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A Method for Ensuring a Load Aggregator's Power Deviations Are Safe for Distribution Networks☆

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ABSTRACT

This paper proposes a method for constraining the control actions of third-party aggregators to ensure safe operation of the distribution network. We design a safety constraint that limits the size of the deviations in power that an aggregator can cause across a network. The constraint's upper limit is the size of the minimum-sized vector of deviations that is unsafe for the network, which is computed by solving a set of optimization problems. We propose two versions of the safety constraint, based on the 2-norm and 1-norm, and find neither is guaranteed to be less conservative than the other. We also derive conditions under which an optimization problem can be eliminated from the set of problems that are necessary to solve. We apply these conditions in a case study and reduce the number of problems by 89% for the test network and 67% for the test network with capacitor banks connected.

1. Introduction

Load aggregations can provide valuable services to the bulk power system, such as frequency regulation, ramping, and spinning reserves [1]. These balancing services will be increasingly important as the share of intermittent renewable generation increases in the generation mix. Residential-load aggregators are able to participate at the scale required by wholesale markets by aggregating thousands of flexible loads, such as air conditioners, water heaters, and electric vehicles.

Many regions in the U.S. allow third-party load aggregators to provide balancing services. Third-party aggregators are distinct from distribution operators and do not have access to distribution network models or measurements. Without feedback from the operator, an aggregator does not know how its control actions affect distribution operation and is unable to adjust its actions to prevent operational issues. Unsafe operation is of particular concern when a large portion of a network's loads are controlled by a third-party aggregator. For example, the distribution simulation study [2] found that an aggregation of residential air conditioners, controlled to track a regulation signal, can result in under-voltages.

The objective of this paper is to develop a method that ensures a third-party aggregator's control actions are safe for distribution networks without requiring the operator and aggregator to share private information. We assume that, to maintain a competitive advantage, third-party aggregators prefer to keep their control algorithms private. Distribution operators protect the privacy of their consumers by keeping network models and load measurements private.

Most prior research on aggregate load control for wholesale services either does not consider distribution network safety or does not consider the privacy needs of third-party aggregators. Many proposed strategies are "grid-agnostic"; these strategies ignore network constraints and cannot ensure safe distribution operation (e.g., [3,4]). A few "grid-safe" strategies have been developed that ensure safe distribution operation. In [5], an AC-OPF is solved to safely provide load frequency control with an aggregation of loads. In [6,7], a gradient-based algorithm that incorporates real-time measurements from the distribution network is proposed; the algorithm controls distributed energy resources to track a time-varying power setpoint while respecting network constraints. However, the strategies in [5–7] do not enable a third-party aggregator to use its own private control algorithm; moreover, the strategies rely on a third-party aggregator having substantial information about the distribution network, which may not be acceptable from a privacy and security perspective.

A few recently proposed strategies are both grid-safe and suitable for a third-party aggregator. The optimization-based method in [8] provides an aggregator with an inner approximation of the safe set of real-power deviations at each bus in a network; future work is necessary to compare the relative conservativeness of this paper's safe set

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approximation with that of [8]. The method in [9] certifies whether a distribution network will operate safely under any set of possible power injections. If a network is certified safe, then any control actions by a third-party aggregator will be safe. However, this approach leaves open the problem for networks that cannot be certified as safe. In our prior work [10], we propose a strategy in which a distribution operator blocks an aggregator's commands to loads when the commands would result in unsafe operation. A drawback of this strategy is that the aggregator is not given an explicit constraint; instead, the aggregator must estimate the operator's blocking behavior to improve its control performance.

In this paper, we design and compute an explicit "safety constraint" that ensures safe distribution operation by limiting the size of the vector of power deviations an aggregator can cause across all load buses in the network, where deviations are with respect to a nominal operating point. The design of the safety constraint is conservative and ensures that the aggregator's vector of deviations is always smaller than the minimum-sized vector of deviations that is unsafe for the network, where size is determined by a vector norm. We compute the safety constraint's limit by solving a set of optimization problems; each problem is for a particular bus in the network and finds the minimum-sized vector of deviations that causes an unsafe voltage at that bus. The minimum size of these minimum-sized vectors is the safety constraint's limit. One benefit of the proposed method is that it can be implemented in a privacy protecting manner: the distribution operator determines the safety constraint given its private network information, and the aggregator ensures the outcome of its private control algorithm satisfies the constraint.

The main contributions of this paper are as follows. First, we propose a method of constraining a third-party aggregator's control actions so that the actions are safe for distribution networks without prescribing a particular control algorithm for the aggregator. Second, through analysis and simulation, we compare the conservativeness of two versions of the safety constraint – one that measures the size of the vector of deviations with a 2-norm, and the other with a 1-norm. Third, we propose and prove two propositions that enable a substantial reduction in the number of optimization problems that must be solved to compute a safety constraint and therefore a reduction in overall computation time.

2. Methods

2.1. Designing the Safety Constraint

There are four desired criteria for the safety constraint: 1) it should ensure the aggregator's actions will not cause unsafe distribution operation, 2) it should not overly restrict the aggregator's control actions, 3) an operator should be able to calculate the constraint without the aggregator's private information, and 4) the aggregator should be able to adhere to the constraint without the operator's private information.

