Age of Information with Soft Updates

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***Abstract*—We consider an information updating system where an information provider and an information receiver engage in an update process over time. Different from the existing literature where updates are countable (hard) and take effect either immediately or after a delay, but *instantaneously* in both cases, here updates start taking effect right away but *gradually* over time. We coin this setting *soft updates*. When the updating process starts, the age decreases until the soft update period ends. We constrain the number of times the information provider and the information receiver meet (number of update periods) and the total duration of the update periods. We consider two models for the decrease of age during an update period: In the first model, the rate of decrease of the age is proportional to the current age, and in the second model, the rate of decrease of the age is constant. The first model results in an exponentially decaying age, and the second model results in a linearly decaying age. In both cases, we determine the optimum updating schemes, by determining the optimum start times and the optimum durations of the updates, subject to the constraints on the number of update periods (number of meetings) and the total update duration.**

I. Introduction

We consider a system where an information provider updates an information receiver (information consumer) over time. We introduce the concept of *soft updates*, where different from the existing literature where updates are countable (hard) and drop the age instantaneously (possibly after a delay), here, updates are soft and begin reducing the age immediately but drop it gradually over time. Our setting models human interactions where updates are soft, and also social media interactions where an update consists of viewing and digesting many small pieces of information posted, that are of varying importance, relevance and interest to the receiver.

Consider a typical information update system as shown in Fig. 1. Starting from time zero, information at the receiver gets stale over time, i.e., the age increases linearly. A time comes when the information source decides to update the information receiver. In the existing literature, this is a hard update, which is contained in an information packet. This hard update *takes effect* and reduces the age instantaneously to the age of the packet itself at the time of its arrival at the receiver. This is denoted as *instantaneous decay* in Fig. 1. The time for the update to take effect (denoted by c_1 for the first update) is either random [1]–[13], or fixed and deterministic [14], [15], or zero [16]–[25]. Essentially, this is the time for the update packet to *travel* from the transmitter to the receiver, and when it arrives, it drops the age instantaneously. This travel time is random if the update goes through a queue, and it is fixed or zero if the update goes through a wireless channel with a non-negligible or a negligible distance, respectively, between the transmitter and the receiver. In contrast, in this work, the soft update begins reducing the age at the time of information source making a decision to update. However, the drop in age is not instantaneous, rather it is *gradual* over time.

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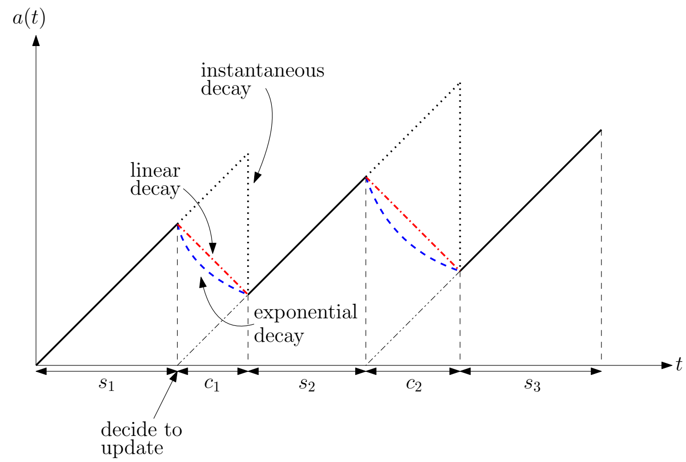

Fig. 1. Evolution of $a(t)$ for different models.

We consider two models for the soft update process: In the first model, the rate of decrease in age is proportional to the current age; see (1). This is motivated by the fact that information is most valuable when it is most aged, i.e., when the new information is most innovative. This model leads to an exponential decay in the age (denoted by *exponential decay* in Fig. 1). Note also that, this is consistent with information dissemination in human interactions as well as in social media feeds, where the most important information is conveyed/displayed first, reducing the age faster initially, and less important information is conveyed/displayed next, reducing the age slower subsequently. In the second model, the rate of decrease in age is constant; see (2). In this case, the age decreases linearly (denoted by *linear decay* in Fig. 1).

In this paper, we determine the optimum updating schemes for soft update systems. In this problem, we are given the total system duration over which the average age is calculated T, the number of update periods (i.e., the number of times information provider and information receiver are allowed to meet) N, and the total allowed update duration T_c. We solve for the optimum start times of the soft updates and their optimum durations in order to minimize the overall age. For the exponentially decaying age model, if T_c is large enough, only one soft update period takes place, even though N periods are allowed, and the system starts updating at time zero, continues updating until T_c, and lets age grow then on until the end. On the other hand, when T_c is low, the optimal policy is to have exactly N soft update periods with equal lengths. For the linearly decaying age model, the optimal policy is to update exactly N times and allocate equal amounts of time for each soft update. In the linear decay model, we also prove that the age after each soft update goes down exactly to zero. In addition, for the linear decay model, we show that the resulting age decreases with N. Finally, we provide numerical results where not only the number of soft update opportunities and the total duration of updates are constrained, but also the time periods during which update encounters may take place.

II. System Model and the Problem

Let $a(t)$ be the instantaneous age at time t. Without loss of generality, let $a(0) = 0$. When there is no update, the age increases linearly with time. We consider two different soft update models. In the first model, the rate of decrease in age is proportional to the current age:

$$\frac{da(t)}{dt} = -\alpha a(t) \tag{1}$$

where α is a fixed constant. In this model, the age decreases exponentially during a soft update period. In the second model, the rate of decrease in age does not depend on the current age, instead it remains constant:

$$\frac{da(t)}{dt} = -\alpha \tag{2}$$

where α is a fixed constant. In this model, the age decreases linearly during a soft update period.

