A given ROL is evaluated against a large number of alternative ROLs, sometimes creating computational burden (e.g., there exist $S!/(S-K)!$ lists ranking $1 \leq K \leq S$ schools).

[6] Analyzing school choice in the United Kingdom, where proximity to schools breaks ties in determining admission to oversubscribed primary schools, Burgess et al. (2014) restrict each student's set of schools to those in close proximity to the student's residence. In the context of admissions to secondary schools in Ghana, where exam scores determine priority, Ajayi (2013) considers a subset of schools with similar selectivity.

of high school admissions in Turkey, the authors estimate preferences based on the assumption that every student is assigned to her favorite feasible school, which amounts to assuming stability of the matching. We formalize and clarify this stability assumption, along with other extensions. Although stability is a rather common identifying assumption in the two-sided matching literature (see the surveys by Fox 2009, Chiappori and Salanié 2016),[7] it is new in empirical studies of school choice and college admissions.

Lastly, estimation of student preferences with college admissions data is under-explored, often due to the decentralized nature of the admission process. Among centralized admissions, however, there are many applications of the DA mechanism (see Table 1).[8] The specifics of the mechanism have led to numerous studies on the causal effects of education (e.g., Hastings, Neilson, and Zimmerman 2015; Kirkebøen, Leuven, and Mogstad 2016), but few on preference estimation. One exception is Kirkebøen (2012) who uses the truth-telling assumption while excluding from a student's choice set every college program at which the student does not meet the formal requirements or is below its previous-year cutoff. Another is Bucarey (2018) who applies our stability-based estimator to evaluate the crowding-out effects of free college tuition for low-income students in Chile.

Organization of the Paper.—Section I presents the model and the theoretical foundation. Section II formalizes the empirical approaches and tests, which are illustrated in Monte Carlo simulations in Section III. School choice in Paris and the empirical results are shown in Section IV. Section V discusses practical considerations for applying the approaches to data and outlines some extensions. We conclude in Section VI.

I. The Model

To study student behavior, we extend the model in Azevedo and Leshno (2016). An economy, as a school choice/college admissions problem, consists of a finite set of schools/colleges, $\mathcal{S} \equiv \{1,\ldots,S\}$, and a set of students. Student i has a type $\theta_i = (u_i, e_i) \in \Theta \equiv [0,1]^S \times [0,1]^S$, where $u_i = (u_{i,1},\ldots,u_{i,S}) \in [0,1]^S$ is a vector of von Neumann-Morgenstern (vNM) utilities of being assigned to schools, and $e_i = (e_{i,1},\ldots,e_{i,S}) \in [0,1]^S$ is a vector of priority indices at schools, a student with a higher index having a higher priority at a school. To simplify notation, we assume that all schools and students are acceptable.[9] Students are matched with schools through a centralized mechanism.

The continuum economy with a unit mass of students is denoted by $E = \{G, q, C\}$, where G is an atomless probability measure over Θ representing the distribution of student types; $q = (q_1,\ldots,q_S)$ are masses of seats available at each school, where $q_s \in (0,1)$ for all s; lastly, C is an application cost, to be specified shortly.

[7] This literature usually considers decentralized matching markets; Agarwal (2015) is an exception.

[8] Some centralized college admissions do not use DA, e.g., Brazil (Carvalho, Magnac, and Xiong forthcoming).

[9] Assuming acceptability of all schools justifies the normalization of $u \in [0,1]^S$. Although we could extend the preference domain to allow for negative values, this would create the possibility that students avoid being assigned to schools with negative vNM utilities when maximizing expected utility.

Probability measure G being atomless implies a measure-zero set of students with indifference in either utilities or priority indices.

A random finite economy of size I is denoted by $F^{(I)} = \left\{G^{(I)}, q^{(I)}, C\right\}$; $F^{(I)}$ is constructed by independently drawing I students, indexed by $i \in \{1, \ldots, I\}$, from the distribution G and adjusting the numbers of seats to integers. Specifically, $G^{(I)}$ is the random empirical distribution of types for a sample of I students;[10] $q^{(I)} = [q \cdot I]/I$ is the supply of seats per student, where $[x]$ is the vector of integers nearest to x (with a rounding down in case of a tie). We use $\hat{F}^{(I)} = \left\{\hat{G}^{(I)}, q^{(I)}, C\right\}$ to denote a realization of $F^{(I)}$.

In the following, we start with $F^{(I)}$ to specify the matching process and to analyze student behavior, because empirical studies deal with finite economies; the extension to the continuum economy E is deferred to Section ID.

In a realization of the random economy, $\hat{F}^{(I)}$, schools first announce their capacities, and every student then submits a rank-order list (ROL) of $1 \leq K_i \leq S$ schools, denoted by $L_i = \left(l_i^1, \ldots, l_i^k, \ldots, l_i^{K_i}\right)$, where $l_i^k \in \mathcal{S}$ is i's kth choice. Note that L_i also represents the set of schools being ranked in L_i. We define $\succ_{L_i}$ such that $s \succ_{L_i} s'$ if and only if school s is ranked above school s' in L_i. The set of all possible ROLs is $\mathcal{L}$, which includes all ROLs ranking at least one school. Student i's true ordinal preference induced by her vNM utilities u_i is denoted by $r(u_i) = \left(r_i^1, \ldots, r_i^S\right) \in \mathcal{L}$.

When submitting an ROL, a student incurs a cost $C(|L|)$, which depends on the number of schools being ranked in L, $|L|$. Furthermore, $C(|L|) \in [0, +\infty]$ for all L and is weakly increasing in $|L|$. To simplify students' participation decisions, we set $C(1) = 0$.

Such a cost function flexibly captures many common applications of school choice mechanisms. If $C(|L|) = 0$ for all L, we are in the traditional setting without costs (e.g., Abdulkadiroğlu and Sönmez 2003); if $C(|L|) = \infty$ for $|L|$ greater than a constant $\bar{K}$, it corresponds to the constrained school choice where students cannot rank more than $\bar{K}$ schools (e.g., Haeringer and Klijn 2009); when $C(|L|) = \max\left\{0, (|L| - \underline{K})c\right\}$, students pay a constant marginal cost c for each choice beyond the first $\underline{K}$ choices, as in Hungarian college admissions (Biró 2011); lastly, the monotonic cost function may simply reflect that it is cognitively burdensome to rank too many schools.

The student-school match is then solved by a mechanism that takes into account students' ROLs and schools' rankings over students. Our main analysis focuses on the student-proposing Gale-Shapley deferred acceptance (DA), leaving the discussion of other variants to Section VB. As a computerized algorithm, DA works as follows.

Round 1.—Every student applies to her first choice. Each school rejects the lowest-ranked students in excess of its capacity and temporarily holds the other students.

Generally, in

Round k.—Every student who is rejected in Round $(k-1)$ applies to the next choice on her list. Each school, pooling together new applicants and those it holds

[10]For a realized economy with realized student types $(\theta_1, \ldots, \theta_I)$, the realized empirical distribution $\hat{G}^{(I)}$ is defined as $\hat{G}^{(I)}(\theta) = \frac{1}{I}\sum_{i=1}^{I} \mathbf{1}(\theta_i \leq \theta), \forall \theta \in \mathbb{R}^{2S}$, where $\mathbf{1}(\cdot)$ is an indicator function.

from Round $(k-1)$, rejects the lowest-ranked students in excess of its capacity. Those who are not rejected are temporarily held by the schools.

The process terminates after any Round k when no rejections are issued. Each school is then matched with the students it is currently holding.

A. *Information Structure and Decision-Making*

In a realization of the finite economy, $\hat{F}^{(I)}$, given its construction, every student's preferences and priority indices are private information, and are i.i.d. draws from G, which is common knowledge (but $\hat{G}^{(I)}$, the realization of $G^{(I)}$, remains unknown).

We start by taking student i's point of view. Conditional on others' submitted ROLs and priority indices (L_{-i}, e_{-i}), as well as student i's (L_i, e_i), her admission outcome is deterministic, given the algorithm. Specifically, i's admission outcome at school s is:

$$a_s(L_i, e_i; L_{-i}, e_{-i})$$

$$\equiv \begin{cases} \mathbf{1}\left(i \text{ is rejected by } l_i^1, \ldots, l_i^k \text{ and accepted by } l_i^{k+1} = s \,|\, L_i, e_i; L_{-i}, e_{-i}\right) & \text{if } s \in L_i \\ 0 & \text{if } s \notin L_i\text{'} \end{cases}$$

where $\mathbf{1}(\cdot \,|\, L_i, e_i; L_{-i}, e_{-i})$ is an indicator function. Moreover, due to the centralized mechanism, a student can receive at most one offer, so $\sum_{s=1}^{S} a_s(L_i, e_i; L_{-i}, e_{-i}) = 0$ or 1.

Of course, L_{-i} and e_{-i} are unknown to i at the time of submitting her ROL, so i takes into account the uncertainty when choosing an action. A pure strategy is $\sigma : \Theta \rightarrow \mathcal{L}$. Given σ, the admission probabilities are $\int a_s(\sigma(\theta_i), e_i; \sigma_{-i}(\theta_{-i}), e_{-i})\, dG(\theta_{-i})$ for all i and s, where $\sigma_{-i}(\theta_{-i}) \equiv \{\sigma(\theta_j)\}_{j \neq i}$. We consider a (type-)symmetric equilibrium σ^* in pure strategies such that σ^* solves the following maximization problem for every θ_i:[11]

$$(1) \quad \sigma^*(\theta_i) \in \underset{\sigma(\theta_i) \in \mathcal{L}}{\arg\max} \left\{ \sum_{s \in \mathcal{S}} u_{i,s} \int a_s(\sigma(\theta_i), e_i; \sigma^*_{-i}(\theta_{-i}), e_{-i})\, dG(\theta_{-i}) - C(|\sigma(\theta_i)|) \right\}.$$

The existence of pure-strategy Bayesian Nash equilibrium can be established by applying Theorem 4 (Purification Theorem) in Milgrom and Weber (1985), although there can be multiple equilibria. For ease of exposition, the following analysis focuses on pure-strategy equilibrium. We note that while economy $F^{(I)}$ is random, a strategy σ is "deterministic" in the sense that it only depends on (G, I, C) but not on the realization of $F^{(I)}$.

We define a realized matching $\hat{\mu}$ as a mapping from Θ to $\mathcal{S} \cup \{\varnothing\}$ such that (i) $\hat{\mu}(\theta_i) = s$ if student i is matched with s; (ii) $\hat{\mu}(\theta_i) = \varnothing$ if student i is unmatched; and (iii) $\hat{\mu}^{-1}(s)$ is the set of students matched with s, while $|\hat{\mu}^{-1}(s)|$ is the number of students matched with s and does not exceed s's capacity.

[11] It is innocuous to focus on symmetric equilibrium, because it does not restrict the strategy of any student given that they all have different priority indices (almost surely).

The terms $\hat{F}^{(I)}$ and σ together lead to an ROL profile as inputs into the DA mechanism and result in a matching, $\mu_{(\hat{F}^{(I)},\sigma)}$, which is uniquely determined by the mechanism. Note that $\mu_{(F^{(I)},\sigma)}$ is a random matching because $F^{(I)}$ is a random economy.

Moreover, the (random) cutoff of school s in random matching $\mu_{(F^{(I)},\sigma)}$ is defined as:

$$P_s\left(\mu_{(F^{(I)},\sigma)}\right) = \begin{cases} \min\left\{e_{i,s} \,|\, \mu_{(F^{(I)},\sigma)}(\theta_i) = s\right\} & \text{if } |\mu^{-1}_{(F^{(I)},\sigma)}(s)| = q_s^{(I)} \\ 0 & \text{if } |\mu^{-1}_{(F^{(I)},\sigma)}(s)| < q_s^{(I)} \end{cases}.$$

That is, $P_s\left(\mu_{(F^{(I)},\sigma)}\right)$ is zero if s does not meet its capacity; otherwise, it is the lowest priority index among all accepted students. The vector of cutoffs is denoted by $P\left(\mu_{(F^{(I)},\sigma)}\right)$, and its realization in $\hat{F}^{(I)}$ is $P\left(\mu_{(\hat{F}^{(I)},\sigma)}\right)$.

B. *Truth-Telling Behavior in Equilibrium*

To assess the plausibility of the truth-telling assumption in empirical studies, we begin by investigating students' truth-telling behavior in equilibrium. A clarification of the concepts is in order. Student i is **weakly truth-telling** (WTT, hereafter) if $\sigma(\theta_i) = \left(r_i^1, r_i^2, \ldots, r_i^{K_i}\right)$ for $K_i \leq S$. That is, i ranks her K_i most-preferred schools by her true preference order but may not rank all schools. If a WTT strategy always truthfully ranks all S schools and thus $\sigma(\theta_i) = r(u_i)$, i is **strictly truth-telling** (STT, hereafter).[12]

We emphasize the difference between WTT and STT because strategy-proofness concerns the latter. However, WTT is often considered in empirical studies because in practice, students rarely rank all available schools, as we shall revisit in Section IIB.

It is known that DA is strategy-proof when there is no application cost (Dubins and Freedman 1981, Roth 1982). That is, when $C(|L|) = 0$ for all $L \in \mathcal{L}$, STT is a weakly dominant strategy for all students. However, strategy-proofness, or weak dominance of STT, leaves open the possibility of multiple equilibria. Even when all others play STT, there may exist multiple best responses for a given student.[13] It is therefore useful to clarify the conditions under which STT is the unique equilibrium. The following example highlights two sources of equilibrium multiplicity in a complete-information environment.

EXAMPLE 1 (Multiple Equilibria under DA without Application Cost): *Consider a finite economy that has two students* (i_1, i_2), *three one-seat schools* (s_1, s_2, s_3), *but no application cost. As common knowledge, all schools rank* i_1 *above* i_2; *student* i_1*'s preference order is* (s_1, s_2, s_3), *but* i_2*'s is* (s_2, s_1, s_3). *There are many equilibria in addition to STT, stemming from two sources:* ***irrelevance at the bottom*** *and* ***skipping the impossible****. Both arise when some admission probabilities are zero.*

[12] Related to the distinction between STT and WTT, the literature on lab experiments on school choice sometimes also defines truth-telling as being different from STT. For example, Chen and Sönmez (2006) call a student truth-telling under the DA mechanism if she ranks her most-preferred schools up to her district school, at which she has guaranteed admission.