We design the safety constraint to prioritize safety (criteria 1) over the aggregator's range of control (criteria 2). The constraint takes the general form

$$||\boldsymbol{\Delta P}^{c}|| < ||\boldsymbol{\alpha}^{\min}||, \quad (1)$$

where $||\cdot||$ represents the 1-norm or 2-norm, variable $\Delta\boldsymbol{P}^{c}$ is the vector of controllable loads' power deviations at each bus, and parameter $\boldsymbol{\alpha}^{\min}$ is the minimum-sized vector of power deviations that causes unsafe operation somewhere on the network. For brevity, we refer to $\boldsymbol{\alpha}^{\min}$ as the minimum unsafe vector of deviations. The controllable loads' power deviations are relative to the power the aggregation would have consumed in the absence of control, referred to as the aggregation's "baseline".

The safety constraint (1) ensures network safety by the definition of $\boldsymbol{\alpha}^{\min}$. The constraint is conservative; that is to say, some *safe* values of $\Delta\boldsymbol{P}^{c}$ will not satisfy the constraint. The benefit of the constraint is that if we are able to compute the minimum unsafe vector of deviations $\boldsymbol{\alpha}^{\min}$, then, by definition, the constraint will guarantee safe distribution operation.

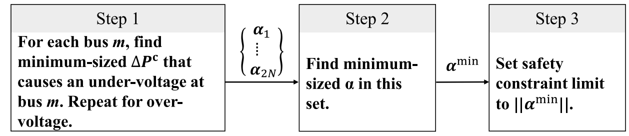

Fig. 1. Process for computing the safety constraint's limit. Vector $\boldsymbol{\alpha}_m$ is equal to the minimum-sized $\Delta\boldsymbol{P}^{c}$ (vector of deviations across all load buses) that causes an under-voltage at bus m, and $\boldsymbol{\alpha}_{m+N}$ is the minimum-sized $\Delta\boldsymbol{P}^{c}$ that causes an over-voltage at bus m.

2.2. Computing the Safety Constraint's Limit

2.2.1. Overview

To determine the safety constraint's limit, we must find the minimum unsafe vector of deviations for a given network at a nominal operating point. We reduce the scope of the problem by considering only the most likely modes of unsafe operation on a network which, for this application, are out-of-range voltage magnitudes [2]. The voltage constraint for bus i is $\underline{V} \le V_i \le \overline{V}$, where V_i is the voltage magnitude, $\overline{V}$ = 1.05 p.u., and $\underline{V}$= 0.95 p.u. Extending our methods to include all network constraints (e.g., over-currents on lines) is future work.

Fig. 1 shows the three main steps to compute the safety constraint limit. In step 1, we find candidates for $\boldsymbol{\alpha}^{\min}$ by solving two optimization problems for each bus. The first problem searches for $\boldsymbol{\alpha}_m$ the minimum-sized $\Delta\boldsymbol{P}^{c}$ that causes an under-voltage at bus m, where vector size is measured by a norm, generically $||\Delta\boldsymbol{P}^{c}||$. The second problem searches for $\boldsymbol{\alpha}_{m+N}$ the minimum-sized $\Delta\boldsymbol{P}^{c}$ that causes an over-voltage at bus m. In step 2, we set $\boldsymbol{\alpha}^{\min}$ equal to the minimum-sized $\boldsymbol{\alpha}$ in the set $\{\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \ldots, \boldsymbol{\alpha}_{2N}\}$. In step 3, we set the safety constraint's limit (the right hand side of (1)) to the value $||\boldsymbol{\alpha}^{\min}||$.

The remainder of Section 2.2 proceeds as follows. First, we define the models that will be used in the optimization problems. Then we formulate the optimization problems used to compute the 2-norm safety constraint's limit. Finally, we formulate the problems used to compute the 1-norm safety constraint's limit.

2.2.2. Modeling

We model an N-bus distribution network with aggregator-controlled loads at each bus. We use a single-phase equivalent line model, which assumes balanced power flow and symmetric lines. The network's $N \times N$ conductance and susceptance matrices are denoted as $\boldsymbol{G}$ and $\boldsymbol{B}$. We aggregate loads at the bus-level and use a constant power model: each bus has real and reactive power consumption P_i and Q_i, respectively. We separate a bus's power consumption into two components: a controllable component (P_i^{c}, Q_i^{c}) that represents aggregator-controlled loads, and an uncontrollable component (P_i^{uc}, Q_i^{uc}) that represents all other loads. Thus, we have that $P_i = P_i^{c} + P_i^{uc}$ and $Q_i = Q_i^{c} + Q_i^{uc}$.

We model the aggregator's control actions in terms of bus-level power deviations. When providing balancing, an aggregator controls the aggregation's total deviation from baseline such that it tracks a balancing signal. In terms of bus-level power deviations, the aggregator controls loads such that the sum of real-power deviations across all buses, $\sum_{i=1}^{N} \Delta P_i^{c}(k)$, tracks the balancing signal with sufficient accuracy in every time step k. The variable ΔP_i^{c} is the deviation in P_i^{c} from the bus's baseline value $\hat{P}_i^{c}$ (i.e., $\Delta P_i^{c}(k) = P^{c}(k) - \hat{P}_i^{c}$). We assume the controllable loads have a constant power factor ζ_i, such that any deviation in real power ΔP_i^{c} is accompanied by a deviation in reactive power given by $\Delta Q_i^{c} = \Delta P_i^{c}\tan(\arccos\zeta_i)$. Finally, the deviations at each bus are naturally constrained by the physical capacities of the loads at that bus; this constraint is given by

$$\underline{P}_i^c \leq (\hat{P}_i^c + \Delta P_i^c) \leq \overline{P_i^c}, \tag{2}$$

where $\underline{P}_i^c$ and $\overline{P_i^c}$ reflect the loads' aggregate physical capacity.

