A Coupling Factor Independent Wireless Power Transfer System Employing Two Nonlinear Circuits

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***Abstract*—Conventional linear resonant wireless power transfer (WPT) systems suffer from a significant performance degradation as the coupling factor between the transmit and receive coils varies. In this paper, the performance of a new WPT circuit that takes the advantage of nonlinear resonant circuit is investigated. It employs passive nonlinear resonators in both the transmitter and receiver sides to regulate the output power without using any active or control circuitry, frequency tuning or complicated coil configurations. A 60 W nonlinear WPT prototype circuit working at 1.5 MHz is designed, fabricated and compared with a similarly designed linear WPT circuit. The nonlinear WPT circuit is capable of maintaining almost a constant output power while the distance between the coils changes from 6 cm to 15 cm, a significant performance improvement as compared to the linear WPT circuit tested under the same conditions.**

***Keywords*—Wireless power transfer, resonant type WPT, nonlinear resonator, distance insensitive, passive nonlinear capacitors, power regulation**

I. Introduction

Resonant-type wireless power transfer (WPT) is an attractive technology that enables contactless power transmission. It has a variety of applications including biomedical implants, portable consumer electronic devices and electric vehicle charging. Fig. 1 presents the schematic of a series-series WPT circuit.

The performance of conventional linear wireless power transfer systems is limited by their high sensitivity to the coupling factor variation between the receiver and transmitter coils (due to distance variation and misalignment). Output power of such systems can only reach the maximum at a certain distance (critical coupling between the transmitter and receiver) when the frequency is fixed and can significantly drop when their distance is increased or decreased. The output voltage gain for a typical series-series resonant WPT circuit is plotted in Fig. 2. There are three working regions for a two-coil WPT system, namely, critically-coupled, over-coupled and under-coupled. The critical coupling point is the maximum distance that energy can be effectively and efficiently transferred. Output power is maximized at resonant frequency in under coupled and critically-coupled regions. In the over-coupled region, frequency splitting effect exists, [1] where the maximum output power is reached at frequencies above or below the original resonant frequency for the system (i.e. even and odd mode frequencies). A mathematical description of the frequency splitting and even and odd mode frequencies are given in [2].

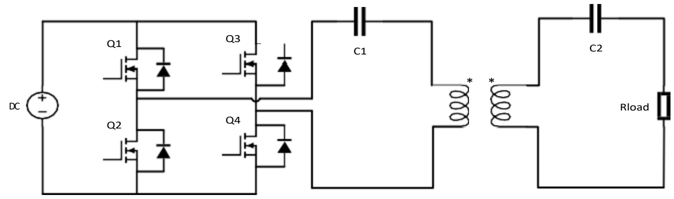


Fig. 1. Schematic of a series-series resonant WPT circuit.

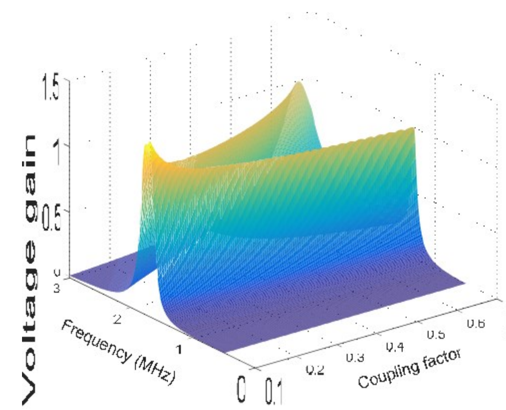


Fig. 2. Voltage gain versus operating frequency and coupling factor, representing the frequency phenomenon in a linear WPT system.

Several methods have been proposed to increase WPT's power transfer capability in the over-coupled region. Two mainstream solutions include: a) frequency tuning and b) impedance matching. In both cases, a series of complex circuits and algorithms are required, such as a phase compensator, phase-lock loops or switched capacitor banks. Additional detection circuits and feedback controls are also needed, which increases the system's complexity, cost, as well as its unreliability. Furthermore, when working at MHz frequencies, the existing ISM band limits the bandwidth, hence reducing the range of frequencies that can be tracked by the circuit. Another approach is to devise coil configurations that are less susceptible to coupling factor variation. However, such solutions often come at the cost of complicated transmitting and/or receiving coils [3] or a reduced power transmission range [4].