[13] Unfortunately, it is impossible to make STT a strictly dominant strategy, because it would require STT to be strictly better than all other strategies against all possible action profiles of other students.

For i_1, the bottom part of her submitted ROL is irrelevant as long as s_1 is top-ranked. In fact, any ROL (s_1, s', s''), for $s', s'' \in \{s_2, s_3\} \cup \{\varnothing\}$, is weakly dominant for i_1, as she is always accepted by s_1.

For student i_2, "skipping the impossible" comes into play. She can omit s_1 from her submitted ROL without affecting her outcome, because s_1 is always taken by i_1 in any equilibrium. Making things worse, how she ranks s_1 is payoff-irrelevant.

One may conjecture that STT might survive as the unique equilibrium when information is incomplete. Indeed, specifying the incompleteness of information, the following proposition provides sufficient conditions and a necessary condition.

PROPOSITION 1:

(*i*) ***Sufficiency:*** *STT is the unique Bayesian Nash equilibrium under DA if* (*i*) *there is no application cost: $C(|L|) = 0, \forall L \in \mathcal{L}$; and* (*ii*) *the joint distribution of preferences and priorities G has full support.*

(*ii*) ***Necessity:*** *For any nonzero application cost, there always exist student types for whom STT is not an equilibrium strategy.*

All proofs can be found in online Appendix A. The no-cost condition is violated if students cannot rank as many schools as they wish, or if they suffer a cognitive burden when ranking too many schools. It should also be emphasized that the cost need not be large to make students deviate from STT, because the marginal benefit of ranking an additional school can be close to zero. When a student considers her admission probability at her *k*th choice, she may face a close-to-one probability of being accepted by at least one of her earlier choices. This is in the same spirit as the "irrelevance at the bottom" in Example 1. When the marginal application cost exceeds marginal benefits, STT is no longer a best response, which implies the necessity of the zero-cost condition.

The full-support condition, also considered in Chen and Pereyra (2017), makes all admission probabilities non-zero by introducing uncertainties, and therefore any deviation from STT is costly. This is more plausible when the priority index is determined by an ex post lottery and when the information on others' preferences over schools is less precise.

REMARK 1: *Proposition 1 specifies when students have incentives to rank all schools truthfully, but this result does not extend to WTT. Although it is sometimes used for identification and estimation, the WTT assumption is not supported as an equilibrium.*[14]

We may take one step back and focus on whether students have incentives to order the *ranked* schools truthfully. We call L_i, $|L_i| \leq S$, a **partial preference order** of

[14] The equilibrium condition, equation (1), implies that a student may "skip the impossible" by omitting her most-preferred school if the admission probability is close to zero, thus violating WTT.

schools if L_i respects the true preference order among those ranked in L_i. That is, L_i ranks s above s', only if $u_{i,s} > u_{i,s'}$; when s is not ranked in L_i, there is no information on how s is ranked relative to any other school according to i's true preferences.

PROPOSITION 2: *Under DA with application cost, if students do not play weakly dominated strategies, a student's submitted ROL is a partial order of her true preferences.*

Proposition 2 can be considered as a corollary of Proposition 4.2 in Haeringer and Klijn (2009), and thus we omit its proof. The key is that a non-partial-preference-order ROL is weakly dominated by the ROL that ranks the same schools according to their true preference order. This result is useful for empirical analysis, as it specifies students' revealed preferences. Section IIE formulates how to use this information in estimation.

C. *Admission Outcome: Stability*

The above results speak to the plausibility of the truth-telling assumptions, WTT and STT, in empirical studies. In particular, WTT is not theoretically supported as a weakly dominant strategy even in DA with no application cost; whenever there is any form of application cost, STT is no longer a dominant strategy.

Taking a different perspective, we note that all equilibria lead to the same matching in Example 1. This motivates us to investigate the properties of equilibrium outcomes of DA. Intuitively, the degree of multiplicity in equilibrium outcomes must be smaller than that in equilibrium strategies. In the two-sided matching literature, stability is the leading concept for equilibrium outcome and the main identifying assumption (Chiappori and Salanié 2016). We investigate whether stability can also be satisfied in all equilibrium outcomes of school choice and college admissions.

Unfortunately, we shall demonstrate that having stability satisfied in all equilibrium outcomes requires similar conditions to those for STT being the unique equilibrium. In fact, whenever there are application costs, stability is not guaranteed in equilibrium either. This is because Bayesian Nash equilibrium implies ex ante optimality of student strategy, while stability requires ex post optimality.

As we study a matching's ex post properties, let us consider $\hat{\mu}$, a realization of the random matching. Say (i,s) forms a **blocking pair** if (i) i prefers s over her matched school $\hat{\mu}(\theta_i)$ while s has an empty seat ($|\hat{\mu}^{-1}(s)| < I \times q_s^{(I)}$), or if (ii) i prefers s over $\hat{\mu}(\theta_i)$ while s has no empty seats ($|\hat{\mu}^{-1}(s)| = I \times q_s^{(I)}$) but i's priority index is higher than its cutoff, $e_{i,s} > \min_{\{j:\hat{\mu}(\theta_j)=s\}}\{e_{j,s}\}$. Matching $\hat{\mu}$ is **stable** if there is no blocking pair. Stability is also known as **elimination of justified envy** in school choice (Abdulkadiroğlu and Sönmez 2003).

Given a realized matching $\hat{\mu}$, school s is ex post **feasible** for i if i's priority index at s is above s's cutoff, $e_{i,s} \geq P_s(\hat{\mu})$. Let $\mathcal{S}(e_i, P(\hat{\mu}))$ be the set of feasible schools for i.

With these definitions, combining Lemmata 1 and 2 in Balinski and Sönmez (1999), we reach an important result: *a realized matching $\hat{\mu}$ is stable if and* *only if every student is matched with her favorite feasible school* (i.e., $\hat{\mu}(\theta_i) = \arg\max_{s \in \mathcal{S}(e_i, P(\hat{\mu}))} u_{i,s}, \forall i$). As the cutoffs of a matching are observed ex post by the researcher, we can define every student's set of feasible schools; stability therefore implies a *discrete choice model with observable*, *personalized choice sets*. We further formalize this in Section IIC.

We are interested in stability being satisfied in an outcome of dominant-strategy equilibrium, which would free us from specifying the information structure and from imposing additional equilibrium conditions. The following lemma provides necessary and sufficient conditions, which are similar to those for STT to be the unique equilibrium.

LEMMA 1: *Under DA*, *a Bayesian Nash equilibrium in dominant strategy always leads to a stable matching if and only if* $C(|L|) = 0$ *for all L. It is the unique equilibrium outcome if additionally G has full support.*

The "if and only if" statement of the lemma is implied by strategy-proofness of DA without application cost, while the uniqueness statement is a result of Proposition 1.

DA is known to produce a stable matching when students are STT (Gale and Shapley 1962), but not when students are only WTT. The following results, clarifying the relationship between WTT and stability, have implications for our empirical approaches.

PROPOSITION 3: *Suppose that every student is WTT under DA*, *which may not be an equilibrium. Given a realized matching,*

(*i*) *whenever a student is assigned*, *she is matched with her favorite feasible school;*

(*ii*) *if everyone who has at least one feasible school is assigned*, *the matching is stable.*

The above results describe the nesting structure of the two assumptions, WTT and stability, although they do not speak to the plausibility of either of them being an equilibrium strategy/outcome. Specifically, WTT is more restrictive, as it implies the no-blocking property among assigned students. We use these results to formulate statistical tests for the choice between WTT and stability in Section IID.

D. *Asymptotic Stability in Bayesian Nash Equilibrium*

So far, we have shown that neither truth-telling (STT and WTT) nor stability can emerge in equilibrium without some potentially restrictive assumptions. Following the literature on large markets, we study whether stability can be asymptotically satisfied.

We now revisit the continuum economy, *E*, and additionally introduce a sequence of random finite economies $\{F^{(I)}\}_{I \in \mathbb{N}}$ that are constructed from *E* as before.

The definitions of matching, DA, and stability can be naturally extended to continuum economies as in Abdulkadiroğlu, Che, and Yasuda (2015) and Azevedo and

Leshno (2016), which are discussed in online Appendix A.2.1. These definitions are similar to their counterparts in finite economies. For example, a matching in E when everyone adopts σ is $\mu_{(E,\sigma)} : \Theta \rightarrow \mathcal{S} \cup \{\varnothing\}$, which satisfies (i) $\mu_{(E,\sigma)}(\theta_i) = s$ when type θ_i is matched with s and (ii) $G\left(\mu^{-1}_{(E,\sigma)}(s)\right) \leq q_s$.

It is known that, generically, there exists a unique stable matching in the continuum economy (Azevedo and Leshno 2016);[15] we impose the conditions for the uniqueness and denote this stable matching in E as μ^{∞} and the corresponding cutoffs as P^{∞}. To continue our exploration, we make the following assumption.

ASSUMPTION 1: *Every Bayesian Nash equilibrium of the continuum economy E results in the unique stable matching*, μ^{∞}.

A sufficient condition for Assumption 1 is $C(2) > 0$ (i.e., it is costly to apply to more than one school), and when $C(2) = 0$, a sufficient and necessary condition is Ergin acyclicity (Proposition A3 in online Appendix A.2.5). An economy is acyclical if no student can block a potential settlement between any other two students without affecting her own match (Ergin 2002). Online Appendix A.2.5 gives its formal definition in continuum economies. This condition is satisfied when all schools rank every student by a single priority index.

Because we are interested in equilibrium outcomes, we augment the sequence of economies with equilibrium strategies, $\{F^{(I)}, \sigma^{(I)}\}_{I \in \mathbb{N}}$, where $\sigma^{(I)}$ is a pure-strategy Bayesian Nash equilibrium in $F^{(I)}$ and satisfies the following assumption.

ASSUMPTION 2: *There exists σ^{∞} such that* $\lim_{I \to \infty} G\left(\left\{\theta_i \in \Theta \,|\, \sigma^{(I)}(\theta_i) = \sigma^{\infty}(\theta_i)\right\}\right) = 1$.

A sufficient condition for Assumption 2 is $C(2) > 0$ (Lemma A5 in online Appendix A.2.4).[16] Although $F^{(I)}$ is a random economy, $\sigma^{(I)}$ is fixed given the size of the economy. In other words, $\sigma^{(I)}$ remains as an equilibrium strategy in any realization of $F^{(I)}$. Assumption 2 regulates how the equilibria evolve with economy size, which is necessary as there are multiple equilibria. By this assumption, in the sequence $\{\sigma^{(I)}\}_{I \in \mathbb{N}}$, fewer and fewer student types need to adjust their optimal

[15] A sufficient condition for the uniqueness of stable outcome in E is that G has full support. Even when G does not have full support, the uniqueness can be achieved when $\sum_{s=1}^{S} q_s < 1$. Let σ^{STT} be the STT strategy. We define the demand for each school in (E, σ^{STT}) as a function of cutoffs, $D_s(P \,|\, E, \sigma^{STT}) = \int \mathbf{1}\left(u_{i,s} = \max_{s' \in \mathcal{S}(e_i, P)} u_{i,s'}\right) dG(\theta_i)$. Let $D(P \,|\, E, \sigma^{STT}) = \left[D_s(P \,|\, E, \sigma)\right]_{s \in \mathcal{S}}$. Note that E admits a unique stable matching if the image under $D(P \,|\, E, \sigma^{STT})$ of the closure of the set

$$\left\{P \in (0,1)^S : D(P \,|\, E, \sigma^{STT}) \text{ is not continuously differentiable at } P\right\}$$

has Lebesgue measure 0.

[16] Allowing $C(2) = 0$, online Appendix A.2.4 investigates the properties of equilibrium strategies. The results, Lemmata A2–A4, imply strong restrictions on the sequence of Bayesian Nash equilibria in the direction of satisfying Assumption 2. Specifically, it is shown that a strategy that does not lead to μ^{∞} in the continuum economy cannot survive as an equilibrium when $I \rightarrow \infty$. This immediately implies that in sufficiently large economies, every student includes in her ROL the school prescribed by μ^{∞}. Moreover, students do not pay a cost to rank more schools in large economies.

actions when the economy enlarges. Moreover, given Assumption 1, the limit strategy σ^{∞} leads to μ^{∞} in E (Proposition A1 in online Appendix A.2.2).

Asymptotic Stability: Definition and Results.—Let the random matching $\mu_{\left(F^{(I)},\sigma^{(I)}\right)}$ be $\mu^{(I)}$, and the associated random cutoffs $P\left(\mu^{(I)}\right)$ be $P^{(I)}$. The following definition formalizes the concept of asymptotic stability.[17]

DEFINITION 1: *A sequence of random matchings, $\left\{\mu^{(I)}\right\}_{I\in\mathbb{N}}$, associated with the sequence of random economies and equilibrium strategies, $\left\{F^{(I)},\sigma^{(I)}\right\}_{I\in\mathbb{N}}$, is asymptotically stable if the fraction of students who are matched with their favorite feasible school in a random finite economy $(F^{(I)})$ converges to* 1, *almost surely*, *or*, *equivalently*,

$$\lim_{I\to\infty} G^{(I)}\left(\left\{\theta_i \in \Theta \,|\, \mu^{(I)}(\theta_i) = \operatorname*{arg\,max}_{s\in\mathcal{S}\left(e_i,P^{(I)}\right)} u_{i,s}\right\}\right) = 1, \quad \textit{almost surely}.$$

We are now ready to introduce our main result.

