On the Use of Utility Theory in Engineering Design

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Abstract—Multiattribute utility theory has a long history of application in engineering and systems design. These applications rely almost exclusively on two types of utility functions: the additive and multiplicative forms. The foundations of utility theory, however, do not place restrictions on the functional form of the utility function, meaning that many methods of representing preferences remain unexplored in systems design. This paper reviews the literature on applications of utility theory in design to bring light to potential new directions for research and to clarify a few subtle misapplications of utility theory. These misuses include the failure to distinguish between direct and indirect value attributes, the use of probability as an attribute, treating costs from different sources differently, and restricting the functional form of utility. This paper introduces a value-based approach that is based on the creation of a deterministic value function for design and the assignment of a 1-D utility function over the value measure. We use a conceptual and numeric example to illustrate the greater flexibility of this approach. We also present the concept of utility transversality in engineering design. We show that several criticisms of utility theory that have appeared in the engineering design literature are actually criticisms of these artificial limitations and that these limitations are overcome by the value-based approach.

Index Terms—Engineering design, multiattribute utility, systems design, utility theory, value-based design.

I. INTRODUCTION

T he important role of decision making has long been recognized in engineering and systems design [1]–[7]. Early texts on the subject describe the need to identify relevant criteria [1], [2], the importance of characterizing uncertainties [3]–[5], and the application of weighting functions [2]–[4]. Tribus [8] underscores the fundamental role of decision making when explaining engineering design:

"Design has been defined by Rosenstein as an 'iterative decisionmaking process' comprising certain well recognized steps. The emphasis should be placed on decision making, for this is the heart of the design process." (page 391)

More recently, the phrase “decision-based design” has been introduced to emphasize the decision-making role of the design engineer [9], [10]. Subsequently, a large volume of work on decision making in engineering and systems design has emerged that uses a variety of terms to reference the role of decisions: decision-based design [11]–[13], decision theory [14], and utility theory [15], [16]. Tomiyama et al. [17] categorize the various approaches decision making into those that are heuristics based and those that are based on decision theory.

The majority of the work in this area applies decision making to the design artifact itself. It is worth noting that defining the scope of the decision, i.e., the decision framing, is an important first step in the analysis. It is possible to define a decision frame to examine other aspects of the design process. Thompson and Paredis, for example, apply decision analysis to the decisions about the design process itself [18], while Shukla et al. examine the problem of aggregating preferences in systems design [19].

Although the design literature has described the application of decision analysis to engineering design [8], we have observed a persistent gap in the understanding of utility theory and the application of decision analytic principles to the design artifact. A literature review is used to identify four concepts in utility theory that have been misapplied in systems design. Clarifying these concepts is the first motivation for this paper. This paper explains each in detail. This clarification is necessary given the stakes involved in systems design; some studies have suggested that the current systems engineering processes contribute to $150 million per day in delay and overrun costs at the Department of Defense alone [20].

This paper also seeks to expand the flexibility of utility theory in systems design by presenting a value-based approach to preference functions. The approach requires the specification of a deterministic value function and a utility function over a singlevalue measure. Tradeoffs are assessed in a deterministic setting. A 1-D utility function is assessed. The approach ensures consistency in interactions among variables and risk attitude toward each attribute.

It is advantageous to acknowledge that doing decisionmaking work within a field of engineering does not make a person an expert in decision analysis. Conversely, knowing decision analysis does not make a person an expert within a field of engineering. However, through appropriate communication and the exchange of ideas, the intersection of the fields can lead to superior designs. An additional motivation for this paper is to facilitate this conversation between academic communities. The contributions of this paper include a literature review of the application of utility theory to systems design, the introduction...
of a value-based approach that allows preference tradeoffs to be made in a deterministic setting, and to spark collaboration between different academic communities.