Let us denote the beginning of the ith soft update period by t_i and the end of the ith soft update period by t'_i. Then, the age evolves as:

$$a(t) \triangleq \begin{cases} a(t'_{i-1}) + t, & t'_{i-1} < t < t_i \\ f(a(t_i), \alpha, t), & t_i < t < t'_i \end{cases} \tag{3}$$

where $f(a(t_i), \alpha, t) = a(t_i)e^{-\alpha(t-t_i)}$ for the exponentially decaying age model, and $f(a(t_i), \alpha, t) = a(t_i) - \alpha(t-t_i)$ for the linearly decaying age model.

Our objective is to minimize the average age of information (AoI) of the system subject to a total of N soft update periods, a total update duration of T_c, over a total time period of T. We formulate the problem as:

$$\begin{aligned} \min_{\{t_i, t'_i\}} \quad & \frac{1}{T}\int_0^T a(t)dt \\ \text{s.t.} \quad & \sum_{i=1}^{N} (t'_i - t_i) \leq T_c \end{aligned} \tag{4}$$

We define the duration of the ith update period as $c_i = t'_i - t_i$, and the ith aging period as $s_i = t_i - t'_{i-1}$.

Let $A_T \triangleq \int_0^T a(t)dt$ be the total age. Note that minimizing $\frac{A_T}{T}$ is equivalent to minimizing A_T since T is a known constant. In the following sections, we provide the optimal policies that minimize the age for the cases of exponentially and linearly decaying age models.

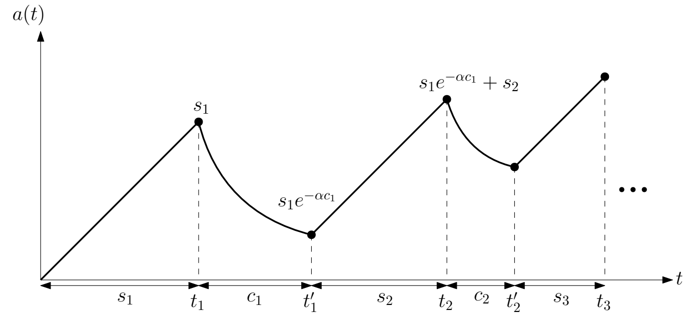

Fig. 2. Evolution of age in the case of exponentially decaying age.

III. Exponentially Decaying Age Model

In the case of exponentially decaying age, the age function evolves as in Fig. 2. Age, in this case, is given as:

$$\begin{aligned} A_T =& \frac{s_1^2}{2} + \frac{1}{\alpha}s_1(1-e^{-\alpha c_1}) + s_2\left(s_1 e^{-\alpha c_1} + \frac{s_2}{2}\right) \\ &+ \frac{1}{\alpha}\sum_{i=2}^{N}\left(\left(s_i + \sum_{j=1}^{i-1} s_j e^{-\alpha \sum_{k=j}^{i-1} c_k}\right)(1-e^{-\alpha c_i}) \right. \\ &\left. + s_{i+1}\left(\sum_{j=1}^{i} s_j e^{-\alpha\sum_{k=j}^{i} c_k}\right) + \frac{s_{i+1}^2}{2}\right) \end{aligned} \tag{5}$$

We minimize A_T in (5) by choosing t_i and t'_i, equivalently, by choosing s_i and c_i, for all i. In the following lemma, we show that the total updating time, T_c, should be fully utilized.

Lemma 1 *In the optimal policy, we must have* $\sum_{i=1}^{N} c_i = T_c$.

Proof: We prove this by contradiction. Assume that there exists an optimal policy such that $\sum_{i=1}^{N} c_i < T_c$. Then, we can obtain another feasible policy by increasing one of the c_i and decreasing one of the s_j. Note that this new policy yields a smaller age. Thus, we reached a contradiction, and $\sum_{i=1}^{N} c_i = T_c$ must be satisfied. ■

In Lemma 1, we observe that the total updating time should be fully used. Then, we need to determine when to start a soft update and the duration of each soft update. In the case of $T_c = T$, the optimal policy is to start updating at $t = 0$ and continue to update until $t = T$. The optimal age in this case is $A_T = 0$. Thus, for the rest of this section, we consider the case where $T_c < T$. Next, we consider the cases of $N = 1$ and $N = 2$ soft update period(s).

For $N = 1$, A_T in (5) becomes:

$$A_T = \frac{s_1^2}{2} + \frac{1}{\alpha}s_1(1-e^{-\alpha c_1}) + s_2 s_1 e^{-\alpha c_1} + \frac{s_2^2}{2} \tag{6}$$

where $s_2 = T - s_1 - c_1$. Due to Lemma 1, $c_1 = T_c$. Thus, we need to find s_1^* that minimizes A_T. By taking the derivative

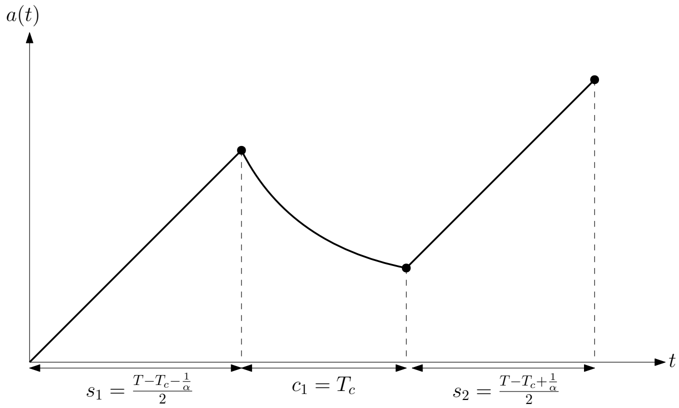

Fig. 3. Optimal evolution of age when $N = 1$ and $T \geq T_c - \frac{1}{\alpha}$.

