Exploration of tensor decomposition applied to commercial building baseline estimation

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***Abstract*—Baseline estimation is a critical task for commercial buildings that participate in demand response programs and need to assess the impact of their strategies. The problem is to predict what the power profile would have been had the demand response event not taken place. This paper explores the use of tensor decomposition in baseline estimation. We apply the method to submetered fan power data from demand response experiments that were run to assess a fast demand response strategy expected to primarily impact the fans. Baselining this fan power data is critical for evaluating the results, but doing so presents new challenges not readily addressed by existing techniques designed primarily for baselining whole building electric loads. We find that tensor decomposition of the fan power data identifies components that capture both dominant daily patterns and demand response events, and that are generally more interpretable than those found by principal component analysis. We conclude by discussing how these components and related techniques can aid in developing new baseline models.**

***Index Terms*—Baseline estimation, demand response, principal component analysis, tensor decompositions.**

I. Introduction

Baselining power profiles is a critical task for commercial buildings participating in demand response programs, which incentivize changes in building electricity consumption to improve grid reliability and economics. The problem is to predict what the power profile would have been had the demand response event not occurred. Baselines are generally used to assess the impact of demand response strategies and are further used in the case of incentive-based demand response programs to calculate financial compensation for participants [1].

Traditionally, baseline models for demand response predict whole building electric loads and fall into three categories. Averaging models take the mean load of several recent baseline days [2], i.e., days without demand response. Regression models use historical data to fit a relationship between some explanatory variables and the load [3], [4]. Control group models use data mining to find a cluster of load curves from baseline days of the same building and/or other buildings to use in baselining [5]–[7]. Some models further incorporate multiplicative or additive adjustments [2].

We consider the problem of baseline modeling for a fast demand response strategy that is expected to primarily impact the fans in the Heating, Ventilation, and Air Conditioning (HVAC) systems [8]. Hence, our goal is to baseline fan power data, but doing so presents new challenges. For example, we have found that fan power does not depend strongly on outdoor temperature, so regression against this variable is less effective. Fan power during demand response events is baselined in [9] by linearly interpolating based on measurements just prior to and some settling time after each event, and [8] employs a low pass filter for signal bandwidth separation.

This paper explores using the tensor decomposition of fan power data to discover dominant patterns that reveal typical behavior and that can inform baselines. We consider submetered fan power data collected at the University of Michigan to assess a demand response strategy [10]. The data naturally forms a three-dimensional time $\times$ fan $\times$ day array, i.e., a third order tensor, making tensor decomposition a natural option. Tensor decomposition is new to baseline modeling, but has been previously used for power system energy consumption breakdowns [11] and model reduction [12]. Its matrix counterpart, principal component analysis (PCA) [13], has also been applied to the highly related problem of short-term load forecasting. Intra-day load variation is captured by PCA in [14] to reduce model complexity, and [15], [16] use PCA to reduce the number of variables in their regression models.

We specifically consider the canonical polyadic (CP) tensor decomposition. Our investigation finds that CP decomposition is able to identify meaningful components in the fan power data that capture both dominant daily patterns and demand response events. Furthermore, the decompositions provide granular per-fan insights into fan power since they work from the full data, rather than reducing to a matrix, e.g., by using total fan power. Comparing with a PCA of total fan power, we also find that the components found by CP decomposition are generally more interpretable, likely because CP does not require factors to be orthogonal. We close the paper with a discussion of how these components and other related tensor techniques might be used for developing new fan power baseline models.

II. Experiments and data

Fan power data were collected at the Rackham Building on the University of Michigan campus in Ann Arbor, MI over the course of 90 days from August 1, 2017 to October 29, 2017 to

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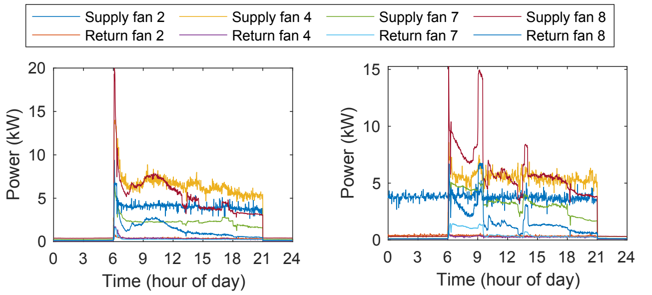

(a) August 8, 2017 (b) September 11, 2017

Fig. 1: Fan power traces from August 8, 2017 (a non-event day) and September 11, 2017 (an event day). Events at 9:00-10:00 and 13:00-14:00 can be seen in (b).