2.2.3. 2-Norm Safety Constraint

In *step 1*, we find $\boldsymbol{\alpha}_m$ the minimum-sized vector of deviations (as measured by the squared 2-norm) that causes an under-voltage at bus m. We solve this problem once for each bus in the network; here the problem is shown for bus m:

$$\text{minimize} \quad \sum_{i=1}^{N} \left(\Delta P_i^c\right)^2 \tag{3a}$$

subject to

$$V_m \leq \underline{V}, \tag{3b}$$

$$\underline{P}_i^c \leq (\hat{P}_i^c + \Delta P_i^c) \leq \overline{P_i^c} \forall\ i \in \mathcal{N}, \tag{3c}$$

$$\Delta Q_i^c = \beta_i \Delta P_i^c \forall\ i \in \mathcal{N}, \tag{3d}$$

$$P_i = \Delta P_i^c + \hat{P}_i^c + P_i^{uc} \forall\ i \in \mathcal{N}, \tag{3e}$$

$$Q_i = \Delta Q_i^c + \hat{Q}_i^c + Q_i^{uc} \forall\ i \in \mathcal{N}, \tag{3f}$$

$$P_i = V_i \sum_{k=0}^{N-1} V_k (G_{ik} \cos\theta_{ik} + B_{ik} \sin\theta_{ik}) \forall\ i \in \mathcal{N}, \tag{3g}$$

$$Q_i = V_i \sum_{k=0}^{N-1} V_k (G_{ik} \sin\theta_{ik} - B_{ik} \cos\theta_{ik}) \forall\ i \in \mathcal{N}, \tag{3h}$$

$$\theta_0 = 0, \tag{3i}$$

$$V_0 = V_{set}. \tag{3j}$$

After solving (3), we set $\boldsymbol{\alpha}_m = [\Delta P_1^{c*}, \Delta P_2^{c*}, ..., \Delta P_N^{c*}]$, where * indicates the optimal solution; note that the optimal objective value is $\|\boldsymbol{\alpha}_m\|_2^2$. In the above problem, the set $\mathcal{N}$ is the set of all buses in the network. The decision variables are $\Delta P_i^c, \Delta Q_i^c, P_i, Q_i, V_i \ \forall \ i \in \mathcal{N}$ and $\theta_{ik} \ \forall \ i, k \in \mathcal{N}$, where θ_{ik} is the voltage angle difference between buses i and k.

The objective function (3a) and constraint (3b) are opposing forces on the size of $\Delta\boldsymbol{P}^c$: the objective function minimizes the size of the deviations, but the deviations must be large enough such that an under-voltage occurs at bus m. A deviation in power is necessary to create an under-voltage at bus m because we assume that the network's voltages are within the operational range $[\underline{V}, \overline{V}]$ at the nominal operating point.

Constraints (3c) and (3d) model the controllable loads. Constraint (3c) restricts the power deviation at each bus according to the controllable loads' physical capacities; the baseline value $\hat{P}_i^c$ is assumed known. Constraint (3d) enforces a constant power factor for controllable loads, where $\beta_i = \tan(\arccos\zeta_i)$.

Constraints (3e)-(3f) sum the controllable and uncontrollable components of the power consumption at each bus. The uncontrollable components P_i^{uc} and Q_i^{uc} are assumed known.

Constraints (3g)-(3j) model the power flow in the network and define the slack bus. Constraints (3g)-(3h) are the standard power flow equations, where the real and reactive power consumption at bus i must be balanced by the sum of all real and reactive power flows into bus i. Constraint (3j) sets the substation bus as the reference for voltage angles. Constraint (3j) fixes the substation bus's voltage magnitude as a constant; we assume the value of parameter V_{set} is set by the distribution operator (e.g., $V_{set} = 1.0$ p.u.).

We also find $\boldsymbol{\alpha}_{m+N}$ the minimum-sized vector of deviations that causes an over-voltage at bus m. We solve this problem once for each bus in the network; here the problem is shown for bus m:

$$\text{minimize} \quad \sum_{i=1}^{N} \left(\Delta P_i^c\right)^2 \tag{4a}$$

subject to constraints (3c)-(3j) and

$$V_m \geq \overline{V}. \tag{4b}$$

After solving (4), we set $\boldsymbol{\alpha}_{m+N} = [\Delta P_1^{c*}, \Delta P_2^{c*}, ..., \Delta P_N^{c*}]$.

The optimization problems (3) and (4) are non-convex because of the non-linear constraints (3g)-(3h) that model the network's AC power flow. In this paper, we use a non-linear programming solver to solve (3) and (4); the solver finds locally optimal solutions, so global optimality is not guaranteed. In future work, we plan to apply a convex relaxation to the AC power flow equations in order to identify a lower bound on the globally optimal objective value of the original problem; this lower bound will make $\boldsymbol{\alpha}_m$ conservative (i.e., smaller than or equal to the minimum-sized vector of deviations that is unsafe for bus m) but will ensure no smaller-sized unsafe vector of deviations exists.

In *step 2*, we find $\boldsymbol{\alpha}^{\min}$ the minimum-sized $\boldsymbol{\alpha}$ from the set of $\boldsymbol{\alpha}$'s found in step 1. We find $\boldsymbol{\alpha}^{\min}$ by simply comparing the problems' optimal objective values:

$$\boldsymbol{\alpha}^{\min} = \underset{\boldsymbol{\alpha}_m \in \mathcal{A}}{\arg\min} \ \|\boldsymbol{\alpha}_m\|_2^2, \tag{5}$$

where $\mathcal{A} = \{\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, ..., \boldsymbol{\alpha}_{2N}\}$.

