A Fast LP-Based Approach for Robust Dynamic Economic Dispatch Problem: A Feasible Region Projection Method

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***Abstract*—The robust optimization based dynamic economic dispatch (DED) model has been extensively studied to address uncertainties of net loads induced by renewable energy resource variabilities and demand fluctuations. However, the robust DED model is computationally expensive, when solved by the column-and-constraint generation (CCG) approach that iterates between a master problem and a max-min subproblem. This letter proposes a feasible region projection-based approach to equivalently reformulate the robust DED as a single-level linear programming (LP) model that can be effectively solved while guaranteeing solution optimality. Numerical studies show the proposed approach is one order of magnitude faster than CCG.**

***Index Terms*—Economical dispatch, feasible region projection, Fourier-Motzkin elimination, robust optimization.**

I. Introduction

Renewable energy resources and flexible demand assets are being proliferated in power systems within the last decade to enhance energy sustainability and efficiency. However, because these resources are volatile in nature, the associated uncertainties could significantly deteriorate forecast accuracy of net loads (i.e., actual load minus renewable output), posing potential threats on operation security of power systems. Robust optimization based dynamic economic dispatch (DED) has been recently studied to derive operation solutions that are immune against uncertainties. Specifically, a robust DED model seeks for optimal generation dispatches to meet forecast loads, while leveraging flexibilities of generation resources to guarantee system security against net load fluctuations. However, robust DED, as a semi-infinite programming in nature, is computational expensive in general.

This letter discusses a feasible region projection-based approach to solve the robust DED problem, with which the robust DED problem is reformulated as a single-level linear programming (LP) problem that can be solved efficiently. Indeed, the proposed approach could be one order of magnitude faster than the widely used column-and-constraint generation (CCG) method [1], which has to compute a mixed-integer linear programming (MILP) counterpart of a bi-level max-min subproblem and perform an iterative procedure between the master problem and the subproblem [2], [3].

The rest of the paper is organized as follows. The robust DED model is discussed in Section II. The proposed feasible region projection based LP solution methodology is presented in Section III. Numerical case studies are conducted in Section IV, and the conclusions are drawn in Section V.

Throughout this letter, buses and lines are collected in sets $\mathcal{I}$ and $\mathcal{L}$, indexed by i and l; $\mathcal{G}$ and $\mathcal{F}$ denote sets of generators and loads, indexed by g and f; $\mathcal{G}_i$ and $\mathcal{F}_i$ denote sets of corresponding assets connected at bus i; scheduling time intervals are collected in $\mathcal{T}$, indexed by t and t'.

II. Robust DED Model

The robust DED model is formulated as in (1)–(12), adopting the shift factor-based DC power flow calculation. The objective is to minimize the total operating cost (1), constrained by system active power balance (2), line flow limits (3), and ramping limits (4) and dispatch ranges (5) of individual generators. Additionally, generator dispatches are restricted within the feasible region $\mathcal{P}_t$ as in (6), while $\mathcal{P}_t$ as defined in (7) describes that any load fluctuation bounded in the box uncertainty set $\mathcal{U}_{f,t}$ (8) can be handled by adjusting dispatches of generators from $p_{g,t}$ to $(p_{g,t} + \Delta p_{g,t})$. Dispatches after adjustment need to meet active power balance (9), line flow limits (10), as well as adjustment ability limits of individual generators (11)–(12). C_g is the energy price of generator g; ΔT is the duration of one time interval; the dispatch of generator g at time t is denoted as $p_{g,t}$; $P_{f,t}$ represents forecast value of load f at time t; $SF_{l,i}$ is the shift factor of bus i to line l, and P_l^{UB} is the flow limit of line l; R_g^{UB}, P_g^{LB}/P_g^{UB}, and ΔP_g^{UB} are respectively the ramping ability, the lower/upper power bound, and the adjustable ability of generator g.