In previous work [5] and [6], a position-insensitive WPT system is designed by employing nonlinear resonance circuits. A single nonlinear capacitor was used in the receiver resonance circuit to maintain the WPT system's output power automatically, compensating for the mutual inductance change between the coils without using any active circuitry, feedback, control systems or imposing any requirement for frequency tuning.

This paper presents a novel nonlinear resonant type WPT topology with two nonlinear elements in both the transmitter and the receiver sides. Again, a constant output power can be maintained with just passive devices and no extra active control circuitry. According to our simulation and experimental results, in comparison to the previous work, this new WPT system provides improved output power regulation as a function of transmit and receive coil distance and misalignment. A 60 W WPT prototype has been designed, fabricated and measured.

II. NONLINEAR RESONANT CIRCUIT ANALYSIS

A. Nonlinear Capacitor Model

Varactor diodes can be used as nonlinear elements, where their capacitance varies as a function of the applied AC signal (devices used here do not require any DC bias). The total stored charge in the varactor diode is dominated by junction charge, given by

$$q(v) = \int_0^V \frac{C_{j0}}{\left(1 - \frac{v}{V_j}\right)^m} dv \qquad (1)$$

where C_{j0} is the zero bias junction capacitance, V_j is the built-in junction voltage, and m the grading coefficient. Since the nonlinear capacitor in WPT introduced here needs to withstand both positive and negative voltage swings, anti-series connected diode pairs are used as the nonlinear capacitors within this work.

As a large amplitude sinusoidal voltage is applied to the anti-series diode pair, during the first quarter cycle, the charge stored by the junction capacitance when varactor voltage reaches its peak value over the cycle, namely $q(V_{peak})$, is injected into the common anode node. This charge is then trapped in the node, keeping both diodes reverse biased over the cycle. This controls the average bias that appears across each junction capacitor. When the input voltage swings from positive to negative, the stored charge moves from the junction capacitor of diode D1 to diode D2. The voltage waveform across each diode is shown in Fig. 3(b).

The equivalent capacitance for the varactor can be defined based on the current and voltage or charge and voltage, as in (2). A capacitance – voltage relationship for the nonlinear element used in the experiments reported herein is shown in Fig. 3(c).

$$C_{eq}(V) = \frac{Q(V)}{V} = \frac{I(V)}{jwV} \qquad (2)$$
$$C_{eq}(Q) = C_{eq}(V) \cdot V$$

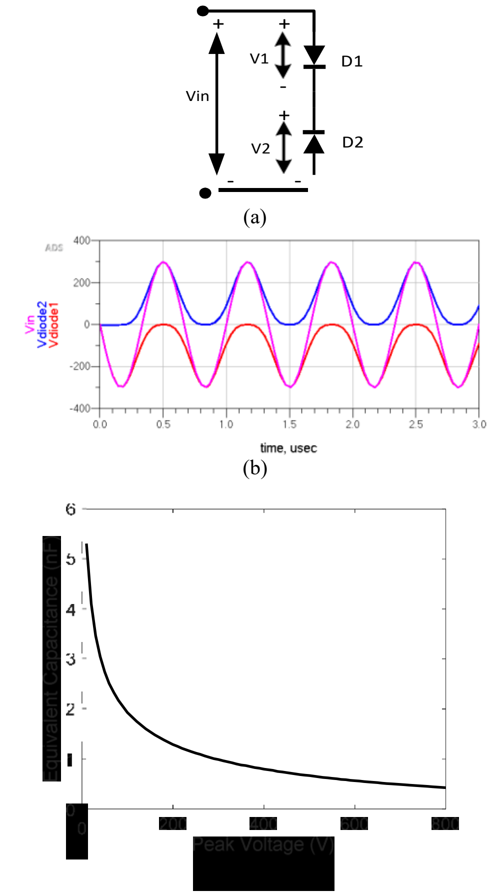


Fig. 3. a) Anti-series diode pair. b) Voltage waveform across diodes. c) Simulated equivalent capacitance versus excitation voltage amplitude (C-V) relationship.

