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**Differential involvement of EEG oscillatory components in sameness
vs. spatial-relation visual reasoning tasks**

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26 **Abstract**

27 The development of deep convolutional neural networks (CNNs) has recently led to great
28 successes in computer vision and CNNs have become de facto computational models of
29 vision. However, a growing body of work suggests that they exhibit critical limitations beyond
30 image categorization. Here, we study one such fundamental limitation, for judging whether
31 two simultaneously presented items are the same or different (SD) compared to a baseline
32 assessment of their spatial relationship (SR). In both human subjects and artificial neural
33 networks, we test the prediction that SD tasks recruit additional cortical mechanisms which
34 underlie critical aspects of visual cognition that are not explained by current computational
35 models. We thus recorded EEG signals from human participants engaged in the same tasks
36 as the computational models. Importantly, in humans the two tasks were matched in terms of
37 difficulty by an adaptive psychometric procedure: yet, on top of a modulation of evoked
38 potentials, our results revealed higher activity in the low beta (16-24Hz) band in the SD
39 compared to the SR conditions. We surmise that these oscillations reflect the crucial
40 involvement of additional mechanisms, such as working memory and attention, which are
41 missing in current feed-forward CNNs.

42
43 **Keywords:** Visual reasoning, spatial relationship, EEG oscillations, ERPs, deep neural
44 networks.

45 **Significance statement**

46 Convolutional neural networks (CNNs) are currently the best computational models of
47 primate vision. Here, we independently confirm prior results suggesting that CNNs can learn
48 to solve visual reasoning problems involving spatial relations much more easily than
49 problems involving sameness judgments. We hypothesize that these results reflect different
50 computational demands between the two tasks and conducted a human EEG experiment to
51 test this hypothesis. Our results suggest a significant difference – both in evoked potentials
52 and in the oscillatory dynamics– of the EEG signals measured from human participants
53 performing these two tasks. We interpret this difference as the signature for the fundamental
54 involvement of recurrent mechanisms implementing cognitive functions such as working
55 memory and attention.

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59 1. Introduction

60 The field of artificial vision witnessed an impressive boost in the last few years, driven
 61 by the striking results of deep convolutional neural networks (CNNs). Such hierarchical
 62 neural networks process information sequentially – through a feedforward cascade of
 63 filtering, rectification and normalization operations. The accuracy of these architectures is
 64 now approaching – sometimes exceeding – that of human observers on key visual
 65 recognition tasks including object (He, Zhang, Ren, & Sun, 2016) and face recognition (P. J.
 66 Phillips et al., 2018). These advances suggest that purely feedforward mechanisms suffice to
 67 accomplish remarkable results in object categorization, in line with previous experimental
 68 studies on humans (VanRullen & Thorpe, 2001) and animals (Hollard & Delius, 1982;
 69 Vogels, 1999). However, despite the remarkable accuracy reached in these recognition
 70 tasks, the limitations of CNNs are becoming increasingly evident (see Serre, 2019 for a
 71 recent review). Beyond image categorization tasks, CNNs appear to struggle to learn to
 72 solve relatively simple visual reasoning tasks otherwise trivial for the human brain (Kim,
 73 Ricci, & Serre, 2018; Stabinger, Rodríguez-Sánchez, & Piater, 2016). A recent study (Kim et
 74 al., 2018) thoroughly investigated the ability of CNN architectures to learn to solve various
 75 visual reasoning tasks, and found an apparent dichotomy between two sorts of problems: on
 76 the one hand, tasks that require judging the spatial relations between items (Spatial
 77 Relationship – SR); on the other, those that require comparing items (Same-Different – SD).
 78 Importantly, Kim and colleagues demonstrated that CNNs can more easily learn the first
 79 class of problems compared to the second one.

80 This prompts the question of how biological visual systems handle such tasks so
 81 efficiently. Kim et al. (2018) suggest that SR and SD tasks tap into distinct computational
 82 mechanisms, thus leading to the prediction that different cortical processes are also involved
 83 when humans perform the two tasks: SR tasks can be successfully solved by feedforward
 84 processes, whereas SD tasks seem to require additional computations, such as working
 85 memory and attention. Here, we tested this hypothesis in two steps: first, we confirmed and
 86 extended Kim's results by comparing the performance of CNNs on an experiment in which
 87 we directly contrasted SD and SR tasks on the same stimulus set. Second, we recorded
 88 electrophysiological responses (EEG) in healthy human participants for the same
 89 experiment, after having matched the difficulty level via an adaptive psychometric procedure.
 90 We hypothesized that the additional computations required by the SD task, as compared to
 91 SR tasks, would elicit differences in evoked potentials (e.g. P300 modulations, which have
 92 been related to attentional mechanisms (Nash & Fernandez, 1996)) and brain rhythms
 93 related to working memory (such as beta-band oscillations (Benchenane, Tiesinga, &
 94 Battaglia, 2011; Lundqvist, Herman, Warden, Brincat, & Miller, 2018)). We found indeed that,

in addition to a variation in evoked potentials, the SD task elicited higher activity in specific beta-band oscillatory components in the occipital-parietal areas, which are typically associated with attention- and memory-related processes. We emphasize that the goal of the present study was not to identify the precise neural computations involved in the two tasks (which would naturally require a broader experimental set-up than a single EEG study), but rather to validate the hypothesis that SD involves additional computations relative to SR (even when the two tasks are equally difficult). We hope that this demonstration can be a first step towards characterizing the processes taking place in visual cortex during visual reasoning tasks, and designing more reliable and more human-like computational models.