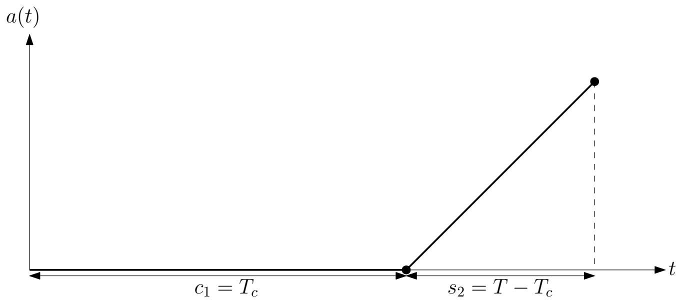

Fig. 4. Optimal evolution of age if $T < T_c - \frac{1}{\alpha}$.

of A_T with respect to s_1, we obtain

$$\left(1 - e^{-\alpha T_c}\right)\left(s_1 + \frac{1}{\alpha} - T + s_1 + T_c\right) = 0 \tag{7}$$

Since $T_c > 0$, we obtain the critical point as follows:

$$s_1^* = \frac{T - T_c - \frac{1}{\alpha}}{2} \tag{8}$$

By checking the second derivative of A_T, we conclude that s_1^* in (8) is the global minimum and gives the minimum age. Evolution of the optimal age is shown in Fig. 3. Note that when $T < T_c + \frac{1}{\alpha}$, we have $s_1^* = 0$ which corresponds to starting the soft update at $t = 0$, continuing to update until $t = T_c$, and then letting age grow till the end. Evolution of the optimal age is shown in Fig. 4.

For $N = 2$, A_T in (5) becomes:

$$\begin{aligned} A_T =& \frac{s_1^2}{2} + \frac{1}{\alpha}s_1\left(1 - e^{-\alpha c_1}\right) + \frac{s_2^2}{2} + s_1 s_2 e^{-\alpha c_1} \\ &+ \frac{1}{\alpha}\left(s_2 + s_1 e^{-\alpha c_1}\right)\left(1 - e^{-\alpha c_2}\right) + \frac{s_3^2}{2} \\ &+ s_3\left(s_2 e^{-\alpha c_2} + s_1 e^{-\alpha T_c}\right) \end{aligned} \tag{9}$$

where $s_3 = T - T_c - s_1 - s_2$. Thus, we need to find s_1^*, s_2^*, c_1^* and c_2^*. Due to Lemma 1, $c_2 = T_c - c_1$. Next, we determine all critical points that satisfy the first order conditions as:

$$\begin{aligned} \left(T - T_c - \frac{1}{\alpha}\right)\left(1 - e^{-\alpha T_c}\right) =& s_2\left(e^{-\alpha c_1} - e^{-\alpha c_2} + 1 - e^{-\alpha T_c}\right) \\ &+ 2s_1\left(1 - e^{-\alpha T_c}\right) \end{aligned} \tag{10}$$

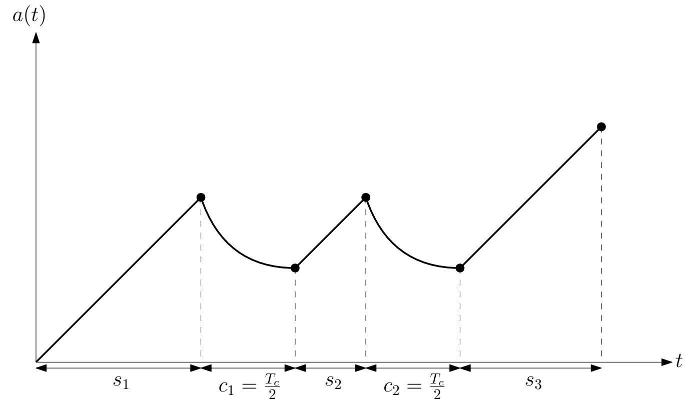

Fig. 5. Optimal evolution of age when $N = 2$ and $T \geq T_c - \frac{1}{\alpha}$.

$$\left(1 - e^{-\alpha c_2}\right)\left(s_1 e^{-\alpha c_1} + \frac{1}{\alpha} - T + T_c + s_1 + 2s_2\right) = 0 \tag{11}$$

$$s_2\left(\alpha s_1 e^{-\alpha c_1} + e^{-\alpha c_2} - \alpha s_3 e^{-\alpha c_2}\right) = 0 \tag{12}$$

By solving (10) and (11), we obtain:

$$s_1^* = \frac{(T - T_c - \frac{1}{\alpha})(1 + e^{-\alpha c_2})}{e^{-\alpha c_1} + e^{-\alpha c_2} - e^{-\alpha T_c} + 3} \tag{13}$$

$$s_2^* = \frac{(T - T_c - \frac{1}{\alpha})(1 - e^{-\alpha T_c})}{e^{-\alpha c_1} + e^{-\alpha c_2} - e^{-\alpha T_c} + 3} \tag{14}$$

Then, by substituting s_1^* and s_2^* into (12), we obtain $c_1^* = c_2^* = \frac{T_c}{2}$. We then substitute c_1^* and c_2^* back to determine s_1^* and s_2^*.

Note that other (degenerate) critical points are: $c_1 = 0$, $c_2 = T_c$ and $c_1 = T_c$, $c_2 = 0$. These degenerate critical points correspond to the optimal solution for $N = 1$. By checking the Hessian, we conclude that $c_1^* = c_2^* = \frac{T_c}{2}$ gives the global minimum, and the critical points at $c_1 = 0$ and $c_1 = T_c$ cannot be optimum. Evolution of the optimum age is shown in Fig. 5. When $T \leq T_c + \frac{1}{\alpha}$, the optimal solution is to utilize only one soft update period even though $N = 2$ is allowed, and start the soft update at $t = 0$, continue the update until $t = T_c$, and let the age grow till the end as shown in Fig. 4.

Thus, we have determined the optimal soft update policies for exponentially decaying age model for $N = 1$ and $N = 2$. We determine the optimal policies for general N in [26].

IV. Linearly Decaying Age Model

In this section, we consider the case of linearly decaying age with $\alpha = 1$ for simplicity. We generalize the result for arbitrary α in Section V. An example evolution of age for this case is shown in Fig. 6. Age, in this case, is given as:

$$\begin{aligned} A_T =& \sum_{i=1}^{N}\left(\left(\sum_{j=1}^{i} s_j - \sum_{j=1}^{i-1} c_j\right)^2 - \left(\sum_{j=1}^{i} s_j - \sum_{j=1}^{i} c_j\right)^2\right) \\ &+ \frac{(T - 2\sum_{i=1}^{N} c_i)^2}{2} \end{aligned} \tag{15}$$

where $c_0 = 0$.