In *step 3*, we set the limit of the 2-norm safety constraint:

$$\sum_{i=1}^{N} \left(\Delta P_i^c\right)^2 < \|\boldsymbol{\alpha}^{\min}\|_2^2. \tag{6}$$

2.2.4. 1-Norm Safety Constraint

We use the same general 3-step method to compute the limit of the 1-norm safety constraint. In *step 1*, the optimization problem formulations are identical to (3) and (4) except the objective functions are in terms of the 1-norm. As before, we solve the under-voltage problem and the over-voltage problem once for each bus in the network. The under-voltage problem for bus m is given by

$$\begin{aligned} &\text{minimize} \ \sum_{i=1}^{N} |\Delta P_i^c| \\ &\text{subject to constraints } (3b) - (3j), \end{aligned} \tag{7}$$

After solving this problem, we set $\boldsymbol{\alpha}_m = [\Delta P_1^{c*}, \Delta P_2^{c*}, ..., \Delta P_N^{c*}]$; note that $||\boldsymbol{\alpha}_m||_1$ is the optimal objective value. To improve solvability, we reformulate the absolute value terms in the objective function such that the objective function is linear (see chapter IX of [11] for details). In *step 2*, we set $\boldsymbol{\alpha}^{\min} = \arg\min \|\boldsymbol{\alpha}_m\|_1$ with the minimization taken over all $\boldsymbol{\alpha}$'s found in step 1. Finally, in *step 3*, we set the limit of the 1-norm safety constraint:

$$\sum_{i=1}^{N} |\Delta P_i^c| < \|\boldsymbol{\alpha}^{\min}\|_1. \tag{8}$$

2.3. Conservativeness of 1 and 2-Norm Safety Constraints

The less conservative of the two constraints should be used because it will allow the aggregator more feasible control actions. However, determining which constraint is less conservative can be a challenge. The conservativeness of a constraint cannot be determined *a priori* because it depends on the minimum unsafe vector of deviations ($\boldsymbol{\alpha}^{\min}$) that has been found. Fig. 2 demonstrates this point with illustrations of the 1-norm and 2-norm approximations of the set of safe power deviations for two different operating points. (Note, for illustration purposes, we consider deviations in only 2 dimensions.) On the right side of Fig. 2, both methods have found the same $\boldsymbol{\alpha}^{\min}$; this is the point where the boundaries of the approximations intersect with the boundary of the actual set. In this case, the 2-norm constraint is uniformly less conservative than the 1-norm constraint. On the left side of Fig. 2, the 1-norm and 2-norm methods have found different $\boldsymbol{\alpha}^{\min}$'s, which we denote as $\boldsymbol{\alpha}^{\min,I}$ and $\boldsymbol{\alpha}^{\min,II}$, respectively. In this case, neither constraint is

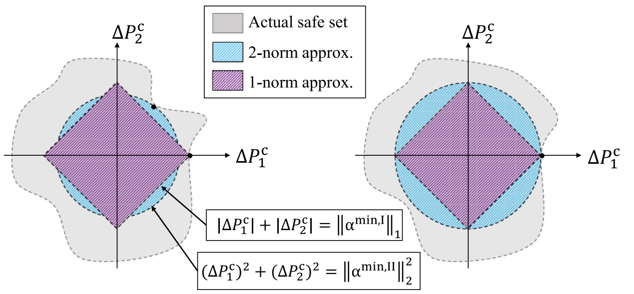

Fig. 2. Illustration of two different sets of safe deviations (gray) and approximations of these sets given by the 1-norm and 2-norm constraints (purple and blue). Left: neither the 2-norm or 1-norm approximation is uniformly less conservative than the other. Right: the 2-norm approximation is uniformly less conservative.

uniformly less conservative than the other.

For cases in which the 1-norm and 2-norm methods find different $\boldsymbol{\alpha}^{\min}$'s, we propose choosing the constraint that allows the larger maximum balancing capacity. In general, an aggregator is compensated for the size of its capacity and would prefer the safety constraint that maximizes its balancing capacity. We find C^{II}, the maximum balancing capacity that the 2-norm safety constraint will allow, by maximizing the capacity $\sum_{i=1}^{N} \Delta P_i^c$ subject to (6). We find that the solution must have all deviations equal (i.e., $\Delta P_i^c = \Delta P_j^c \;\; \forall \;\; (i, j) \in \mathcal{N}$). Setting both sides of (6) equal and all deviations equal gives us the following result

$$C^{\mathrm{II}} = \sqrt{N}\; \|\boldsymbol{\alpha}^{\min,\mathrm{II}}\|_2, \tag{9}$$

The value of C^{I}, the maximum balancing capacity that the 1-norm constraint will allow, is clear upon inspection and is given by

$$C^{\mathrm{I}} = \|\boldsymbol{\alpha}^{\min,\mathrm{I}}\|_1. \tag{10}$$

To maximize balancing capacity, if $C^{\mathrm{II}} > C^{\mathrm{I}}$, the 2-norm constraint should be used; otherwise, the 1-norm constraint should be used.

2.4. Reducing Problem Size

Solving $2N$ optimization problems to compute a safety constraint may be too computationally intensive for real-time operations. We propose reducing computation time by reducing the number of buses for which an optimization problem must be solved. We can eliminate a problem if we can guarantee it is infeasible or its solution will not have the minimum objective value of the full set of problems.

