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Robust component: a robustness measure that incorporates access to critical facilities under disruptions

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The objective of this paper is to integrate the post-disaster network access to critical facilities into the network robustness assessment, considering the geographical exposure of infrastructure to natural hazards. Conventional percolation modelling that uses generating function to measure network robustness fails to characterize spatial networks due to the degree correlation. In addition, the giant component alone is not sufficient to represent the performance of transportation networks in the post-disaster setting, especially in terms of the access to critical facilities (i.e. emergency services). Furthermore, the failure probability of various links in the face of different hazards needs to be encapsulated in simulation. To bridge this gap, this paper proposed the metric robust component and a probabilistic link-removal strategy to assess network robustness through a percolation-based simulation framework. A case study has been conducted on the Portland Metro road network during an M9.0 earthquake scenario. The results revealed how the number of critical facilities severely impacts network robustness. Besides, earthquake-induced failures led to a two-phase percolation transition in robustness performance. The proposed robust component metric and simulation scheme can be generalized into a wide range of scenarios, thus enabling engineers to pinpoint the impact of disastrous disruption on network robustness. This research can also be generalized to identify critical facilities and sites for future development.

1. Introduction

Infrastructure failures are often inevitable following either natural or man-made disasters including hurricanes [1], earthquakes [2] and ensuing tsunamis [3], ice storms [4], and terrorism incidents [5]. The economic prosperity, security and public health of our society are extremely vulnerable to these accidental, weather-related and human-instigated events [6,7]. Events such as 11 September 2001, hurricane Katrina in 2005, the Haiti earthquake in 2010, etc., showed the cataclysmic aftermath of such hazards. Notably, US Pacific Northwest is highly prone to an M9.0 Cascadia subduction zone earthquake [8,9]. The last mega-earthquake occurred in 1700, and there is a 40% chance of recurrence within the next 50 years [10]. This fact calls for our understanding of network robustness behaviour and action on critical infrastructure protection.

Roadway networks play important roles in transporting people and goods efficiently and safely, evacuating people from the site of a disaster and importing critical resources to affected sites. Patuelli *et al.* [11] suggested that physical constraints are likely to restrict the topology of road networks and make them nearly planar, which also makes them extremely sensitive to failures. Therefore, systematic understanding and accurately measuring network robustness under disruptions is of great significance in achieving a resilient critical infrastructure system. Various studies [12–16] defined their own metric in analysing network robustness. From a network science perspective, the robustness of a network is

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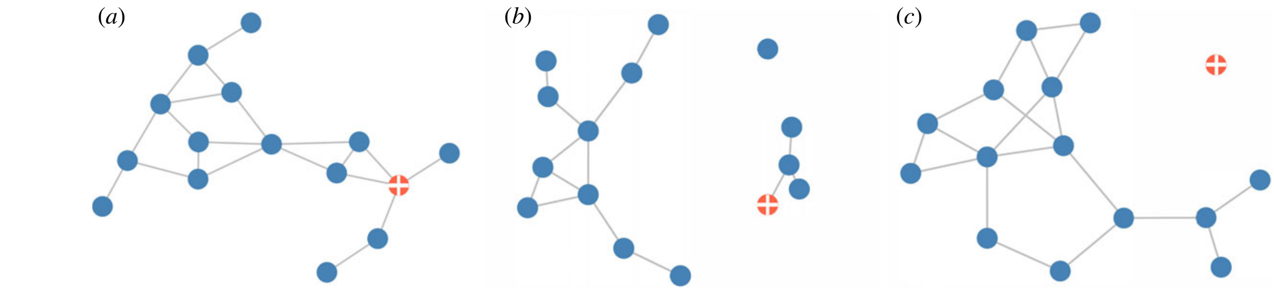

Figure 1. Network access to critical facilities. Blue nodes represent the ordinary nodes in the network, while the red node represents the critical facility. Critical facilities can be defined as the facilities that contain essential resources which support the normal functionality or restoration of the system, e.g. a lifeline warehouse, hospital, fire station and restoration centre. In (*a*), the network is fully connected to the critical facility. In this case, the largest connected component (or giant component) contains the critical facility. In (*b*), the network is broken into different clusters. The largest connected component contains most of the nodes; however, it does not include the critical facility, only the second largest component connects to the critical facility. In (*c*) the critical facility is isolated; without access to critical resources, the network would fail. (Online version in colour.)

often characterized by the value of the critical threshold analysed using percolation theory and defined as the largest connected cluster size during the entire attack process [17]. Essentially, network robustness describes the ability of a system to maintain its performance after a disruption, and accessibility to the critical facilities (i.e. shelters, hospitals and police/fire stations), all of which are an essential part of post-disaster roadway performance. Therefore, in this paper, transportation network robustness is measured by the integrated size of the clusters that are connected to critical facilities, which is formulated as a robust component. With this definition, we assume that if a node is contained in a robust component, the cars can reach the critical facilities. Empirically, it represents the ability of the transportation system to withstand hazard-induced infrastructure failure without reducing access to critical facilities.

Assessing robustness generally consists of determining the system behaviour that results from each possible network disruption state. Engineering models use reliability [18–20] or travel cost [21,22] to measure network robustness under disturbance. However, these models use daily operational travel demand in the analysis, which provides limited insights on post-disaster transportation network robustness assessment. Besides, this paper focuses more on the existence of post-disaster network access to critical facilities rather than normal travel performance. In addition, link disruption in the model is arbitrary, which is far from the reality of infrastructures exposed to different levels of hazard risk. Hazard vulnerability should be included in the model in order to provide an accurate measurement of post-disaster network robustness. Furthermore, access to critical facilities is essential in a network's functionality and therefore, should be included in the network robustness assessment framework. Existing research [23] uses centrality measures to quantify link importance in accessing specified emergency services. However, such a model omits the impact of collective network disruption on access to critical facilities. Also, the centrality measure focuses on the shortest path to the destination, while in reality, all paths should be considered in evaluating network access to critical facilities.

In post-disaster network robustness assessment, disruptive events are described by removing one or more network components from the system, which can be modelled with a percolation approach [24,25]. Extensive research has been primarily focusing on the percolation modelling of network robustness [17,26–31]. However, there are several limitations in conventional percolation modelling of infrastructure network robustness. First, existing research mainly uses generating function methods to derive network robustness. However, due to the degree correlation in spatial networks, generating function methods fail to measure the robustness of the infrastructure network accurately Dong *et al.* [32]. Second, network disruption is commonly described by random failures, localized failure or targeted attack. However, the infrastructures' vulnerability to natural hazards varies across the network. For example, low-lying roads in a valley and roads on steep hills are more vulnerable to landslides than roads on flat ground. Thus, a probabilistic link-removal scheme should be considered to provide a realistic post-disaster road network robustness assessment. Third, the traditional giant component assumes the largest connected component of the network functions after the disruption. However, nodes in infrastructure networks such as water, gas and electricity, are not autonomous but rather rely on resources feeding them, without which network performance will deteriorate. In a disastrous event such as an earthquake or a deliberate attack, the network may be broken into different components and having access to critical facilities in their local neighbourhood is essential for individuals in each component [20]. For example, if a community's access to a hospital is cut off by an earthquake, injured people cannot receive timely medical treatment and their health and safety will be at risk. Therefore, investigating network robustness and considering the network's access to critical facilities is essential to avoid functional isolation in disasters, and accessibility to critical facilities needs to be reflected in the robustness measurement. Figure 1*a* shows that when a network is fully connected, the giant component has access to critical facilities. When the network is disrupted, although the left giant component is more connected and contains the most nodes (as in figure 1*b*), it does not connect to the critical facilities, and therefore may likely fail without access to critical resources. When critical facilities are entirely isolated from all other components (as in figure 1*c*), the network will paralyse due to lack of resources regardless of the size of the cluster.