PROPOSITION 4: *In the sequence of random economies and equilibrium strategies, $\left\{F^{(I)},\sigma^{(I)}\right\}_{I\in\mathbb{N}}$, if Assumptions 1 and 2 are satisfied*, *then*

(*i*) *the random cutoffs converge to those of the stable matching in the continuum economy:* $\lim_{I\to\infty} P^{(I)} = P^{\infty}$, *almost surely;*

(*ii*) *the sequence of random matchings, $\left\{\mu^{(I)}\right\}_{I\in\mathbb{N}}$, is asymptotically stable.*

Part (ii) implies that the fraction of students who are matched with their favorite feasible school, or not in any blocking pair, converges to one almost surely, as the economy grows. This provides justifications for the stability assumption in large markets.[18]

Probability of Being in a Blocking Pair for a Given Student.—To assess if a matching is likely to be stable, we investigate how the probability that a given student is in a blocking pair changes with economy attributes. The following proposition shows how economy size, application cost, and other factors play a role.

[17] We define the probability space, $(\Omega,\mathcal{F},\mathcal{P})$. Specifically, $\Omega = \prod_{I\in\mathbb{N}}\Theta^{I}$, and an element in Ω is denoted by $\omega = (\omega_1,\omega_2,\ldots)$, where ω_I is a possible realization of student types in the random economy $F^{(I)}$. Further, $\mathcal{F}$ is a Borel σ-algebra of Ω, and $\mathcal{P}$ is a probability measure from $\mathcal{F}$ to $[0,1]$.

[18] This result, however, does not mean that the probability of a matching being stable converges to one as the market grows. As long as there is at least one blocking pair, a matching is not stable.

PROPOSITION 5: *Suppose student i exists in all economies in the sequence* $\{F^{(I)}\}_{I\in\mathbb{N}}$ *which is associated with a sequence of Bayesian Nash equilibria in pure strategies* $\{\sigma^{(I)}\}_{I\in\mathbb{N}}$.

(*i*) *Let* $\sigma^{(I)}(\theta_i) = L^{(I)}$; *then* $L^{(I)}$ *is a partial order of i's ordinal preferences. If ex post i forms a blocking pair with s, s must not be included in* $L^{(I)}$, $s \notin L^{(I)}$. *The probability that i is in a blocking pair with any school in the random matching* $\mu^{(I)}$, *denoted by* $B_i^{(I)} = \Pr\Big(\exists s \in \mathcal{S}, u_{i,s} > u_{i,\mu^{(I)}(\theta_i)}$, *and* $e_{i,s} \geq P_s^{(I)}\Big)$, *satisfies:*

(*ii*) $B_i^{(I)}$ *is bounded above:* $B_i^{(I)} \leq |\mathcal{S}\backslash L^{(I)}| \dfrac{C(|L^{(I)}|+1) - C(|L^{(I)}|)}{\max_{s\in\mathcal{S}\backslash L^{(I)}} u_{i,s}}$;

(*iii*) *if* $\{\sigma^{(I)}\}_{I\in\mathbb{N}}$ *satisfies Assumptions* 1 *and* 2, $B_i^{(I)}$ *converges to* 0 *almost surely.*

Because in equilibrium student *i* reports a partial order of her true preferences, she can only form a blocking pair with a school that she did not rank (part i). Therefore, the probability that *i* is in a blocking pair decreases whenever it is less costly to rank more schools (part ii). Together, Proposition 5 shows that stability is more plausible when the cost of ranking more schools is lower and/or the economy is large. Moreover, in the case of constrained/truncated DA where there is a limit on the length of ROLs, the higher the number of schools that can be ranked, the more likely stability is to be satisfied.

II. Empirical Approaches

Building on the theoretical results, we formalize the estimation of student preferences under different sets of assumptions and propose a series of tests to guide the selection of the appropriate approach. To be more concrete, we consider a logit-type random utility model, although our approaches can be extended to other specifications.

This section focuses on a random finite economy $F^{(I)}$ in which I students compete for admissions to S schools. Each school has a positive capacity, and students are assigned through a version of the student-proposing DA. Besides submitted ROLs and admission outcomes, the researcher observes priority indices, student characteristics, and school attributes. Given these observables, we discuss the probability of a student submitting a given ROL or being matched with a given school from the researcher's perspective.

A. *Model Setting and Revealed Preferences*

As is traditional and more convenient in empirical analysis, we let the student utility functions take any value on the real line.[19] With some abuse, we still use the

[19] In the theoretical discussion, the utility functions are restricted to be in $[0,1]$. One can use the inverse of the standard normal distribution, Φ^{-1}, to transform them to be on the real line. Note that the expected utility theory cannot be applied to the transformed utility functions; indeed, we do not.

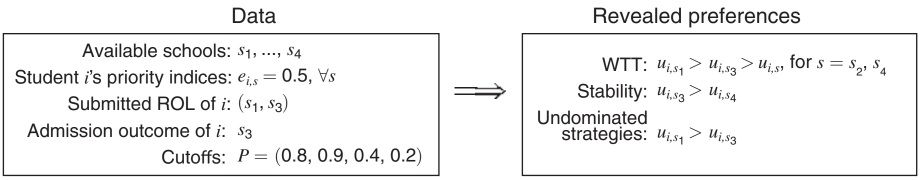

Figure 1. Revealed Preferences under Different Assumptions: An Example

same notation for utility functions. To facilitate the analysis, student i's utility from attending schools s is parameterized:

$$(2) \qquad u_{i,s} = V_{i,s} + \epsilon_{i,s} = V(Z_{i,s}, \beta) + \epsilon_{i,s},$$

where $V(\cdot,\cdot)$ is a known function, taking as arguments $Z_{i,s}$, a vector of observable student-school characteristics, and β, a vector of unknown parameters to be estimated; $\epsilon_{i,s}$ is the unobservable student heterogeneity.

We further define $Z_i \equiv \{Z_{i,s}\}_{s=1}^{S}$, and $\epsilon_i \equiv \{\epsilon_{i,s}\}_{s=1}^{S}$. It is assumed that $\epsilon_i \perp Z_i$ and that $\epsilon_{i,s}$ is i.i.d. over i and s with the type-I extreme value (Gumbel) distribution. Such a formulation rules out outside options, although this assumption can be relaxed.

We also assume that a student's preferences are not affected by other students' school assignments (no peer effects) and that statistics associated with the realized matching, such as cutoffs, do not enter the utility function. This is consistent with the theoretical model in Section I and implies that Z_i does not include variables that depend on the ex post observed matching.

The estimation relies on revealed student preferences in the data, and what information is revealed crucially depends on the imposed assumption—WTT, stability, or undominated stategies. Figure 1 shows an example. WTT takes the submitted ranking as truthful and assumes unranked schools being the least preferable. Stability dictates that a student is matched with her favorable feasible school. Lastly, a submitted ROL reveals the student's partial preference order if no one plays dominated strategies. We now detail how to use this information in the estimation.

B. *Truth-Telling*

In the literature on school choice with lotteries, some empirical approaches are based on the truth-telling assumption (Hastings, Kane, and Staiger 2008; Abdulkadiroğlu, Agarwal, and Pathak 2017). As similar mechanisms are commonly used in our strict-priority setting, we extend these approaches to our setting and clarify the assumptions embedded within.

We start with WTT instead of STT because in practice students in school choice and college admissions rarely rank the same number of choices (Abdulkadiroğlu, Agarwal, and Pathak 2017; He 2015; Artemov, Che, and He 2017). Under the assumption of truth-telling without outside option, this can only be consistent with WTT but not STT, because STT requires everyone to rank all schools. We discuss STT with outside options in online Appendix A.4.

For notational convenience, we make it explicit that student i's type θ_i is described by (u_i, e_i). Let $\sigma^W : \mathbb{R}^S \times [0,1]^S \to \mathcal{L}$ be a WTT pure strategy.[20] More precisely, the WTT assumption amounts to the following.

ASSUMPTION 3 (Characterization of Weak Truth-Telling):

WTT1. Suppose $\sigma^W(u_i, e_i) = L = (l^1, \ldots, l^{K_i})$. *Here, L ranks i's top K_i preferred schools according to her true preferences*: $u_{i,l^1} > \cdots > u_{i,l^{K_i}} > u_{i,s'}$ *for all s' not ranked in L*;

WTT2. The number of schools ranked by a student is exogenous: $u_i \perp |\sigma^W(u_i, e_i)|, \forall i.$

We are interested in the choice probability of L conditional on observables, where the uncertainty from the researcher's perspective is due to the utility shocks (ϵ_i). Note that

$$\Pr\big(\sigma^W(u_i, e_i) = L \,|\, Z_i; \beta\big)$$
$$= \Pr\big(\sigma^W(u_i, e_i) = L \,|\, Z_i; \beta; |\sigma^W(u_i, e_i)| = K\big) \times \Pr\big(|\sigma^W(u_i, e_i)| = K \,|\, Z_i; \beta\big),$$

which is calculated by integrating out the unobservables (ϵ_i) in u_i. Assumption WTT2 implies that $\Pr\big(|\sigma^W(u_i, e_i)| = K | Z_i; \beta\big)$ does not depend on preferences. Thus, in the estimation, it suffices to focus on the following conditional probability:

$$\Pr\big(\sigma^W(u_i, e_i) = L \mid Z_i; \beta; |\sigma^W(u_i, e_i)| = K\big)$$
$$= \Pr\big(u_{i,l^1} > \cdots > u_{i,l^K} > u_{i,s'} \; \forall s' \in \mathcal{S} \backslash L \mid Z_i; \beta; |\sigma^W(u_i, e_i)| = K\big)$$
$$= \prod_{s \in L} \left(\frac{\exp(V_{i,s})}{\sum_{s' \nsucc_L s} \exp(V_{i,s'})} \right),$$

where $s' \nsucc_L s$ indicates that s' is not ranked before s in L, including s itself and those excluded from L. This rank-ordered (or "exploded") logit model can be seen as a series of conditional logits: one for the top-ranked school (l^1) being the most preferred, another for the second-ranked school (l^2) being preferred to all schools except l^1, and so on.

Let $|\sigma^W|$ be the vector of lengths of all submitted ROLs. The model can be estimated by maximum likelihood estimation (MLE) with the log-likelihood function:

$$\ln L_{TT}\big(\beta \mid \mathbf{Z}, |\sigma^W|\big) = \sum_{i=1}^{I} \sum_{s \in \sigma^W(u_i, e_i)} V_{i,s} - \sum_{i=1}^{I} \sum_{s \in \sigma^W(u_i, e_i)} \ln\left(\sum_{s' \nsucc_{\sigma^W(u_i, e_i)} s} \exp(V_{i,s'}) \right).$$

[20] Because the preference space is transformed from $[0,1]^S$ to $\mathbb{R}^S$, a strategy is now defined on the transformed type space. Moreover, it will be clear that σ^W does not depend on priority indices, e_i.

The estimator is denoted by $\hat{\beta}_{TT}$. Alternatively, the generalized method of moment (GMM) can be employed, for which the moment conditions are derived as in Section IIE. WTT implies additional restrictions beyond standard discrete choice models (see details in Section IID). Thus, the discrete-choice literature (e.g., Matzkin 1993) implies that student preferences are nonparametrically identified (also see Agarwal and Somaini 2018).

C. *Stability*

We now assume that the matching is stable, which is different from, but in large samples justified by, asymptotic stability. The following analysis abstracts away from the matching mechanism and ignores how stability is obtained. We formulate a stable matching as the outcome of a discrete choice model and clarify the conditions that are needed for identification and estimation.

Consider the matching μ and the associated cutoffs $P(\mu)$, which are random variables determined by the unobserved utility shocks (ϵ). Matching μ is the outcome of a discrete choice model with personalized choice set, $\mathcal{S}(e_i, P(\mu))$ (i.e., the set of i's feasible schools). The probability that i is matched with s, or chooses s in $\mathcal{S}(e_i, P(\mu))$, is

$$\Pr\left(s = \mu(u_i, e_i) = \underset{s \in \mathcal{S}(e_i, P(\mu))}{\arg\max}\, u_{i,s} \,|\, Z_i, e_i, \mathcal{S}(e_i, P(\mu)); \beta\right).$$

To proceed, we impose the following assumptions.

ASSUMPTION 4 (Exogeneity of Priority Index and Feasible Set):

EXO1. For all i, $e_i \perp \epsilon_i \,|\, Z_i$: *Conditional on observables* Z_i, *student preferences and priority indices are independent.*

EXO2. For all i *and* s, $\mathbf{1}(e_{i,s} < P_s(\mu)) \perp \epsilon_i \,|\, Z_i$, *or* $\mathcal{S}(e_i, P(\mu)) \perp \epsilon_i \,|\, Z_i$: *Conditional on observables* Z_i, *a student's preferences and her set of feasible schools are independent.*

Assumption EXO1 implies that, when priority indices (e_i) are determined by test scores, no student intentionally under-performs or over-performs in exams.

Assumption EXO2 deserves some discussion. Most importantly, it *does not require* that cutoffs $P(\mu)$ are conditionally independent of preferences shocks ϵ_i. Instead, it only assumes that the personalized choice set, $\mathcal{S}(e_i, P(\mu))$, is exogenously given, which is necessary for identification in a discrete choice model with personalized choice sets. For instance, if instead $\mathcal{S}(e_i, P(\mu))$ is endogenous and only includes school s when s is i's most preferred school, we lose the identification of i's preferences, because there is no variation in i's choice whenever s is in $\mathcal{S}(e_i, P(\mu))$. Online Appendix A.5 details such an example, along with a discussion and an example in which the assumption is satisfied.

One may argue that, in a finite market, a student can affect some cutoffs by applying to a school or not, and thus can change the feasibility of some schools. Another concern is that given student preferences, there can be multiple stable matchings. If a single student can select among the stable matchings, Assumption EXO2 is also violated.

These concerns diminish as the economy grows large, because the potential influence on cutoffs by any student decreases and there tends to be a unique stable matching. For instance, part (i) of Proposition 4 implies that a single student's impact on cutoffs diminishes to 0, almost surely. Moreover, even in small markets, Assumption EXO2 can be satisfied, because the assumption does not require $P(\mu) \perp \epsilon_i | Z_i$. An example is when every school ranks students in the same way, or $e_{i,s} = \bar{e}_i$ for all s and i.[21]

Given the parametric assumptions on utility functions, the corresponding (conditional) log-likelihood function is

$$\ln L_{ST}\big(\beta | \mathbf{Z}, \mathbf{e}, \mathcal{S}(e_i, P(\mu))\big) = \sum_{i=1}^{I}\sum_{s=1}^{S} V_{i,s} \times \mathbf{1}\big(\mu(u_i, e_i) = s\big) - \sum_{i=1}^{I} \ln\Bigg(\sum_{s' \in \mathcal{S}(e_i, P(\mu))} \exp\big(V_{i,s'}\big)\Bigg). \tag{3}$$

This estimator is denoted by $\hat{\beta}_{ST}$. Similarly, GMM can be applied, as in Section IIE.