We propose two conditions – one for the under-voltage problem and one for over-voltage problem – that if satisfied enable the problem to be eliminated. To derive these conditions, we find the loading conditions that guarantee two adjacent buses in a radial network, referred to as a "bus-pair", have decreasing voltage magnitudes in the downstream direction (i.e., away from the substation). We represent a generic bus-pair with the two-bus equivalent system in Fig. 3. (Note we use vector notation $\vec{X}$ and polar notation $X\angle\theta_X$ interchangeably to represent a vector in the complex plane.) In Fig. 3, $\vec{S}_{\mathrm{eqv}}$ is the apparent power of the equivalent load at the downstream bus and is equal to the sum of all loads connected to and downstream of the bus in the actual network, as well as their associated line-losses. Variables $\vec{V}_{\mathrm{up}}$ and $\vec{V}_{\mathrm{dn}}$ are the voltages at the upstream and downstream buses, respectively. Parameter $\vec{Z}$ is the line's impedance, and $\vec{I}$ is the line's current flow.

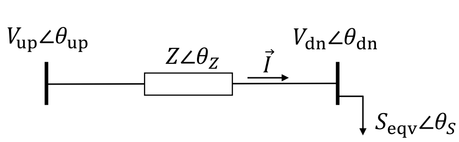

Fig. 3. Two-bus equivalent system. Every pair of adjacent buses in a network can be represented by this two bus system.

We derive the loading conditions that ensure $V_{\mathrm{dn}} \le V_{\mathrm{up}}$ as follows. By Ohm's law we have that $\vec{V}_{\mathrm{dn}} = \vec{V}_{\mathrm{up}} - \vec{I}\vec{Z}$. After a few simple operations, we transform this expression into

$$V_{\mathrm{dn}}^2 = V_{\mathrm{dn}} V_{\mathrm{up}} \cos(\theta_{\mathrm{up}} - \theta_{\mathrm{dn}}) - Z S_{\mathrm{eqv}} \cos(\theta_Z - \theta_S) \tag{11a}$$

$$0 = V_{\mathrm{dn}} V_{\mathrm{up}} \sin(\theta_{\mathrm{up}} - \theta_{\mathrm{dn}}) - Z S_{\mathrm{eqv}} \sin(\theta_Z - \theta_S), \tag{11b}$$

where the voltage angle of the downstream bus has been defined as the reference angle. By replacing the $\cos(\theta_{\mathrm{up}} - \theta_{\mathrm{dn}})$ term in (11a) with 1, and given that $\cos(\theta_{\mathrm{up}} - \theta_{\mathrm{dn}}) \le 1$, we derive the inequality

$$V_{\mathrm{dn}}^2 \le V_{\mathrm{dn}} V_{\mathrm{up}} - Z S_{\mathrm{eqv}} \cos(\theta_Z - \theta_S). \tag{12}$$

If the last term in (12) satisfies

$$Z S_{\mathrm{eqv}} \cos(\theta_Z - \theta_S) \ge 0, \tag{13}$$

then $V_{\mathrm{dn}} V_{\mathrm{up}} - Z S_{\mathrm{eqv}} \cos(\theta_Z - \theta_S) \le V_{\mathrm{dn}} V_{\mathrm{up}}$. Combining this inequality with (12), we have $V_{\mathrm{dn}}^2 \le V_{\mathrm{dn}} V_{\mathrm{up}}$ and thus $V_{\mathrm{dn}} \le V_{\mathrm{up}}$, since V_{dn} is positive by definition.

We have derived the following loading condition: if (13) is satisfied, then $V_{\mathrm{dn}} \le V_{\mathrm{up}}$. Since the magnitudes S_{eqv} and Z are positive by definition, (13) is satisfied if $\cos(\theta_Z - \theta_S) \ge 0$. Thus, the loading condition simplifies to: if

$$-\frac{\pi}{2} < \theta_Z - \theta_S < \frac{\pi}{2}, \tag{14}$$

then $V_{\mathrm{dn}} \le V_{\mathrm{up}}$.

We state the first of two propositions for reducing the overall problem size:

Proposition 1. *If a bus-pair satisfies (14) for all possible operating points, then the under-voltage problem for the upstream bus in the pair can be eliminated from the set of problems that are necessary to solve.*

The proof follows. Let us assume (14) is satisfied for a given bus-pair for all operating points. According to the loading condition, we have $V_{\mathrm{dn}} \le V_{\mathrm{up}}$ for all operating points. Thus $V_{\mathrm{dn}} \le V_{\mathrm{up}}$ for the globally optimal solution of the under-voltage problem for the upstream bus. This solution is also a feasible solution for the downstream bus's under-voltage problem because $V_{\mathrm{dn}} \le V_{\mathrm{up}} \le \underline{V}$, which satisfies constraint (3b). Thus $||\boldsymbol{\alpha}_{\mathrm{up}}||$, the globally optimal objective value to the under-voltage problem for the upstream bus, must be greater than or equal to $||\boldsymbol{\alpha}_{\mathrm{dn}}||$, the globally optimal objective value to the under-voltage problem for the downstream bus. If $||\boldsymbol{\alpha}_{\mathrm{up}}|| \ge ||\boldsymbol{\alpha}_{\mathrm{dn}}||$, then $\boldsymbol{\alpha}_{\mathrm{up}}$ is not needed as a candidate for $\boldsymbol{\alpha}^{\min}$ and the under-voltage problem for the upstream bus can be eliminated. We note that this proof relies on an assumption that the globally optimal solution to the under-voltage problem will be found, but this is not guaranteed since we are solving a non-convex optimization problem using a non-linear programming solver. In future work, we plan to extend Proposition 1 such that it applies to a convex relaxation of the under-voltage problem; the relaxed problem will provide lower bounds on the globally optimal objective values $||\boldsymbol{\alpha}_{\mathrm{dn}}||$ and $||\boldsymbol{\alpha}_{\mathrm{up}}||$.