In the pursuit of understanding and academic dialogue, Section II explains the rationale for the expected utility criterion and why it is not appropriate to use arbitrary measures in a decision problem. Section III discusses common misuses of utility theory in engineering design, and Section IV explains the value-based approach to decision-based design. Section V summarizes the main insights and explores future directions and opportunities for research.

II. RATIONALE FOR EXPECTED UTILITY

Utility theory as a foundation for decision making is based on a set of axioms. If a decision maker agrees with the concept described by axiom, then he or she should make decisions consistent with utility theory in order to be consistent with the axiomatic principles. Decisions that are inconsistent with utility theory imply that the decision maker is violating at least one of the foundational concepts. Thus, clarity in understanding these axioms is crucial to understanding the usefulness—and limitations—of applying decision analysis to engineering design, and to understanding the distinctions discussed in this paper.

These axioms, as originally stated by von Neumann and Morgenstern [21], are completeness, transitivity, continuity, and independence. Howard [22] and others [23], [24] present these axioms in the form of five rules of decision making. To facilitate understanding utility theory and provide a basis for the discussion in this paper, the axioms as formulated in five rules are as follows.

1) The probability rule: this rule states that:
   a) I can characterize each decision alternative I face in terms of the set of possible prospects that may occur; and
   b) I can assign a probability of receiving each prospect for every alternative I choose. Put very generally, this rule states that we can draw a decision tree with all of its nodes and numbers.

2) The order rule: this rule states that I can rank order the deterministic prospects that I have characterized in a list from the most preferred to the least preferred. Ties are allowed but preferences must be transitive, i.e., if I prefer prospect A to prospect B, and if I prefer prospect B to prospect C, then I must prefer prospect A to prospect C.

3) The equivalence rule (von Neumann Morgenstern utility rule): this rule states that given three-ordered prospects with strict preference, A > B > C, I can assign a preference probability, pB, that would make me indifferent between receiving prospect B for certain, or a binary deal having a probability pB of receiving A and a 1 − pB probability of receiving C.

4) The substitution rule: this rule states that whenever I face a prospect B, for which I have stated a preference probability pB of receiving A and a probability 1 − pB of receiving C in the equivalence rule, I would be indifferent between receiving this binary deal and receiving prospect B. This rule allows us to make various substitutions of a binary deal (with prospects A and C) and the deterministic prospect, B, whenever either occurs in the decision tree.

5) The choice rule: this rule states that if I face two binary decision alternatives, “L1” and “L2,” both yielding the same prospects A and B (prospect A is preferred to B), and

   if “L1” has a higher probability of getting A, then I should choose “L1” over “L2.”

Fig. 1(a) shows a set of prospects, R1,...,Rn that has been characterized by the probability rule and the corresponding probabilities, p1,..., pn, for a given decision alternative in the form of a tree. The figure also shows the order rule ranking (it assumes that R1 is the best prospect and Rn is the worst). Fig. 1(b) shows an example of the equivalence rule, where a preference probability, Uj, is assigned to prospect Rj in terms of the best and worst prospects, R1 and Rn. Fig. 1(c) shows the substitutions made for the prospects in terms of their utilities (preference probabilities) according to the substitution rule, and Fig. 1(d) shows the equivalent lottery after multiplying the probabilities in Fig. 1(a) by the von Neumann and Morgenstern utilities (preference probabilities). Note that the probability of the best prospect in Fig. 1(d) is equal to the expected utility of the original alternative. A decision maker who follows the five rules mentioned earlier will be indifferent between receiving the lotteries of Fig. 1(a)
and (d). Thus, any decision alternative can be reduced into an equivalent binary alternative using von Neumann and Morgenstern utility (preference probability) assessments. Furthermore, the expected utility of the original alternative is equal to the probability of getting the best prospect in the equivalent binary alternative. Since any other alternative can also be reduced into a binary alternative with the same prospects, \( R_t \) and \( R_s \) (but with different probabilities of achieving them), the choice rule determines the best decision alternative: we choose the decision alternative (lottery), with the highest expected utility.