2. Materials & Methods

2.1 Participants and pilot experiment

Twenty-eight participants (aged 21–34 years old with a mean age of 26.6 ± 3.7 , 11 women, 5 left-handed), volunteered to join the experiment. All subjects reported normal or corrected to normal vision and had no history of epileptic seizures or neurological disorders. Participants were pooled in two groups of 14 each: one group performed a pilot experiment, while the second one was tested on a final version of the task. The only difference between the pilot and the main study was the QUEST adaptive procedure used to match the difficulty level between conditions, which was not implemented in the pilot experiment. However, in both studies we found the very same result (see below, specifically fig. 4 and 5). In the main experiment, we kept the same number of participants to replicate the effect, after having removed the behavioral difference in task difficulty via the QUEST algorithm. This study complies with the guidelines of the research center where it was carried out, and the protocol was approved by an external committee (ethics approval number N° 2016-A01937-44). All participants gave written informed consent before starting the experiment, in accordance with the Declaration of Helsinki, and received monetary compensation for their participation.

2.2 Experimental design

The experiment was composed of 16 experimental blocks of 70 trials each, with a total duration of about 1 hour. Each trial lasted ~ 2 seconds (Fig. 1A): 350ms after the onset of a black fixation cross (0.6° width), 2 shapes were displayed for 30ms on opposite sides of the screen, distant $2 \cdot p$ from each other with an angle of $\pm(45^\circ + \theta)$ with respect to the horizontal midline (p being the distance from the center of the screen, and θ the angular difference with the diagonal; see Fig. 1B). Each shape was selected from a subset of 36 hexominoes, a geometric figure composed of 6 contiguous squares (see Fig. 1B). One second after the onset of the hexominoes, the fixation cross turned blue, cuing participants to respond. In half of the blocks, participants had to report whether the two shapes were the

130 same or different (Same-Different – SD condition); in the remaining blocks participants had to
 131 judge whether the two stimuli were aligned more horizontally or vertically (Spatial Relation –
 132 SR condition). Shapes were displayed at opposite sides of the screen along two main
 133 possible orientation axes sampled at random for every trial (either 45° and 225° or -45° and -
 134 225°). Both stimuli positions were jittered by a random offset Δx and Δy in both the x and y
 135 axis and a rotation θ from the main axis. The same offsets were applied to both shapes, so
 136 they did not affect the angle between stimuli. The aim of such offsets was to prevent
 137 participants in the SR condition from determining the configuration of the two stimuli
 138 (orientation task) by merely judging the position of a single stimulus: without the random
 139 offsets, considering for example the top-right corner position, if the item were below/above
 140 the (imaginary) screen diagonal line, the overall orientation would be horizontal/vertical,
 141 without the need to consider the position of the corresponding bottom-left item. The offset
 142 then compelled participants to consider the relative position of both hexominoes at once.
 143 Importantly, in the main experiment (compared to the pilot experiment) the difficulty of the
 144 two tasks was controlled by an adaptive psychometric procedure (QUEST method, Watson &
 145 Pelli, 1983), which varied the eccentricity of the two stimuli p (in the SD blocks) or θ (in the
 146 SR blocks) to maintain an overall accuracy level of 80% throughout the whole experiment. In
 147 fact, larger (smaller) values of p made the stimuli more (less) eccentric and the task more
 148 (less) difficult; similarly, smaller (larger) values of θ set the stimuli closer to (farther from) the
 149 45° diagonal line, making the task more (less) difficult. We modified one parameter per
 150 condition (i.e., per block), while the other was kept constant (using the same value as in the
 151 preceding block). After participants responded, they received feedback on their performance:
 152 the fixation cross turned green (red) in case of a correct (incorrect) answer. Throughout the
 153 experiment the condition blocks were alternated, the first block being the SD condition for all
 154 participants. Before starting the first block, participants performed one training block per
 155 condition. The purpose of this training was 1) to familiarize participants with the experimental
 156 conditions, 2) to initialize the p and θ parameters in the QUEST method for the first
 157 experimental block (initial values were respectively $p = 5.4^\circ$ of visual angle and $\theta = 6^\circ$ of
 158 rotation). All experiments were performed on a cathode ray monitor, positioned 57 cm from
 159 the subject, with a refresh rate of 160 Hz and a resolution of 1280 × 1024 pixels. The
 160 experiment was coded in MATLAB using the Psychophysics Toolbox (Brainard, 1997). The
 161 stimuli were presented in black on a gray background. Throughout the experiment we
 162 recorded EEG signals.