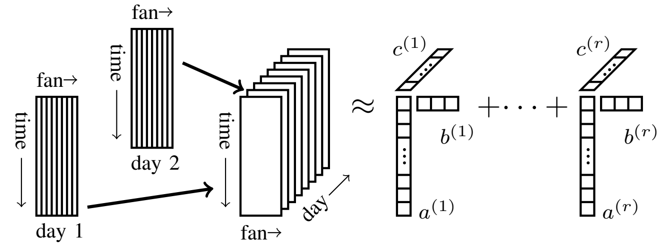

Fig. 2: Forming a three-way tensor from fan power data and its rank r tensor decomposition.

assess the impact of a demand response strategy. Each day has measurements taken by sensors on 8 fans (4 supply, 4 return) at a sampling rate of one measurement per minute. The Rackham Building is a $157\,957\,\mathrm{ft}^2$ office/auditorium space with 8 AHUs, 8 supply fans, and 8 return fans. Its 2016 energy consumption was $972\,\mathrm{MWh}$ with a peak demand of $226\,\mathrm{kW}$.

Researchers carried out demand response experiments on 16 September weekdays. Each event day consists of two events, one at 9:00-10:00 and another at 13:00-14:00. In each event, 109 temperature setpoints in the 4 instrumented AHU zones were modulated in one of two ways:

1) the setpoints are decreased below their usual value for 30 minutes then increased symmetrically above their usual value for 30 minutes, or
2) the reverse, i.e., setpoints are increased then decreased.

Fig. 1 plots the 8 fan power traces for August 8, 2017 (a non-event day) and September 11, 2017 (an event day); events can be seen as spikes in Fig. 1b.

The goal was to assess whether buildings consume more energy when demand response is used to shift load over short ($< 1\,\mathrm{hr}$) timescales. A baseline is necessary to calculate the energy impact of the demand response event. Previous analysis with simple baseline models found that buildings generally consume more energy when doing this type of short time scale shifting, but the quantitative results are imprecise since the baseline model error is potentially large [10]. This paper explores tensor methods as a tool for developing new, more reliable models. See [10] for further discussion of the experiments and data collected.

III. Overview of Tensor Decompositions

In this paper, a *tensor* refers to a multi-dimensional array. For example, the fan power data is naturally expressed as a three-dimensional time $\times$ fan $\times$ day array, as shown in Fig. 2. Namely, it forms a third *order* or three-*way* tensor with three *modes*: a) time, b) fan, and c) day. A matrix can be formed from the same data, e.g., by vertically concatenating the matrices for each day to form a time/day $\times$ fan matrix. Doing so makes it possible to apply matrix methods such as PCA, but this representation does not capture correlation across days as naturally. One could also sum across the fans to form a time $\times$ day matrix of total fan powers, but this representation loses the per-fan breakdown of power usage. Tensor representations instead capture and preserve these underlying structures in the data. We focus on third order data tensors in this paper, but the techniques we discuss generalize to arbitrary order tensors.

Power usage over the course of the day is likely to share some common overall patterns across both fans and days, and these patterns may be useful for estimating baselines. The canonical polyadic (CP) tensor decomposition[1] finds such underlying patterns by fitting the data tensor $\mathcal{X} \in \mathbb{R}^{n_1 \times n_2 \times n_3}$ with a rank r tensor, i.e., a sum of r outer products

$$\mathcal{M} = a^{(1)} \circ b^{(1)} \circ c^{(1)} + \cdots + a^{(r)} \circ b^{(r)} \circ c^{(r)}, \qquad (1)$$

where

- $a^{(1)}, \ldots, a^{(r)} \in \mathbb{R}^{n_1}$ are the r factors for mode one,
- $b^{(1)}, \ldots, b^{(r)} \in \mathbb{R}^{n_2}$ are the r factors for mode two,
- $c^{(1)}, \ldots, c^{(r)} \in \mathbb{R}^{n_3}$ are the r factors for mode three,

and $\circ$ denotes an outer product, as illustrated in Fig. 2. Written in terms of its entries, (1) is equivalently

$$\mathcal{M}_{ijk} = a_i^{(1)} b_j^{(1)} c_k^{(1)} + \cdots + a_i^{(r)} b_j^{(r)} c_k^{(r)}.$$