The application of these five rules results in maximizing the expectation of the utility function as the objective function for the selection of the best decision alternative. Not any arbitrary numeric measure representing some form of preferences can be used in this formulation. The measure whose expected value is to be maximized over the set of alternatives must be the von Neumann Morgenstern utility (that reduces the lottery into an equivalent binary lottery). If we choose to optimize any other arbitrary measure, such as minimizing the “value at risk,” or maximizing the “maximum regret,” or any arbitrary “risk measure” or “score,” then we would be violating one of these rules. Further, we make the underscore the observation that these rules place no restrictions on the functional form of the utility (or multiattribute utility) function.

Only those decision-making methods that follow these five rules are consistent with utility theory. This paper focuses specifically on applications of utility theory. We distinguish utility theory and decision analysis from other methods that do not follow utility theory such as the analytic hierarchy process [25] or the quality function deployment method [26]. Comments and critiques on these methods may be found elsewhere [27]–[30], and are not the focus of this paper.

III. SOME COMMON MISUSES OF UTILITY THEORY

The application of utility theory requires attention to some important distinctions that may not be readily apparent to experts in fields outside decision analysis. A literature search has shown four common misuses of utility theory. This section provides a literature review and to bring clarity to the seemingly small details that have a large effect on the validity of an analysis.

A. Difference Between Direct and Indirect Value Attributes

For any given decision or design, there may be numerous factors important to the decision maker. A business may care about market share, brand recognition, product quality, and profit. A closer inspection reveals, however, that the business cares about market share, brand recognition, and product quality because they all update the information about profit. If the profit were known in advance for all future time periods, information about market share would no longer have value as it would not affect the deterministic profit. This example illustrates that not all factors in a decision should be treated the same. The distinction between direct and indirect attributes must be made [23], [31], [32].

Direct value attributes are those items that a decision maker cares about after all uncertainties are resolved. For profit-maximizing firms, a direct value is profit [10], [13], [33], [34]. An indirect value is something that the decision maker cares about because it updates his state of information about the direct values or affects the probability distribution of the direct value attributes. A question that can be asked to help distinguish between two is: would I accept a lower profit in exchange for a better measure on this variable? If the answer is yes, then the variable represents a direct value attribute. If the answer is no, then the variable is likely an indirect value attribute.

This distinction is represented in a decision diagram by the presence (direct value attribute) or absence (indirect value attribute) of an arrow into a final value node. Consider the two different formulations of a design decision shown in Fig. 2. Square nodes represent decisions; ovals represent uncertainties; double-lined ovals are deterministic; hexagons are values. The diagram on the left presents product quality as an indirect value attribute. Quality affects both revenue and cost but is not valued by itself. The diagram on the right shows how the analysis changes if quality is treated as a direct value. In addition to profit, another value node is needed to represent how the decision maker values both profit and quality. In this case, some tradeoff exists between quality and profit.

Treating a variable as an indirect versus a direct value attribute changes the objective function of the analysis. If quality is an indirect value attribute, then any quality-related decision variables affect the distribution over profit. On the other hand, if quality is a direct value attribute, then it appears as an argument in the utility function, and is an additional variable over which the expectation is calculated.

In the absence of a clear distinction between indirect and direct value attributes, it seems logical to include indirect value attributes in a multiattribute utility function. In a probabilistic setting, these variables are in fact important in affecting outcomes. However, the way in which they affect the outcomes must be correctly modeled for consistency with utility theory.

![Diagram](image-url)
Multiattribute utility functions should be constructed to include only direct value attributes in their arguments. Indirect value attributes contribute to the expectation portion of the objective function.

The specification of direct versus indirect attributes also highlights an important distinction about whose preferences should be represented by the utility function. If the design decision is being made by a company, then the company’s utility function should be optimized. Some analyses in the literature use the customer’s utility function [35], [36]. Although customer preferences are important in determining the willingness to pay and thus the distribution over profit, optimizing the customer’s utility function does not optimize the profit realized by the company.