This paper is largely motivated by the fact that network robustness assessment based on the conventional percolation approach alone is not sufficient to represent the performance of transportation networks in the post-disaster setting and that hazard-induced link failure probability is lacking in the engineering studies. To bridge the gap between engineering simulation methods and theoretical analysis, this paper contributes to the state-of-art design by proposing a new robustness metric: robust component—the component that has access to critical facilities—in incorporating post-disaster access to critical facilities in network robustness measurement. Further, in order to overcome the limitation of applying generating function in infrastructure network robustness characterization due to degree correlation, and also integrate hazard vulnerability into the post-disaster transportation network robustness assessment, we proposed a probabilistic network disruption percolation modelling and simulation framework to accurately measure network-level robustness behaviour in the face of disaster-induced massive disruption. This paper enables a comprehensive assessment of network robustness, one that considers network access to critical facilities in the face of disastrous events. The result allows decision-makers to devise mitigation strategies to different hazards by focusing on the critical area to counter the disastrous effects, especially in areas such as emergency response planning, evaluation of the location of additional critical facilities and infrastructure prioritization.

The remainder of the paper is organized as follows: §2 presents a literature review of the related research. In §3, the methods adopted in this paper are discussed in detail, and a comparison study is conducted on a simulated ER network and road network. Section 4 presents network robustness in considering access to hospitals by using earthquake-induced probabilistic failure scenarios. Following that, §5 provides the major findings of this paper. Finally, the paper concludes with a discussion in §6.

2. Literature review

2.1. Post-disaster transportation network robustness modelling

Investigating the robustness of disrupted networks is often associated with the measurement of vulnerability, reliability and accessibility [33]. Vulnerability refers to the degree of inability of a system to function due to disruption [33]. Essentially, the concept can be considered as the reciprocal of robustness [34]. Poorzahedy & Bushehri [19] define the network reliability for the (k, s) origin–destination (OD) pair user group as the probability that the network may hold a suitable condition for those users after an incident. Murray & Grubesic [35] define reliability as the probability that a given element in a critical infrastructure system is functional at any given time. Mattsson & Jenelius [36] refer to reliability as the probability that a system can maintain its satisfactory operation over the long run. Murray and Grubesic [35] suggest that reliability analysis mainly focuses on the possibility of maintaining the performance of critical infrastructure elements. Reliability analysis is often classified into three categories: connectivity reliability (the probability that a node remains connected), travel time reliability (the probability that a trip between nodes is made within a specified time interval) and capacity reliability (the probability that a network can successfully accommodate a given level of travel demand) [20,35]. Poorzahedy and Bushehri [19] proposed a measure of link importance based on consumer surplus for solving the problem of network performance in case of incidents. This study incorporated the link survival probability after catastrophic events and provided a heuristic solution to solve large-scale network problems. Chen *et al.* [18] introduced capacity reliability to measure the performance of a transportation network under traffic disturbance. The proposed performance measure took network capacity and traffic demand into consideration. Chen *et al.* [21] further extended the research by providing a network accessibility measure that considered the consequences of link failure expressed in terms of travel time and calculated them based on the travel cost increase. However, the methodology was tested on a rather small hypothetical network (five nodes). To assess a transportation network of tens of thousands of nodes, the performance of the method is not guaranteed. Overall, these reliability analyses require the travel demand from the OD pair. However, such real post-disaster demand data are limited and therefore are impractical to implement in a real-life study.

Network robustness measures a system's capability to withstand an unexpected internal or external event or change without degradation in performance [37,38]. Chopra *et al.* [39] presented a resilience analysis on the London metro system that considered network topology, spatial organization and passenger flow. The results identified the particular sources of structural and functional vulnerabilities that needed to be mitigated for improving the resilience of the London metro network. Nagurney & Qiang [22] proposed two relative total cost indices to assess road network robustness when the links are disrupted or travel behaviour is altered. Measures such as link travel cost [22], traffic delay [40] and the ratio of pre- and post-disaster condition [41] are also commonly used to investigate the transportation network robustness behaviour. It is worth noting that, depending on the performance that we focus on, e.g, network connectivity, efficiency, travel time, etc., the metric that is used to characterize the network robustness would change. For example, Albert *et al.* [42] introduced connectivity loss in a study of the structural vulnerability of the North American power grid. Similarly, Dueñas-Osorio & Vemuru [43] adopted connectivity loss in investigating cascading failure in power systems. Hines *et al.* [44] used connectivity loss to measure vulnerability in an electric power blackout risk analysis. In addition, Crucitti *et al.* [13] used the network efficiency measure to analyse the structural vulnerability of the Italian GRTN power grid. Kinney *et al.* [14] also adopted the efficiency concept to measure the impact of cascading failure on the North American power grid. Although network connectivity and people's commute behaviour are studied extensively [45–48], disrupted network connectivity, especially network access to critical facilities, is rarely studied.

However, post-disaster access to critical facilities including hospitals, police stations, fire stations and rescue is essential to the health, safety, post-disaster response and recovery of our society. As the roadway infrastructure provides essential access to these facilities, it should be included in the measurement of network performance. Investigating access to critical facilities in a disrupted network is often associated with accessibility analysis. Accessibility is generally defined as 'the relative ease of reaching various services, destinations and/or activities from a particular origin [49,50]. Redondi *et al.* [51] studied the accessibility of the airport by using the shortest path length, the minimum number of non-stop flights in their case, to measure airport network connectivity. Bigotte *et al.* [52] formulated a mixed-integer model for integrated urban hierarchy and transportation network planning. The model enables identification of the links that should be improved or new links that should be built to increase network accessibility. Grubesic & Murray [53] explored network interconnection by removing the vital nodes, and Church & Scaparra [54] studied operation efficiency loss as the identified facilities are destroyed, which are all similar to a targeted attack scenario. Network disruption in natural hazards, however, is determined by the network's geographical exposure instead of its geographical or functional importance. Novak & Sullivan [23] introduced the critical closeness accessibility measure for evaluating the accessibility of emergency services on a road network. It can also identify the critical links that are important in terms of facilitating system-wide access to emergency services. It measures accessibility on a link-by-link basis, which essentially assigns an accessibility value to the individual links in the network that accounts for the empirical information such as the spatial distribution of critical link/node, road network topology and road characteristics such as road type, capacity, volume and travel speed. Nevertheless, it fails to capture network-level accessibility behaviour, which is the key factor of state, regional and municipal transportation agencies' funding and policy decision-making. Dong [55] used a total accessibility matrix to measure the topological structural robustness of the supply chain network. However, the calculation of the accessibility matrix does not include the impact of the collective network disruptions such as earthquake-induced failure. Although it provides an index for each node, it does not show where this node can access, which is of great importance in emergency rescue. Therefore, these methods provide limited insight into the post-disaster network access to critical facilities.

On the other hand, there is rich literature on transportation network robustness assessment. Sullivan *et al.* [56] introduced a scalable system-wide performance measure called network trip robustness to compare networks of different sizes, topology and connectivity levels. Erath *et al.* [57] presented a framework that investigates the robustness of the transportation system to natural hazards. This approach can accommodate transportation-related failure consequences, including congestion effects. In addition, Chen *et al.* [21] used network-based measures to assess disrupted transportation networks, which consider the consequences of one or more link failures in terms of network travel time or generalized travel cost increases as well as the behavioural responses of users due to the failure in the network. However, existing research on transportation network robustness analysis faces several challenges. First, the travel demand under normal operation is used in the model to characterize the degraded network performance. As the travel behaviour in a post-disaster setting is expected, the existing analysis offers limited insights into post-disaster network accessibility to critical facilities. Second, the road disruptions considered in these studies are selected arbitrarily. However, in real disaster scenarios, the geographical exposure of infrastructures to hazards are different and the condition of infrastructures due to ageing varies across the network. In a post-disaster network access analysis, these ought to be incorporated into the analysis. Third, the centrality measures in nature, such as closeness and betweenness, measures the shortest distance between an OD pair [58]. However, in a post-disaster scenario, we focus on investigating whether or not an individual site has access to any of the critical facilities.