Identification.—The above discussion transforms the matching game into a discrete choice model.[22] Therefore, the nonparametric identification arguments for discrete choice models still apply (Matzkin 1993). An important feature in the stability-based estimation is that students face *personalized* choice sets. As long as the choice sets are determined exogenously (Assumption EXO2), the identification goes through.

Another concern is that a student's priority index may enter her utility functions directly, when, for example, priority indices are determined by test score or student ability. In this case, the stability assumption does not reveal information about low-scoring students' preferences over popular schools, because such schools are often infeasible to them. This may lead to a failure of identifying how test scores determine student preferences.

[21] In this case, DA is equivalent to serial dictatorship in which students choose among the remaining schools one by one in the order determined by their priority indices. There is a unique stable matching for each realization of student types. Moreover, the set of feasible schools for student i is determined by the students with higher priority indices. Because preferences are independent across students by assumption, we have $\mathcal{S}(e_i, P(\mu)) \perp \epsilon_i | Z_i$, or $\mathbf{1}(e_{i,s} < P_s(\mu)) \perp \epsilon_i | Z_i$ for all s.

It should be noted that $P(\mu) \not\perp \epsilon_i | Z_i$ even in this case. For example, when i chooses s among the feasible schools, the cutoff of s will possibly increase; similarly, i may decrease s's cutoff by choosing a different school. However, we always have $\mathbf{1}(e_{i,s} < P_s(\mu)) \perp \epsilon_i | Z_i$, because s will remain feasible to i either way.

[22] A simplification is that we ignore the restrictions implied by the cutoffs $P(\mu)$, which may lead to efficiency loss in estimation. That is, even when the sets of feasible schools are exogenous to every single student's preferences, $P(\mu)$ is endogenously determined by the model's parameters. However, the additional information in these restrictions may be negligible, since we use the information on the whole matching already. An earlier version of the paper relaxes this assumption and uses the restrictions implied by the cutoffs. Our estimation results from simulated data and school choice data from Paris show that using the cutoff restrictions makes a negligible difference in the estimation results.

This problem is mitigated if we have another measure of student ability, as in our empirical exercise. We assume that conditional on student ability, priority indices do not determine preferences and only affect school feasibility. If, additionally, priority indices have full support (i.e., can take any possible value) at each given level of student ability, we can observe some low-ability students having all schools feasible. This restores nonparametric identification in discrete choice models as in Matzkin (1993).

Relative to WTT, the stability assumption uses unambiguously less information from the data (see Figure 1 for an example). WTT utilizes all information implied by the submitted ROLs, while stability only imposes restrictions on admission — outcome. One may expect that the stability-based approach leads to a loss of information; in particular, we may lose some precision in estimating the substitution patterns when we allow for more flexible random utility models (Berry, Levinsohn, and Pakes 2004; Abdulkadiroğlu, Agarwal, and Pathak 2017). Indeed, as we shall see in our Monte Carlo simulations and the analysis of the school choice data from Paris, there is a clear bias-variance tradeoff: stability tolerates non-truth-telling behavior at the cost of yielding less precise estimates.

Estimation with Asymptotic Stability.—When taking the above results to real-life data, one may be concerned that the matching may not be exactly stable. Indeed, our theoretical results only prove *asymptotic* stability. This raises the question of whether the estimator is still consistent. In online Appendix A.3, we show that the MLE with asymptotic stability is consistent (Proposition A4). In a finite economy, the stability-based estimation is incorrectly specified, because some students may not be assigned to their favorite feasible school and their revealed preferences are mis-classified when stability is imposed. However, the fraction of students who are not assigned to their favorite feasible school converges to zero at an exponential rate (part iii of Proposition A2), implying that the mis-classification in revealed preferences vanishes with economy size. By verifying the conditions in Theorem 2.1 of Newey and McFadden (1994), we show that the stability-based estimator is consistent even when the matching is only asymptotically stable.

D. *Testing Truth-Telling against Stability*

Having two distinct estimators, $\hat{\beta}_{TT}$ and $\hat{\beta}_{ST}$, makes it possible to test the truth-telling assumption against stability. Maintaining the assumption of identification given stability, we shall see shortly that WTT provides over-identifying restrictions.

Before we present the tests, a few caveats are in order. First, one should check that the conditions for identification (for example, those in Matzkin 1993) are satisfied before conducting the tests. Second, because the tests are essentially about joint restrictions on the parametric assumptions and the behavioral assumptions, one should be aware of the consequence of model misspecification. Rejecting truth-telling in favor of stability may not provide definitive proof against truth-telling, since the proposed tests are conditional on the model's parametric assumptions. In light of these limitations, it is often useful to provide additional empirical results, such as reduced-form results on student behavior (see, e.g., Section IVB) and goodness of fit of the estimates (see, e.g., Section IVD).

Over-Identifying Restrictions.—As summarized in Proposition 3, if every student is WTT and is assigned to a school, the matching is stable. Stability, however, does not imply that students are WTT and is therefore a less restrictive assumption.

To see the additional restrictions from WTT, let us consider student i who submits a K-choice list L and is matched with school s. Therefore, s must be ranked in L. WTT implies the following conditions on the choice probability:

$$\begin{aligned}(4)\quad & \Pr\Big(\sigma^W(u_i,e_i) = L \,\Big|\, Z_i;\beta;\big|\sigma^W(u_i,e_i)\big| = K\Big) \\ & = \Pr\Big(u_{i,l^1} > \cdots > u_{i,l^K} > u_{i,s'}, \forall s' \in \mathcal{S}\backslash L \mid Z_i;\beta; \\ & \qquad\qquad \big|\sigma^W(u_i,e_i)\big| = K; s = \underset{s\in\mathcal{S}(e_i,P(\mu))}{\arg\max}\, u_{i,s}\Big) \\ & \quad \times \Pr\Big(s = \mu(u_i,e_i) = \underset{s\in\mathcal{S}(e_i,P(\mu))}{\arg\max}\, u_{i,s} \mid Z_i;\beta;\mathcal{S}\big(e_i,P(\mu)\big)\Big).\end{aligned}$$

This equality uses the fact that the event, $\big(u_{i,l^1} > \cdots > u_{i,l^{K_i}} > u_{i,s'}, \forall s' \in \mathcal{S}\backslash L\big)$, implies $\big(s = \arg\max_{s\in\mathcal{S}(e_i,P(\mu))} u_{i,s}\big)$ but not the reverse.[23] This is because i's feasible schools are either ranked below s in L or are omitted from L; in either case, WTT requires that s is preferred to any other feasible school. Therefore, the first conditional probability on the right-hand side of the equality cannot always be one. As the restrictions implied by stability are just

$$\Pr\Big(s = \mu(u_i,e_i) = \underset{s\in\mathcal{S}(e_i,P(\mu))}{\arg\max}\, u_{i,s} \mid Z_i;\beta;\mathcal{S}\big(e_i,P(\mu)\big)\Big),$$

the additional restrictions from WTT are summarized in the first term. When the model is identified under stability, equation (4) summarizes the over-identifying restrictions.

Hausman Test.—Our estimator $\hat{\beta}_{TT}$ uses all the restrictions implied by WTT. Therefore, under the null hypothesis that students are WTT, both estimators $\hat{\beta}_{TT}$ and $\hat{\beta}_{ST}$ are consistent but only $\hat{\beta}_{TT}$ is asymptotically efficient. Under the alternative that the matching is stable but not all students are WTT, only $\hat{\beta}_{ST}$ is consistent.

[23] We also make use of the exogeneity of the set of feasible schools (Assumption EXO2) and the exogeneity of the length of submitted ROL (Assumption WTT2). Therefore,

$$\Pr\Big(s = \mu(u_i,e_i) = \underset{s\in\mathcal{S}(e_i,P(\mu))}{\arg\max}\, u_{i,s} \mid Z_i;\beta;\mathcal{S}\big(e_i,P(\mu)\big)\Big) = \Pr\Big(s = \underset{s\in\mathcal{S}(e_i,P(\mu))}{\arg\max}\, u_{i,s} \mid Z_i;\beta;|\sigma^W(u_i,e_i)| = K\Big),$$

In this setting, the general specification test developed by Hausman (1978) can be applied by computing the following test statistic:

$$T_H = \left(\hat{\beta}_{ST} - \hat{\beta}_{TT}\right)'\left(\hat{\mathbf{V}}_{ST} - \hat{\mathbf{V}}_{TT}\right)^{-1}\left(\hat{\beta}_{ST} - \hat{\beta}_{TT}\right),$$

where $\left(\hat{\mathbf{V}}_{ST} - \hat{\mathbf{V}}_{TT}\right)^{-1}$ is the inverse of the difference between the asymptotic covariance matrices of $\hat{\beta}_{ST}$ and $\hat{\beta}_{TT}$.[24] Under the null hypothesis, $T_H \sim \chi^2(d_\beta)$, where d_β is the dimension of β. If the model is correctly specified and the matching is stable, the rejection of the null hypothesis implies that WTT is violated in the data.

Testing Over-Identifying Restrictions.—The above Hausman test requires that we have a consistent and efficient estimator, $\hat{\beta}_{TT}$. When relying on MLE or GMM, this calls for strong parametric assumptions. An alternative is to construct a test for over-identifying restrictions (Hansen 1982), which is made feasible because of the nesting structure of WTT and stability due to Proposition 3. Instead of requiring $\hat{\beta}_{TT}$ to be asymptotically efficient, the test for over-identifying restrictions only requires that $\hat{\beta}_{TT}$ utilizes more restrictions than $\hat{\beta}_{ST}$. With equation (4), we can separate out the additional restrictions and test whether they are satisfied based on Hansen (1982).

No-Blocking among Assigned Students.—The above estimation and tests can be applied even if stability is violated. Part (i) of Proposition 3 states that whenever a WTT student is assigned, she is matched with her favorite feasible school and thus is not in any blocking pair. However, this no-blocking condition can be violated among unassigned students, implying the violation of stability. We can thus re-formulate the above tests as WTT against "no-blocking among assigned students." The estimation based on "no-blocking among assigned students" will exclude unassigned students; it does not create selection bias under the null hypothesis, because the length of every submitted ROL, which determines the probability of being unassigned, is exogenous under WTT.

E. *Undominated Strategies and Stability*

The stability-based approach described above is only valid when the matching is stable. However, as we have shown theoretically, stability can fail. Without stability, one may consider the undominated-strategies assumption, under which observed ROLs are students' true partial preference orders. That is, a submitted ROL, L_i, respects i's true preference order among the schools ranked in L_i (see, for an example, Figure 1).

These partial orders provide information that can be used to identify student preferences, but only partially, because the econometric structure is now incomplete (Tamer 2003). In other words, for a student with type (u_i, e_i), the assumption of undominated strategies does not predict a unique ROL for the student. As we shall see, undominated strategies lead to a set of inequality restrictions that can be

[24] Since exact stability is assumed, the calculation of $\hat{\mathbf{V}}_{ST}$ does not take into account the sampling variance of cutoffs in a finite economy.

satisfied by a set of βs, instead of a unique vector of β. Therefore, we lose point identification.

Moment Inequalities.—Students' submitted ROLs can be used to form *conditional moment inequalities*. Without loss of generality, consider two schools s_1 and s_2. Since not everyone ranks both schools, the probability of i, who adopts the strategy $\sigma(u_i, e_i)$, ranking s_1 before s_2, i.e., $s_1 \succ_{\sigma(u_i,e_i)} s_2$, is

$$\Pr\left(s_1 \succ_{\sigma(u_i,e_i)} s_2 \,|\, Z_i; \beta\right) = \Pr\left(u_{i,s_1} > u_{i,s_2} \text{ and } s_1, s_2 \in \sigma(u_i, e_i) \,|\, Z_i; \beta\right) \leq \Pr\left(u_{i,s_1} > u_{i,s_2} \,|\, Z_i; \beta\right). \tag{5}$$

The first equality is due to undominated strategies, and the inequality defines a lower bound for the conditional probability of $u_{i,s_1} > u_{i,s_2}$. Similarly, an upper bound is

$$\Pr\left(u_{i,s_1} > u_{i,s_2} \,|\, Z_i; \beta\right) \leq 1 - \Pr\left(s_2 \succ_{\sigma(u_i,e_i)} s_1 \,|\, Z_i; \beta\right). \tag{6}$$

Inequalities (5) and (6) yield the following conditional moment inequalities:

$$\Pr\left(u_{i,s_1} > u_{i,s_2} \,|\, Z_i; \beta\right) - E\left[\mathbf{1}\left(s_1 \succ_{\sigma(u_i,e_i)} s_2\right) \,|\, Z_i; \beta\right] \geq 0;$$

$$1 - E\left[\mathbf{1}\left(s_2 \succ_{\sigma(u_i,e_i)} s_1\right) \,|\, Z_i; \beta\right] - \Pr\left(u_{i,s_1} > u_{i,s_2} \,|\, Z_i; \beta\right) \geq 0.$$

Similar inequalities can be computed for any school pair and can be generalized to any n schools in $\mathcal{S}$, for $2 \leq n \leq S$. In the simulations and empirical analysis, we focus on inequalities for pairs. The bounds become uninformative if $n \geq 3$, because not many schools are simultaneously ranked by the majority of students. We interact Z_i with the conditional inequalities and obtain M_1 unconditional moment inequalities, $\left(m_1, \ldots, m_{M_1}\right)$.