P1: Robust DED Model

$$\min \sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}} (C_g \cdot p_{g,t} \cdot \Delta T) \tag{1}$$

$$\sum_{g\in\mathcal{G}} p_{g,t} = \sum_{f\in\mathcal{F}} P_{f,t} \qquad t \in \mathcal{T} \tag{2}$$

$$-P_l^{UB} \le \sum_{i\in\mathcal{I}} SF_{l,i} \cdot \left(\sum_{g\in\mathcal{G}_i} p_{g,t} - \sum_{f\in\mathcal{F}_i} P_{f,t}\right) \le P_l^{UB}; \qquad l \in \mathcal{L},\ t \in \mathcal{T} \tag{3}$$

$$-R_g^{UB} \le p_{g,t} - p_{g,t-1} \le R_g^{UB}; \qquad g \in \mathcal{G},\ t \in \mathcal{T} \tag{4}$$

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$$P_g^{LB} \le p_{g,t} \le P_g^{UB}; \qquad g \in \mathcal{G},\ t \in \mathcal{T} \quad (5)$$

$$\{p_{g,t} | g \in \mathcal{G}\} \in \mathcal{P}_t; \qquad t \in \mathcal{T} \quad (6)$$

$$\mathcal{P}_t := \{p_{g,t} |\ \forall u_{f,t} \in \mathcal{U}_{f,t}, \exists \Delta p_{g,t} \textit{ such that } (9)\text{–}(12) \text{ are met}; \ f \in \mathcal{F},\ g \in \mathcal{G}\}; \qquad t \in \mathcal{T} \quad (7)$$

$$\mathcal{U}_{f,t} := \{u_{f,t} | U_{f,t}^{LB} \le u_{f,t} \le U_{f,t}^{UB}\};\ f \in \mathcal{F},\ t \in \mathcal{T} \quad (8)$$

$$\sum_{g \in \mathcal{G}} (p_{g,t} + \Delta p_{g,t}) = \sum_{f \in \mathcal{F}} (P_{f,t} + u_{f,t});\ \ t \in \mathcal{T} \quad (9)$$

$$-P_l^{UB} \le \sum_{i \in \mathcal{I}} SF_{l,i} \cdot \left[\sum_{g \in \mathcal{G}_i} (p_{g,t} + \Delta p_{g,t}) - \sum_{f \in \mathcal{F}_i} (P_{f,t} + u_{f,t}) \right] \le P_l^{UB}; \qquad l \in \mathcal{L},\ t \in \mathcal{T} \quad (10)$$

$$-\Delta P_g^{UB} \le \Delta p_{g,t} \le \Delta P_g^{UB}; \qquad g \in \mathcal{G}, t \in \mathcal{T} \quad (11)$$

$$P_g^{LB} \le p_{g,t} + \Delta p_{g,t} \le P_g^{UB}; \qquad g \in \mathcal{G},\ t \in \mathcal{T} \quad (12)$$

III. Feasible Region Projection Based Solution Methodology for Robust DED

A. Feasible Region Projection Method

We consider a system of linear constraints with respect to two sets of variables X and Y as in (13), where M^X and M^Y are coefficient matrices and N is a vector of constants.

$$M^X \cdot X + M^Y \cdot Y \le N \quad (13)$$

Definition: The feasible region projection of (13) onto the space of X is to identify a set of linear constraints (14) that only involve X, for which if $\hat{X}$ is feasible to (14), there must exist $\hat{Y}$, such that $\hat{X}$ together with $\hat{Y}$ is feasible to (13).

$$A^X \cdot X \le B \quad (14)$$

In (14), coefficient matrix A^X and constant vector B are generated via the feasible region projection algorithm. Indeed, it is always desirable to generate (14) as compact as possible, i.e., containing as fewer redundant constraints as possible.

For the specified robust DED problem, with respect to a certain pair of solutions to $p_{g,t}$ and $u_{f,t}$, constraints (9)–(12) focuses more on the existence of $\Delta p_{g,t}$ instead of its exact solution. To this end, we take $\Delta p_{g,t}$ as variables to be projected off, namely Y in (13).