Proposition 2. *If all bus-pairs on the path between bus m and the substation satisfy (14) for all possible operating points, then the over-voltage problem for bus m can be eliminated from the set of problems that are necessary to solve.*

The proof follows. Let us assume (14) is satisfied for all bus-pairs between bus m and the substation. The voltage magnitude is necessarily non-increasing along this path since $V_{\mathrm{dn}} \le V_{\mathrm{up}}$ for all of the bus-pairs. Thus the voltage magnitude of bus m must be less than or equal to that of the substation. Since the voltage at the substation is regulated within operational limits (i.e., $\underline{V} \le V_0 \le \overline{V}$), bus m's voltage magnitude cannot be greater than $\overline{V}$. Thus constraint (4b) cannot be satisfied and the over-voltage problem is infeasible for bus m. Because the problem is infeasible, it can be eliminated.

3. Case Study

3.1. Study Setup

In the case study, we use a 56-bus distribution feeder model that is a modified version of the IEEE 123-bus test feeder. The 56-bus model has balanced loads and symmetric lines, enabling a single-phase equivalent model. Full details of the model are provided in [12]. Fig. 4 shows the network's radial topology and its range of voltage magnitudes at the nominal operating point. In most of the case study, we assume there are no capacitor banks on the network and set the substation voltage to 1.02 p.u., which ensures that there are no under-voltage violations at the nominal operating point. We also assume there are no voltage regulators except at the substation. Extending the optimization problem (3) to include in-line voltage regulators is future work. We solve the proposed optimization problems using the non-linear programming solver Ipopt. We initialize the solver at the network's nominal operating point.

We use the network's nominal loading data to determine the operating points for our model's uncontrollable and controllable loads. At each bus, we assume 50% of the nominal real-power consumption is controllable and set $\hat{P}_i^c$ equal to it. We assume the controllable load's power factor ζ_i is 0.95 lagging for all buses, which is within the range of power factors for residential and commercial loads (see Table A.2 in [13]). At each bus, the remaining nominal power consumption is assigned to the uncontrollable load (P_i^{uc}, Q_i^{uc}). When the controllable loads are at baseline, the network's loading matches that of the nominal data. Finally, we assume the physical capacity of the controllable loads at each bus is $\pm$ 80% of their baseline power (i.e., $\underline{P}_i^c = 0.2\hat{P}_i^c$ and $\overline{P}_i^c = 1.8\hat{P}_i^c$).

3.2. Demonstration of 2-Norm Method

We demonstrate step 1 of the 2-norm method by showing the optimal solution of the under-voltage optimization problems for a particular bus. The top plot of Fig. 5 shows the network's voltage magnitudes for the optimal solution to the under-voltage, 2-norm problem for bus 20. Voltages at and around bus 20 are close to or under the lower voltage limit, as indicated by dark red. Fig. 5 (middle) shows the exact voltage magnitudes at each bus. At its nominal operating point, bus 20's voltage is 0.957 p.u. and decreases to 0.950 p.u for the optimal solution. Because bus 20 is not a terminal bus, under-voltages also occur at downstream buses 21-26, as well as at adjacent buses 27-32. Fig. 5 (bottom) shows the components of $\boldsymbol{\alpha}_{20}$, the minimum-sized vector of power deviations that causes an under-voltage at bus 20. Power consumption increases at all load-buses: the largest increases occur at or downstream of bus 20, and the smallest increases occur close to the substation (buses 1-4). This pattern shows which buses' power deviations have the most influence over bus 20's voltage, with larger deviations indicating larger influence.

We demonstrate step 2 of the method by selecting the minimum-sized $\boldsymbol{\alpha}$ from the set of $\boldsymbol{\alpha}$'s found in step 1. We select the $\boldsymbol{\alpha}_m$ that corresponds to the optimization problem whose optimal objective value $||\boldsymbol{\alpha}_m||$ is the least of all of the problems with feasible solutions. Fig. 6 shows the optimal objective value of each under-voltage problem for the network. Problem 32 has the minimum objective value, and corresponds to bus 32, a terminal bus far from the substation. For buses close to the substation, no feasible solution is found, and likely none exists, because of the controllable loads' physical capacity limits (see (3c)). In addition, no feasible solutions were found for any of the over-voltage problems, which is unsurprising since there are no positive power injections in the network (e.g., from capacitor banks or photovoltaic systems). Among all of the problems with feasible solutions, the under-voltage problem for bus 32 has the minimum optimal objective value, so we set $\boldsymbol{\alpha}^{\min} = \boldsymbol{\alpha}_{32}$. The last step of the method is simply to set the limit of the 2-norm safety constraint such that .