Estimation with Moment Inequalities.—For estimation with moment inequalities, one can follow the approach of Andrews and Shi (2013), which is valid for both point and partial identifications. The objective function is a test statistic, $T_{MI}(\beta)$, of the Cramer-von Mises type with the modified method of moments (or sum function). With the unconditional moment inequalities, it is constructed as follows:

$$T_{MI}(\beta) = \sum_{j=1}^{M_1} \left[\frac{\bar{m}_j(\beta)}{\hat{\sigma}_j(\beta)}\right]_{-}^{2}, \tag{7}$$

where $\bar{m}_j(\beta)$ and $\hat{\sigma}_j(\beta)$ are the sample mean and standard deviation of the jth moment, $m_j(\beta)$, respectively; and $[\cdot]_-$ is such that $[a]_- = \min\{0, a\}$. One can then follow Bugni, Canay and Shi (2017) to construct *marginal confidence intervals*. For a given coordinate β_k of β, the authors test the hypothesis H_0: $\beta_k = \beta_0$, for a given $\beta_0 \in \mathbb{R}$. The confidence interval for β_k's true value is the convex hull of all β_0s at which H_0 is not rejected.

While the assumption of undominated strategies seems plausible, it should be noted that the above approach often leads to uninformative confidence intervals of parameters of interest, constrained by the available econometric techniques. However, one can integrate the inequalities with the restrictions implied by stability, when stability is also plausible.

Integrating Stability with Undominated Strategies.—An important advantage of the stability-based approach is that it only requires data on the admission outcomes. However, submitted ROLs are often observed and can be used to improve estimation efficiency. Under the assumption that stability provides point identification of student preferences, these ROLs provide over-identifying information that can be used together with stability in estimation.

The potential benefits can be illustrated in a simple example. Consider a constrained/truncated DA where students are only allowed to rank up to three schools out of four. With personalized sets of feasible schools under the stability assumption, the preferences over two schools, say s_1 and s_2, are estimated mainly from the sub-sample of students who are assigned to either of these schools while having priority indices above the cutoffs of both. Yet it is possible that all students include s_1 and s_2 in their ROLs, even if these schools are not ex post feasible for some students. In such a situation, all students could be used to estimate the preference order of s_1 and s_2, rather than just a sub-sample. As shown below, this argument can be extended to the case where two or more schools are observed being ranked by a subset of students.

Moment Equalities.—To integrate the above over-identifying information in ROLs with that from stability, we reformulate the likelihood function described in equation (3) into *moment equalities*. The choice probability of the matched school can be rewritten as a moment condition by equating theoretical and empirical probabilities:

$$\sum_{i=1}^{I} \Pr\left(s = \underset{s' \in \mathcal{S}(e_i, P)}{\arg\max}\left(u_{i,s'}\right) \mid Z_i, P(\mu); \beta\right) - E\left(\sum_{i=1}^{I} \mathbf{1}\left(\mu\left(u_i, e_i\right) = s\right)\right) = 0, \quad \forall s \in \mathcal{S},$$

where $\mathbf{1}\left(\mu\left(u_i, e_i\right) = s\right)$ is an indicator function taking the value of one if and only if $\mu\left(u_i, e_i\right) = s$. We again interact the variables in Z with the above conditions, leading to M_2 moment equalities, $\left(m_{M_1+1}, \ldots, m_{M_1+M_2}\right)$.

Estimation with Moment (*In*)*equalities*.—To obtain consistent point estimates with both equality and inequality moments (henceforth, moment (in)equalities), we augment the test statistic in equation (7) to incorporate the M_2 unconditional moment equalities:

$$T_{MEI}(\beta) = \sum_{j=1}^{M_1} \left[\frac{\bar{m}_j(\beta)}{\hat{\sigma}_j(\beta)}\right]_-^2 + \sum_{j=M_1+1}^{M_1+M_2} \left[\frac{\bar{m}_j(\beta)}{\hat{\sigma}_j(\beta)}\right]^2. \tag{8}$$

We denote the point estimate $\hat{\beta}_{MEI}$, which minimizes $T_{MEI}(\beta)$, and we can take the same approach as in Bugni, Canay, and Shi (2017) to construct marginal confidence intervals for β.

F. *Testing Stability against Undominated Strategies*

Given the identification of student preferences under stability, the moment inequalities add over-identifying information. This constitutes a test of stability, provided that students do not play dominated strategies. More precisely, if both assumptions are satisfied, the moment (in)equalities in Section IIE should yield a point estimate that fits the data relatively well; otherwise, there should not exist a point β that satisfies all moment (in)equalities. Formally, we follow the specification test in Bugni, Canay, and Shi (2015).

It should be noted that, for the above test, we maintain the undominated-strategies assumption, which may raise concerns, because students could make mistakes as documented in several real-life contexts; moreover, untrue partial preference ordering is not dominated under the school-proposing DA. We revisit these issues in Section VB.

The discussion in Section IIE provides another test of the undominated-strategies assumption, which also relies on the non-emptyness of the identified set under the null hypothesis (Bugni, Canay, and Shi 2015). That is, if there is no value of β satisfying the moment inequalities, the undominated-strategies assumption is not satisfied. Unfortunately, the available methods of moment (in)equalities tend to result in conservative confidence sets of parameters, which implies that this test may lack power.

III. Results from Monte Carlo Simulations

To illustrate the proposed estimation approaches and tests, we carry out Monte Carlo simulations, the details of which are consigned to online Appendix C.

Bayesian Nash equilibrium of the school choice problem is simulated in two settings where I students compete for admission to 6 schools with per capita capacities $\{q_s\}_{s=1}^{6} = \{0.1, 0.1, 0.05, 0.1, 0.3, 0.3\}$.[25] The first is the **constrained/truncated DA** where students are allowed to rank up to K schools $(K < 6)$. The second setting, labelled as **DA with cost**, allows students to rank as many schools as they wish but imposes a constant marginal cost c per additional school in the list after the first choice.

Student preferences over schools follow a random utility model:

$$u_{i,s} = \alpha_s - d_{i,s} + 3(a_i \cdot \bar{a}_s) + \epsilon_{i,s}, \tag{9}$$

[25] Online Appendix C.2 describes the details on solving equilibrium. In general, there are multiple equilibria. We focus on the one that is found by an algorithm iterating over the following steps: (i) for each candidate ROL (a true partial preference order of the schools), every student calculates the admission probability at each school by comparing her priority indices to the cutoff distribution; (ii) each student selects the ROL that maximizes her expected utility; (iii) the matchings across M simulation samples jointly lead to an updated cutoff distribution; (iv) students update the admission probabilities based on the updated distribution. The initial cutoff distribution is the empirical cutoff distribution with strictly truth-telling students, and steps (i)–(iv) are repeated until a fixed point in the cutoff distribution is found.

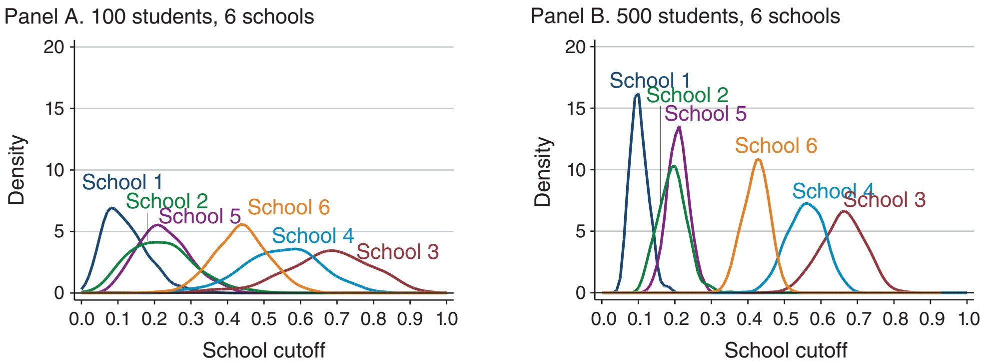

Figure 2. Monte Carlo Simulations: Impact of Economy Size on the Equilibrium Distribution of Cutoffs (Constrained/Truncated DA)

Notes: This figure shows the marginal distribution of school cutoffs in equilibrium under the constrained/truncated DA (ranking 4 out of 6 schools) when varying the number of students, I, who compete for admission to 6 schools with a total enrollment capacity of $I \times 0.95$ seats. Using 500 simulated samples, the line fits are from a Gaussian kernel with optimal bandwidth using MATLAB's ksdensity command. See online Appendix C for details on the Monte Carlo simulations.

where α_s is school s's fixed effect; $d_{i,s}$ is the distance from student i's residence to s; a_i is i's ability; $\bar{a}_s$ is school s's quality; and $\epsilon_{i,s}$ is randomly drawn from the type-I extreme value distribution. Student priority indices are constructed such that (a) i's priority index at each school is correlated with her ability a_i (correlation coefficient 0.7) and (b) i's priority indices at any two schools s and s' are also correlated (correlation coefficient 0.7).

Several lessons can be drawn from these simulations. The first is that in both settings, the distribution of school cutoffs is close to jointly normal and degenerates as school capacities and the number of students increase proportionally while holding constant the number of schools (Figure 2); the matching is almost stable (i.e., almost every student is assigned to her favorite feasible school) even in moderately sized economies. By contrast, WTT is often violated among the majority of the students, even when they can rank four out of six schools (constrained DA) or when the cost of including an extra school is negligibly small (DA with cost).[26] When the application cost increases, equilibrium strategies may prescribe that many students rank fewer than six schools even though they are allowed to rank all of them. Based on these results, observing that only a few students make full use of their ranking opportunities may not be viewed as a compelling argument in favor of truth-telling when the application cost is a legitimate concern.

The second insight is that stability leads to estimates much closer to the true values than WTT. Table 2 reports the results from estimation under each of the following assumptions: (i) weak truth-telling (columns 2–4); (ii) stability (columns 5–7); and (iii) stability and undominated strategies (columns 8 and 9). Panel A is for

[26] Consistent with Proposition 5, our simulations show that the fraction of students who are matched with their favorite feasible school decreases with the application cost. However, students with justified envy are rare unless students face very large application costs (see Figure C4 in online Appendix C.3).

Table 2—Monte Carlo Results (500 Students, 6 Schools, 500 Samples)

		Identifying assumptions							
		Weak Truth-telling			Stability of the matching			Stability and undominated strategies	
	True value	Mean	SD	CP	Mean	SD	CP	Mean	SD
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Panel A. Constrained/truncated DA (ranking up to 4 out of 6 schools)									
Parameters									
School 2	0.50	−0.13	0.06	0.00	0.51	0.29	0.94	0.50	0.28
School 3	1.00	−2.08	0.14	0.00	1.05	0.58	0.96	1.02	0.57
School 4	1.50	−1.29	0.12	0.00	1.54	0.52	0.96	1.52	0.50
School 5	2.00	0.56	0.07	0.00	2.02	0.31	0.96	2.01	0.29
School 6	2.50	0.23	0.12	0.00	2.53	0.45	0.96	2.51	0.43
Own ability × school quality	3.00	9.40	0.64	0.00	2.97	2.29	0.96	3.05	2.26
Distance	−1.00	−0.71	0.08	0.08	−1.01	0.20	0.95	−1.01	0.20
Summary statistics (averaged across Monte Carlo samples)									
Average length of submitted ROLs			4.00						
Fraction of weakly truth-telling students			0.39						
Fraction of students assigned to favorite feasible school			1.00						
Model selection tests									
Truth-telling (H_0) versus stability (H_1):		H_0 rejected in 100 percent of samples (at 5 percent significance level)							
Stability (H_0) versus undominated strategies (H_1):		H_0 rejected in 0 percent of samples (at 5 percent significance level)							
Panel B. DA with application cost (constant marginal cost $c = 10^{-6}$)									
Parameters									
School 2	0.50	0.41	0.09	0.88	0.51	0.29	0.94	0.49	0.28
School 3	1.00	0.57	0.16	0.23	1.05	0.58	0.96	1.00	0.53
School 4	1.50	1.17	0.15	0.37	1.54	0.52	0.96	1.49	0.48
School 5	2.00	1.74	0.11	0.32	2.02	0.30	0.96	1.99	0.29
School 6	2.50	2.24	0.14	0.50	2.54	0.45	0.96	2.48	0.41
Own ability × school quality	3.00	2.19	0.72	0.77	2.96	2.29	0.96	3.16	2.29
Distance	−1.00	−0.93	0.09	0.88	−1.01	0.20	0.95	−1.00	0.20
Summary statistics (averaged across Monte Carlo samples)									
Average length of submitted ROLs			4.60						
Fraction of weakly truth-telling students			0.79						
Fraction of students assigned to favorite feasible school			1.00						
Model selection tests									
Truth-telling (H_0) versus stability (H_1):		H_0 rejected in 37 percent of samples (at 5 percent significance level)							
Stability (H_0) versus undominated strategies (H_1):		H_0 rejected in 0 percent of samples (at 5 percent significance level)							

Notes: This table reports Monte Carlo results from estimations under different sets of identifying assumptions: (i) weak truth-telling; (ii) stability; (iii) stability and undominated strategies. 500 Monte Carlo samples of school choice are simulated under two data generating processes for an economy in which 500 students compete for admission to 6 schools: a constrained/truncated DA where students are allowed to rank up to 4 schools out of 6 (panel A); an unconstrained DA where students can rank as many schools as they wish, but incur a constant marginal cost $c = 10^{-6}$ for including an extra school in their ROL beyond the first choice (panel B). Under assumptions (i) and (ii), student preferences are estimated using maximum likelihood estimation. Under assumption (iii), they are estimated using Andrews and Shi (2013)'s method of moment (in)equalities. Column 1 reports the true parameter values. The mean and standard deviation (SD) of point estimates across the Monte Carlo samples are reported in columns 2, 5, and 8, and in columns 3, 6, and 9, respectively. Columns 4 and 7 report the coverage probabilities (CP) for the 95 percent confidence intervals. The confidence intervals in estimations (i) and (ii) are the Wald-type confidence intervals obtained from the inverse of the Hessian matrix. The marginal confidence intervals in estimation (iii) are computed using the method proposed by Bugni, Canay, and Shi (2017). The CP in estimation (iii) is 100 percent for every parameter. Truth-telling is tested against stability by constructing a Hausman-type test statistic from the estimates of both approaches. Stability is tested against undominated strategies by checking if the identified set of the moment (in)equality model is empty, using the test proposed by Bugni, Canay, and Shi (2015). See online Appendix C for details on the Monte Carlo simulations.

the constrained/truncated DA where students are allowed to rank up to 4 schools; panel B is for the DA with a marginal application cost equal to 10^{-6}.