We perform the feasible region projection of (9)–(12) from the space of $\{p_{g,t}, \Delta p_{g,t}, u_{f,t}\}$ to the space of $\{p_{g,t}, u_{f,t}\}$ via two steps: (i) equality constraint (9) is used to eliminate variable $\Delta p_{\hat{g},t}$ of an arbitrarily selected unit $\hat{g}$, by substituting $\Delta p_{\hat{g},t}$ in (10)–(12) via $\sum_{f \in \mathcal{F}} (P_{f,t} + u_{f,t}) - \sum_{g \in \mathcal{G}/\{\hat{g}\}} (p_{g,t} + \Delta p_{g,t}) - p_{\hat{g},t}$; (ii) *Fourier-Motzkin elimination* [4] is used to eliminate remaining $\Delta p_{g,t}$ variables. Finally, a set of linear constraints (15) can be obtained, where $A_{h,g,t}^G$ and $A_{h,f,t}^U$ are generated coefficients and $\Delta p_{g,t}$ is eliminated. We collect all generated constraints in $\mathcal{H}_t$, indexed by h.

$$\sum_{g \in \mathcal{G}} A_{h,g,t}^G \cdot p_{g,t} + \sum_{f \in \mathcal{F}} A_{h,f,t}^U \cdot u_{f,t} \le B_{h,t}; \quad h \in \mathcal{H}_t \quad (15)$$

With (15) being the projection of constraints (9)–(12) onto the space of $\{p_{g,t}, u_{f,t}\}$, constraint (7) can be equivalently reformulated as in (16), i.e., guaranteeing that for $\forall u_{f,t} \in \mathcal{U}_{f,t}$, $\exists \Delta p_{g,t}$, together with the given $p_{g,t}$, is feasible to (9)–(12). That is, (7) is met. To this end, model **P1** can be equivalently reformulated as model **P1'**.

$$\tilde{\mathcal{P}}_t := \{p_{g,t} |\ \ \forall u_{f,t} \in \mathcal{U}_{f,t}, (15) \textit{ is met}; f \in \mathcal{F}, g \in \mathcal{G}\}; \quad t \in \mathcal{T} \quad (16)$$

P1': Robust DED Model With Feasible Region Projection

Objective: (1)

Subject to: Constraints (2)–(5), (8), (15)–(16)

$$\{p_{g,t} | g \in \mathcal{G}\} \in \tilde{\mathcal{P}}_t;\ \ t \in \mathcal{T} \quad (17)$$

B. LP Based Solution Methodology

Solving model **P1'** remains challenging, as it still involves an infinite number of constraints as described in (16). Indeed, according to the definition of $\tilde{\mathcal{P}}_t$, as constraint (15) must be satisfied for $\forall u_{f,t} \in \mathcal{U}_{f,t}$ while $p_{g,t}$ is independent of $u_{f,t}$, $\tilde{\mathcal{P}}_t$ can be equivalently reformulated via a finite number of constraints as in (18), where $D_{h,f,t}$ is defined as in (19). The basic idea is that constraint (15) is satisfied for $\forall u_{f,t} \in \mathcal{U}_{f,t}$ if and only if it is satisfied against the worst-case realizations of $u_{f,t}$ [5]. In fact, at each time t, for each constraint h in (15), the worst-case realization of $u_{f,t}$ for positive/negative $A_{h,f,t}^U$ is its upper/lower bound, namely $U_{f,t}^{UB}/U_{f,t}^{LB}$, while the corresponding maximum possible value is $\sum_{f \in \mathcal{F}} D_{h,f,t}$.

Finally, the robust DED model with feasible region projection **P1'** can be equivalently reformulated as model **P2** by identifying the worst-case situation of individual constraints referring [5]. Model **P2** is an LP problem that can be solved efficiently via commercial solvers.