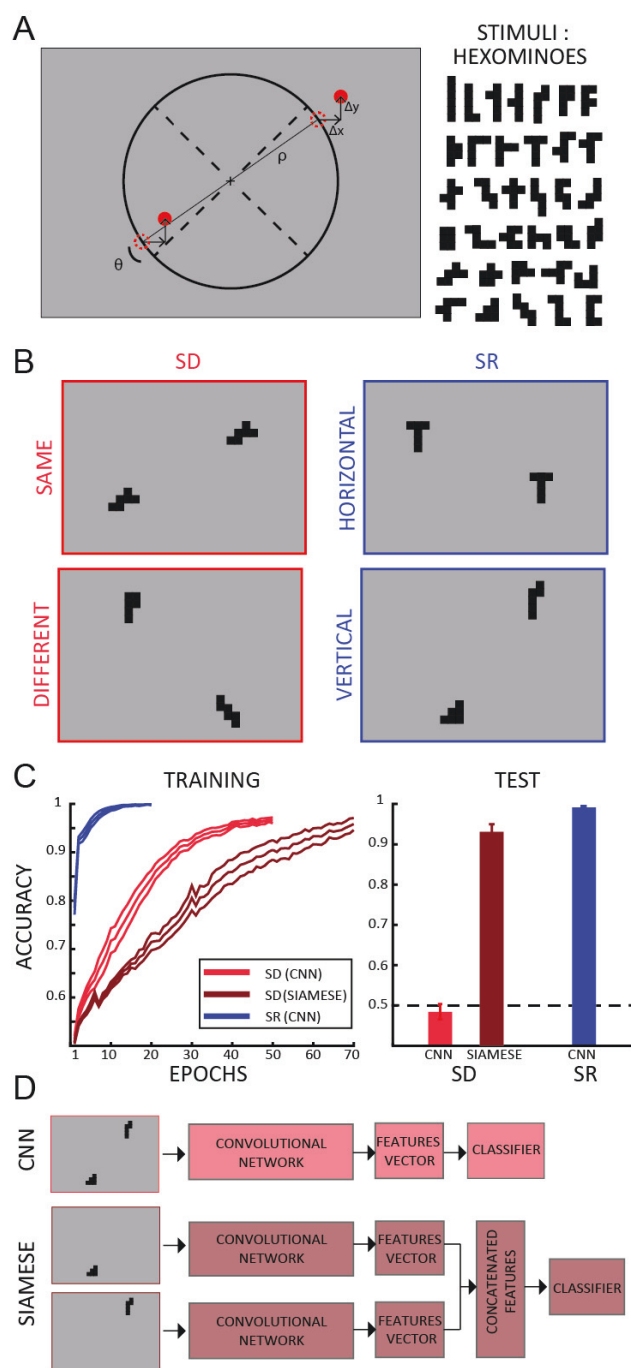
163

164 2.3 EEG recording and pre-processing

165 We recorded brain activity using a 64-channel active BioSemi electro-
 166 encephalography (EEG) system (1,024 Hz digitizing rate, 3 additional ocular electrodes). The
 167 pre-processing was performed in MATLAB using the EEGLab toolbox (Delorme & Makeig,
 168 2004). First, the data was downsampled to 256 Hz. A notch filter [47Hz - 53Hz] was then
 169 applied to remove power line artifacts. We applied an average-referencing and removed slow
 170 drifts by applying a high-pass filter (>1 Hz). We created the data epochs aligning the data to
 171 the onset of the fixation cross. Finally, we performed an ICA decomposition in order to
 172 remove components related to eye movements and blink artifacts: we visually inspected the
 173 data and removed from 2 to 5 components per subject with a conservative approach (we
 174 removed only components in the frontal regions clearly related to eye movements' activity).

175 **2.4 Computational modeling and code accessibility**

176 We extended a previous computational study (Kim et al., 2018) from which we chose
 177 the parameters of the convolutional feedforward network trained on the SD and SR tasks.
 178 Each task was run 10 times, randomly initializing the networks' parameters and the stimuli
 179 used in the training and test set. The network was fed with 50x80 pixel images. Two
 180 hexominoes (width and height of 2 to 5 pixels) were placed at opposite sides of the screen
 181 (see Fig. 1A and 'Experimental design'). The dictionary of hexominoes was composed of 35
 182 items, which were randomly split between a training (30 items) and a test set (5 items) at
 183 each iteration. Both the training, validation and test sets were composed of 1,000 stimuli (i.e.
 184 different combinations of the hexominoes, with slightly different eccentricity and/or offset
 185 relative to the diagonal). The network consisted of 6 convolutional layers. Each layer
 186 contained 4 channels of size 2x2, with stride of 1. All convolutional layers used a ReLu
 187 activation function with stride of 1 and were followed by pooling layers with 2x2 kernels and a
 188 stride of 1. Eventually, two fully connected layers with 128 units preceded a two-dimensional
 189 classification layer with a sigmoid activation function. As a regularizer we set a dropout rate
 190 of 0.3 in each layer of the network. We used binary cross-entropy as a loss function, the
 191 Adaptive Moment Estimation (Adam) optimizer (Kingma & Ba, 2015) and a learning rate of
 192 $10e-4$. Each simulation was run over 70 epochs with batch size of 50. All simulations were
 193 run in TensorFlow (GoogleResearch, 2015). The Siamese network had the same exact
 194 convolutional architecture as described above; additionally, the difference between features-
 195 vectors of each separate item (computed on an input image where this item was shown
 196 alone) was fed to the classifier to perform the SD task. All networks count $\sim 7e06$ parameters.
 197 All the code and data required to replicate the simulations are available at a github repository
 198 (<https://github.com/artipago/SD-SR>). The code has been run on a Window PC on Python
 199 using the "Tensorflow", "Keras", "Scipy" and "Numpy" libraries.



200

201 **Fig.1: Stimuli and simulation results.** A) The stimuli were the same in the simulations and in the human
 202 experiments. The items were displayed at opposite sides of the screen (either 45° and 225° or -45° and -225°).
 203 Both item positions were jittered by a random amount in both the x and y axis (Δx and Δy in the picture) to
 204 make the task non-trivial for human participants (i.e. preventing participants from performing the SR task
 205 considering only the position of one item, thus ignoring the spatial relationship between the two items). The
 206 items used are hexominoes (right panel). Minimum and maximum item height and width are 1.2° – 3.6° and

1.2° – 2.7° of visual angle respectively, and 2 to 5 pixels used for the simulations (image size was 50 x 80 pixels). B) Example of stimuli position for the same-different task (SD - left column) and spatial relation task (SR - right column). For the sake of illustration the ratio between the screen and hexominoes size has been modified (stimuli here look bigger than in the real experiment). C-D) Accuracy of the CNN network on the same-different (SD; light red) and spatial relationship (SR; blue) tasks, and of a Siamese network trained on the SD task (dark red). The Siamese network mimics segmentation in a feedforward network, by separating the items in two distinct channels of the network (see panel D). The left panel shows the training curves for each network (accuracy over epochs during training); we stopped the training when the validation accuracy reached 90%. In the right panel we show the training accuracy at the last epoch and the test accuracy. The latter was evaluated using novel items never used for training, and it reveals that the CNN seems to only learn the required rule for the SR but not for the SD task, as shown in a previous study. Conversely, the Siamese network (CNN with segmentation) can solve the SD task, demonstrating that segmentation can allow the CNN to successfully accomplish this task. In both panels we show average values \pm SE over 10 repetitions using different random initializations.