We call each outer product a *component*, and $\mathcal{M}$ is rank (at most) r since it can be written as the sum of r components. Each component of $\mathcal{M}$ captures correlations across all three modes simultaneously. For example, *fibers* of the first component along the time mode are of the form

$$(a^{(1)} \circ b^{(1)} \circ c^{(1)})_{:jk} = a^{(1)} b_j^{(1)} c_k^{(1)} \in \mathbb{R}^{n_1},$$

i.e., they are a single temporal pattern $a^{(1)}$ modulated by fan weights $b_j^{(1)}$ and day weights $c_k^{(1)}$. Likewise, fan mode fibers are a single fan profile $b^{(1)}$ modulated by time weights $a_i^{(1)}$ and day weights $c_k^{(1)}$, and similarly for day mode fibers.

CP decomposition seeks a *least-squares* fit to $\mathcal{X}$, namely a low-rank tensor $\mathcal{M}$ that minimizes

$$\|\mathcal{X} - \mathcal{M}\|_F^2 = \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} \sum_{k=1}^{n_3} (\mathcal{X}_{ijk} - \mathcal{M}_{ijk})^2. \qquad (2)$$

CP decompositions are typically computed via an alternating least-squares algorithm. Minimizing (2) with respect to the factors is a nonconvex optimization problem, so running alternating least-squares from different initializations can produce

[1]Strictly speaking, this paper uses *approximate* tensor decompositions.

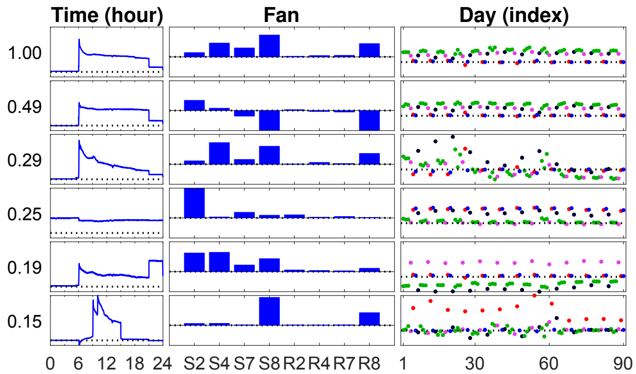

Fig. 3: Factors from rank-6 CP tensor decomposition of 3-way power tensor. In the day factors, Fridays are purple, Saturdays are red, Sundays are blue, Mondays are dark blue, and all other days are green.

different factors (e.g., due to local minima). One often runs several times with random initializations to verify that the patterns identified appear consistently, finally taking the run with the best fit. We use 20 runs with the implementation of alternating least squares in the Tensor Toolbox for Matlab [17], [18]. See [19, Section 3] for a further discussion and overview of CP tensor decompositions.

IV. Initial CP decomposition compared to PCA

This section applies CP tensor decomposition to the full 1440 time steps $\times$ 8 fans $\times$ 90 days data tensor $\mathcal{X}$ described in Section II. This tensor contains both days with and without demand response events, and we carry out a rank-6 CP decomposition. For this data, lower rank decompositions captured too few interesting patterns; higher rank decompositions capture more diverse behaviors but eventually begin to find redundant factors. A rank-6 decomposition yields the factors $a^{(1)}, \ldots, a^{(6)} \in \mathbb{R}^{1440}$, $b^{(1)}, \ldots, b^{(6)} \in \mathbb{R}^{8}$, and $c^{(1)}, \ldots, c^{(6)} \in \mathbb{R}^{90}$, shown in Fig. 3 as the first, second, and third columns of plots, respectively. Each row of plots in Fig. 3 corresponds to one of the six components. Since $a^{(1)}, \ldots, a^{(6)}$ are time factors, we plot their 1440 values versus the corresponding times (0 and 24 correspond to midnight, and 12 is noon). Likewise, $b^{(1)}, \ldots, b^{(6)}$ are fan factors so we plot their 8 values as bars labeled by fan name. Finally, $c^{(1)}, \ldots, c^{(6)}$ are day factors so we plot their 90 values as scatter plots. The relative scaling of factors in each component is arbitrary, e.g., doubling $a^{(\ell)}$ and halving $b^{(\ell)}$ has no overall impact, so we normalize all factors to have unit norm for plotting and print each component's relative magnitude on left.