P2: LP Based Robust DED Model

Objective: (1)

Subject to: Constraints (2)–(5)

$$\sum_{g \in \mathcal{G}} A_{h,g,t}^G \cdot p_{g,t} + \sum_{f \in \mathcal{F}} D_{h,f,t} \le B_{h,t};\ \ h \in \mathcal{H}_t,\ \ t \in \mathcal{T} \quad (18)$$

$$D_{h,f,t} = \begin{cases} A_{h,f,t}^U \cdot U_{f,t}^{UB};\ A_{h,f,t}^U \ge 0 \\ A_{h,f,t}^U \cdot U_{f,t}^{LB};\ A_{h,f,t}^U < 0 \end{cases};\ \ f \in \mathcal{F},\ h \in \mathcal{H}_t,\ t \in \mathcal{T} \quad (19)$$

Theorem 1: **P1** and **P2** are equivalent, i.e., the optimal solution to **P2** is also optimal to **P1**.

Proof: We first claim that **P1** and **P1'** are equivalent, which can be naturally proved via Theorem 1 of reference [6]. Next, we show the equivalence of **P1'** and **P2**. The difference between **P1'** and **P2** is that constraints (8) and (15)–(17) are replaced by (18)–(19). Based on the reformulation approach in [5], we can obtain that (15) is met for $\forall u_{f,t} \in \mathcal{U}_{f,t}$ if and only if $\max_{u_{f,t} \in \mathcal{U}_{f,t}} (\sum_{g \in \mathcal{G}} A_{h,g,t}^G \cdot p_{g,t} + \sum_{f \in \mathcal{F}} A_{h,f,t}^U \cdot u_{f,t})$ is met, while the maximum value can be calculated via (18)–(19). This concludes that **P1** and **P2** are equivalent. Q.E.D.

C. Discussions on Computational Burden and Practical Applicability of Feasible Region Projection

A well-known weakness of the *Fourier-Motzkin elimination* is its theoretically exponential complexity. The Imbert's acceleration theorems [4] can be used to accelerate computational performance by filtering out redundant inequality constraints generated during an iterative elimination process, thereby mitigating the rapid growth of constraints. Beyond that, we apply a strategy to identify non-binding constraints by leveraging known boundaries of variables $p_{g,t}$, $u_{f,t}$, and $\Delta p_{g,t}$. That is, for a constraint in the standard "$\leq$" form, its maximum possible value can be calculated by substituting a variable with a positive/negative coefficient via its upper/lower bound. If the maximum possible value is strictly less than the right-hand-side, this constraint is redundant and can be dropped. In this letter, we refer to this computationally inexpensive strategy as boundary filtering. More details about boundary filtering can be seen in [7]. Other advanced filtering strategies with stronger filtering ability could be applied to further squeeze redundancy as needed [8], at the cost of higher computational complexity.

Moreover, in practice, the number of line flow constraints (10) to be monitored is limited, and the number of generators that are able to provide adjustment (11)–(12) against uncertainties is also limited. This could further contribute to reducing computational burden of feasible region projection.

As the robust DED is executed in a rolling manner by leveraging the most recent load forecasts $P_{f,t}$ to instruct real-time operation, the robust DED execution, including model building and solving, is time restricted. Therefore, it is preferable to construct $\tilde{\mathcal{P}}_t$ offline and apply it online to repeatedly solve robust DED in real-time. To achieve this, we further pick $P_{f,t}$ as retained variables in the projection, which further modifies (15) as (20). In this way, (20) can be prepared offline without knowing exact values of $P_{f,t}$, and becomes reusable in multiple executions of the robust DED in real-time. To apply the boundary filtering, boundaries of $P_{f,t}$ and $u_{f,t}$ can be enlarged to enclose all possible forecast values. In each execution of robust DED, $P_{f,t}$ in (20) can be substituted by the most recent load forecast value, after which (20) degenerates to the form of (15).

$$\sum_{g\in\mathcal{G}} A^{G'}_{h,g,t}\cdot p_{g,t} + \sum_{f\in\mathcal{F}} A^{U'}_{h,f,t}\cdot\left(u_{f,t}+P_{f,t}\right) \leq B'_{h,t};\quad h\in\mathcal{H}_t \qquad (20)$$