Here, at first, we assume that we stop the soft update when the age goes down to zero. This will imply that $\sum_{i=1}^{k} s_i - c_i \geq$

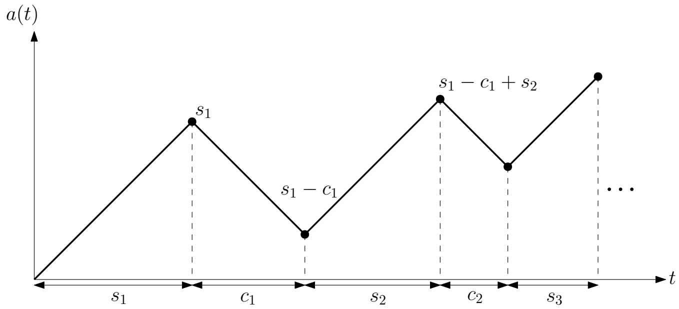

Fig. 6. Evolution of age in the case of linearly decaying age.

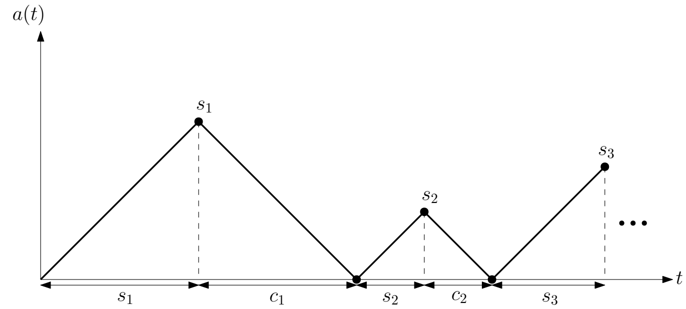

Fig. 7. Evolution of age in the case of $N = 2$ and $s_i = c_i$, $\forall i$.

0 for all $k \leq N$. Thus, we formulate the problem as follows:

$$\begin{aligned} \min_{\{s_i,c_i\}} \quad & A_T \\ \text{s.t.} \quad & \sum_{i=1}^{N} c_i \leq T_c \\ & \sum_{i=1}^{k} s_i - c_i \geq 0, \quad \forall k \end{aligned} \tag{16}$$

Next, we identify some important properties of the optimal solution for the problem in (16). First, the following lemma states that, in the optimal solution, if the total updating time T_c is not fully utilized, then the age should be equal to zero at the end of each soft update period.

Lemma 2 *In the optimal policy, if* $\sum_{i=1}^{N} c_i < T_c$*, then* $a(t_i') = 0$ *for all* i*. Additionally,* $s_i = c_i = \frac{T}{2N+1}$*, and we must have* $T_c > \frac{NT}{2N+1}$.

Proof: We prove this by contradiction. Assume that $\sum_{i=1}^{N} c_i < T_c$ and the update policy is such that the age at the end of each update period is not zero. Let us choose the smallest index, j, such that $a(t_j') > 0$. We can decrease the age further by increasing c_j. This policy is still feasible since the total update time constraint is not tight. Thus, we continue to increase c_j until either $a(t_j') = 0$ or $\sum_{i=1}^{N} c_i = T_c$. If $a(t_j') = 0$ and $\sum_{i=1}^{N} c_i < T_c$, we move to the second smallest index such that the age at the end of the update period is not zero and apply the same procedure. We apply this procedure until $a(t_i') = 0$ for all i. At the end, we obtain $\sum_{i=1}^{N} c_i < T_c$ and $a(t_i') = 0$ for all i. This new policy has smaller age at each step, implying we have reached a contradiction. Thus, in the optimal policy, if $\sum_{i=1}^{N} c_i < T_c$, then $a(t_i') = 0$ for all i.

Since $a(t_i') = 0$ for all i, this implies that $s_i = c_i$ for all i. Thus, evolution of the age will be as in Fig. 7. Then, the problem becomes:

$$\begin{aligned} \min_{\{s_i,c_i\}} \quad & \sum_{n=1}^{N} s_i^2 + \frac{1}{2} s_{N+1}^2 \\ \text{s.t.} \quad & \sum_{i=1}^{N} 2s_i + s_{N+1} = T \end{aligned} \tag{17}$$

We solve this problem using a Lagrangian:

$$L = \sum_{i=1}^{N} s_i^2 + \frac{1}{2} s_{N+1}^2 - \lambda \left(\sum_{i=1}^{N} 2s_i + s_{N+1} - T \right) \tag{18}$$

Taking the derivative with respect to s_i and equating to zero, we obtain $s_i - \lambda = 0$, for all i. Thus, the optimal solution is $s_1 = s_2 = \cdots = \lambda$. Since $\sum_{i=1}^{N} 2s_i + s_{N+1} = T$, the optimal solution is $s_i = c_i = \frac{T}{2N+1}$. Since $c_i = \frac{T}{2N+1}$ for all i, we must have $\frac{NT}{2N+1} < T_c$. ■

Next, the following lemma states that, in the optimal solution, if the total updating time T_c is fully utilized, the optimal policy decreases the age down to 0 after each update and lets the age grow till the end after T_c is totally utilized.

Lemma 3 *In the optimal policy, if* $\sum_{i=1}^{N} c_i = T_c$*, then* $a(t_i') = 0$ *for all* i*. Additionally, the optimal policy is* $c_i = \frac{T_c}{N}$ *for all* i*, and we must have* $T_c \leq \frac{NT}{2N+1}$.