2.5 Statistical analysis – behavior

We analyzed both accuracy and reaction times (RT) by means of Bayesian ANOVA, considering the block condition (SR and SD, see above) as independent variables and the trial condition (whether the stimuli were same or different, or more horizontally or vertically aligned). The result of such analysis provides a Bayes Factors (BF), which quantifies the ratio between statistical models given the data. Throughout the paper, all BFs reported correspond to the probability of the alternative hypothesis over the null hypothesis (indicated as BF_{10}). Practically, a large BF ($\sim BF > 5$) provides evidence in favor of the alternative hypothesis (the larger the BF the stronger the evidence), whereas low BF ($\sim BF < 0.5$) suggests a lack of effect (Masson, 2011; Smith, 2001). We performed all Bayesian analyses in JASP (JASP Team, 2018; Love et al., 2015).

2.6 Statistical analysis – electrophysiology

Regarding the EEG recording we performed 2 analyses: one in the time domain measuring Evoked Related Potentials – ERPs, and the other one in the frequency domain using a time-frequency transform. In the first case, we considered the ERPs recorded from 7 midline electrodes (i.e., Oz, POz, Pz, CPz, Cz, FCz and Fz). After subtracting the baseline activity recorded during the 350ms before stimuli onset, we averaged the signals from the SD and SR blocks respectively (i.e., 8 blocks for each condition). Finally, we tested whether the difference between these signals differed from 0 by means of a point-by-point 2-tailed t test with a false discovery rate (FDR) correction for multiple comparisons (Hochberg, 1995). Regarding the time-frequency analysis, we computed the power spectra by means of a wavelet transform (1–50 Hz in log-space frequency steps with 1–20 cycles). After baseline correction (i.e., dividing by the averaged activity of the 350ms prior to the onset of the fixation cross), for each participant, we computed the difference in decibel of the two conditions point by point, averaging over all electrodes. As in the ERP analysis, we performed a point-by-

point 2-tailed t test to identify the time-frequency regions which were significantly different. We applied a cluster-based permutation to correct for multiple comparisons (Maris & Oostenveld, 2007). First, we identified clusters composed of t values $t > 3.5$ ($p < 0.01$), and for each one we computed the respective global sum. In order to estimate the null distribution over the combined t values, we performed the same procedure 500 times after shuffling the subject by subject SD-SR assignment. Eventually, we obtained the p values for each non-shuffled cluster given the null distribution. All EEG analyses were performed in Matlab; the wavelet transform was performed using the EEGLab toolbox (Delorme & Makeig, 2004).

3. Results

3.1 Computational modeling

We first extended the results by Kim et al. (2018) for our novel stimulus set: we trained two separate Convolutional Neural Networks (CNN) architectures to solve an SD and an SR task using a single stimulus set (Methods). The input to these networks was an image (50x80 pixels) in which two hexominoes (width and height of 2 to 5 pixels) were displayed at opposite sides of the screen (see Fig. 1A). The networks were trained to classify whether the two hexominoes were the same or not (SD task) or whether they were aligned more vertically or more horizontally with respect to the midline (SR task).

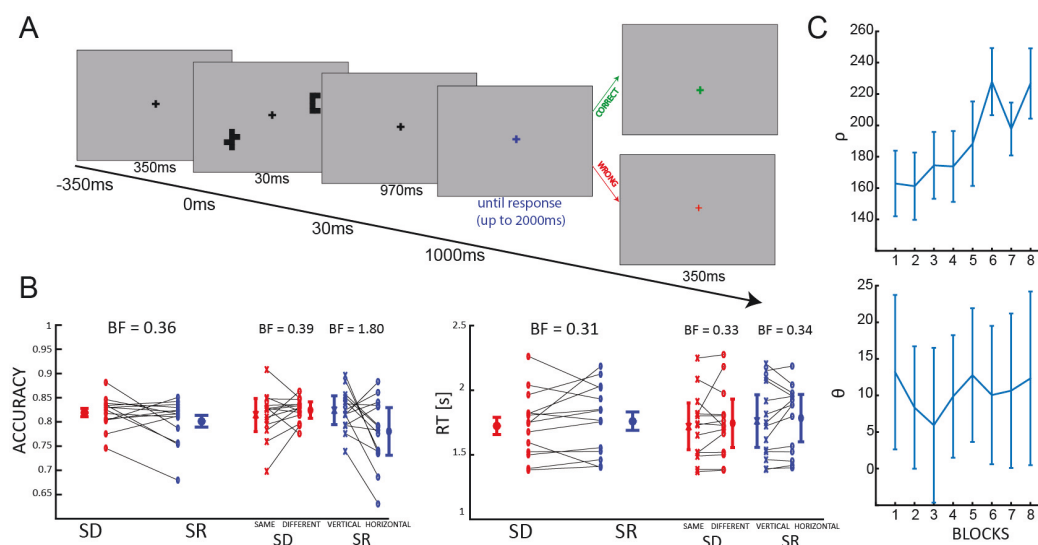


Fig.2: Experimental design and human behavioral results. A) At the beginning of each trial a black fixation cross was displayed for 350ms. After 2 stimuli were shown for 30ms, participants waited an additional 970ms before providing the answer. The response was cued by the fixation cross turning blue. After the response, the color of the fixation cross provided feedback: green if the response was correct, red otherwise. B) Humans performed the SD and SR tasks with comparable levels of performance. In the left and right panels are shown

the averages \pm SE for accuracy and reaction times, respectively. Each pair of connected markers represent an individual subject. The results for the same-different (in red) and spatial relationship (in blue) conditions are further broken down for each condition separately (same-different and vertical-horizontal). BF indicates the Bayes factor against the null hypothesis (difference between the two conditions). C) Changes over blocks of p (the distance between the stimuli - left panel) and θ (the angle between the stimuli and the meridian - right panel) as adjusted by the QUEST algorithm.