In order to accurately measure the performance of a transportation network post-earthquake considering the network's access to critical facilities, we propose to use a network topology-based metric robustness component to assess post-disaster network robustness. We define network robustness in this paper as a transportation system's capability to withstand hazard-induced infrastructure failure without degradation in providing access to critical facilities. The robustness investigated in this paper attempts to tackle the question of how earthquake-induced failure will impact a network's access to critical facilities such as hospitals. Despite missing traffic information, the comprehensive topological and geometrical data can provide engineers with a comprehensive first-step assessment of network robustness in the face of a catastrophic event such as an earthquake, hurricane, ice storm, flooding or other natural disasters. In addition, we integrated infrastructure probabilistic geographical exposure to network failure in order to generate a realistic post-disaster network robustness assessment. More importantly, instead of only considering a predefined OD pair, we examine the individual site's access to all of the designated critical facilities on the network. This enables a comprehensive evaluation of network-level robustness behaviour. From a network science perspective, we are investigating whether there is a path between a site to any selected critical nodes. Intuitively speaking, the robust component can be interpreted as 'given that ϕ proportion of the network is disrupted, the *RCS* part of the network still maintains its performance in terms of having access to critical facilities'.

2.2. Percolation modelling of network robustness

Existing research in assessing the robustness of real-world networks shares some common features [59]: (1) simulating or obtaining empirical data for a network (e.g., generating a network from random graph, mapping the real network to obtain the data); (2) measuring the investigated network's structural features; (3) conducting random failure or a targeted attack on the network and (4) assessing the aftermath performance (static, dynamic) of the network. In particular, road network robustness analysis normally models road infrastructure as a network with links (roads) and nodes (intersections) in order to investigate the network disruption and its impacts on society [60]. A wide variety of network robustness measures have emerged from recent research. Figure 2 presented the major types.

The largest connected component (i.e. the giant component) in percolation theory is commonly used in physics and computer science [70]. Cohen *et al.* [71] studied the robustness of scale-free networks with power-law degree distribution, i.e. the Internet, by measuring the giant component (spanning cluster) after network breakdown. Solé *et al.* [16] assessed network robustness by measuring the giant component on the European power grid under targeted attacks and determined the transition threshold. Motter & Lai [15] used the largest connected component method to measure network robustness after cascading failure occurs. Li *et al.* [72] measured the largest connected component in spatially constrained Erdös–Rényi networks to determine the impact of spatial constraints on network robustness. The above studies assume the largest component will be functioning after the disruption. However, we argue that accessibility to critical facilities is essential for post-disaster survival. For

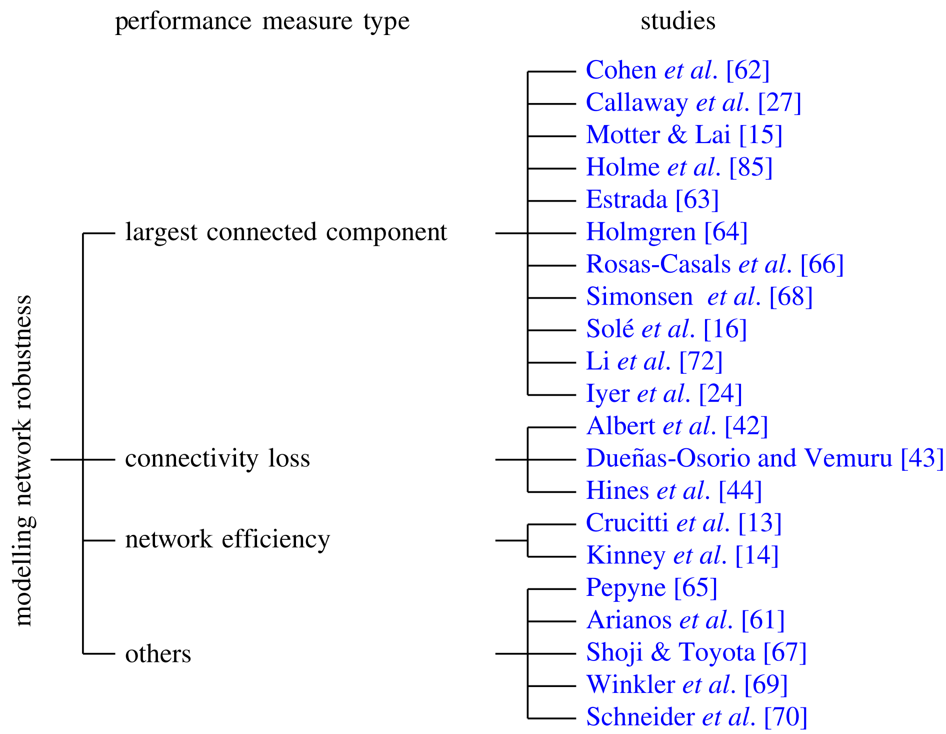

Figure 2. Selected performance measure types for modelling network robustness [61–69]. (Online version in colour.)

example, a giant component without access to medical services cannot be considered as functioning from an emergency response perspective. The proposed robust component measure overcomes this challenge by including important nodes into the critical component measuring.

Inspired by the similarities between human and insect infrastructures, Middleton & Latty [73] reviewed the literature on resilience in three key social insect infrastructure systems: transportation networks, supply chains and communication networks, and then described how systems invest in three pathways to resilience: resistance, redirection or reconstruction. This finding demonstrates that we can learn from social insect research and then develop analytical and simulation tools to study human infrastructure resilience based on their findings. Percolation theory is a powerful tool that allows the analysis of network robustness. There is a rich body of literature focusing on percolation modelling of network robustness through generating function methods. Most of the studies focus on theoretical networks such as the ER network and scale-free network [29,74–77]. However, due to the degree correlation in the infrastructure network, the generating function method is incapable of capturing network robustness behaviour [32]. Since this degree correlation is inevitable, because our infrastructure network is spatially embedded, a simulation-based method is more desirable in characterizing the percolation process in infrastructure network robustness analysis. Furthermore, most theoretical methods assume an infinite size of the investigated network, which is not the case in infrastructure network robustness research [74]. Although Radicchi [29] studied percolation in the real interdependent network, his model still focuses on the theoretical networks. This paper proposes a percolation modelling and simulation framework that captures the spatial complexity of the road network in order to generate an accurate assessment of network robustness behaviour.

Regarding another modelling spectrum, extensive research has focussed on the random failures [32,78], localized attacks [79–81] and targeted attacks [16,82,83]. However, in reality, the probability of multiple road failures is largely dependent on the built environment it is exposed to. For example, landslides are widespread in regions that have steep slopes, weak soil and significant precipitation or storm events. Probabilistic failure based on the link's exposure to hazards is desired for a case study on the real-life disaster. Therefore, this paper integrates infrastructure failure probability into the percolation modelling framework and presents a study on network robustness under the influence of earthquake-induced probabilistic failure.

3. Percolation modelling of the robust component

The percolation process is parametrized by the probability, p, that a node or an edge is present or functioning in the network. The functional nodes/vertices are considered occupied and p is called the occupation probability. When p is large, the network tends to be more connected. As p decreases, there comes a point where the giant component breaks apart. This point is called the percolation threshold. The formation or dissolution of a giant component is called a percolation transition [28]. Here, we use the notation $\phi = 1 - p$ to represent the proportion of links that are removed from the network.

Giant component size (GCS) is commonly used as a measure of network robustness because percolation assumes that the largest connected cluster will maintain its functionality after network disruption [28]. However, this is not a valid assumption since the largest connected component is not guaranteed to be functioning if there is no access to the necessary resources. For example, if an injured person has no access to a hospital, a cluster containing said hospital cannot be considered functional from a healthcare point of view, even if it maintains the largest possible size. In terms of hazards, robustness represents the degree to which a system is able to withstand an unexpected internal or external event or change without degradation in performance [37,38]. Therefore, we propose a new network robustness measurement: robust component.

Robust Component. In a graph G, two vertices u and v are considered connected if there is a path from u to v, which is denoted as $\rho(u, v) = 1$. Given a network of size N, containing K critical facilities, the connected component of k can be represented as $C_k = \{v_i \mid \rho(k, v_i) = 1,\ \forall i = 0, 1, \ldots, N\}$. Following this, the robust component of a network with occupation probability ϕ can be defined as

$$\Re_\phi = \bigcup_{k=0,1,\ldots,K} C_k \tag{3.1}$$

Intuitively, the robust component is the union of the nodes that connect to at least one critical facility. In other words, since the original network is fully connected, every node has connections to all of the critical facilities. After the imposed link failure on the network, a node is considered failed when it loses connection to all of the critical facilities. A robust component will significantly help to maintain network performance in an unexpected disruptive event. A percolation illustration of the robust component is presented in figure 3. Algorithm 1 shows the designed algorithm for calculating the robust component size (RCS).