The components reveal some interesting patterns. Looking at the fan and day factors for the first two components indicate that they capture weekday patterns across fans; negative values in the second fan factor result in some cancellation when added to the first component in (1). Component 3 appears to differentiate August and September from October. Component 4 captures a pattern arising Saturday-Monday that primarily involves supply fan 2, and component 5 separates Fridays from the other weekdays. Finally, component 6 captures a pattern in zone 8 that appears primarily on Saturdays between 9:00am and 3:00pm in August-September.

In general, the time factors are fairly continuous as is natural to expect for temporal traces. The channel factors indicate that supply fans typically consume more power than return fans, with the notable exception of return fan 8. The day factors show general separation between weekdays and weekends. Notably, the tensor decomposition identified these inherent properties from the data alone. The method is otherwise "blind" to which days correspond to weekdays and was not explicitly regularized to encourage any smoothness in time. The factors provide a view into the underlying patterns of the data, yielding insights about fan power behavior that can inform baseline estimates.

Conventionally, one might identify dominant patterns like these from total fan power (a 1440 times $\times$ 90 days matrix) by using matrix methods such as PCA. Fig. 4 shows the first six principal components for total fan power; we use singular value decomposition (without standardization) and show the scaled singular vectors in the same way as Fig. 3. Component 1 again captures an overall weekday trend. Components 2 and 3 appear to roughly correspond with a combination of components 5 and 6 in Fig. 3. They separate Fridays from the other weekdays like component 5 in Fig. 3 with similar activity after 9:00pm, and they capture activity between 9:00am and 3:00pm on Saturdays like component 6 in Fig. 3. Component 4 reflects the demand response events in its time factor, but the extent of its day factor is not limited to the event days. The interpretation of components 5 and 6 is not as clear, but they seem to be somewhat more active August-September. For this data, the PCA factors are generally more challenging to interpret, likely because PCA requires factors to be orthogonal. CP decomposition, on the other hand, does not enforce any constraints on the factors and hence can more flexibly find interpretable patterns. Furthermore, these PCA factors do not provide per-fan insights since they come from total fan power. One can instead consider PCA factors from a concatenated time/day $\times$ fan matrix, but in our experiments these factors were also less interpretable than those from CP. CP decomposition gives granular insights while utilizing the natural structure of the data.

V. Baseline & demand response CP decompositions

Section IV applied tensor decomposition to the full data that contains both days with and without demand response events. Next we carry out separate rank-6 CP tensor decompositions on data from just baseline days and just event days. Fig. 5 shows the factors obtained for the 1440 time steps $\times$ 8 fans $\times$ 74 days tensor formed from only the 74 days without any events. This provides a view into baseline only behavior. The components capture similar patterns as those in Fig. 3. Namely, component 1 roughly corresponds to a combination of components 1 and 2 in Fig. 3, and a combination of components 4 and 6 seems to correspond to component 3 in Fig. 3. Finally, components 2, 3 and 5 roughly correspond to components 4, 5 and 6, respectively, in Fig. 3. This similarity

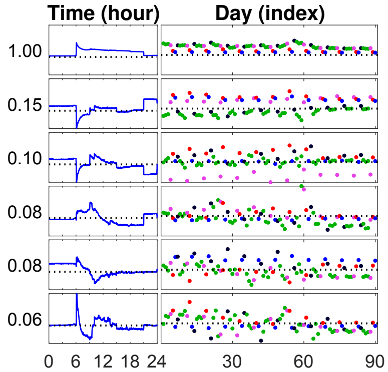

Fig. 4: First six principal components for total fan power. Days are colored as in Fig. 3.