We trained and tested the network on different sets of items (a training and test set, respectively) to assess the networks' ability to generalize beyond training data. We trained and tested the networks 10 times – randomly initializing networks parameters and training – test set split each time. We report the mean accuracy and standard deviation over these 10 repetitions in Fig. 1B. Our results are consistent with those from Kim et al. (2018): a CNN appears to be able to learn the abstract rule (as measured by the network's ability to generalize beyond the shapes used for training) for SR tasks much more easily than SD tasks. The effortless ability of humans and other animals (Daniel, Wright, & Katz, 2015; Wasserman, Castro, & Freeman, 2012) to learn SD tasks suggest the possible involvement of additional computations that are lacking in CNNs, possibly achieving items identification or segmentation (e.g. via attention and working memory). In order to verify that segmentation could be a missing ingredient for the SD task, we implemented a variant of the CNN with built-in segmentation properties, and tested it on the SD task (it is not necessary to test it on the SR task, because generalization performance is already at ceiling). The new network used a Siamese architecture (Bromley et al., 1994) in which each item is processed separately and eventually combined before being passed to a classifier. Therefore this model mimics the effect of selective attention and item segregation by feeding to the network each item separately. The Siamese network could achieve the same training performance on the SD task as the standard CNN (even though the training took more epochs), however the network was able to generalize to the test set, while the standard CNN test accuracy was at chance. This supports the idea that item segmentation or individuation abilities are needed to achieve the SD task. Next, we test the prediction that SD tasks in humans also require additional computational mechanisms than SR tasks, by recording EEG signals from a pool of 28 participants (14 of which were tested on a pilot experiment –fig. 5) performing the same SD and SR tasks.

3.2 Human behavior

A first pilot group of 14 participants performed the SD and SR tasks as described in Figure 2A, but without any procedure for adjusting task difficulty (i.e. the QUEST method). The same EEG oscillatory differences between the two tasks as in the main experiment were observed (fig 5); however, concomitant differences in behavioral task performance left open the possibility that the oscillatory effects were caused by differences in task difficulty (fig. 5A).

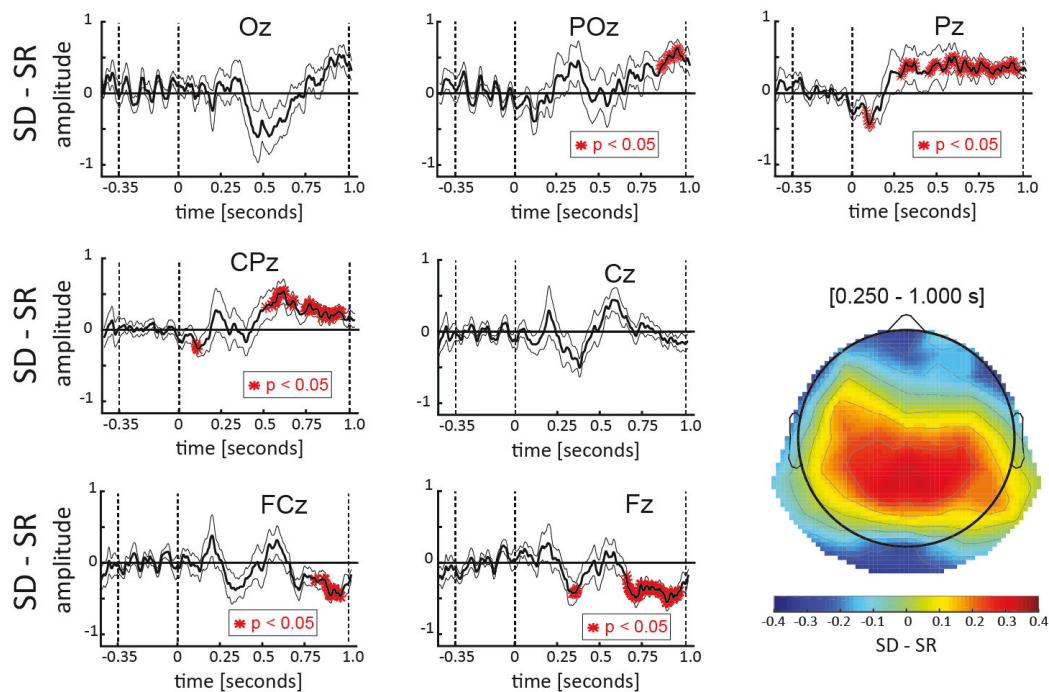
308 Therefore, we replicated the experiment on another group of 14 subjects, this time with an
 309 adaptive procedure to equate behavioral performance between the SD and SR tasks.

310 Participants (N=14) in this main experimental task completed 16 blocks using the
 311 same stimuli as those used to train CNNs (Fig.1): in half of the blocks they were asked to
 312 report whether the two hexominoes were the same or not (SD conditions), in the other half
 313 whether the hexominoes were more vertically or horizontally aligned (SR conditions). The
 314 two conditions were interleaved in a block design. Participants were required to answer after
 315 one second from stimulus onset in order to disentangle motor from visual components in the
 316 EEG recordings (Fig. 2A). The QUEST algorithm was used to assure that participants'
 317 accuracy was matched between the two tasks and remained constant throughout the whole
 318 experiment. This was done by adjusting two experimental parameters trial by trial (i.e., the
 319 hexominoes eccentricity in SD blocks, ρ , and the angle from the diagonal in SR blocks, θ ;
 320 see Fig. 1A and 2C). Maintaining a comparable accuracy between the two tasks reduces the
 321 potential for confounds in the electrophysiological analysis due to differences in performance,
 322 vigilance or motivation. We confirmed the absence of any substantial behavioral difference
 323 between the SD and SR tasks (Fig. 2B) with a Bayesian ANOVA on both accuracy ($BF_{10} =$
 324 0.361 , error $< 0.001\%$) and RT ($BF_{10} = 0.317$, error $< 0.89\%$). In addition, we also
 325 investigated each condition separately (Fig. 2B), comparing the difference between 'same'
 326 and 'different' trials (in SD blocks) and 'vertical' and 'horizontal' trials (in SR blocks) in both
 327 RT and accuracy. All comparisons revealed overall no differences between tasks, except for
 328 the accuracy of vertical and horizontal trials in the SR condition, in which the BF proved
 329 inconclusive (accuracy: SD - $BF_{10} = 0.39$, error $< 0.012\%$; SR - $BF_{10} = 1.80$, error $< 0.001\%$;
 330 RT: SD - $BF_{10} = 0.333$, error $< 0.01\%$; SR - $BF_{10} = 0.34$, error $< 0.01\%$).