B. Money Is Money

Money, whether in the form of revenue, cost, or profit, or something else, is a special case among direct value attributes which deserves attention. The reason for the special consideration is that money itself has an inherent value equal to its magnitude. Therefore, money from different sources must be treated equally. For example, costs may arise from the purchase of materials and the time required for assembling those materials. But the overall cost is the sum of the two.

This clarification may seem trivial, but it is important. While some authors correctly sum costs from multiple sources to create a single cost attribute [37], others treat costs from different sources as multiple cost attributes. This issue can appear in subtle ways. For example, slowed productivity is a cost that is affected by factors such as load capacity and work speed. If there are costs associated with smaller load capacity and costs associated with slower work speed, then these costs should be summed and represent a single monetary attribute. This approach requires the specification of how factors such as work speed affect cost, but if a detailed decision analysis is to be conducted, such a task should be considered an important component of the analysis. Some studies in the literature, however, do not consider how such factors affect cost and therefore risk treating costs due to load capacity differently from costs due to work speed [38], [39]. Other examples include treating costs from the usable life of a part differently from other costs [40], [41] or treating costs from disassembly of a product differently than costs from reassembly [42].

Another subtle way sources of money are treated differently is in the inclusion of environmental impacts. In some cases, a direct value may be placed on environmental impact. For example, a nonprofit entity dedicated to environmental protection clearly places value directly on the environment. In most cases of a profit-maximizing, publicly traded company as the decision maker, such a direct value does not exist. The company ensures that all applicable environmental regulations are met, but does not value them directly. This type of company is described by Fitzgerald et al. [43]:

Product development organizations are unwilling to compromise product functionality, unit cost, or time to market in order to create products that have less environmental impact than that required by regulations.

The actions required to meet environmental regulations have costs associated with them. Thus, when considering a set of decision alternatives, the environmental impact of each alternative is important because of the costs it represents and any effect it may have on the probability distribution over customer purchasing. In other analyses in the literature, the cost of the environmental impact of a decision alternative is not treated as a cost at all. Rather, the environmental impact is treated as a unique direct value attribute [44], [45].

C. Using Probability as an Argument in the Utility Function

Another common misuse of utility theory occurs when probability is itself used as an attribute. On occasion, probability has been explicitly defined as an attribute [42], but more often, the practice is much more subtle. For example, reliability is important to many designs. But if reliability is defined as a measure of the probability of a breakdown, then treating reliability as an attribute means that probability has become an attribute even if not explicitly stated as such [46], [47]. Such a treatment is no longer consistent with the axiomatic basis of utility theory. An analysis consistent with utility theory would have probability contribute to the objective function in the expectation, i.e., the probability distribution, only.

We do not mean to say that reliability is not an important concept or that reliability should not be included in the analysis. Rather, we wish to highlight that a decision analytic treatment of reliability requires a different approach to the concept. Utility theory demands the decision maker consider why reliability matters and model its consequences. For example, if a breakdown results in additional costs, then the probability of the breakdown should be included in the expectation, and the costs associated with it should be included within the measure of money. If a breakdown affects consumer demand for the product, then the appropriate probability distribution over demand should be specified for those variables that affect the reliability. Thus, while we understand that it is tempting to define reliability as an attribute, the resulting analysis will be inconsistent with utility theory.

D. Using Only Traditional Representations of Preferences

After the indirect and direct value attributes are specified, the application of utility theory generally requires the specification of some utility function to represent the decision maker’s preferences under uncertainty. The utility function must represent the decision maker’s preferences in a way that is consistent with the equivalence rule explained in Section II. The form of the utility function is neither specified nor restricted by the axioms of von Neumann and Morgenstern [21].