Proof: Here, again, we want to solve the optimization problem in (16) with the objective function in (15). This is not a convex optimization problem as the objective function is not convex. Our approach will be to lower bound the objective function, minimize this lower bound, and then show that this minimized lower bound can be achieved with a certain feasible selection of the variables. We first note that A_T in (15) can equivalently be written as:

$$A_T = \sum_{i=1}^{N} c_i^2 + 2 \sum_{i=1}^{N} (s_i - c_i) \left(\sum_{j=i}^{N} c_j \right) + \frac{(T - 2\sum_{i=1}^{N} c_i)^2}{2} \tag{19}$$

We next note that, even though we do not know the sign of each $(s_i - c_i)$ in (19) at this point, we know that the entirety of the middle term in (19) is always non-negative since:

$$\sum_{i=1}^{N} (s_i - c_i) \left(\sum_{j=i}^{N} c_j \right) = \sum_{i=1}^{N} \left(\sum_{j=1}^{i} s_j - c_j \right) c_i \tag{20}$$

where the right hand side is non-negative due to the constraints in (16). Thus, we lower bound (19) by setting the middle term as zero by choosing $s_i = c_i$ for all i. We also note that the last term in (19) is constant by the hypothesis of the lemma, where $\sum_{i=1}^{N} c_i = T_c$. Then, minimizing the lower bound becomes equivalent to minimizing $\sum_{n=1}^{N} c_i^2$ subject to $\sum_{i=1}^{N} c_i = T_c$,

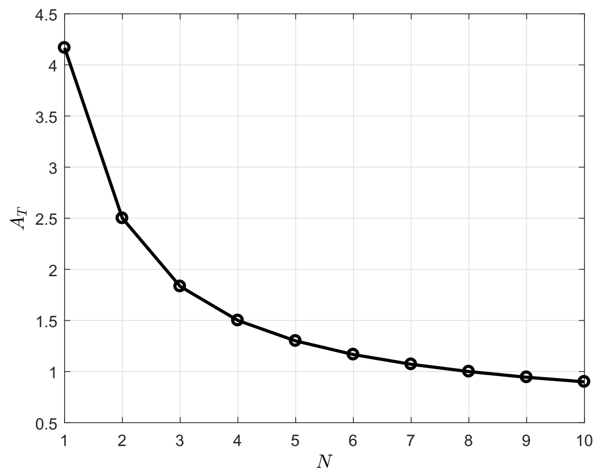

Fig. 8. Minimum age A_T^* as a function of N for $T=5$ and $T_c=2$.

whose solution is $c_i = \frac{T_c}{N}$. Note that, we can choose $s_i = c_i$ and $c_i = \frac{T_c}{N}$ for all i since $T_c \leq \frac{NT}{2N+1}$, which makes s_i selection feasible. Finally, note that $c_i = s_i$ for all i implies that after each soft update, the age goes down to zero. ■

In summary, if we have $T_c > \frac{NT}{2N+1}$, we are in Lemma 2, where the total updating time T_c is large enough in comparison to T, that it is not a limiting factor, and it is not fully utilized. In this case, the optimal policy is to choose $s_i = c_i = \frac{T}{2N+1}$. On the other hand, if we have $T_c \leq \frac{NT}{2N+1}$, we are in Lemma 3, where the total updating time T_c is the limiting factor, and it is fully utilized. In this case as well, the optimal policy is to choose $s_i = c_i = \frac{T_c}{N}$. Note that, in both cases, the age at the end of each update period drops down to zero.

A. Effect of N on the Age

In this sub-section, we investigate how the final age varies as a function of N, the number of soft update opportunities. Intuitively, if we are in the case of Lemma 2, since T_c is not fully utilized, additional soft update opportunities may help utilize T_c more and decrease the age. Additionally, even if we are in the case of Lemma 3, where T_c is fully utilized, an increased N may help utilize T_c more efficiently. We note that the minimum age is expressed in terms of N for a given T_c and T as follows: If $T_c > \frac{NT}{2N+1}$, then we are in Lemma 2, and the minimum age is given as,

$$A_T^* = \frac{1}{2}\left(\frac{T}{2N+1}\right)^2 (2N+1) = \frac{1}{2}\frac{T^2}{2N+1} \tag{21}$$

whereas if $T_c \leq \frac{NT}{2N+1}$, then we are in Lemma 3, and the minimum age is given as,

$$A_T^* = \left(\frac{T_c}{N}\right)^2 N + \frac{(T-2T_c)^2}{2} = \frac{T_c^2}{N} + \frac{(T-2T_c)^2}{2} \tag{22}$$

We note that in both cases A_T^* decreases in N. As an example, the minimum age A_T^* as a function of N is plotted in Fig. 8 for $T=5$, $T_c=2$.

B. Assumption of Stopping Soft Updates When $a(t)=0$

In this sub-section, we discuss the implications of the fact that we have stopped the soft update process when the age has reached zero. As a result of that, the age started to increase linearly right away until the next soft update opportunity. We observe that this assumption affects the optimal solution when T_c is not fully utilized. In fact, in this case, we can continue the soft update process and as a result keep the age at the level of zero, i.e., not allow it to increase.

With this modification, the solution changes as follows: If $T_c \leq \frac{NT}{2N+1}$, then the solution remains the same as in Lemma 3, and the minimum age is given as in (22). However, if $T_c > \frac{NT}{2N+1}$, then we solve for x, such that

$$\frac{T-x}{2N+1} = \frac{T_c - x}{N} \tag{23}$$

and then apply the solution either in Lemma 2 or Lemma 3 with the new $T' = T - x$ and $T_c' = T_c - x$ (either lemma can be used as now the differentiating inequality is satisfied with equality). Note that this essentially decreases the total duration T by moving some of unused T_c from the total duration. Note also that, in the case that total T_c is not utilized, the minimum age in (21) depends only on T, and it increases with it. Therefore, this reduction in T by an amount x reduces the achievable minimum age further.