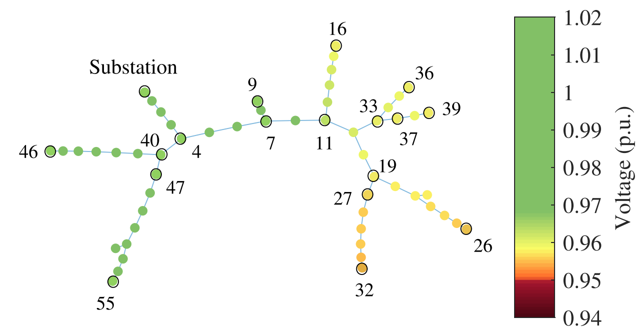

Fig. 4. 56-Bus distribution network used in case study. Voltage magnitudes are shown for the nominal operating point.

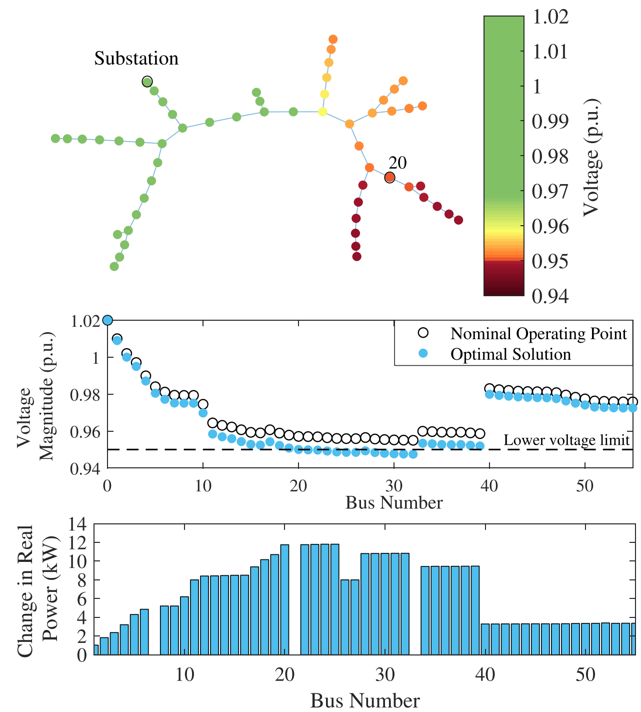

Fig. 5. Solution to the under-voltage, 2-norm problem for bus 20. Top and middle plots: the voltage at bus 20 is exactly at the lower limit (0.95 p.u.); voltages downstream of bus 20 are below the limit. Bottom plot: Each load-bus contributes some change in real power to achieve the under-voltage at bus 20; change is relative to the nominal operating point.

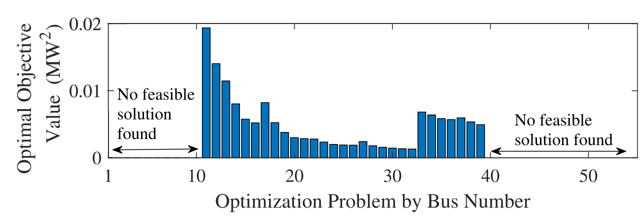

Fig. 6. Optimal objective values for all 2-norm, under-voltage problems. Minimum value is at bus 32. No feasible solutions are found for buses closer to the substation, i.e., buses 1-10 and 40-55 (see Fig. 4 for numbering).

3.3. Comparing and Testing the 1-Norm and 2-Norm Methods

We compare the 1-norm and 2-norm methods for the under-voltage optimization problems. First, we consider an example problem, again the under-voltage problem at bus 20. Fig. 7 shows the two solutions ($\boldsymbol{\alpha}_{20}$) found by both the 1-norm and 2-norm problems, with deviations reported as a percentage of the nominal operating point at each bus. As

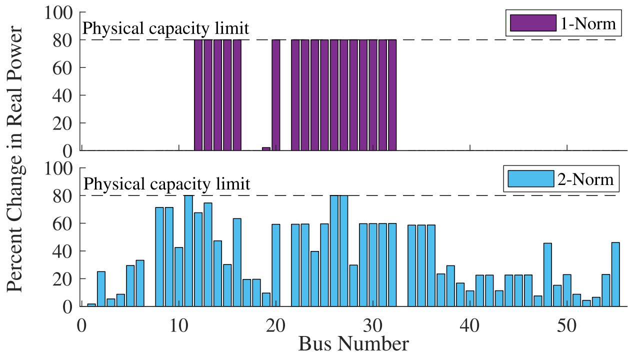

Fig. 7. Comparison of 1-norm and 2-norm solutions to the under-voltage problem for bus 20. The 1-norm and 2-norm methods find different optimal solutions. Note the physical capacity limit of controllable loads is 80% of their baseline for all buses.

the figure shows, the two solutions are strikingly different. The solutions differ because of the methods' different objective functions. The 2-norm objective function penalizes an incremental increase to a large deviation more than to a small deviation; this preference for small deviations causes the 2-norm problem to distribute the deviations across all load-buses. In contrast, the 1-norm objective function penalizes all incremental increases equally; this causes the 1-norm problem to concentrate the deviations among buses with the largest influence over the constrained bus's voltage.

Next, we compare the maximum balancing capacities that the two methods allow. For both methods, the under-voltage problem for bus 32 has the minimum optimal objective value; the values are $\|\boldsymbol{\alpha}^{\min,\mathrm{II}}\|_2^2 = 0.0013$ MW^2 and $\|\boldsymbol{\alpha}^{\min,\mathrm{I}}\|_1 = 0.163$ MW for the 2-norm and 1-norm methods, respectively. Using (9) and (10), we find that the maximum balancing capacities for the 2-norm and 1-norm methods are $C^{\mathrm{II}} = 0.260$ MW and $C^{\mathrm{I}} = 0.163$ MW, respectively. Thus, for this operating point, the 2-norm method is preferred because it allows for a larger balancing capacity.