Within the engineering design literature, however, utility theory has largely become synonymous with the application of
just two-attribute functions: the weighted sum and the multiplicative utility function. See, for example, analyses using the weighted sum [37], [48]–[54], or the multiplicative utility function [15], [16], [42], [46], [55]–[60]. The extensive number of cases using these two utility functions makes them a traditional choice. However, these functional forms place additional restrictions on the types of preferences that can be modeled. These restrictions appear to be largely unappreciated in the literature. But when they are mentioned, they seem to be cited as shortcomings of utility theory [61]–[65]. This section explains the preference restrictions implied by these functional forms. Further, we highlight that utility theory is not limited to these functions and absolutely does not require these restrictions.

Analyses employing the additive or multiplicative utility functions are generally formulated with a set of n attributes. A utility function over each attribute is specified. The utility of attribute i is denoted u_i. A weighting factor is assessed for each attribute and is denoted k_i. A scaling factor, K, is calculated. The multiplicative utility function for the decision is then

\[ U = \frac{1}{K} \prod_{j=1}^{n} (K k_i u_i + 1) - 1 \]  

In the case that \( \sum k_i = 1 \), the multiplicative utility function simplifies to the weighted sum

\[ U = \sum_{i=1}^{n} k_i u_i . \]  

Keeney and Raiffa [64] introduced the multiplicative form of (1) that applies under specific conditions that are more restrictive than the axioms of utility theory. These functions ought to be employed only when these conditions do apply. A careful analysis will check that these conditions are valid for the specific case in question.

To use (1), mutual utility independence must hold. This requirement means that your preferences under uncertainty for one attribute are not affected by the level of any other attribute. To illustrate, consider a company that values both profit (x_1) and safety as measured by number of injuries (x_2). The company specifies the probability p that makes it indifferent between a fixed (deterministic) outcome and a lottery over these two attributes.

The call for more general types of multiattribute utility functions in engineering design becomes even more important given the physical relationships between many of the decision variables in an engineering design problem. Specifying the relationship between these design attributes using the additive or multiplicative forms can lead to misrepresentations of the decision maker’s preferences. It seems more prudent for the case of engineering design problems to use a value function. The value function approach for applying utility theory has not received attention in the engineering design community. The next section explains this approach and its benefits in an engineering design context.

IV. VALUE-BASED DESIGN: IMPLICATIONS FOR THE UTILITY FUNCTION

The presence of uncertainty motivates the use of utility theory for normative decision making. However, the very presence of uncertainty has been cited as a factor that contributes to difficulty in specifying preferences [62]. Fortunately, a value-based approach exists in which the decision maker first specifies her preferences for deterministic outcomes in terms of a value function and then specifies a utility function over value [65], [66]. While the engineering design community has suggested correlating objective functions with value functions [67], the concept of constructing a single-dimensional utility function over value as a first step in the construction of a multiattribute utility function has not been explored. While this value-based approach seems to be the most appropriate for constructing multiattribute utility functions in engineering design, we have not found many of its applications in the literature.

Before explaining the steps of the value-based approach, an important distinction between utility and value must be made. Utility describes preferences under conditions of uncertainty while value describes preferences under conditions of certainty. This distinction is not always made clear in the engineering design literature. Some authors use value and utility interchangeably [36], [68], while others reference the worth of a design without clarifying the meaning to be a value or utility [69]. Failure to clarify this distinction can lead to confusion in the literature, for example, fixing certain variables in a multiattribute utility function and calling the resulting 1-D utility function a value function [16], using a measure of cost as a utility function [70], or using a utility function to construct an estimate of costs [71].
Fig. 3. Value-based design requires that utility (risk preference) and value models are consistent with the underlying system structural model [32], [72].

The value-based approach [65], [66] is predicated on a structural model of the system that includes both state variables and decision variables. This model is necessary to understand how the system performs under different configurations. A value function is then determined, and this function may include performance measures. If the system value will not be realized for some time period, a time preference model can be included. Finally, a utility or risk preference model over value is defined. This process is illustrated in Fig. 3 which shows that each step provides the foundation for the next part of the analysis, a conceptualization that originally appeared in [72]. This approach provides a consistent link between the underlying state and decision variable, the system value, and the utility function.