V. Linearly Decaying Age Model with General α

So far, we have considered the case where $\alpha = 1$. However, in general, α does not have to be 1, implying that the aging process can be slower or faster than the updating process. In this section, we consider the most general case where the slope in the soft update policy $\alpha \neq 1$ is arbitrary. In this case, the instantaneous age becomes:

$$a(t) \triangleq \begin{cases} a(t_{i-1}') + t, & t_{i-1}' < t < t_i, \\ a(t_i) - \alpha(t - t_i), & t_i < t < t_i' \end{cases} \tag{24}$$

Then, A_T is given as:

$$A_T = \frac{\alpha+1}{2}\left(\alpha \sum_{i=1}^N c_i^2 + 2\sum_{i=1}^N \left(\sum_{j=i}^N c_j\right)(s_i - \alpha c_i)\right) \tag{25}$$

$$+ \frac{(T - (\alpha+1)\sum_{i=1}^N c_i)^2}{2} \tag{26}$$

and the optimization problem becomes:

$$\begin{aligned} \min_{\{s_i, c_i\}} \quad & A_T \\ \text{s.t.} \quad & \sum_{i=1}^N c_i \leq T_c \\ & \sum_{i=1}^k s_i - \alpha c_i \geq 0, \quad \forall k \end{aligned} \tag{27}$$

If we substitute $c_i' = \alpha c_i$, we see that minimizing (26) in (27) becomes equivalent to minimizing (19) in the proof of Lemma 3. Thus, the optimal solution depends on whether T_c

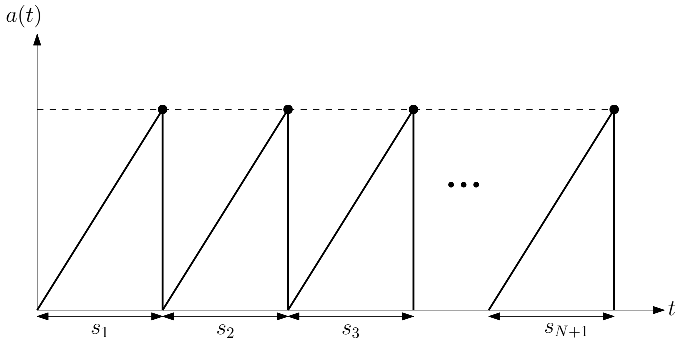

Fig. 9. Evolution of the optimal age when $\alpha \to \infty$.

is fully utilized or not in this case, which can be determined by comparing T_c with the following term:

$$\frac{NT}{N(\alpha+1)+\alpha} \tag{28}$$

which reduces to previously used $\frac{NT}{2N+1}$ when $\alpha = 1$.

If $T_c > \frac{NT}{N(\alpha+1)+\alpha}$, then T_c is not the limiting factor i.e., increasing T_c does not decrease the age further. In this case, we are in Lemma 2 and the age expression in (21) becomes:

$$A_T^* = \left(\frac{T}{N(\alpha+1)+\alpha}\right)^2 \left(N\frac{(\alpha+1)\alpha}{2} + \frac{\alpha^2}{2}\right) \tag{29}$$

If $T_c \leq \frac{NT}{N(\alpha+1)+\alpha}$, we are in Lemma 3 and the age expression in (22) becomes:

$$A_T^* = \frac{(\alpha+1)\alpha}{2}\frac{T_c^2}{N} + \frac{(T-(\alpha+1)T_c)^2}{2} \tag{30}$$

Finally, we note that, when $\alpha \to \infty$, T_c is not a limiting factor, and the optimal age can be calculated as follows:

$$\lim_{\alpha\to\infty} A_T = \frac{1}{2}\left(\frac{T}{N+1}\right)^2 (N+1) \tag{31}$$

In this case, the optimal age is as shown in Fig. 9, which corresponds to the optimal age with instantaneous drops as in the existing literature, i.e., *hard* updates.

VI. Numerical Results

In this section, we give simple numerical examples to illustrate our results. In the first example, we consider the exponentially decaying age model with $T = 5$, $T_c = 3$, $N = 2$ and $\alpha = 1$. Since $T > T_c - \frac{1}{\alpha}$, the optimal update policy is to update $N = 2$ times with equal time allocated to each update, i.e., $c_1 = c_2 = 1.5$. The evolution of $a(t)$ is shown in Fig. 10.

In the second example, we consider the exponentially decaying age model with $T = 6$, $T_c = 5$, $N = 2$ and $\alpha = 1$. Since T_c is large enough, i.e., $T \leq T_c - \frac{1}{\alpha}$, only one soft update period occurs, even though $N = 2$ periods are allowed, and the system starts updating at $t = 0$, continues updating until T_c, and lets age grow then on until the end. The evolution of $a(t)$ is shown in Fig. 11.

In the following three examples (third, fourth and fifth), we consider the linearly decaying age model with $\alpha = 1$. In the third example, we see the case where T_c is fully used and T_c

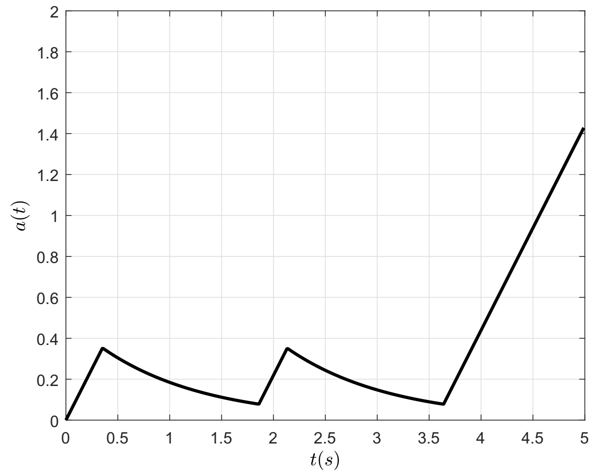

Fig. 10. Evolution of $a(t)$ in the case of exponentially decaying age model when $N = 2$, $T = 5$, and $T_c = 3$.