IV. Case Studies

The robust DED with a 5-minute time interval at a 2-hour look-ahead timeframe (i.e., 24 time intervals) is implemented on a modified IEEE 30-bus system, to evaluate the proposed approach in terms of solution quality and computational performance. Modifications on this system include increasing the load level and reducing line flow limits. In the test system, all generators are considered adjustable, and 29 out of the 41 lines are monitored with line flow limits (10). Uncertainty boundaries are set as symmetrical (i.e., $-U^{LB}_{f,t} = U^{UB}_{f,t}$) and proportional to the corresponding load of each time interval. The detailed test system data can be found in [7].

The feasible region projection program, including *Fourier-Motzkin elimination* and boundary filtering strategies, is implemented via Python. Robust DED models are implemented in MATLAB, and solved by Gurobi 8.0.1. Furthermore, MILP problems are solved to the zero MIP gap for fair comparison. All numerical simulations are executed on a PC with i7-3.6 GHz CPU and 16 GB RAM.

Constraints (9)–(12) are projected to eliminate variables $\Delta p_{g,t}$ and obtain constraint (15)/(20), which further derives constraint (18). Specifically, adopting (15) to individual time intervals leads to different sets of constraints (18). For instance, the number of constraints (18) at $t = 1$ is 2,577; while adopting (20) and treating $P_{f,t}$ as variables, the same set of 3,457 constraints is generated for individual time intervals. Indeed, feasible region projection only needs to be executed once to generate (20), while it is conducted multiple times to generate (15) for individual time intervals. However, more constraints are generated when (20) is used, because in order to fully cover variation ranges of $P_{f,t}$ and $u_{f,t}$ for all time intervals, their upper and lower bounds have to be enlarged and the performance of boundary filtering is weakened.

TABLE I
Comparison of Computational Performance

Approach		Objective ($)	Number of Iterations	Execution time (seconds)
CCG Approach		3862.37	5	16.92
Proposed Approach	adopting (15)	3862.37	-	1.16
	adopting (20)	3862.37	-	1.27

The average processing time to generate (15) for each time interval is about 60.62 seconds, and is about 78.74 seconds to generate (20). However, since (15) and (20) can be prepared offline before **P2** is executed, it is reasonable to exclude this processing time from the execution time of robust DED in the following comparison analysis.

We solve the proposed LP-based robust DED model **P2** with constraints (15) and (20), respectively, and compare them with model **P1** solved via the CCG approach, which employs an iterative procedure between an LP master problem and an MILP subproblem that reformulates the bi-level max-min problem [2]. We note that computational cost of repeatedly solving MILP subproblems is dramatically high, especially when seeking for small MIP gaps. In addition, LP/MILP models have to be built repeatedly during the iterative procedure, which also increases the actual execution time.

Table I shows that the proposed approach for solving **P2**, either through (15) or (20), and the CCG approach, yield the identical optimal solution. However, computational efficiency of the proposed approach dramatically outperforms the CCG approach. The major saving in execution time comes from avoiding repeatedly building and solving MILP subproblems involved in the CCG approach. To be more specific, execution time of the proposed approach by adopting (20) includes the model building time of 0.95 seconds and the model solving time of 0.32 seconds. By contrast, for the CCG approach, the average model building and solving time of subproblems are 1.27 and 0.98 seconds, and the total time spent on subproblems over 5 iterations is about 11.25 seconds, representing 67% of the whole execution time. In addition, it can be seen from Table I that (15) shows negligible improvement over (20) in terms of computational performance, although it has around 25% fewer constraints.

V. Conclusion

This paper proposes a feasible region projection based approach to reformulate the robust DED model as a single-level LP problem that can be solved effectively while guaranteeing the same solution optimality. Numerical studies show the proposed approach is one order of magnitude faster than CCG, by avoiding solving the MILP counterpart of bi-level max-min subproblems as well as the iterative procedure between the master problem and subproblem.

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