A. Power Generation Example

As a simple introduction of the utility over value approach, consider a contrived example of power generation using the basic formula

\[ \text{Power} = i^2R \]  

(3)

where \( i \) is the current and \( R \) is the resistance. If the direct value attribute is money, then dollars or value produced can be represented as

\[ \text{Value} = a (i^2R) \]  

(4)

where \( a \) is a coefficient equal to the unit dollars per power generation. If the decision maker exhibits constant risk aversion (where risk aversion follows the Arrow–Pratt definition [75], [76]), then the decision maker has an exponential utility function. Thus, his utility over money, in this situation, becomes

\[ U_V (\text{Value}) = -\exp(-\gamma ai^2R) \]  

(5)

where \( \gamma \) denotes the risk aversion.

This illustration results in two observations. First, the utility function is neither an addition nor a multiplication of the utility functions of the system variables. Although this example uses an exponential utility function over power, any function that represents the decision maker’s preferences could be used. Second, the utility over resistance is exponential for a fixed level of current, and the utility over current is power exponential for a fixed resistance. Once the decision maker assigns an exponential utility function over money, he does not have a degree of freedom to state a different utility function for current or resistance if he wishes to be consistent with the structural model for power (3).

This value-based approach ensures a consistency of preferences that is not guaranteed with other methods. For example, consider the result if a decision maker constructs the multiattribute utility function by multiplying individually assessed utility functions over current and resistance. Suppose the utility function over current is assessed for 1 Ω of resistance and yields

\[ U_i(i) = -\exp(-\gamma i) \]  

(6)

while assessing the utility function over resistance for 1 A of current yields

\[ U_R(R) = -\exp(-\gamma R). \]  

(7)

If the multiattribute utility function is taken to be the negative product of these two utility functions over individual attributes, the result is

\[ U_{i,R}(i,R) = -\exp(-\gamma R - \gamma i). \]  

(8)

If the direct value attribute is power, all points with equal values of power should have the same utility. Yet it is clear that (8) does not maintain this structural consistency for all points, while (5) does. The set of points for which (8) is consistent with the structural model consist of those points that satisfy \((-\gamma R - \gamma i) = -\gamma ai^2R\). In terms of resistance, equivalence will only be reached for

\[ R = \frac{\gamma ai}{\gamma i + \gamma a^2i^2}. \]  

(9)

For all other points, the use of (8) will imply a different deterministic tradeoff between current and resistance than the one specified by (3).

B. High-Speed Machining Example

We next discuss the utility over value approach in the context of a real-life example. Consider a company that sells products
manufactured through high-speed machining. Given its expertise in the area, the company has a well-defined structural model of the machining process.

1) Deterministic Analysis: After a structural model of the system is well defined, the first step of the value-based approach is to construct a deterministic value function that relates design variables to an attribute of direct value that can be used within the utility function. Note that the arguments in the value function may include indirect value attributes that are important to the analysis because they contribute to value in some way.

For the high-speed machining company, an important component of the value function relates to a specialized milling tool that is required to cut the product being manufactured. The tool is costly and has a limited usable life. Abbas et al. [73] derive a value function that accounts for the costs of manufacture and includes the physical relationships between the variables. For the current illustration, we use a simplified version of this function

\[ V(l, c, t) = 1200 - 0.45\frac{ce}{l} - 60t \]  

where \( t \) is the time spent manufacturing the part, \( c \) is the cost of the specialized tool, and \( l \) is the life of the tool. The time to manufacture the part contributes to costs at a constant marginal rate, and the time spent cutting the part represents 45% of the total manufacturing time. Tool life is related to time spent cutting the part, not total manufacturing time. Selecting parameter values inconsistent with the tradeoffs specified with the value function will result in lost value or a value gap [74].