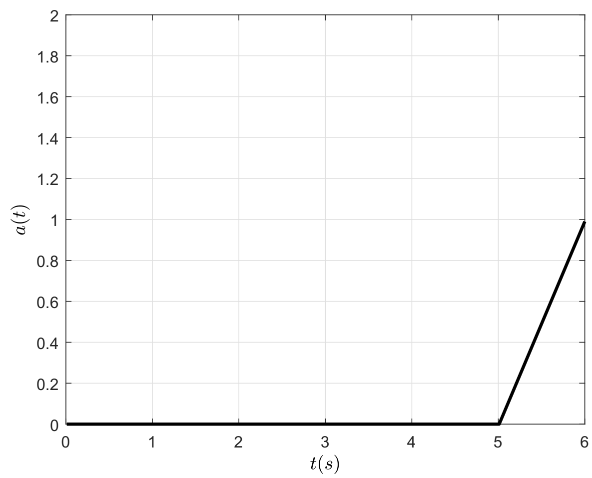

Fig. 11. Evolution of $a(t)$ in the case of exponentially decaying age model when $N = 2$, $T = 6$, and $T_c = 5$.

is not the limiting factor, i.e., if we have additional updating time, it would not decrease the age further. The evolution of $a(t)$ is shown in Fig. 12.

In the fourth example, we consider the case in Lemma 2, where T_c is not fully used. We observe that even though the system has additional time for updating, the optimal solution is to update periodically and age should be equal to 0 after each update which is shown in Fig. 13.

In the fifth example, we consider the case in Lemma 3, where T_c is tight and increasing T_c can reduce the age further. In this case, the optimal policy is to keep updating periodically. When total updating time is fully utilized, the age grows till the end. The evolution of $a(t)$ is shown in Fig. 14.

So far, we have provided examples of the linear case with $\alpha = 1$. In the following examples, cases with $\alpha > 1$ and $\alpha < 1$ are considered. In the first case, we consider $\alpha = 2$, $N = 2$, $T = 6$, and $T_c = 2$, and in the second case, we consider $\alpha = 0.5$, $N = 2$, $T = 8$, and $T_c = 4$. The optimal policies are shown in Fig. 15 and Fig. 16, respectively.

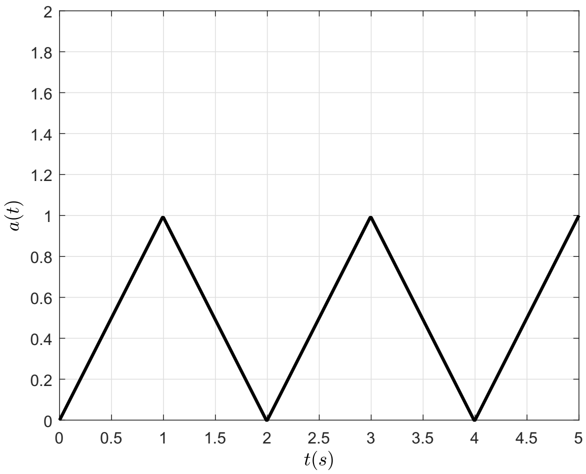

Fig. 12. Evolution of $a(t)$ in the case of $N = 2$, $T = 5$, and $T_c = 2$.

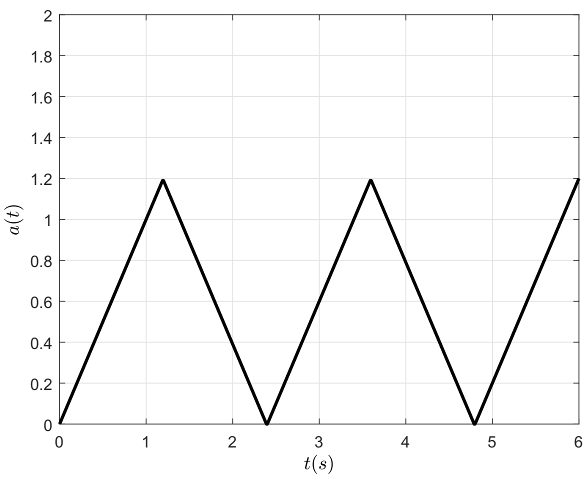

Fig. 13. Evolution of $a(t)$ in the case of $N = 2$, $T = 6$, and $T_c = 4$.

VII. Conclusion and Future Directions

In this paper, we introduced the concept of soft updating which is relevant in systems with human interactions and social media settings, where the decrease in age occurs gradually over soft update periods. We studied two soft update regimes, where in the first one, age decays exponentially and in the second one age decays linearly during the soft update period. In the case of exponentially decaying age model, we showed that T_c should be fully utilized, the update periods must be chosen equally $c_i = \frac{T_c}{N}$. In the case of linearly decaying age model, if we terminate an update process when the age reaches zero, the optimal solution depends on the size of T_c with respect to T. If T_c is large enough, the optimal policy is to allocate $s_i = c_i = \frac{T}{2N+1}$ for all i. Otherwise, the optimal solution is to choose $s_i = c_i = \frac{T_c}{N}$ for all $i \in [1, N]$ and $s_{N+1} = T - 2T_c$. We observed that for the linear case, age decreases with N. If we do not terminate the update process when the age reaches zero, but rather continue with the update process, then we can further decrease the final age by effectively decreasing the total time. Finally, we showed

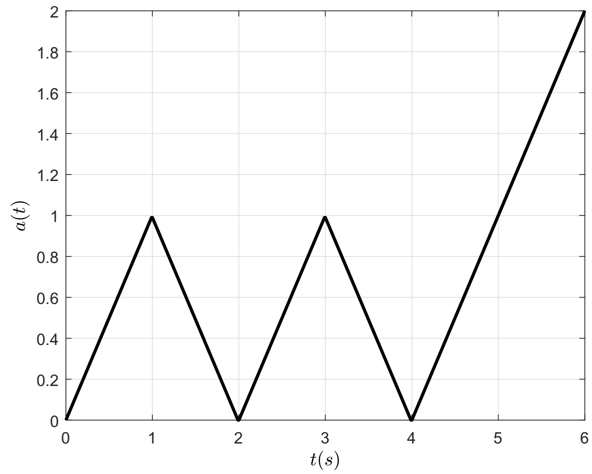

Fig. 14. Evolution of $a(t)$ in the case of $N = 2$, $T = 6$, and $T_c = 2$.

that even if $\alpha \neq 1$, the optimal solution structure is the same as in the case with $\alpha = 1$.