331 **3.3 Human electrophysiology: evoked potentials**

332 After having confirmed that performance was equal in the two tasks, we characterized
 333 the evoked potentials (EP) in each task. First, we estimated the difference between SR and
 334 SD conditions considering 7 midline electrodes (Fig.3). The results of a point-by-point t-test
 335 corrected for multiple comparisons revealed a significant difference in central and posterior
 336 electrodes (mostly Pz and CPz) between 250ms after the onset of the stimuli and the
 337 response cue, and the opposite effect in frontal electrodes (FCz and Fz) from 750ms to
 338 1000ms, as confirmed by the topography (Fig.3). Overall these results indicate larger
 339 potentials in visual areas during the SD task than in the SR. Previous studies have shown a
 340 relation between EP amplitude (particularly P300 and late components) with attention
 341 (Itthipuripat, Cha, Byers, & Serences, 2017; Itthipuripat, Cha, Deering, Salazar, & Serences,
 342 2018; Krusemark, Kiehl, & Newman, 2016; Van Voorhis & Hillyard, 1977) and visual working
 343 memory (Fabiani, Karis, & Donchin, 1986; Kok, 2001; McEvoy, 1998). Our results are thus

344 consistent with a larger involvement of executive functions in the SD vs. SR task. In the
 345 following, we investigated whether this hypothesis is corroborated by corresponding
 346 oscillatory effects in the time-frequency domain in the main experiment.



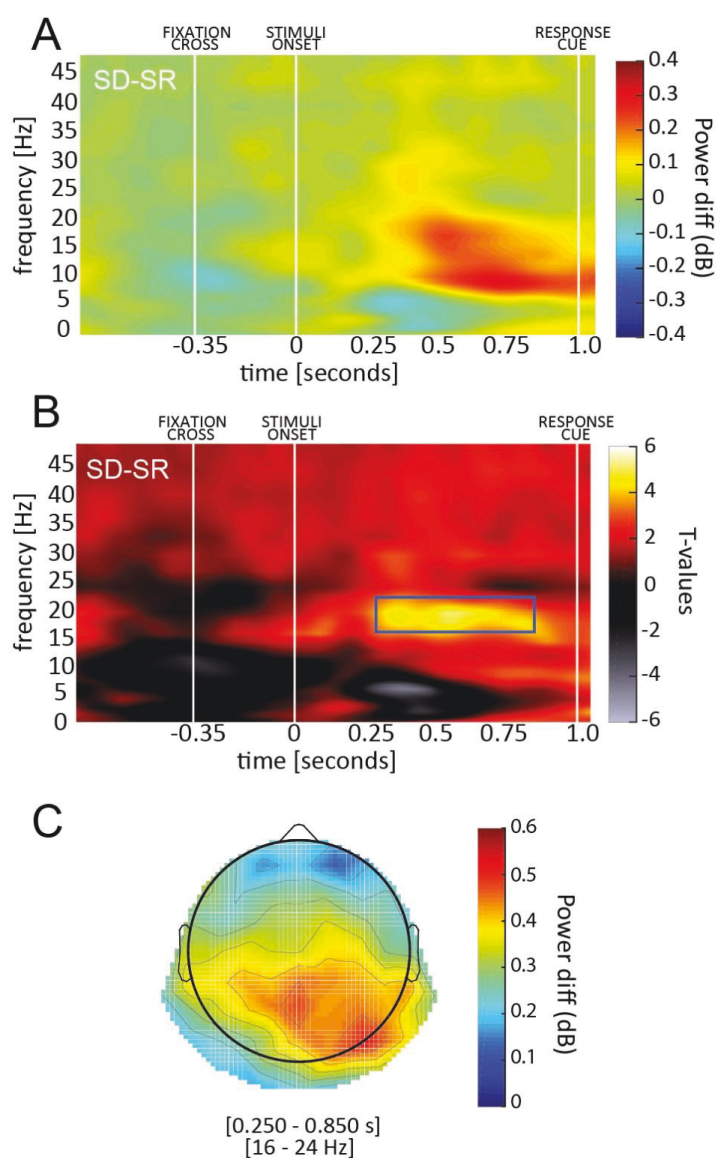
347 **Fig.3: ERPs results.** Each panel represents the difference between ERPs elicited in the SD and SR conditions for the 7 midline electrodes (average \pm SE). Shown in red are the points for which a significant difference was
 348 found against zero. The results reveal a significant difference from 250ms after stimuli onset until the response
 349 cue (at 1000ms) in central parietal regions, and an opposite effect after 750ms in frontal regions. In the
 350 bottom-right panel the topography, computed over the 250ms – 1000ms interval, confirmed a larger activity in
 351 the SD than in the SR condition (positive difference, warmer colors) in the central-parietal regions, and an
 352 opposite effect (negative difference, colder colors) in the frontal regions (which –although not significantly–
 353 also included occipital regions).