2) Uncertainty Analysis: The multiattribute utility function is then found by assessing a single-dimensional utility over value

\[ U(x_1, \ldots, x_n) = U_V(V(x_1, \ldots, x_n)) \]  

For the risk neutral company, the utility function equals the value function. For the company with constant risk aversion \( \gamma \), utility over value may be represented by the exponential function

\[ U_V(V) = -e^{-\gamma V} \]  

\[ U(l, c, t) = -e^{-\gamma(1200 - 0.45\frac{ce}{l} - 60t)} \]  

The previous analysis is quite straightforward and guarantees that the tradeoffs implied by the multiattribute utility surface are the same as those provided by the value function. Surprisingly, much of the design literature does not construct a utility function using these steps and does not even acknowledge these physical dependences in relation to a value function. Rather, much of the literature describes utility among the attributes as a function of preferences only [12], [54]. For example, Thurston [12] writes:

\[ \ldots the two independence conditions of utility analysis have nothing to do with the physical design artifact, but rather with preferences for attributes. \]

This view neglects to consider how each attribute contributes to value. Ignoring the physical connections between the attributes as mentioned in the previous quotation leads to inconsistencies. Constructing a multiattribute utility function for the previous example by making direct utility assessments of \( c, l, t \) and then combining them into a multiplicative form creates an inconsistent assessment of tradeoffs. Assessing individual utility functions and then combining them into an arbitrary form result in arbitrary tradeoffs that are inconsistent with those implied by the value function.

The value-based approach highlights the implications of using a particular functional form of utility for the value function and the structural model. In this example, if the value function was additive over the attributes and the company had an exponential utility function over value, the multiattribute utility

\[ t = 2 \text{ hours} \quad c = \$125 \quad l = 5 \text{ hours} \]

\[ t = 6 \text{ hours} \quad c = \$125 \quad l = 5 \text{ hours} \]

\[ t = 10 \text{ hours} \quad c = \$125 \quad l = 5 \text{ hours} \]

\[ \$1057.50 \quad \$487.50 \]

\[ \$772.50 \quad \$772.50 \]

would be equivalent to the product of three individual utility functions, and directly assessing preferences over individual attributes could have yielded the same multiattribute utility function. Even in this case, however, consistency is not guaranteed and the relationships among the variables within the structural model and the value function are limited to being additive in nature.

3) Analyzing Utility Independence: Analyzing utility independence in the presence of a value function highlights the flexibility of the value-based approach. Recall that the multiplicative utility function requires utility independence. Any inconsistencies of using the multiplicative utility function result when this requirement does not hold. We continue the analysis of the high-speed machining company with value function (3) and examine the assessment of utility independence.

Fig. 4. Utility independence lottery in terms of (a) attributes and (b) value.
If the company were asked lottery questions to determine the existence of utility independence, the company would make judgments in terms of the attributes as shown in Fig. 4(a). The same question can be posed in terms of equivalent value as shown in Fig. 4(b). Utility independence for $ requires that the preference probability remains the same and are unchanged.

Due to the linear nature of the value function (10), a risk neutral decision maker will assert utility independence. Thus, the risk neutral decision maker could assess individual utility functions over individual attributes and combine them in the multiplicative utility function and yield the same results as the value-based approach. If, however, the decision maker exhibits any degree of risk aversion, assertions of utility independence will result in inconsistent preferences. Suppose the decision maker exhibits a constant risk aversion $\gamma = 0.01$ and therefore has an exponential utility function. The certain equivalent for this decision maker is given by the inverse of the utility function

$$CE(E[U_V]) = \frac{-1}{\gamma} \ln (1 - E[U_V]).$$ (14)

For this decision maker, the probability $p$ that makes him indifferent between the certain deals and the lottery in Fig. 4(a) and in Fig. 4(b) is $p = 0.945$.