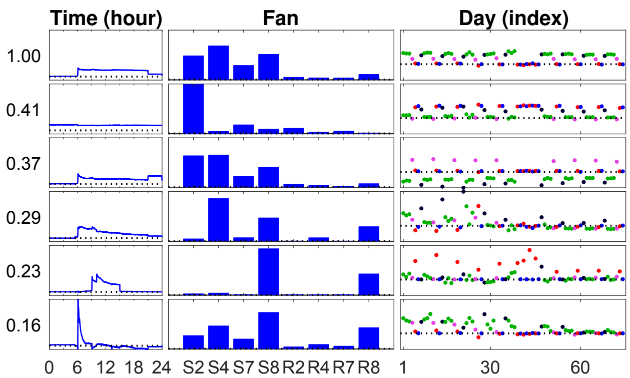

Fig. 5: Factors from rank-6 CP tensor decomposition of 3-way baseline days power tensor. Days are colored as in Fig. 3.

is likely because most days are baseline days (only 16 of 90 are event days) and the variations among them (e.g., weekday versus weekend) appear to be fairly dominant.

Fig. 6 shows the factors obtained for the 1440 time steps $\times$ 8 fans $\times$ 16 days tensor formed from only the 16 event days in September (all weekdays). Component 1 roughly corresponds to component 1 in Fig. 5 and captures overall power behavior in the data. A combination of components 2 and 3 appears to capture other temporal shapes that varied over the course of September. Like component 3 in Fig. 5, component 4 appears to separate Fridays from the other weekdays with activity after 9:00pm. Similarly, component 6 separates Mondays from the other weekdays.

Finally, component 5 appears to primarily capture the demand response events. The time factor shows a down-up event at 9:00am followed by an up-down event at 1:00pm. For the day factor of this component, we color each point by whether the events that day were down-up followed by up-down (red) or vice versa (blue). Notably, this factor cleanly identifies and separates the two experimental protocols even though the decomposition is unaware of them. Moreover, the fan factor indicates that zone 8 fans respond the most, followed by zone 7 and then zone 4.

In both cases, the tensor decompositions identify patterns in the data that can be used to inform baseline estimates.

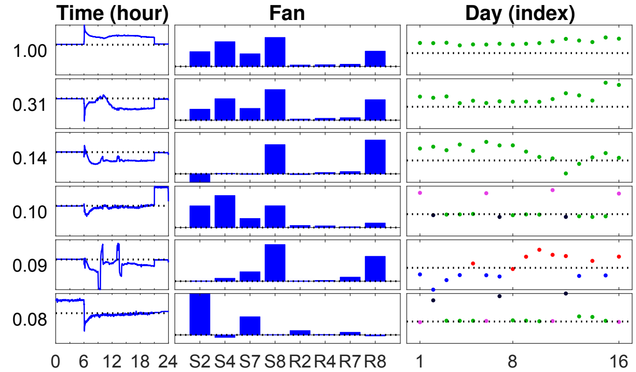

Fig. 6: Factors from rank-6 CP tensor decomposition of 3-way event days power tensor. Day factors for components 4 and 6 are colored as in Fig. 3. In component 5, down-up followed by up-down days are red, and up-down followed by down-up days are blue. Note how this component cleanly splits them.

VI. Discussion

This paper explored the use of tensor decomposition for identifying dominant components in fan power data. We found that the components can capture dominant daily patterns and demand response events, including how much each fan contributes. The extracted factors lend new insights that can be used to guide the development of new baseline models. Baselines might also be formed by regressing measurements outside event windows against the factors from baseline days, i.e., by using them as compact representations of baseline data.

Another possibility is to approach baseline estimation as a tensor completion problem, where measurements in the event window are considered missing. Since low-rank CP decomposition appears to effectively capture interpretable patterns and structure in fan power data, one might impute event entries by seeking a low-rank tensor that closely matches fan power data on the non-event entries. Doing so corresponds to minimizing a version of (2) that sums only over the event entries.

One can also try other tensor decompositions. For example, non-negative CP decompositions [19, Section 5.6] can be even more interpretable (since everything adds) and generalized CP decompositions [20] can be configured for outlier-robustness.

In all these cases, choosing the rank is challenging. Note, e.g., that CP decompositions do not nest; rank-6 factors may not be part of higher rank decompositions. This paper found rank-6 decompositions to extract interesting factors, but one might instead select for sufficient fit, i.e., sufficiently small residual (2). Developing general rank selection strategies for baseline models is a topic for future work.

Finally, further work is needed to assess how incorporating tensor methods in baseline estimation may impact analyses that use these estimates, e.g., to calculate compensation in incentive-based demand response programs.

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