The WTT-based estimator ($\hat{\beta}_{TT}$) is severely biased (column 2). Particularly in panel A, we note that low-ability students' valuation of the most popular schools (e.g., School 6) tends to be underestimated, because such schools are more likely to be omitted from these students' ROLs due to their low admission probabilities. This bias is also present among small schools (e.g., Schools 3 and 4), which are often left out of ROLs because their cutoffs tend to be higher than those of equally desirable but larger schools.

By contrast, the average of the stability-based estimates ($\hat{\beta}_{ST}$) is reasonably close to the true values. Its standard deviations, however, are larger than those obtained under WTT. This efficiency loss is a direct consequence of ignoring the information content of ROLs.[27] The Hausman test strongly rejects WTT in favor of stability.

The estimator based on moment (in)equalities ($\hat{\beta}_{MEI}$), which integrates stability with information in ROLs, is also consistent (column 8). Moreover, the test based on moment (in)equalities never rejects the null hypothesis that stability is consistent with undominated strategies. A limitation of this approach, however, is that the currently available methods for conducting inference based on moment (in)equalities are typically conservative. As a result, the 95 percent marginal confidence intervals based on moment (in)equalities cover the true values too often (coverage probability, or CP, is 100 percent for every parameter, although not shown in Table 2).

IV. School Choice in Paris

Since 2008, the Paris Education Authority (*Rectorat de Paris*) assigns students to public high schools based on a version of the school-proposing DA called AFFELNET (Hiller and Tercieux 2014). At the district level, student priority indices are not school-specific (as detailed below) and the mechanism is equivalent to a serial dictatorship.

Towards the end of the Spring term, final-year middle school students who are admitted to the upper secondary academic track (*Seconde générale et technologique*)[28] are requested to submit an ROL of up to eight public high schools to the Paris Education Authority. Students' priority indices are determined as follows:

(i) Students' academic performance during the last year of middle school is graded on a scale of 400 to 600 points.

(ii) Paris is divided into four districts. Students receive a "district" bonus of 600 points at each school located in their home district. Thus, students applying to a within-district school have full priority over out-of-district applicants to the same school.

[27] Online Appendix C.4 further quantifies the efficiency loss in simulations with strictly truth-telling students.

[28] In the French educational system, students are tracked at the end of the final year of *collège* (equivalent to middle school), at the age of 15, into vocational or academic upper secondary education.

(iii) Low-income students are awarded a bonus of 300 points.[29] As a result, these students are given full priority over all other students from the same district.

The DA algorithm is run at the end of the academic year to determine school assignment for the following academic year. Unassigned students can participate in a supplementary round of admissions by submitting a new ROL of schools among those with remaining seats, the assignment mechanism being the same as for the main round.

Note that the mechanism would be strategy-proof if there were no constraints on the length of ROLs, because it is equivalent to serial dictatorship. Nonetheless, under the current mechanism, it is still a dominated strategy to submit an ROL that is not a partial order of true preferences (Proposition 2).

A. *Data*

For our empirical analysis, we use data from Paris' southern district (*Sud*) and study the behaviors of 1,590 within-district middle school students who applied for admission to the district's 11 public high schools for the academic year 2013–2014. Owing to the 600-point "district" bonus, this district is essentially an independent market.[30]

Along with socio-demographic characteristics and home addresses, our data contain all the relevant variables to replicate the matching algorithm, including the school capacities, the submitted ROLs, and the priority indices (converted into percentiles between 0 and 1). Individual examination results for the *Diplôme national du brevet* (DNB)—a national exam that all students take at the end of middle school—are used to construct different measures of academic ability (French, math, and composite score), which are normalized as percentiles between 0 and 1. Note that the DNB exam scores are not used in the computation of the student priority index, which is based on the grades obtained throughout the final year of middle school. The DNB scores therefore provide additional measures of student ability.[31] Table 3 reports students' characteristics, choices, and admission outcomes. Almost half of the students are of high socioeconomic status (SES), while 15 percent receive the low-income bonus. Ninety-nine percent are assigned to a within-district school in the main admission round, but only half obtain their first choice.

Table 4 presents summary statistics for the 11 high schools. Columns 1–4 show a high degree of stratification among the schools, both in terms of the average ability of students enrolled in 2012 and of their social background (measured by the fraction of high SES students). Columns 5–8 describe school choice in 2013. The district's total capacity (1,692 seats) is unevenly distributed across schools: the smallest school has 62 seats while the largest has 251. School cutoffs in 2013 are strongly correlated with

[29] The low-income status is conditional on a student applying for and being granted the means-tested low-income financial aid in the last year of middle school. A family with two children would be eligible in 2013 if its taxable income was below 17,155 euros. The aid ranges from 135 to 665 euros per year.

[30] Out-of-district applicants could affect the availability of school seats in the supplementary round, but this is of little concern since, in the district, only 22 students were unassigned at the end of the main round (for the comparison between assigned and unassigned students, see online Appendix Table E1).

[31] See online Appendix B for a description of the data sources and online Appendix Figure E1 for a map.

TABLE 3—HIGH SCHOOL APPLICANTS IN THE SOUTHERN DISTRICT OF PARIS: SUMMARY STATISTICS

	Mean	SD	Min	Max	Observations
Panel A. Student characteristics					
Age	15.0	0.4	13	17	1,590
Female	0.51	0.50	0	1	1,590
French score	0.56	0.25	0.00	1.00	1,590
Math score	0.54	0.24	0.01	1.00	1,590
Composite score	0.55	0.21	0.02	0.99	1,590
High SES	0.48	0.50	0	1	1,590
With low-income bonus	0.15	0.36	0	1	1,590
Panel B. Choices and outcomes					
Number of choices within district	6.6	1.3	1	8	1,590
Assigned to a within-district school	0.99	0.12	0	1	1,590
Assigned to first choice school	0.56	0.50	0	1	1,590
Panel C. Attributes of first-choice school					
Distance (km)	1.52	0.93	0.01	6.94	1,590
Mean student French score	0.62	0.11	0.32	0.75	1,590
Mean student math score	0.61	0.13	0.27	0.78	1,590
Mean student composite score	0.61	0.12	0.31	0.77	1,590
Fraction high SES in school	0.53	0.15	0.15	0.71	1,590
Panel D. Attributes of assigned school					
Distance (km)	1.55	0.89	0.06	6.94	1,568
Mean student French score	0.56	0.12	0.32	0.75	1,568
Mean student math score	0.54	0.14	0.27	0.78	1,568
Mean student composite score	0.55	0.13	0.31	0.77	1,568
Fraction high SES in school	0.48	0.15	0.15	0.71	1,568

Notes: This table provides summary statistics on the choices of middle school students from the southern district of Paris who applied for admission to the district's 11 public high schools for the academic year starting in 2013, based on administrative data from the Paris Education Authority (*Rectorat de Paris*). All scores are from the exams of the *Diplôme national du brevet* (DNB) in middle school and are measured in percentiles and normalized to be in $[0,1]$. The composite score is the average of the scores in French and math. The correlation coefficient between French and math scores is 0.50. School attributes, except distance, are measured by the average characteristics of students enrolled in each school in the previous year (2012).

school quality. The last column shows the fraction of submitted ROLs in which each school is ranked. The least popular three schools are each ranked by less than 24 percent of students, and two of them remain under-subscribed (Schools 1 and 3) and thus have a 0 cutoff. Consistent with our Monte Carlo results, smaller schools are omitted by more students, even if they are of high quality. Likewise, a sizeable fraction of students (20 percent) do not rank the best-performing school (School 11) in their ROLs.

Enrollment data further reveals a high level of compliance with the assignment outcome. Among the assigned students, 96 percent attend the school they were matched with (online Appendix Table E1), about 1 percent attend a public high school different from their assignment school, and less than 3 percent opt out to enroll in a private school.

B. *Evaluating the Assumptions: Reduced-Form Evidence*

To evaluate the WTT and stability assumptions, we investigate if students are less likely to rank schools at which they expect low admission probabilities. Similar to "skipping the impossible" as in Example 1, this behavior would be inconsistent with WTT.

Table 4—High Schools in the Southern District of Paris: Summary Statistics

	School attributes (2012)				Admission outcomes (2013)			
	Mean French score (1)	Mean math score (2)	Mean composite score (3)	Fraction high SES students (4)	Capacity (5)	Count (6)	Admission cutoffs (7)	Fraction ROLs ranking it (8)
School 1	0.32	0.31	0.31	0.15	72	19	0.000	0.22
School 2	0.36	0.27	0.32	0.17	62	62	0.015	0.23
School 3	0.37	0.34	0.35	0.16	67	36	0.000	0.14
School 4	0.44	0.35	0.39	0.46	140	140	0.001	0.59
School 5	0.47	0.44	0.46	0.47	240	240	0.042	0.83
School 6	0.47	0.46	0.46	0.32	171	171	0.069	0.71
School 7	0.58	0.54	0.56	0.56	251	251	0.373	0.91
School 8	0.58	0.66	0.62	0.30	91	91	0.239	0.39
School 9	0.65	0.62	0.63	0.66	148	148	0.563	0.83
School 10	0.68	0.66	0.67	0.49	237	237	0.505	0.92
School 11	0.75	0.78	0.77	0.71	173	173	0.705	0.80

Notes: This tables provides summary statistics on the attributes of high schools in the southern district of Paris and on the outcomes of the 2013 assignment round, based on administrative data from the Paris Education Authority (*Rectorat de Paris*). School attributes in 2012 are measured by the average characteristics of the schools' enrolled students in 2012–2013. All scores are from the exams of the *Diplôme national du brevet* (DNB) in middle school and are measured in percentiles and normalized to be in $[0,1]$. The composite score is the average of the scores in French and math. The correlation coefficient between school-average math and French scores is 0.97.

Figure 3 focuses on the district's 4 most selective schools (as measured by their cutoffs). For each school, we separately plot the fraction of students who rank it in their ROL as a function of their distance to the school cutoff, measured by the difference (using the original scale in points) between the student's priority index and the cutoff.[32] Each plot shows that almost all students with a priority index above a school's cutoff include that school in their ROL, whereas the fraction of students ranking the school decreases rapidly when the priority index falls below the cutoff. Irrespective of strategic considerations, one might expect high priority students to have a stronger preference for the most selective schools—since priorities are positively correlated with academic performance—and hence to rank them more often. However, the kink around the cutoffs is consistent with students omitting the most selective schools from their ROL because of the low admission probabilities. In online Appendix D.1, we show that the kink-shaped relationship between student priority index and their ranking behavior is robust to controlling for potential determinants of preferences, including distance to school and the student's DNB exam scores in French and math. Recall that DNB scores are not used to calculate the priority indices. These results cannot be easily reconciled with truth-telling behavior.

The evidence in Figure 3 suggests the potential influence of expected admission probabilities on student ranking behavior. At the time of application, students know their academic grades and low-income status but not their priority ranking nor the ex post cutoffs.[33] They can, however, gather information on past cutoffs to assess admission probabilities. While we do not have direct information on students' beliefs, Figure 4 shows that the current year (2013) cutoffs are similar to those

[32] We restrict the sample for a school to students whose score is no more than 50 points away from its cutoff. Due to the low-income bonus of 300 points, low-income students' priority indices are always well above the cutoffs. They are therefore not considered in the analysis.

[33] This uncertainty in both priority ranking and cutoffs may explain why some students find it optimal to rank multiple schools, given that the cost of ranking up to eight choices is arguably negligible.

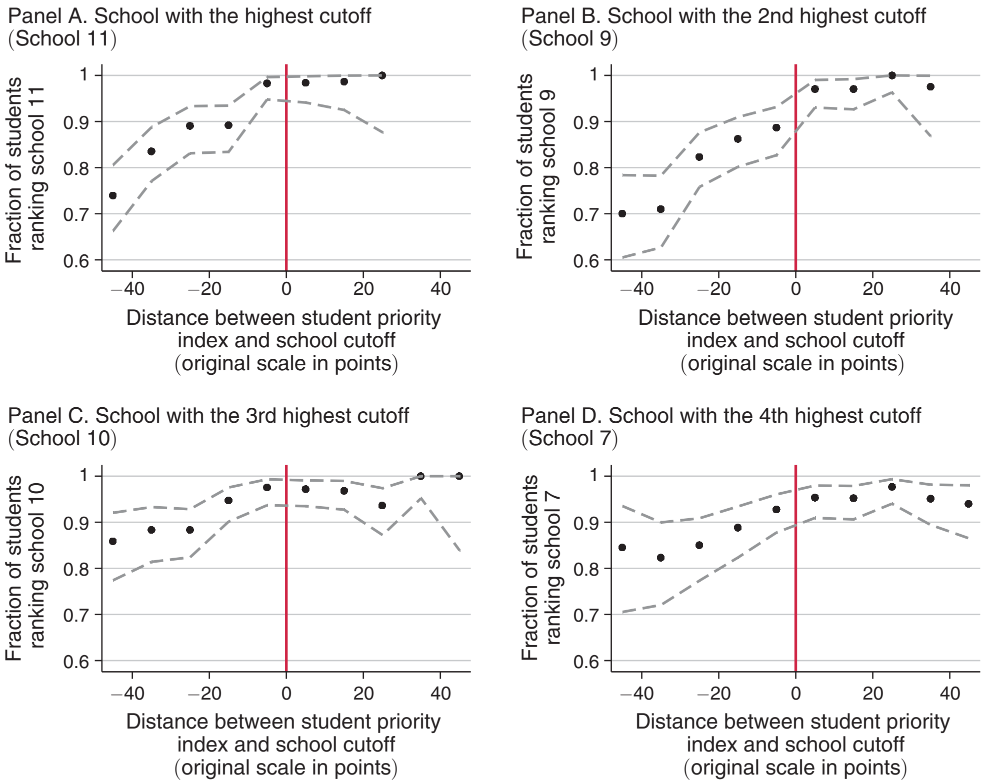

Figure 3. Fraction of Students Ranking Each of the Four Most Selective Schools in the Southern District of Paris, by Distance to School Cutoff

Notes: The results are calculated with data from the Paris Education Authority on students who applied to the 11 high schools of the southern district in 2013. The figure shows the ranking behavior of students as a function of the distance (using the original scale in points) between each school's cutoff and students' priority index. For each school, the sample only includes students with a priority index within −50 and +50 points of the cutoff, and students are grouped into bins of 10-point width. Bins with less than ten observations are excluded. Each point represents the fraction of students in a given bin who rank the school in their list. The dotted lines show the 95 percent confidence interval. Low-income students are not included because the low-income bonus of 300 points places them well above the cutoffs.

from the previous year (2012).[34] This lends support to the assumption that students have some ability to predict their admission probabilities. Although not a necessary condition for the matching to be stable, this feature makes the stability assumption more likely to be satisfied.