![Fig. 5. Value assigned by a decision maker with constant risk aversion $\gamma = 0.01$ to the certain deal and lottery with a constant probability $p = 0.945$.](image)

To assert that this probability in Fig. 4 remains constant as either tool cost (c) or tool life (l) is varied requires that the decision maker be indifferent between the certain deal and lottery curves shown in Fig. 5 at all points. Thus, the assertion of utility independence would require the decision maker to be indifferent between $615$ and $515$ at a tool life of 1.5 h, or to be indifferent between $678$ and $519$ if the tool cost is changed to $300$.

Because of the different contributions of the attributes to value, the indifference probability must change in order for the decision maker to remain consistent in his values. Utility independence is inextricably linked to the value function and utility over value. If utility independence is assessed in the absence of a value function, these relations may not be recognized. The analyst then risks using the multiplicative utility function when it is not warranted; see, for example, an analysis of a high-speed machining case in which value is not considered [77].

C. Implications for Risk Attitude

An interesting and useful implication of the value-based approach is the implication for the risk attitude toward the different attributes. Matheson and Abbas show that in the presence of a value function and a risk attitude toward value, the risk attitudes toward each attribute must be consistent with the utility and value functions [65]. This consistency results in the derivation of utility transversality, a closed-form expression that relates the risk aversion functions of each attribute to the value tradeoffs between them [65].

Although a full derivation and explanation of utility transversality is beyond the scope of this paper, it is worthwhile to point out a few implications for constructing multiattribute utility functions. First, the presence or absence of utility independence can be checked by deriving the risk attitude toward a single attribute following the Pratt [75] and Arrow [76] definition of risk attitude and using the partial derivatives of the utility function over value. If this expression contains any of the other attributes in it, then utility independence does not hold between the attributes. Second, the utility transversality relation provides a useful way to reconstruct preference relations in a consistent manner when the decision maker can specify preferences over one attribute but has difficulty specifying them over another attribute.

The concept of utility transversality has not been previously examined in the context of engineering design. However, we find it to be a helpful tool in understanding and constructing multiattribute utility functions.

V. CONCLUSION

This paper seeks to clarify utility theory and its application to engineering design. Toward this end, we describe the basis of utility theory and survey how it has been commonly applied in engineering design. Through clear examples, we seek to highlight some of the misconceptions about utility theory and open doors to new research and dialogue between the engineering design and decision analysis communities.

A major misconception about utility theory is that it is limited to the multiplicative and additive functional forms. In reality, utility theory places no restrictions on the functional form used to represent preferences. The limitations imposed by these forms are underscored by the comparison of the use of these approaches to the use of the value-based approach introduced by this paper.

The value-based approach that is introduced in this paper is based on the creation of a deterministic value function for design and the assignment of a 1-D utility function over the value measure. We use a conceptual example and a numeric
example to illustrate the consistency of the method with underlying, structural models and the flexibility of themethod to hand cases where utility independence may or may not hold.

Overall, this paper highlights the need for new ways to represent preferences without placing arbitrary restrictions on them. Both the decision analysis literature [65, 78, 79] and the engineering design literature [80]–[82] describe new and ongoing work in representing preferences. This research is motivated by the need to incorporate possible dependences among attributes, as discussed by several authors [83]–[87]. Exciting opportunities for future research may be realized once it is recognized that these new and developing ideas in engineering design need not be at odds with utility theory. For example, several authors have examined preference consistencies and inconsistencies [81], [88]. This paper does not invalidate the use of utility theory or decision analysis. On the contrary, concepts from decision analysis such as the value of information can be used in conjunction with these ideas to derive additional insights. Recent work has also explored the implications of setting targets and requirements on the selection of design projects [89]–[91].

Because of the generality of utility theory, there are great opportunities for research in the engineering design community in the areas of utility theory and preference representation. There is a need for ways to help the decision maker contemplate and specify preferences. There is also a need for ways to represent those preferences accurately, accounting for possible dependences and nonlinearities that may exist among the attributes.

REFERENCES


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