Algorithm 1. Calculate *rcs*.

```
K ⇐ Critical Facility List
network ⇐ Roads(v_i, v_j)(i, j = 1, …, N)
link_failure [φ_1, …, φ_m](m = 1, …, M) ⇐ Network Failure Scenario
m ⇐ 1
While m ≤ M do
    disrupted_network = network.remove_edges(φ_m)
    for k in K do
        rcs_m = ∑ v_i, ∀v_i | ρ(k, v_i) = 1, i = 1, …, N
    end for
    m+ = 1
end while
```

To demonstrate the performance of the robust component in measuring the network robustness considering the network access to critical facilities, two types of network are used to experiment: The ER network and the Portland road network as shown in figure 4. The ER network is generated through Networkx module, and the Portland road network is provided by Portland Metro, the road network GIS shapefile can also be obtained from Metro [84]. Two networks are of the same size (5147 nodes) and same mean degree (2.97). There are two ways to present a graph: the prime approach (vertices are intersection/joints, edges are links/interactions) and the dual approach (vertices represent links/interactions, edges represents intersections/joints). Both approaches are investigated through the ER and Portland road networks, and their degree distributions are presented in figure 4. As we can observe, the road network shows a strong spatially embedded feature. A peak shows at the degree of 3 and 4. The road network here only contains major arterial roads and minor roads are not included in the Portland Metro's traffic analysis, which results in some four-way intersections turning into a T-intersection. Also, the massive amount of ramps that connect arterial roads to highways contribute to the high frequency of degree-3 intersections. On the other hand, the random network shows a smooth degree distribution which fits into a Poisson distribution with a mean degree of 2.97.

According to the two aforementioned methods of constructing the network, the percolation process can be classified into two categories: node percolation, which removes the node, and edge percolation, which conducts edge removal. In a road network failure percolation case, roads are the objects that tend to be destroyed. Therefore, in this paper, network percolation is conducted through derived link removal based on the failure strategies. In particular, a node is considered failed only when it loses all alternative connections to all the critical facilities, in other words, all the possible routes from the node to critical facilities are impassible.

Since the size of the robust component is dependent on the network's connection to designated critical facilities, the number of critical facilities will impact the robust component's performance during link percolation. To investigate network robustness, we increased the number of critical facilities (k) and recorded the size of the robust component in each case. At each k, 100 simulations are conducted and each simulation represents a random failure scenario. The average RCS is calculated and presented in figure 5. To compare with the conventional robustness measures, the giant component, the size of the largest connected cluster, is recorded as well.

Comparing network robustness under both GCS and RCS in figure 5, we can see that in the case when access to critical facilities is of significance to disaster recovery, GCS gives an incorrect assessment of network robustness, which can lead to further false disaster mitigation strategy planning. The value of the critical percolation transition threshold, where the RCS diminishes to zero, is normally used to define the robustness of the network [24,72,77,85]. As shown in figure 5, the critical threshold ϕ_c varies between the ER network and the Portland Metro road network. Low ϕ_c means that very few link removals are required to destroy a network's functionality. In other words, the network is more vulnerable to failure. Looking at $k = 1$, we can conclude that under the random removal scenario, the Portland road network is more vulnerable to the loss of critical facilities than the ER network. As k increases, the value of ϕ_c becomes nearly identical for the ER and Portland road networks. However, there still exists a difference in the percolation process. Purely using the critical percolation transition threshold ϕ_c neglects the case when the network is severely damaged but not completely destroyed [77]. To complement this, Schneider *et al.* [70] proposed a systematic measure, called the robustness measure R, to estimate the robustness of the network.

$$R = \frac{1}{N}\sum_{Q=1}^{N} s(Q), \tag{3.2}$$

where N is the total number of the nodes in the network, and $s(Q)$ is the fraction of nodes in the largest connected component after removing $Q = Nq$ nodes. The $1/N$ normalizes

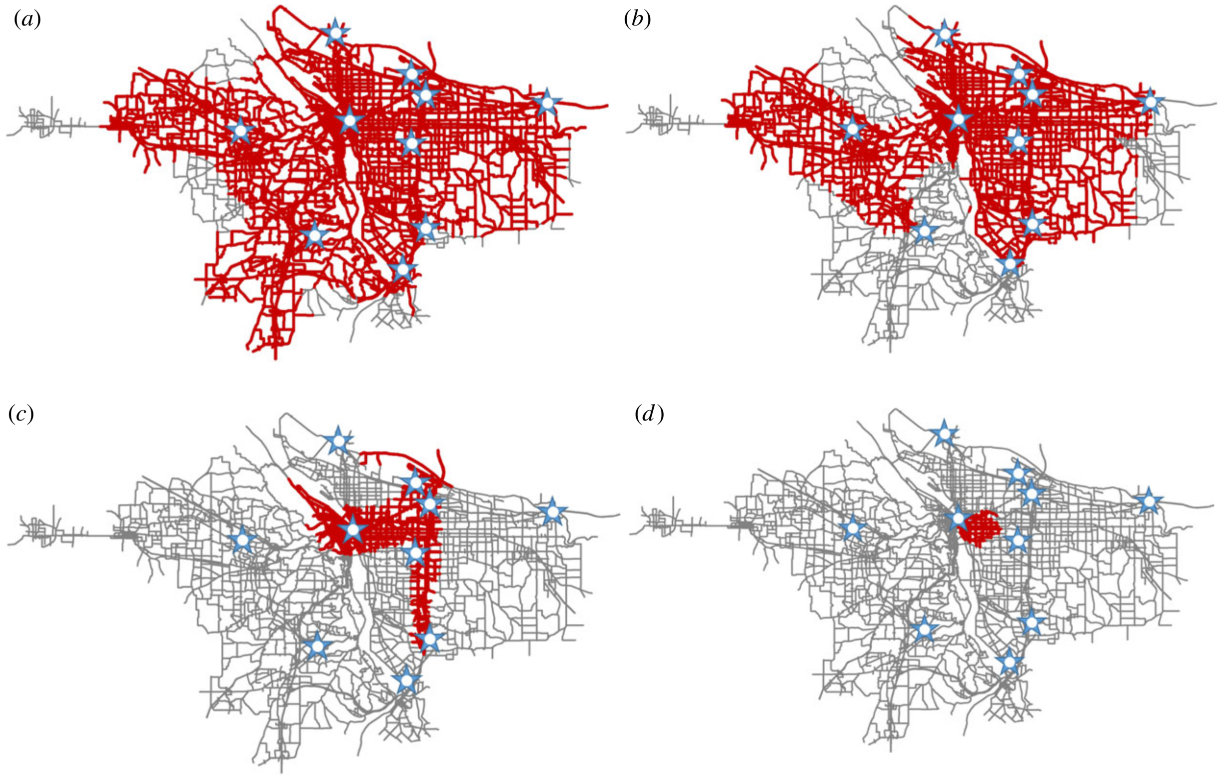

Figure 3. Percolation simulation of the robust component under varying attack sizes. Green dots represent the selected critical facilities (randomly generated for demonstration), and red links represent the links that are connected to the critical facilities. In (*a*), as 10% of the edges are destroyed ($\phi = 0.1$), 85% of the network still has access to the critical facilities. In (*b*), an increase of 5% more edge failure ($\phi = 0.15$) would lead to 80% of the network being within reach of critical facilities. In (*c*,*d*), as nodes continue to be removed from the network, the destruction effect escalates. As 40% of the edges are removed, the majority of the network loses access to the critical facilities. (*a*) $\phi = 0.1$, (*b*) $\phi = 0.15$, (*c*) $\phi = 0.25$, (*d*) $\phi = 0.4$. (Online version in colour.)