356

357 3.4 Human electrophysiology: time-frequency analysis

358 We performed a time-frequency analysis to try to identify differences between conditions
 359 observed in specific frequency bands commonly related to executive functions (e.g., visual
 360 working memory). For this purpose, we computed a baseline-corrected log-scale ratio
 361 between the two conditions (as shown in Fig. 4A), averaging over all electrodes.
 362 Remarkably, a point-by-point 2-tailed t-test corrected with cluster-based permutation test
 363 (Maris & Oostenveld, 2007) revealed a significantly larger activity in the low beta-band (16-
 364 24Hz) in the SD condition between 250 and 950ms after stimuli onset (Fig. 4B). We further
 365 quantify the magnitude of the effect by computing the effect size of a one sample t-test

366 against zero averaging per each participant the values within the significant region
 367 ($t(13)=2.571$, $p=0.023$, Cohen's $d=0.687$). The topography of the effect spread mostly over
 368 parietal and occipital regions (Fig. 4C), mimicking the topography of the EPs analysis. As
 369 previously, these results confirm the prediction that the SD task may involve additional
 370 computational mechanisms beyond feedforward computations, possibly indexed by the beta-
 371 band oscillatory processes identified here. As previously, these results confirmed those from
 372 the pilot experiment (figure 5D,E), confirming the robustness of the effect also in the
 373 oscillatory domain. Below, we contextualize and substantiate our results in light of the
 374 relevant literature.

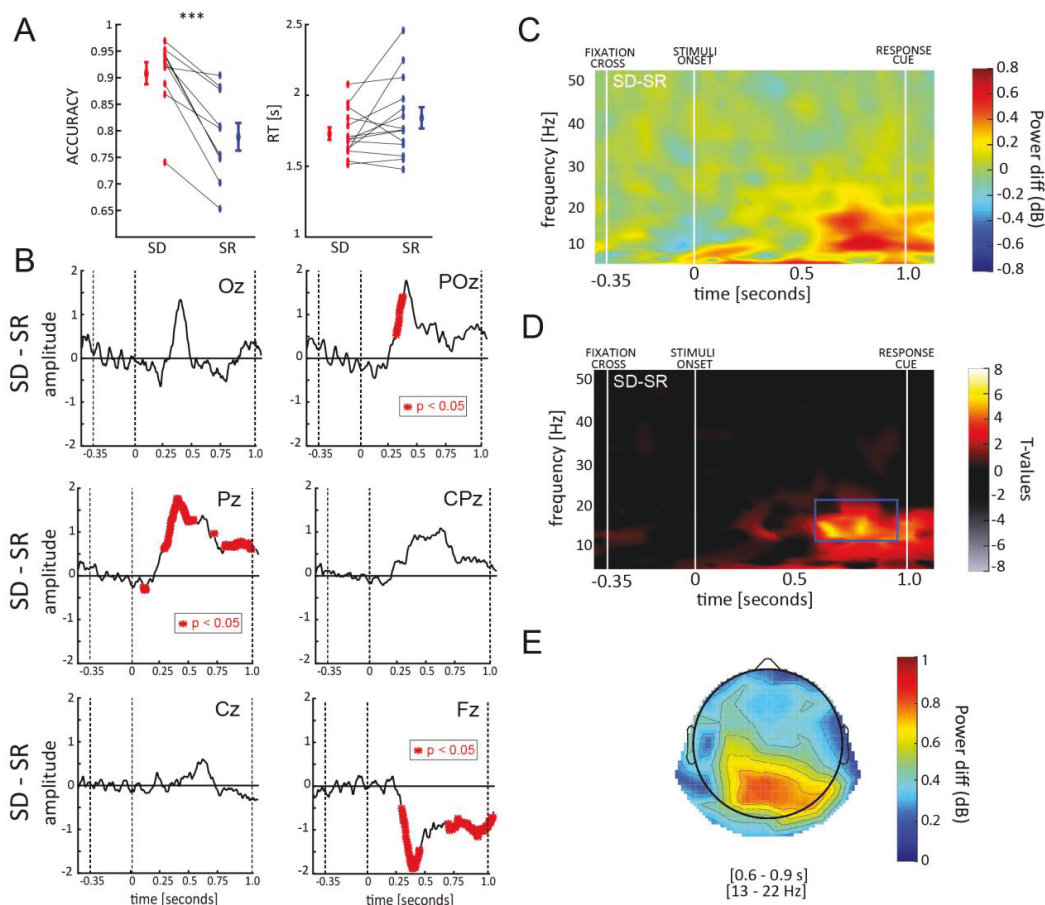


375

376 **Fig.4: Time-frequency results.** A) The difference between SD and SR power spectra is shown in the first panel.
 377 White lines indicate the onset of the fixation cross, the stimuli and the response cue. B) The second panel
 378 shows the corresponding t values (when testing the difference against zero). We observed a significant region
 379 in the low beta band (16-24Hz), between 250ms and 950ms after stimulus onset. C) The topography of the
 380 significant time-frequency window reveals the involvement of occipital-parietal regions.

381

382



383

384 **Fig.5: Pilot experiment results.** A) Behavioral results of the pilot experiment: left and right panel shows
 385 accuracy and reaction times for SD (red) and SR (blue) tasks. Differently than in the main task, in the pilot
 386 experiment participants performed significantly better in the SD than in the SR task (compare the accuracy
 387 between figure 5A and 2B). B) Difference between SD and SR evoked potentials. Red asterisk indicate time
 388 window significantly different than zero C) Difference between SD and SR power spectra: white lines indicate
 389 the stimulus onset and the response cue. D) Testing the SD-SR difference against zero reveals a significant
 390 region in the low beta band (13-21Hz), before the response cue, in agreement with the results of the main
 391 experiment –figure 4. We reported a large effect size for this effect (one sample t-test against zero averaging

per each participant the values within the significant region, $t(13)=7.049$, $p<0.001$, Cohen's $d=1.820$ E) As in the main experiment, the SD-SR difference mostly involves occipital-parietal regions.