We numerically test the 1-norm and 2-norm safety constraints with the optimal solutions of the other method. The 2-norm safety constraint should exclude the set of $\boldsymbol{\alpha}$'s found by the 1-norm problem, and the 1-norm safety constraint should exclude those found by the 2-norm problem. Fig. 8 shows the results of these tests. The y-axis values are calculated by evaluating the objective function of the method being tested at the optimal solutions (i.e., set of $\boldsymbol{\alpha}$'s) found by the other method. As shown in the left plot of Fig. 8, the 1-norm safety constraint passes its test: all of the 2-norm method's solutions lie outside of the safe region defined by the 1-norm safety constraint. Similarly, as shown in the right plot of Fig. 8, the 2-norm safety constraint passes its test.

3.4. Reducing Problem Size

We apply Propositions 1 and 2 to reduce the problem size for the 56-bus network, as well as a modified version of the network. The modified network has capacitor banks located at buses 26, 28, 29, and 30 that

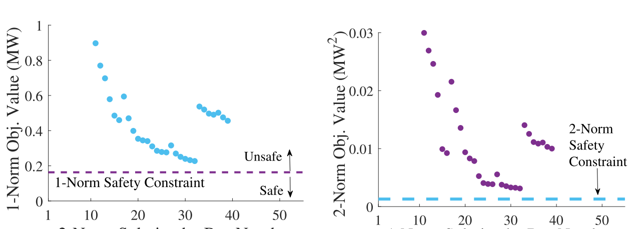

Fig. 8. Verification of safety constraints. The 1-norm safety constraint correctly excludes the solutions (i.e., set of $\boldsymbol{\alpha}$'s) found by the 2-norm method (left); the 2-norm safety constraint passes a similar test (right).

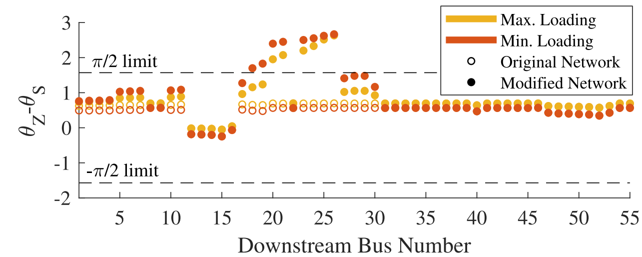

Fig. 9. Loading conditions for each bus-pair. Bus-pairs with $\theta_Z - \theta_S > \pi/2$ do not satisfy (14). Fewer optimization problems can be eliminated when this condition is not met.

cause a voltage rise along the line from bus 19 to 26 (see [12] for details). Because of this voltage rise, fewer problems for the modified network should qualify for elimination. For Propositions 1 and 2 to hold for a given bus-pair, constraint (14) must be satisfied by both the minimum and maximum possible values of θ_S for the bus-pair. In this analysis, we evaluate (14) at two extreme operating points that approximate the operating points that minimize and maximize θ_S. These extreme points are: 1) "maximum loading" in which the controllable loads consume maximum power and 2) "minimum loading" in which they consume minimum power. Finding the actual operating points with maximum and minimum θ_S is future work.

Fig. 9 shows which bus-pairs satisfy (14) and indicates which problems can be eliminated. Points that lie above the π/2 limit indicate bus-pairs that do not satisfy the constraint. In the original network, all of the bus-pairs satisfy the constraint. By Proposition 1, we can eliminate the under-voltage problems for the upstream bus of each of these bus-pairs; after this elimination, only under-voltage problems for the network's 11 terminal buses remain. By Proposition 2, we can eliminate all over-voltage problems because voltages are always decreasing. In the modified network, eight bus-pairs do not satisfy (14). As expected, these buses are where the voltage rise occurs due to capacitor banks. By Proposition 1, we can eliminate under-voltage problems for all non-terminal buses except for the identified eight upstream buses; after the allowed eliminations, under-voltage problems for 11 terminal buses and 8 non-terminal buses remain. (Note that here we apply Proposition 1 despite not having a guarantee of global optimality.) To apply Proposition 2 to the modified network, we identify bus 18 as the downstream bus closest to the substation for which (14) is not satisfied. By Proposition 2, all buses – except for bus 18 and those downstream of 18 – can be eliminated. Thus, the over-voltage problem must be solved for 15 buses.

Using the proposed reduction techniques, we are able to drastically reduce the size of the overall problem. For the original network, the number of total optimization problems decreased from 104 to 11 for the original network and from 104 to 34 for the modified network.

4. Conclusion

We have proposed a method for constraining a load aggregator's control actions to ensure the safe operation of the distribution network. The proposed safety constraint can be computed by a distribution operator and adhered to by a third-party aggregator with minimal sharing of private information between the two entities. The safety constraint is, by design, conservative. We compared two versions of the constraint to determine if one was uniformly less conservative than the other. Although we were unable to draw general conclusions, in a case study we found that the 2-norm constraint allowed for a larger balancing capacity than the 1-norm constraint. To reduce the time it takes to compute the safety constraint, we proposed two conditions under which optimization problems can be eliminated from the set of necessary problems. In the case study, we found this reduction technique to be very effective. Because the method's conservatism and computational intensity is network dependent, some networks will be better suited for the safety-constraint method than others.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

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