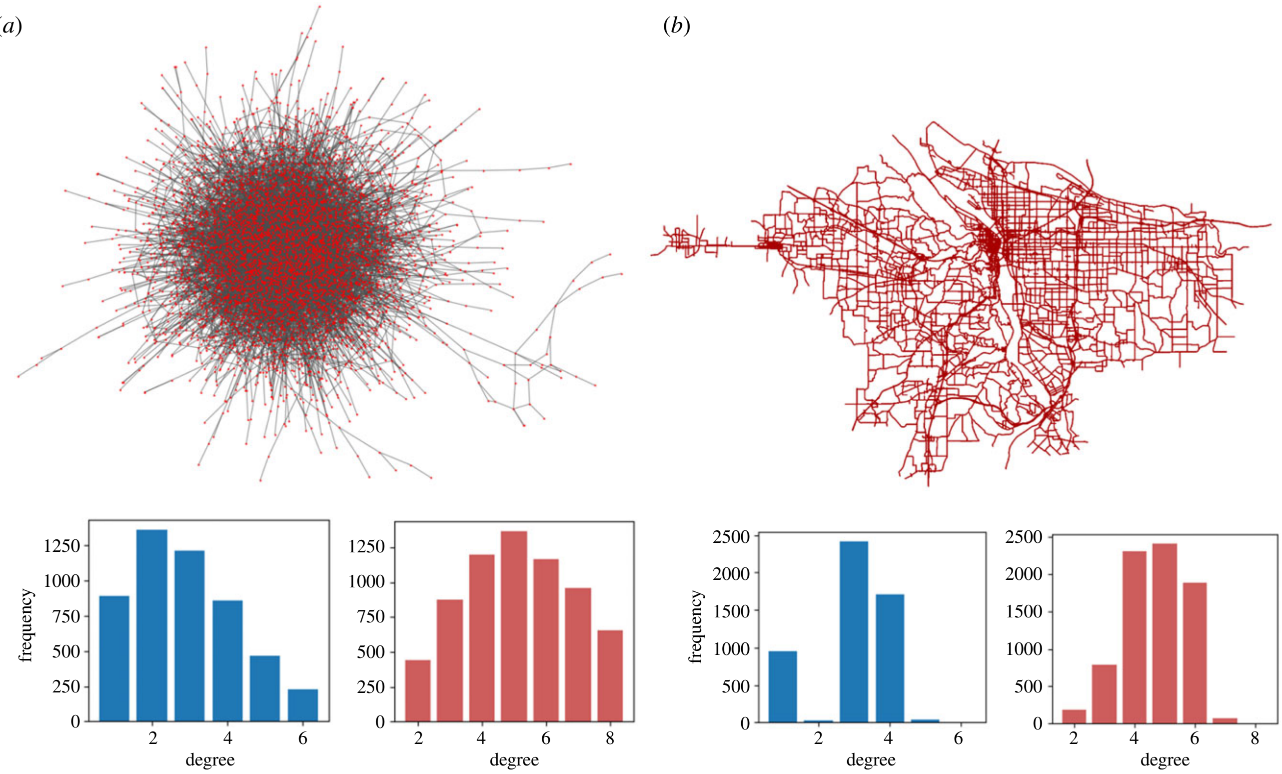

Figure 4. Network configuration and degree distribution of random network and spatially embedded road network. (*a*) ER network, (*b*) Portland Metro road network. (Online version in colour.)

the result so that the robustness of networks of different sizes can be compared [24,77]. From a geometry perspective, R describes the area under the percolation curve.

Figure 6 shows the comparison between the ER network and the Portland road network. The simulation results show that when the number of critical facilities is very low, the ER

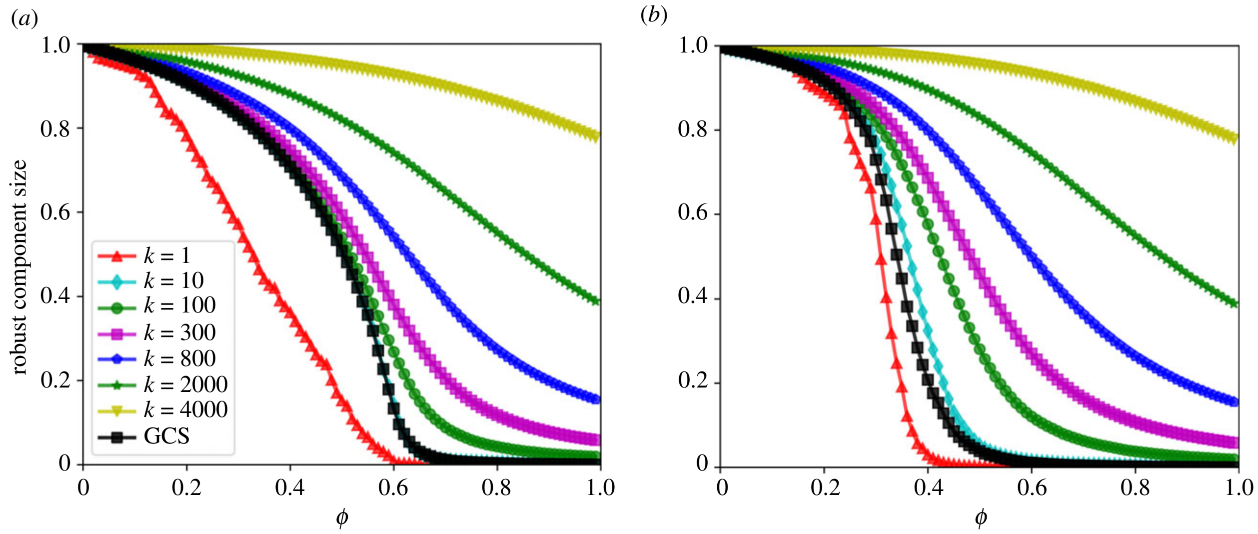

Figure 5. Degree distribution of the (*a*) ER network and (*b*) Portland Metro road network in prime and dual approach. (Online version in colour.)

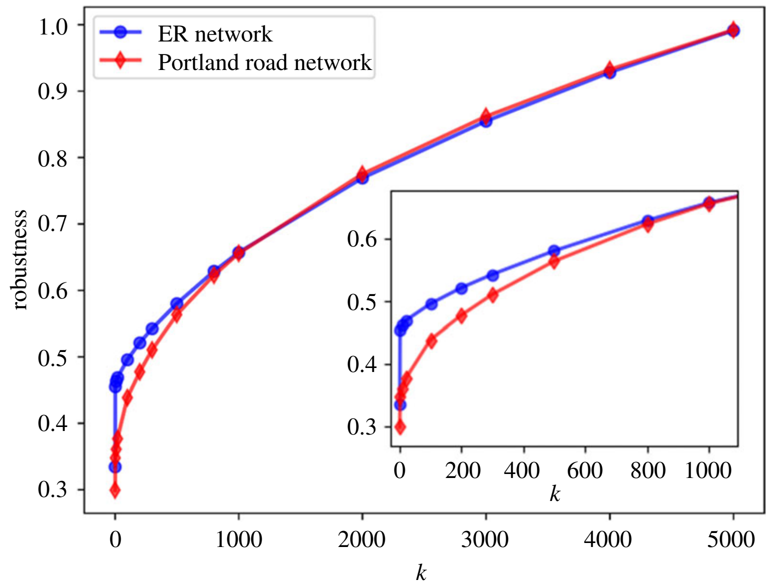

Figure 6. Robustness comparison between the ER network and Portland Metro road network with varying numbers of critical facilities. (Online version in colour.)

network exhibits more robust behaviour than Portland road network, despite the fact that they have the same mean node degree. The difference in robustness behaviour is largely due to the spatially embedded feature of the road network as the random network possesses a better-balanced degree node/link distribution and the nodes have more redundancy in coping with the link removal. The comparison shows that at $k = 1000$, the robustness behaviour of ER network and Portland Metro road network are very similar. This comparison shows the extent to which the infrastructure network is different from the theoretical network and suggests that when the network evolves to a certain level, the spatial network can also obtain similar properties to the theoretical networks.

4. Impact of earthquake-induced failure on network access to hospitals

A Cascadia subduction zone earthquake posts a great threat to the Pacific Northwest region as an estimated M9.0 earthquake will severely damage the infrastructure and affect community' access to the necessary resources for post-disaster recovery. In this paper, we use Portland, OR, as the study site to investigate the impact of earthquake-induced infrastructure failure on the network's access to hospitals. It is worth mentioning that the proposed methodology can be applied to road networks in other cities in different disaster scenarios.