4. Discussion

In this study, we confirmed in a series of two experiments a prediction from the computational study by Kim et al. (2018) that there exists an important dichotomy between visual reasoning tasks: While spatial relation (SR) tasks can be solved by modern deep convolutional neural networks (CNNs), same-different (SD) tasks pose a significant challenge for these architectures, suggesting the need for additional computations beyond a feedforward cascade of filtering, rectification and normalization operations. Importantly, the result of these simulations does not allow us to formulate any prediction about the specific cortical processes involved in the two tasks. Nonetheless, it demonstrates a fundamental computational difference, which can be tracked in terms of its human brain neural correlates while subjects solve SD vs. SR tasks (with difficulty objectively matched by an adaptive psychometric procedure). Remarkably, in both the pilot and the main experiment we found higher activity in the former task, in both evoked potentials and oscillatory components. We interpret these differences as reflecting additional computations required by the SD task. We can speculate that these additional computations involve working memory and attention processes, which are lacking in feedforward architectures such as CNNs.

Additionally, it is possible to interpret our results in a broader context, by considering other tasks supposed to involve spatial attention, such as visual search. Previous experimental work suggested the need for re-entrant processes (Treisman & Gelade, 1980; Wolfe, Cave, & Franzel, 1989), and how increased activity in specific oscillatory components (i.e. low [22-34Hz] and high [36-56Hz] gamma bands) are characteristic of these processes (Buschman & Miller, 2007; S. Phillips & Takeda, 2009). Accordingly, state-of-the-art computational models performing visual search and related tasks (e.g. instance segmentation) also employ attentional or recurrent mechanisms (Linsley, Ashok, Govindarajan, Liu, & Serre, 2020), supporting the hypothesis that convolutional feedforward networks can benefit from recurrent mechanisms in solving visual reasoning tasks (Kreiman & Serre, 2020).

Computational evidence for the hypothesis that the SD task requires additional computational mechanisms beyond those needed to solve the SR task is provided by the results of the Siamese network simulations (Bromley et al., 1994). This feedforward network processes each stimulus item in a separate (CNN) channel and then passes the processed items to a single classifier network. Since each item is processed separately (the network is

427 fed two images with only one item represented in each), this 'oracle' architecture performs
 428 the task with item-segmentation processes automatically provided. Our results (as previously
 429 shown on another dataset by Kim and colleagues (Kim et al., 2018) demonstrate that such a
 430 feedforward network, once endowed with object individuation using the Siamese
 431 architecture, can easily learn to solve the SD task. In other words, this model simulates the
 432 beneficial effects of attentional selection, individuation and working memory by segregating
 433 the representations of each item. Our EEG results are compatible with this interpretation,
 434 with higher activity in the SD compared to the SR task, visible in both evoked potentials and
 435 oscillatory frequency bands that have been previously related to attention and working
 436 memory (Benchenane et al., 2011; Lundqvist et al., 2018; Nash & Fernandez, 1996).

437 Previous work has shown that modulations of beta-band oscillations can be related to
 438 selective attention mechanisms (Benchenane et al., 2011; Buschman & Miller, 2007; Lee,
 439 Whittington, & Kopell, 2013; Richter, Coppola, & Bressler, 2018). Different attentional
 440 mechanisms may indeed be involved in the two tasks: the SR task could be solved by first
 441 grouping items and then determining the orientation of the group (Franconeri, Scimeca, Roth,
 442 Helseth, & Kahn, 2012), whereas the SD task requires the individuation of the two items
 443 before comparison. In addition, our results are also consistent with differences in memory
 444 processes between the two tasks (de Fockert, G., Frith, & Lavie, 2001). One common
 445 assumption is that items that are grouped together (as in the SR task) occupy only one
 446 working memory slot (Clevenger & Hummel, 2014; Franconeri, Alvarez, & Cavanagh, 2013),
 447 whereas non-grouped items would each hold one slot, resulting in a larger working memory
 448 load. Previous literature showed that working memory can also be characterized by neuronal
 449 oscillatory signatures. Recent studies, for example, have demonstrated an interplay between
 450 beta and gamma band frequencies during working memory tasks (Lundqvist et al., 2016,
 451 2018). Similarly, alpha and low beta bands, not only increase with working memory load
 452 (Babiloni et al., 2004; Pesonen, Hämäläinen, & Krause, 2007), but also in conjunction with
 453 the inhibition of competing visual memories in selective memory retrieval (Park, Min, & Lee,
 454 2010; Waldhauser, Johansson, & Hanslmayr, 2012). Besides, previous studies have
 455 reported that increased oscillatory activity in the alpha band is a signature of attentional
 456 processes, and it can predict the likelihood of successful trials in many tasks (Händel,
 457 Haarmeier, & Jensen, 2011; Klimesch, 2012; Nelli, Itthipuripat, Srinivasan, & Serences,
 458 2017); however, in our current study we did not investigate differences between correct and
 459 incorrect trials, but between different types of tasks (involving spatial relationship or
 460 sameness judgment), after controlling for task difficulty. This could explain why alpha-band
 461 amplitude differences were less prominent in our study. All considered, several lines of
 462 evidence point towards beta oscillations as crucially involved in both attention and working

memory related processes. These processes, therefore, might be part of the additional computational mechanisms required for SD tasks compared to SR tasks. Future work could more directly compare the attention and memory dependence of each task in human subjects.

That feedforward neural networks are limited in their ability to solve simple visual reasoning tasks is already being recognized by the computer vision and neuroscience communities (Kar, Kubilius, Schmidt, Issa, & DiCarlo, 2019; Rajalingham, Issa, Schmidt, Kar, & DiCarlo, 2017; Yamins, Hong, Cadieu, & Dicarlo, 2013). Current CNN extensions include modules for integrating local and global features (Chen et al., 2018) as well as recurrent neural architectures (Yang et al., 2018). Our results suggest that the human visual system also deploys additional computations beyond feedforward processes to successfully solve visual reasoning tasks. Rhythmic cortical oscillations in the beta-band represent the signatures of these additional computations, which may involve selective attention and working memory.

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