For future work, further restrictions on the update times can be considered, e.g., restrictions on the time intervals in which meetings may take place. We provide two numerical results for this case. In the first example, $T = 5$, $T_c = 2$ and $N = 2$, and we restrict updates to take place only in the intervals $t \in [0, 2]$ and $t \in [4, 5]$. We recall that if there is no further restriction on the update processes, the optimal age evolution is given in Fig. 12. Since updating is not allowed in $t \in (2, 4)$, the optimal age evolution is different and is shown in Fig. 17. In this case, we see that T_c is fully utilized and age becomes zero at the end of the first update period which seems to follow the optimal policy structure with no restrictions.

In the second example, we consider the same case except this time, updating is not allowed in $t \in (3, 4)$. In Fig. 18, we see the optimal age evolution in this case. Note that even though the system can use T_c fully, the optimal policy in this case is different and T_c is not fully utilized. Since the number of soft updates is restricted, the system chooses to use them at the beginning and the age becomes zero after each update. Note that in the first example, age is not equal to zero after each update because utilizing T_c reduces the age further. It seems that there is a point below which utilizing T_c is more important whereas after this point, reducing age to zero after each update yields an optimal solution.

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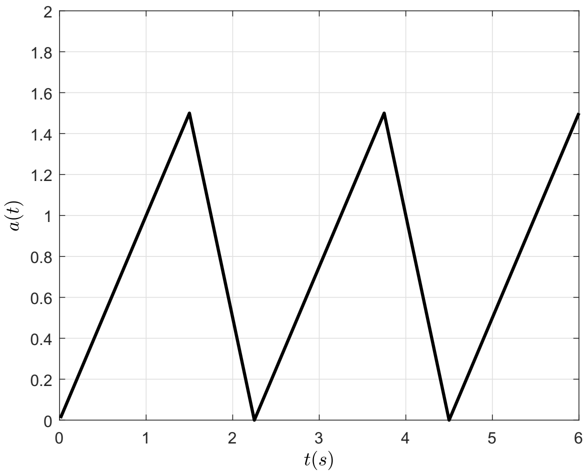

Fig. 15. Evolution of $a(t)$ in the case of linear model with $\alpha = 2$, $N = 2$, $T = 6$, and $T_c = 2$.

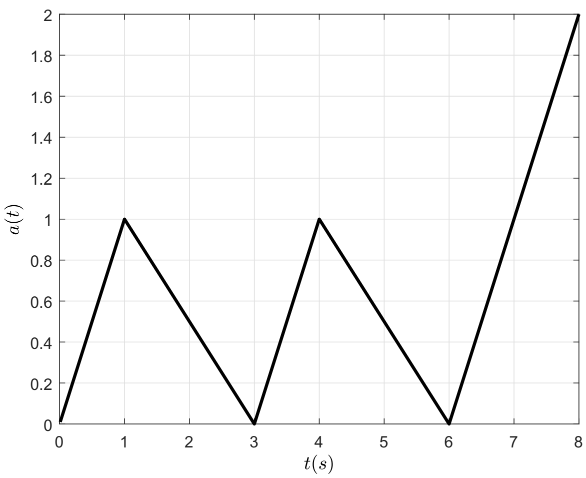

Fig. 16. Evolution of $a(t)$ in the case of linear model with $\alpha = 0.5$, $N = 2$, $T = 8$, and $T_c = 4$.

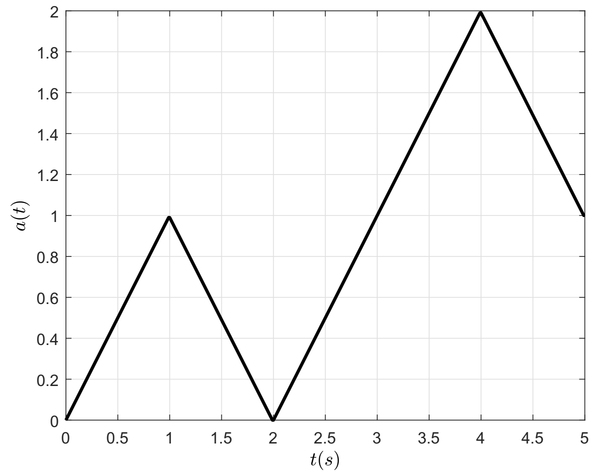

Fig. 17. Evolution of $a(t)$ in the case of $N = 2$, $T = 5$, and $T_c = 2$. Updates are allowed only in $t \in [0, 2]$ and $t \in [4, 5]$.

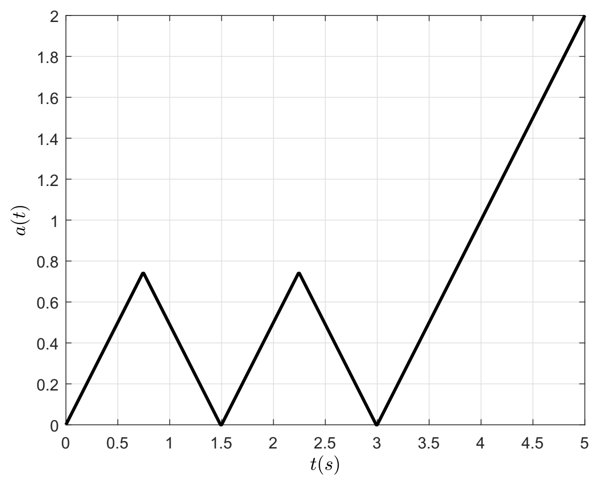

Fig. 18. Evolution of $a(t)$ in the case of $N = 2$, $T = 5$, and $T_c = 2$. Updates are allowed only in $t \in [0, 3]$ and $t \in [4, 5]$.

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