We have previously investigated the network robustness in a random critical facility and random failure scenario. However, in real life, the location of critical facilities is normally decided based on the geographical features or the needs of the surrounding areas. Emergency medical services (EMS) personnel and hospitals are the community-based resources that are responsible for injuries during the initial disaster response. Robustness towards disasters varies from community to community and is dependent on the availability of EMS and hospital resources. Therefore, we evaluate network robustness considering the access of communities to hospitals post-disaster. Figure 7 shows the

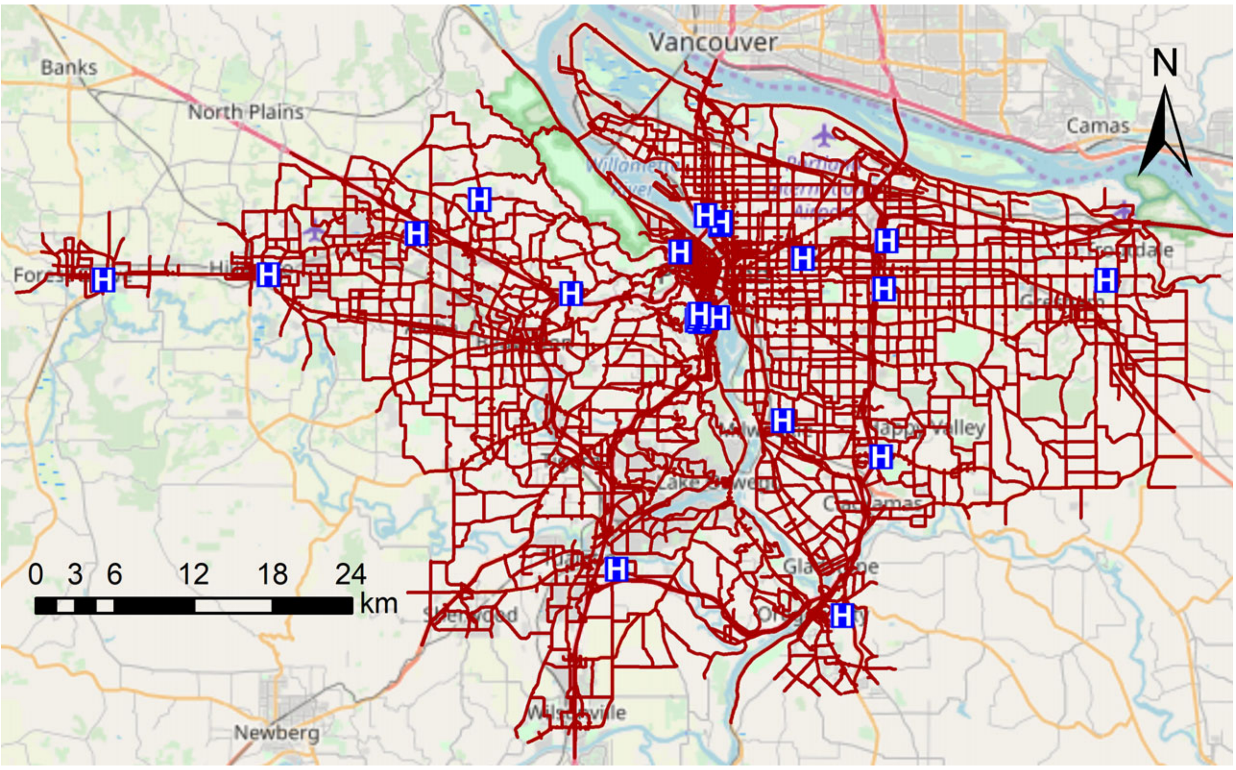

Figure 7. Portland Metro area transportation network. (Online version in colour.)

Portland Metro network and area with hospitals. There are 5147 nodes and 7646 links included in the network. Initially, there were 22 hospitals in our scope of study [84]. To reduce overlapping, 20 hospitals were used for the simulation. It is worth mentioning that in reality, hospitals are not built at the intersection of the roads, but close to a major intersection. Therefore, the location of the closest intersection to the hospital is used to represent the hospital's location.

4.1. Probabilistic network failure: an M9.0 earthquake scenario

Because road networks are spatially embedded, their vulnerability relies heavily on their structural nature and surrounding geographical features. For example, bridges are more vulnerable during an earthquake and roads on top of faults are prone to landslides. Figure 8 shows the geographical exposure of the Portland Metro network to natural hazards, i.e. earthquake-induced landslides and liquefaction. This information can be obtained from O-HELP [86]. Since the probability that the links that are most prone to failure varies across the network, uniform link removal in a traditional percolation process is inadequate in characterizing the stochastic nature of network disruption. In this paper, we propose a probabilistic approach in removing links in order to approximate the destructive effect of natural hazards on the network.

Link failure probability (P_f) in this model is comprised of three elements: probability of failure by landslide P_l (figure 8*a*), probability of failure by liquefaction P_q (figure 8*b*), and the probability of failure if the link is a bridge P_b (figures 8*c* and 9). It is worth mentioning that all of the earthquake-induced hazards considered in this paper can severely damage the road. When roads are disrupted, they are likely to be impassible. Restoration time is variable and depends on many different factors. In this paper, we specifically focus on the immediate impact of network failure. First, we determine the probability that a bridge will be damaged. As each bridge has four possible damage states with different probabilities, we consider a bridge as failed only when it experiences moderate or complete damage. Then, the overall link failure probability $P_f = P_l + P_q + P_b - P_l * P_q - P_q * P_b - P_l * P_b + P_l * P_q * P_b$. Once we obtained the link failure probability, we used that probability to conduct a weighted selection to determine the probabilistic failure sequence at each iteration. First, we constructed a cumulative probability range based on the probability we obtained. Then we randomly generated a number in the range of $[0, \sum(P_f)]$, and it will fall into one of the intervals in the constructed cumulative probability range. The corresponding link of the range will be considered as failed. In this case, the larger the P_f, the wider the range, and the more likely it will be selected. Once all the links are iterated through, we will have the probabilistic failure sequence. Based on ϕ, we can select the links to be removed at each step and simulate the percolation process on the Portland Metro network. Using the proposed robust componentequation (3.1) and algorithm 1, the size of the robust component is recorded throughout the simulation and presented in figure 10.

Since link removal is probabilistic, a Monte–Carlo simulation was conducted to capture the overall performance. The Monte–Carlo simulation allows the generation of different link failure scenarios and produces more comprehensive and accurate results. Here, ϕ is increased in intervals of 1%, and at each ϕ, 100 simulations were conducted. Figure 10 shows a two-phase transition in the RCS. First, we need to note that robustness variation amplifies in the range of [0, 0.49], which results from the uncertainty in links' exposure to natural hazards. This is because 48% of the nodes on the network are exposed to earthquake-induced hazards, i.e. landslide, liquefaction, and bridge failure. Based on the