C. *Estimation and Test Results*

We parameterize student i's utility of being matched with school s as follows:

$$u_{i,s} = \alpha_s - d_{i,s} + Z'_{i,s}\gamma + \lambda\epsilon_{i,s}, \quad s = 1,\ldots,11; \tag{10}$$

[34] The comparison could not be performed for earlier years due to the modifications in the computation of the priority index and the small changes in the set of available schools.

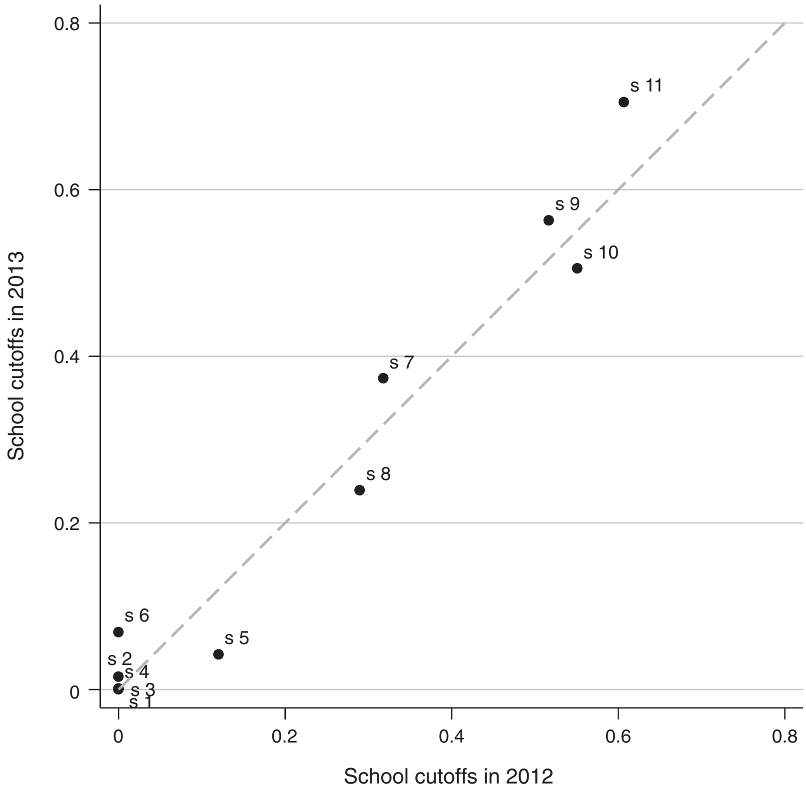

Figure 4. School Cutoffs in 2012 and 2013

Notes: The results are calculated with administrative data from the Paris Education Authority. Each dot represents a school, with its cutoff in 2013 on the y-axis and the one in 2012 on the x-axis. The dashed line denotes the 45-degree line.

where α_s is the school fixed effect, $d_{i,s}$ is the distance to s from i's residence, and $Z_{i,s}$ is a vector of student-school-specific observables. As observed heterogeneity, $Z_{i,s}$ includes 2 variables that capture potential non-linearities in the disutility of distance and control for potential behavioral biases towards certain schools: "closest school" is a dummy variable equal to one if s is the closest to student i among all 11 schools; "high school colocated with middle school" is another dummy that equals one if high school s and the student's middle school are colocated at the same address.[35] To account for students' heterogeneous valuation of school quality, interactions between student scores and school scores are introduced separately for French and math, as well as an interaction between own SES and the fraction of high SES students in the school. These school attributes are measured among the entering class of 2012, whereas our focus is on students applying for admission in 2013. We normalize the variables in $Z_{i,s}$ so that each school's fixed effect can be interpreted as the mean valuation, relative to School 1, of a non-high-SES student who has median

[35] There are five such high schools in the district.

scores in both French and math, whose middle school is not colocated with that high school, and for whom the high school is not the closest to her residence.

The error term $\epsilon_{i,s}$ is assumed to be an i.i.d. type-I extreme value, and the variance of unobserved heterogeneity is λ^2 multiplied by the variance of $\epsilon_{i,s}$. The effect of distance is normalized to -1, and, therefore, the fixed effects and γ are all measured in terms of willingness to travel. As a usual position normalization, $\alpha_1 = 0$. We do not consider outside options because of students' almost perfect compliance with the assignment outcome.

Using the same procedures as in the Monte Carlo simulations (described in online Appendix C), we obtain the results summarized in Table 5, where each column reports estimates under a given set of identifying assumptions: (i) weak truth-telling (column 1); (ii) stability (column 2); and (iii) stability with undominated strategies (column 3).[36]

The results provide clear evidence that the WTT-based estimates (column 1) are rather different from the others. Specifically, a downward bias is apparent for popular schools that are not ranked by many students, such as Schools 8 and 11. School 8, which is omitted by 61 percent of students, is deemed by WTT to be less desirable than all the schools included in the ROL, which leads to a low estimated fixed effect. Similarly, the fixed effect estimate of School 11, one of the most popular schools, varies substantially across the identifying assumptions. The under-estimation is mitigated when the model is estimated under a different assumption (columns 2 and 3). Provided that the model is correctly specified, the Hausman test rejects WTT in favor of stability (p-value < 0.01); the test based on moment (in)equalities does not reject the null hypothesis that stability is consistent with undominated strategies at the 5 percent significance level.

The results show that "closest school" has no significant effect, but students significantly prefer colocated schools. Compared with low-score students, those with high French (math) scores have a stronger preference for schools with higher French (math) scores. Moreover, high SES students prefer schools that have admitted a larger fraction of high SES students in the previous year (2012).

Although the WTT-based estimates of the coefficients of covariates (panel B) are not markedly different from the stability-based estimates, one cannot conclude that the WTT assumption produces reasonable results, as shown by the estimates of fixed effects. To provide a better evaluation, we now compare the estimators by their model fit.

[36] For the estimates in column 3, we use the method of moment (in)equalities where inequalities are constructed as described in Section IIE. Determined by $Z_{i,s}$, we interact French score, math score, and distances to Schools 1 and 2 with the conditional moments. Although one could use more variables, e.g., SES status and distance to other schools, they provide little additional variation. In principle, the assumption of undominated strategies alone implies partial identification (Section IIE). Because stability is not rejected by our test, we do not present results based on this approach (available upon request). We note that the marginal confidence intervals from moment inequalities only turn out to be wide in our empirical setting, and hence are relatively uninformative. The possible reasons are that the empirical bounds for the probability of a preference ordering over a pair of schools are fairly wide, and that the available methods to conduct inference based on moment inequalities are typically conservative.

Table 5—Estimation Results under Different Sets of Identifying Assumptions

	Identifying assumptions		
	Weak Truth-telling	Stability of the matching	Stability and undominated strategies
	(1)	(2)	(3)
Panel A. School fixed effects			
School 2	−0.71	1.46	1.21
	[−1.17, −0.24]	[0.64, 2.28]	[0.14, 2.29]
School 3	−2.12	1.03	0.84
	[−2.66, −1.58]	[0.19, 1.86]	[-0.56, 2.01]
School 4	3.31	2.91	2.90
	[2.75, 3.86]	[2.07, 3.76]	[2.36, 3.39]
School 5	5.13	4.16	4.16
	[4.41, 5.84]	[3.22, 5.10]	[3.71, 4.49]
School 6	4.87	4.24	4.30
	[4.21, 5.54]	[3.29, 5.18]	[3.73, 4.82]
School 7	7.33	6.81	6.24
	[6.47, 8.18]	[5.65, 7.98]	[5.76, 7.28]
School 8	1.59	4.46	4.27
	[1.10, 2.08]	[3.46, 5.47]	[2.98, 5.26]
School 9	6.84	7.77	6.57
	[6.07, 7.61]	[6.55, 8.99]	[5.84, 7.26]
School 10	7.84	7.25	6.44
	[6.94, 8.75]	[6.01, 8.49]	[5.87, 7.05]
School 11	5.35	7.28	5.61
	[4.62, 6.08]	[6.06, 8.51]	[4.98, 7.33]
Panel B. Covariates			
Closest school	−0.37	−0.19	−0.15
	[−0.63, −0.11]	[−0.47, 0.10]	[−0.75, 0.57]
High school colocated with middle school	2.54	1.76	1.54
	[2.02, 3.07]	[1.19, 2.32]	[0.17, 3.12]
Student French score [$\times$ 10] $\times$ school French score [$\times$ 10]	0.20	0.18	0.23
	[0.16, 0.23]	[0.13, 0.24]	[0.10, 0.35]
Student math score [$\times$ 10] $\times$ school math score [$\times$ 10]	0.30	0.27	0.30
	[0.26, 0.34]	[0.21, 0.32]	[0.18, 0.40]
High SES $\times$ fraction high SES in school	6.79	4.92	8.12
	[5.62, 7.97]	[3.31, 6.54]	[4.18, 12.55]
Scaling parameter (λ)	3.09	1.33	1.50
	[2.79, 3.38]	[1.16, 1.50]	[1.20, 1.64]
Number of students	1,590	1,568	1,590

Notes: This table reports the estimates of the parameters in equation (10) for the southern district of Paris, with the coefficient on distance being normalized to -1. The point estimates in columns 1 and 2 are based on maximum likelihood, whereas those in column 3 are based on moment equalities and inequalities, with 95 percent confidence intervals in brackets. *Model selection tests:* A Hausman test, testing weak truth-telling against stability (or columns 1 against 2), rejects WTT in favor of stability (p-value < 0.01); a test based on moment equalities and inequalities does not reject the null hypothesis that stability is consistent with undominated strategies at the 5 percent significance level.

D. *Goodness of Fit*

In three dimensions (cutoffs, assignment, and revealed preferences), we compare the observed values to those predicted by the estimates from Table 5. This comparison reveals that the stability-based estimates fit the data well, as opposed to those based on WTT (see online Appendix D.2 for computational details).

Specifically, Figure 5 and online Appendix Table D2 show that the stability-based estimates (with or without undominated strategies) predict cutoffs close to the observed ones.[37] By contrast, WTT substantially under-predicts the cutoffs of the most popular schools.

Panel A of Table 6 compares each student's predicted assignment to the observed one. The stability-based estimates have 33–38 percent success rates, whereas the WTT-based estimates accurately predict only 22 percent of the assignments. In panel B, we take as given the schools that a student has included in her submitted ROL, and compute the probability of observing this particular preference order among the ranked schools. The observed order of students' top two choices has a mean predicted probability of 60 or 62 percent based on the stability-based estimates, higher than the 55 percent achieved by the WTT-based estimates. We next consider the observed order of a student's full list of choices. Again, the stability-based estimates outperform those based on WTT, with an average predicted probability between 2.2 and 2.5 percent for the former versus 1.2 percent for the latter. The predictive power of the stability-based estimates along the two measures in panel B is noteworthy because the prediction is partly out of sample.[38]

V. Summary and Discussion

As a summary of the results, we clarify when each approach is more appropriate for empirical analysis. We also discuss whether the results can be extended to the school-proposing DA, the case with non-equilibrium behavior, and settings beyond school choice.

A. *Choosing among the Approaches: A Summary*

In the preference estimation with real-life data from centralized school choice and college admissions, some practical considerations should be taken into account. Recall that we focus on the strict-priority setting in which students are ranked based on strict priority indices that are ex ante known privately. Building on the results from our theoretical and empirical analyses, this section emphasizes some of the key market features that deserve careful examination when one decides which approach to use in a given context.

The Nesting Structure of Identifying Assumptions.—Our results imply that the identifying assumptions follow a nesting structure, as depicted in Figure 6.

Truth-telling is a natural candidate identifying assumption because of DA's strategy-proofness. However, strict truth-telling (i.e., students truthfully rank all schools) is not an equilibrium, if students cannot rank all schools at no cost (Proposition 1). In real-life data, students seldom rank all schools, which calls for a

[37] It should be emphasized that the stability-based estimation does not try to fit cutoffs directly, neither does it restrict a student's preferences over infeasible schools. The difference in predicted cutoffs between stability and WTT is solely due to their differences in predicting preferences.

[38] In the data, 54 percent of students ranked at least 1 infeasible school among their top 2 choices (34 percent ranked 1 infeasible school, while 20 percent ranked 2). The average fraction of infeasible schools among all submitted choices is 30 percent.