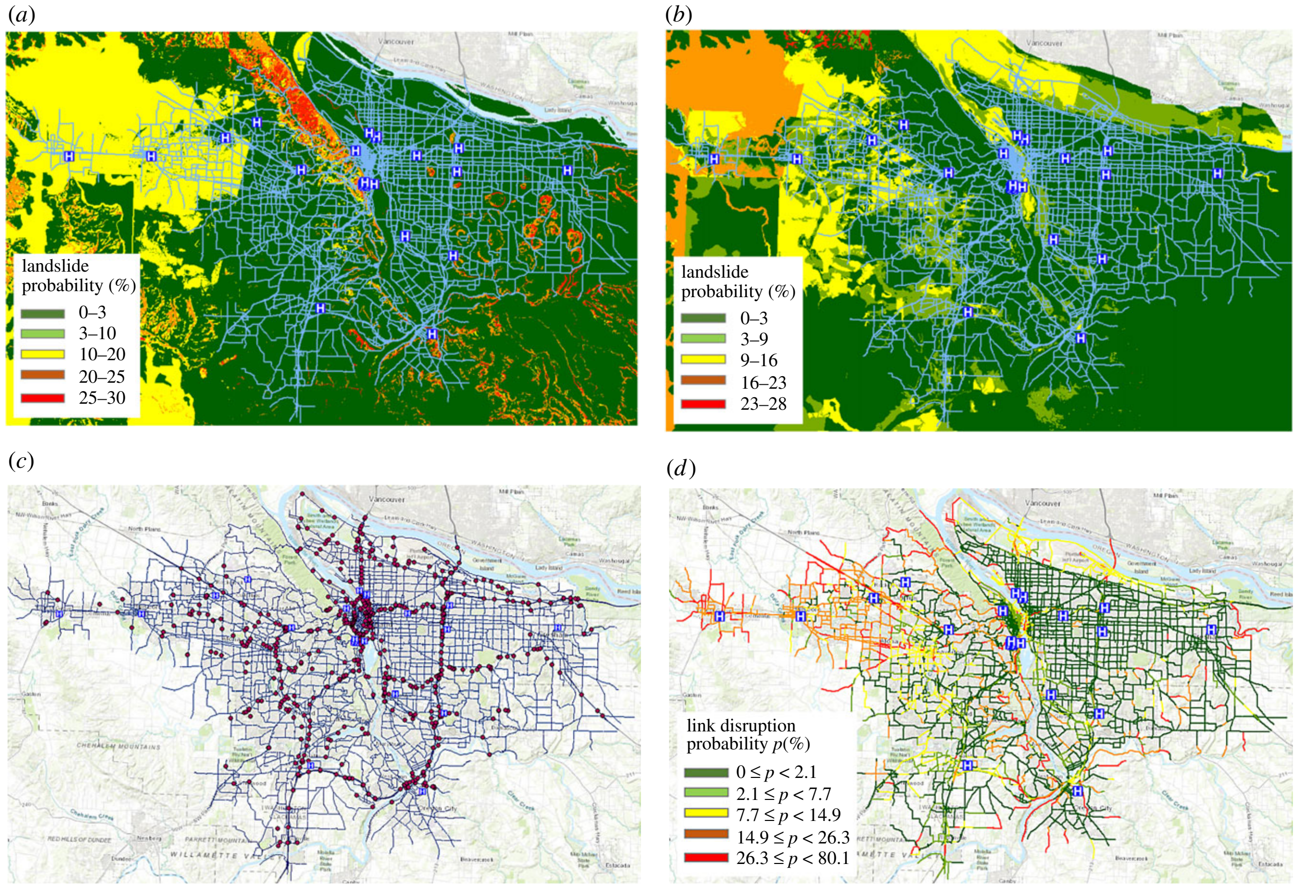

Figure 8. Geographic exposure of Portland Metro transportation routes to natural hazards. (*a*) Landslide probability. (*b*) Liquefaction probability. (*c*) Bridge location. (*d*) Link disruption probability integration illustration. (Online version in colour.)

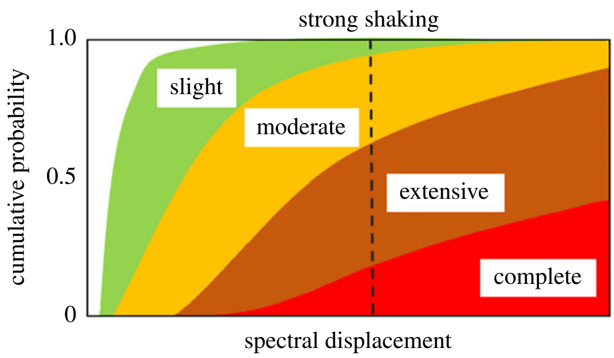

Figure 9. Bridge fragility curve illustration. (Online version in colour.)

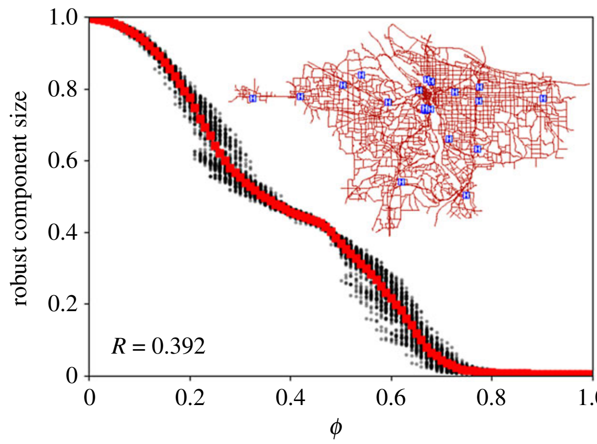

Figure 10. Network robustness of Portland Metro network under earthquake-induced probabilistic failure. Black dots represent the 100 simulation conducted, and red squares represent the average performance. (Online version in colour.)

generating scheme of probabilistic failure, the links exposed to these hazards are more likely to be removed from the network. Therefore, the simulation first converges at $\phi = 0.49$. The high variance at the range of $\phi \in [0.18, 0.38]$ suggests that the impact of earthquakes at this scale is hard to predict. The corresponding hazard mitigation plan and post-disaster recovery effort should prepare for the worst scenario so that post-earthquake access to hospitals can be maintained at its highest level. Similarly, when all the hazard-prone links are removed, the percolation at a range of $\phi \in [0.49, 1.0]$ is analogous to the random failure on the road network as the rest of the links have equal probability of being removed. The network achieved an overall robustness $R = 0.392$. Although network failure is less likely to reach 80%, monitoring of RCS percolation helps to evaluate network access to hospitals in the face of an M9.0 earthquake and in devising mitigation strategies accordingly.

4.2. Robustness-driven identification of critical hospitals and future sites for new hospitals

The robustness component characterizes the contribution of the ability of critical facilities to provide resources and maintain the functionality of the network. Removing one critical

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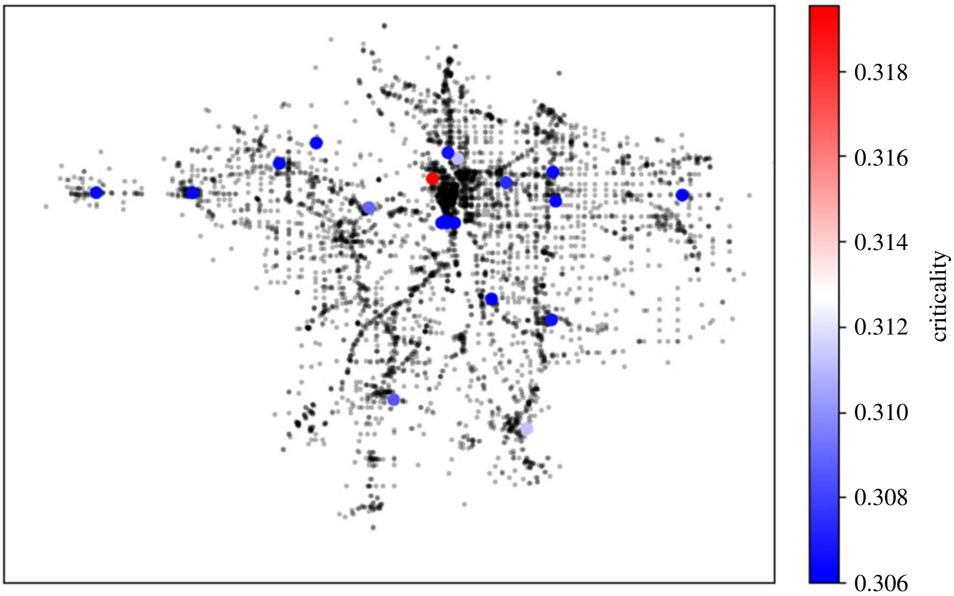

Figure 11. Hospital criticality under earthquake-induced failure. (Online version in colour.)

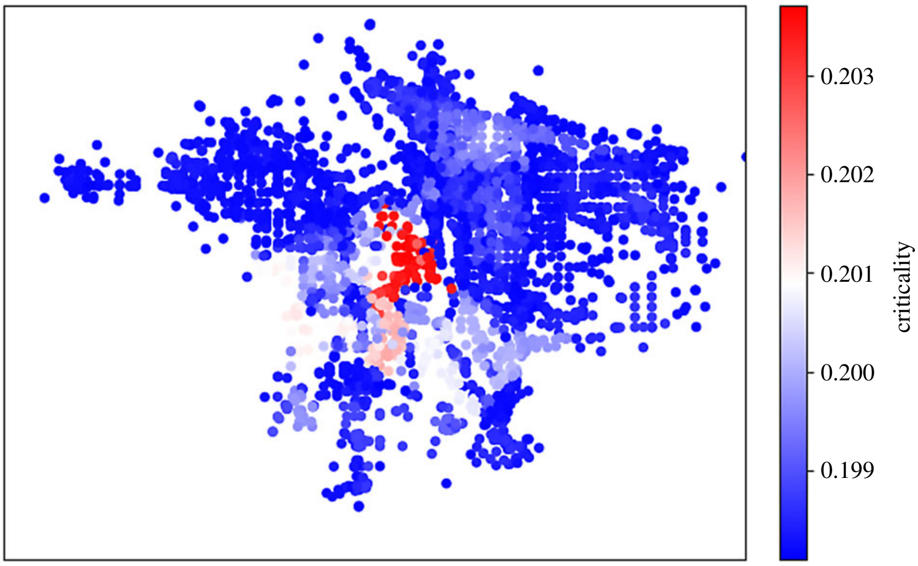

Figure 12. Identified optimal location for the future hospital. (Online version in colour.)

facility will lead to the degradation of robustness performance. This decrease in robustness, in another way, highlights the importance of maintaining network access to the critical facilities and enhancing network robustness, and it can be used to identify a component's criticality. The lower the robustness (the higher the vulnerability) when the identified critical facility is missing, the more critical it is to the network's robustness. Using this logic, we iterated through the list of hospitals in the Portland Metro region and identified the most critical hospital in terms of ensuring network access to EMS after an earthquake.

Figure 11 shows the hospital's criticality when the selected hospital is removed. The nodes represent the crossing of the roads, and the colour indicates the criticality of the node to the overall network robustness. We can observe that the hospital located in northern downtown (Legacy Good Samaritan) is the most critical. This makes sense because figure 8*a* shows that Legacy Good Samaritan is located at a region (Portland hill fault) that is highly prone to landslides, and that its failure will lead to a severe shortage of medical care in the surrounding neighbourhoods. In designing a critical infrastructure protection plan, the route to Legacy Good Samaritan should be retrofitted to make sure that part of the community can have access to the hospital.

Network robustness if a new node is included (i.e. a new hospital is built) can be used to determine the significance of the node. Iterating through the network, figure 12 presents the identified optimal placement of a future hospital. The nodes are the intersections of roads, and colour identifies the hot spots for the future site of the hospital. As we can observe, southwest of Portland is one potential location for a future hospital. This is because figure 7 shows that no hospital is built in this region. Despite the fact that the southwest is an earthquake-prone area, the construction of a new hospital that can withstand high magnitude earthquakes will enhance network robustness.

5. Conclusion

Post-disaster network access to a critical facility can significantly impact the robustness of a network. In this paper, we proposed a robust component measurement using a percolation simulation framework to assess network robustness by encapsulating the impact of network access to critical facilities. Two types of network are used: the random network and the Portland Metro road network. The results demonstrate that without considering network access to critical facilities, the conventional giant component will falsely estimate network robustness and lead to inefficient hazard mitigate strategies. In addition, increasing the number of critical facilities will enhance network robustness, but the marginal benefit decreases after a certain threshold is reached. Also, when the number of critical facilities reaches 1000, Portland Metro network shows a very similar robustness performance to a random network of the same size and mean degree.

Rather than random failures, network disruptions in real life are often influenced by the geometric features of the network. Therefore, we conducted a probabilistic earthquake-induced failure of the Portland Metro road network and used hospitals as a case study. The percolation of RCS shows a two-phase transition decided by the proportion of the links exposed to earthquake-induced hazards. The robustness shows great variation between $\phi \in [0.18,\ 0.38]$ and $\phi \in [0.5,\ 0.8]$. The depicted transition in robustness can help us to create effective mitigation plans and informed policies to minimize the loss of network access to hospitals, in different scenarios under an M9.0 earthquake.

To transform the research findings to network design and inform the stakeholders about critical infrastructure protection, we used the current framework in devising a strategy to protect existing infrastructures and to allocate resources on newly built infrastructure. To maximize network robustness, we iterated through the hospitals and derived the robustness that reveals the criticality of each hospital. We found that Legacy Good Samaritan is the most critical hospital for maintaining network robustness and should be particularly protected. Furthermore, we also identified the optimal placement for a future hospital. We identified the region that will enhance network robustness by providing community access to a hospital. Through the simulation, we identified that southwest of Portland city centre is the potential location for future hospital facilities.

6. Discussion

Limited access to critical facilities will make a poorly connected network vulnerable to network disruption. This suggests that the number and location of critical facilities can significantly influence network robustness. Therefore, network robustness should be characterized based on interdependencies with critical facilities and not as an inherent property of the transportation network alone. In this paper, we incorporated post-disaster access to critical facilities into network robustness to accurately assess the network condition, and to provide future critical infrastructure protection strategies and new development schemes.

It is worth mentioning that the simultaneous link-removal on the network is to approximate network disruption resulting from catastrophes such as an M9.0 earthquake or flooding like Hurricane Harvey. The network will suffer from extensive loss of connectivity due to the link failure. To counter such damage on the network, different strategies can be applied, for example, emergency response planning, evaluation of the location of additional critical facilities, and infrastructure prioritization. The proposed simulation framework can not only assess network robustness in consideration of post-disaster access to critical facilities but also can identify critical infrastructure and future infrastructure sites. The results show us the infrastructure criticality hot spot on a network. With this information, we can emphasize hazard mitigation planning in the critical areas, prioritize protection of critical facilities, set up a temporary emergency response centre for effective post-disaster recovery and build a new facility or relocate existing facilities to mitigate the disastrous effect.

Network robustness can be analysed via a number of different approaches. However, comprehensive empirical data are hard to obtain, which limits the implementation of a majority of the methods. This paper enables an assessment of network robustness in considering post-disaster network access to critical facilities through the use of topological and geometrical data. The parameter in calculating robustness involves network structure, critical facilities, and natural hazards mapping, which can all be obtained from agencies and state departments of transportation. For example, spatial networks from GIS files can be extracted into node/link graphs. Hazards such as the hurricanes, ice-storms and flooding can also be explored by calculating the link exposure to disruption through the use of the proposed simulation framework. Despite the use of hospitals as the representative critical facilities, other types of critical infrastructure can also be investigated in the future, such as material warehouses, resource repositories and equipment centres. Therefore, the proposed robust component can be generalized in a wide range of scenarios and help cities to evaluate the robustness of their road networks. Dong *et al.* [32] discovered that most cities share very similar road structures using a giant component as the robustness metric. Therefore, although the exact percolation transition threshold would vary, the robustness patterns are very similar. Beyond the road network, the proposed assessment framework can be also applied to other critical infrastructure networks such as electricity networks or water distribution networks. For example, power lines have to connect to the generator and distributor in order to maintain functionality. Failure of the water pumps would lead to water shortage in the communities. These phenomena can all be investigated through the proposed robust component-based percolation modelling approach.

Transportation planning can not only identify the existing or future critical facilities and inform resource allocation but can also be beneficial by applying a robustness component lens to assess post-disaster accessibility to resources and services. For example, in the event of urban flooding, flooded neighbourhoods will certainly lose transportation to essential services such as food and pharmacy. On the other hand, neighbourhoods that survived flooding may also be isolated from the critical services as roads become inundated. The proposed metric and framework allow us not only to measure the direct impact of network disruption but also to identify the network components that are indirectly affected by disasters. We can thus identify the critical roads for future infrastructure development and hazards mitigation planning to alleviate the societal impact of disastrous network disruption. In addition, different subpopulations of a community use, access, and rely on the infrastructure and respond to disaster impacts in different ways. The proposed infrastructure network robustness assessment framework can also be combined with social vulnerability to identify the focal area for urban planning and critical infrastructure protection.

Data accessibility. The network data used in this paper can be obtained from OpenStreetMap, and the natural hazards data can be obtained from OHELP (https://ohelp.oregonstate.edu/).

Authors' contributions. S.D. and H.W. conceived the study; S.D collected and analysed the data, conducted the experiment, and drafted the manuscript; H.W. and J.G. provided constructive comments on the methodology and paper revision; H.W., J.G. and A.M. advised and edited the study. All authors participated in the design of the study, helped draft the manuscript and gave final approval for publication.

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