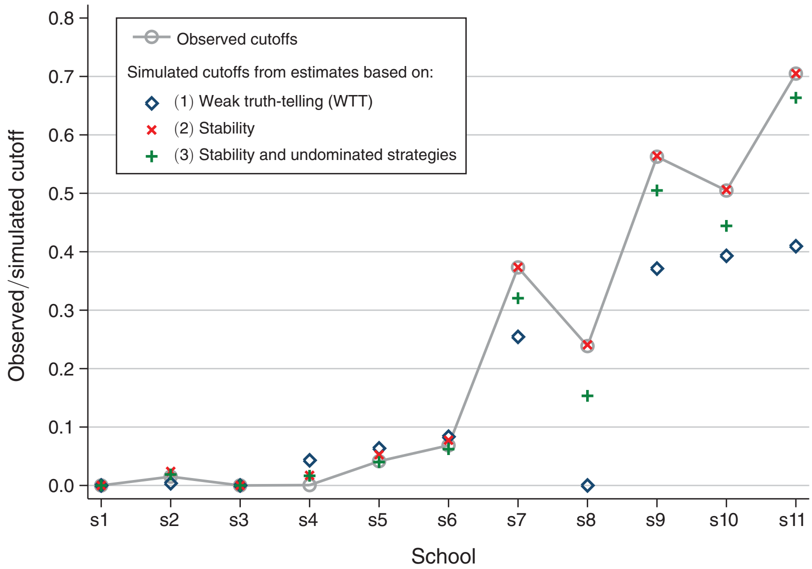

Figure 5. Goodness of Fit: Observed versus Simulated Cutoffs

Notes: This figure compares the cutoffs observed for the 11 high schools of Paris' southern district in 2013 to those simulated with the three sets of estimates in Table 5. The simulated cutoffs are averaged over 300 simulated samples. See online Appendix D.2 for details.

Table 6—Goodness-of-Fit Measures Based on Different Sets of Identifying Assumptions

	Estimates from		
	Weak Truth-telling (1)	Stability of the matching (2)	Stability and undominated strategies (3)
Panel A. Simulated versus observed assignment (300 simulated samples)			
Mean predicted fraction of students assigned to observed assignment	0.220 (0.011)	0.383 (0.010)	0.326 (0.012)
Panel B. Predicted versus observed partial preference order of given schools			
Mean predicted probability that a student prefers the top-ranked school to the 2nd-ranked in her submitted ROL	0.553	0.615	0.595
Mean predicted probability that a student's partial preference order among the schools in her ROL coincides with the submitted rank order	0.012	0.025	0.022

Notes: This table reports two sets of goodness-of-fit measures comparing the observed outcomes to those predicted under the different sets of identifying assumptions as in Table 5, for the high school assignment of students in the southern district of Paris. Panel A compares students' observed assignment with their predicted assignment in 300 simulated samples. In all simulations, we vary only the utility shocks, which are kept common across columns 1–3 (see online Appendix D.2 for details). Predicted and observed assignments are compared by computing the average predicted fraction of students who are assigned to their observed assignment school, with standard deviations across the simulation samples reported in parentheses; in other words, this is the average fraction of times each student is assigned to her observed assignment in the 300 simulated samples. Panel B uses two measures to compare students' observed partial preference order of given schools (revealed in their submitted ROL) with the prediction, among students who rank at least two schools: (i) mean predicted probability that a student prefers the top-ranked school to the second-ranked in her submitted ROL, which is averaged across students; and (ii) mean predicted probability that a student's partial preference order among the schools in her ROL coincides with the submitted rank order. Due to the logit specification, those probabilities can be calculated without simulation.

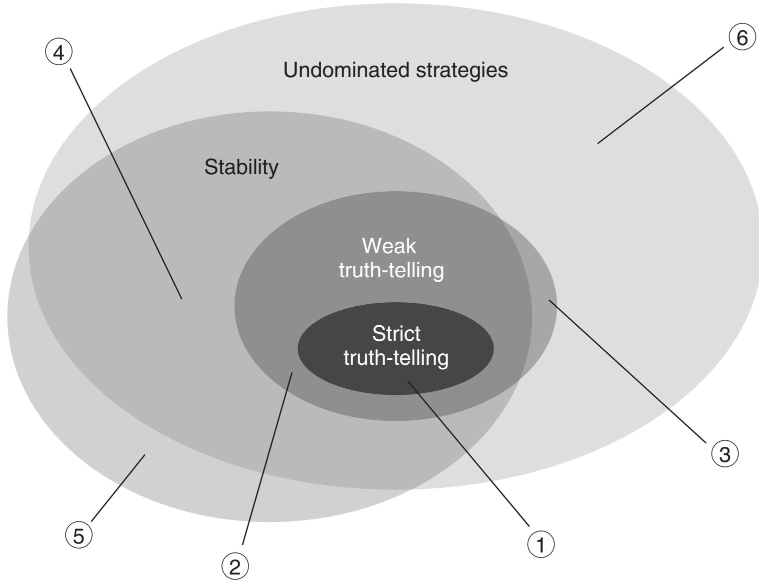

Figure 6. Nesting Structure of Identifying Assumptions

Notes: This figure shows the nesting structure of the identifying assumptions that can be used to analyze data generated by DA and its variants in the strict-priority setting. The numbered areas correspond to different combinations of identifying assumptions: ① strict truth-telling; ② weak truth-telling and stability; ③ weak truth-telling without stability; ④ stability and undominated strategies; ⑤ stability without undominated strategies; ⑥ undominated strategies without stability.

weaker version of the truth-telling assumption. As clarified in the theoretical analysis, weak truth-telling (i.e., students truthfully rank their most preferred schools and omit some least preferred ones) does not follow directly from strategy-proofness, since it requires additional assumptions such as the length of ROLs being independent of preferences.

Stability is an even weaker assumption on student behavior, while still allowing for the identification of preferences. It states that every student is assigned to her favorite ex post feasible school, and is always satisfied when students are strictly truth-telling. Although stability is not guaranteed in all Bayesian Nash equilibria, even when students are weakly truth-telling, it is asymptotically satisfied when the economy grows large (Proposition 4).

The third candidate identifying assumption is that students do not play dominated strategies (Proposition 2), so that submitted ROLs reveal students' partial preference orders of schools. Weak truth-telling is a special case of this more general assumption, whereas stability may hold even if students play dominated strategies.

The Choice of Empirical Approaches.—When choosing among the candidate identifying assumptions, consideration should be given to the features of the problem under study, as well as the available data. For each assumption, Table 7 summarizes the features making it more plausible, the required data, and some discussion about identification and estimation.

Truth-telling is more likely to be satisfied when students can rank as many schools as they wish at no cost, and face large uncertainty about each school's exact ranking of students. Conditional on students' submitted ROLs being observed, preferences can be estimated using either MLE or GMM. The choice between weak truth-telling and strict truth-telling depends on whether students rank all schools (Section IIB) and on the importance of outside options (online Appendix A.4).

When students face some cost of ranking more schools (e.g., if the length of submittable ROLs is restricted), stability can be a more plausible assumption than truth-telling. This assumption is more likely to hold when the market is larger (i.e., many students and many seats per school), when students are less constrained in applying to multiple schools (e.g., longer ROLs), when they are less uncertain about each school's ranking of all students at the time of application, when they know more about others' preferences, or when it is easier for them to predict school cutoffs (Proposition 5). Our Monte Carlo simulations additionally provide numerical evidence suggesting that stability is a plausible assumption even when students face non-negligible application costs (online Appendix C.3).

Estimating preferences based on stability uses information on the admission outcome, the school capacities, and the priority indices, but has the advantage of not requiring data on submitted ROLs. However, it is necessary to assume the conditional independence between priority index and unobserved preference heterogeneity. Compared to truth-telling, the main cost of the stability-based approach is its limited power to identify rich substitution patterns, because the information content of ROLs is discarded.

Weak truth-telling does not always imply stability, but it does imply no-blocking among all *assigned* students (Proposition 3). Therefore, weak truth-telling can be tested against stability (or no-blocking among assigned students) using the Hausman (1978) and Hansen (1982) tests. It should be emphasized that these tests do not provide definitive proof against truth-telling unless the model is correctly specified and identified.

If it is believed that neither truth-telling nor stability is likely to be satisfied, preferences can still be partially identified under the assumption that students do not play dominated strategies. This assumption is more plausible when no school is either "safe" or "impossible" for students, making it less likely that students rank some schools in an arbitrary manner. Submitted ROLs can then be used to form conditional moment inequalities that partially identify preferences.

When the conditions for both stability and undominated-strategies assumptions are jointly satisfied, the moment inequalities from the latter assumption provide over-identifying information that can be integrated with the stability assumption to estimate preferences based on all of the available data (ROLs, matching outcome, school capacities, and priority indices). Additionally, the stability assumption can be tested against the undominated-strategies assumption using the specification test in Bugni, Canay, and Shi (2015).

B. *Discussion and Extension*

The School-Proposing DA.—Our main results can be extended to the school-proposing DA, which is also commonly used in practice (see Table 1). Under this mechanism, schools "propose" to students following the order of student priority indices. Proposition 2 no longer holds; that is, students might have incentives not to report a

TABLE 7—SUMMARY OF EMPIRICAL APPROACHES

Identifying assumption	What makes the assumption more plausible?	Required data	Identification and estimation
Weak truth-telling: Schools in a submitted ROL are ranked in true preference order and omitted ones are less preferred	(a) No cost of ranking more schools, e.g., no restriction on the length of submittable ROLs and choice set not being too large (b) At the time of application, each student knows her own priority index but not others', and the distribution of priority indices has a large variance	Submitted ROLs	Point identification Estimation by, e.g., MLE/GMM
Stability of the matching: Every student is assigned to her favorite feasible school. Priority indices and unobserved preference heterogeneity are conditionally independent	Stability is satisfied if truth-telling holds and (almost) everyone is assigned. Otherwise, it is more likely to be true when (a) market is large (many students, big schools); (b) students are less constrained when applying to more schools; (c) students face limited uncertainty about how schools rank them at the time of application; (d) students know more about others' preferences; or (e) cutoffs are easy to predict	Admission outcome, school capacities, priority indices	Point identification Estimation by, e.g., MLE/GMM
Undominated strategies: Submitted ROLs are true partial preference orders	(a) No "safety school" so that "irrelevance at the bottom" of one's ROL is less likely. (b) No "impossible school" so that students do not rank impossible school arbitrarily.	Submitted ROLs	Partial identification Estimation with moment inequalities
Stability and undominated strategies: See the conditions laid out separately for stability and undominated strategies	See the conditions laid out separately for stability and undominated strategies	Submitted ROLs, admission outcome, school capacities, priority indices	Point identification Estimation with moment equalities and moment inequalities

Notes: This table describes the empirical approaches to analyses of data generated by DA and its variants in the strict-priority setting. In addition, there are two tests available: (i) weak truth-telling can be tested against stability (H_0: both weak truth-telling and stability are satisfied; H_1: only stability is satisfied), e.g., using the Hausman (1978) or Hansen (1982) tests; (ii) stability can be tested against undominated strategies (H_0: both stability and undominated strategies are satisfied; H_1: only the undominated-strategies assumption is satisfied) using the approach in Bugni, Canay, and Shi (2015).

true partial preference order (Haeringer and Klijn 2009). Nonetheless, the asymptotic stability result (Proposition 4) is still valid, as its proof does not rely on Proposition 2. Indeed, the matching can be stable in equilibrium (Haeringer and Klijn 2009).

To summarize, if the market under the school-proposing DA has features making the matching stable (see Table 7), we can formulate identification and estimation of student preferences based on stability. However, the truth-telling assumption no longer has theoretical support, as the school-proposing DA is not strategy-proof for students (Roth 1982). The approach based on undominated strategies does not apply either, since there are no dominated strategies under this mechanism (Haeringer and Klijn 2009).

Non-Equilibrium Strategies.—We have thus far assumed that everyone plays an equilibrium strategy with a common prior. More realistically, some students could have different information and make mistakes when strategizing.

Indeed, a growing number of studies find that strategic mistakes are common even in strategy-proof environments. Laboratory experiments show that a significant fraction of subjects do not report their preferences truthfully in strategy-proof mechanisms (Chen and Sönmez 2006). More relevantly, mistakes occur in real-world contexts, e.g., the admissions to Israeli graduate programs in psychology (Hassidim, Romm, and Shorrer 2016), the medical resident match in the United

States (Rees-Jones 2018), and the Australian university admissions (Artemov, Che, and He 2017). Without estimating preferences, these studies show that a non-negligible fraction of participants make unambiguous mistakes in their ROLs.

However, the vast majority of these mistakes are not payoff relevant. In other words, although some students play dominated strategies, the matching is still close to stable, corresponding to area ⑤ in Figure 6. Based on these observations, the results in Artemov, Che, and He (2017) imply that, as identifying restrictions, assuming stability can be more robust and more plausible than the assumption of undominated strategies.

Beyond School Choice and College Admissions.—Although the analysis has focused on school choice and college admissions, our results can apply to certain assignment procedures based on DA. Let us call agents on the two sides "applicants" and "recruiters," respectively. The key requirement is that when applying, applicants have sufficiently precise information on how recruiters rank them and that researchers observe how recruiters exactly rank applicants.[39] Examples include the assignment of teachers to schools in France (Combe, Tercieux, and Terrier 2017) and the Scottish Foundation Allocation Scheme matching medical school graduates with training programs (Irving 2011). The estimation approaches discussed in Section II could be implemented in these settings.

VI. Conclusion

We present novel approaches to estimating student preferences with school choice or college admissions data generated by the popular deferred acceptance mechanism when applicants are ranked strictly by some ex ante known priority index. We provide theoretical and empirical evidence showing that, in this commonly observed setting, it can be rather restrictive to assume that students truthfully rank schools when applying for admission. Instead, stability (or justified-envy-freeness) of the matching provides rich identifying information, while being a weaker assumption on student behavior. Assuming that students do not play dominated strategies, we also discuss methods with moment inequalities, which can be useful with or without stability. A series of tests are proposed to guide the selection of the appropriate approach.

The estimation and testing methods are illustrated with Monte Carlo simulations. When applied to school choice data from Paris, our results are more consistent with stability than with truth-telling. Reduced-form evidence on ranking behavior suggests that some students omit the most selective schools from their list because of low admission probabilities. Provided that the model is correctly specified, our proposed tests reject truth-telling but not stability. Compared with our preferred estimates based on stability (with or without imposing undominated strategies), assuming truth-telling leads to an under-estimation of preferences for popular or small schools. Moreover, the stability-based estimators outperform the truth-telling-based estimator in predicting matching outcomes and student preferences.

[39] Without information on how either side ranks the other, it becomes the classical two-sided matching, and additional assumptions are needed for identification and estimation (Chiappori and Salanié 2016).

Our approaches are applicable to many school choice and college admissions systems around the world, as well as to other matching schemes such as teacher assignment in France and medical matching in Scotland.

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