B. Coupled Nonlinear Resonator Description

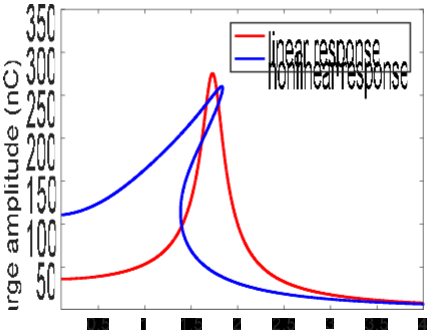
The forced oscillation of a single RLC resonator using a nonlinear capacitor is modeled by the Duffing differential equation (3),

$$\ddot{q} + \frac{R}{L}\dot{q} + \frac{1}{La_1}q + \frac{1}{La_3}q^3 = \frac{V_s}{L} \qquad (3)$$

where a_1 is the linear capacitance value and V_s is the excitation voltage. In [5] and [6], the nonlinearity of the capacitor is approximated by the third order term $(1/a_3)q^3$ when the voltage-charge relation of the capacitor follows $V_c = (1/a_1)q + (1/a_3)q^3$. Multiple scale method is applied to solve the Duffing differential equation, and an equivalent linear capacitance $C_{eff}(Q) = 4a_3/3Q^2$ is defined to quantify the restoring force contribution due to the third order term. The total equivalent capacitance is the series combination of C_{eff} and linear capacitance. The nonlinear equation (3) can be rewritten in the frequency domain as shown in (4), where its amplitude – frequency response is presented in Fig. 4, along with the frequency response of a linear resonance circuit.

$$(jw)^2QL + jwQR + \left(\frac{1}{a_1} + \frac{1}{C_{eff}(Q)}\right)Q = V_s \qquad (4)$$

Anti-series diode pairs are used as nonlinear capacitors, and the average restoring force over the cycle still holds. The average capacitance as a function of applied voltage or charge can be obtained by simulation [Fig. 3(c)] or through measurements. Based on this relationship, the amplitude – frequency response of a nonlinear series resonator can be calculated and plotted as in Fig. 4. The response is similar to the response of the Duffing resonance circuit, with a tilted resonant peak.



[redacted]

Fig. 4. Amplitude – frequency [redacted]gle series resonator with anti-series diode pair as nonlinear capacitor.

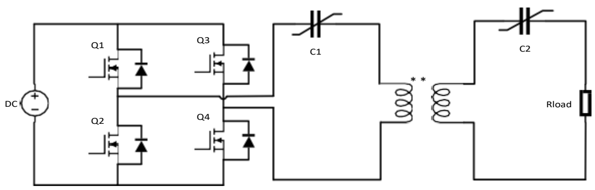


Fig. 5. Circuit schematic of the two nonlinear resonant WPT system.

A basic series-series resonant WPT topology, as shown in Fig. 5, is adopted in this design. The system has two nonlinear resonators coupled through the mutual inductance between the transmitter and receiver coils. The primary side resonator is driven by a converter and the secondary side is loaded by a 15 Ω resistor. The system can be modeled in the frequency domain by (5).

$$
\begin{aligned}
&((jw)^2Q_1L_1 + jwQ_1R_1 + \frac{Q_1}{C_1(Q_1)} - (jw)^2Q_2M = V_s \\
&1\,(jw)^2Q_2L_2 + jwQ_2R_2 + \frac{Q_2}{C_2(Q_2)} - (jw)^2Q_1M = 0
\end{aligned}
\qquad (5)
$$

Equation (5) can be numerically solved, where the secondary resonator charge amplitude response under different excitation frequencies and coupling is determined. With the aid of this tool, a two nonlinearity WPT circuit is designed and optimized. Fig. 6 presents circuit's simulated output power as a function of the coupling factor at a frequency of 1.5 MHz. As it can be seen, the output power delivered to a 15 Ohm load remains quite constant as the coupling factor between the transmit and receive coils significantly varies.

Circuit simulation indicates that the WPT circuit employing two nonlinear devices performs better than a similar WPT circuit which only has one nonlinear device. Specifically, the WPT circuit that uses two nonlinear devices provides a constant output power over a larger range of coupling factor. Furthermore, it has smaller power variation within the range as compared to the two nonlinearity circuit.

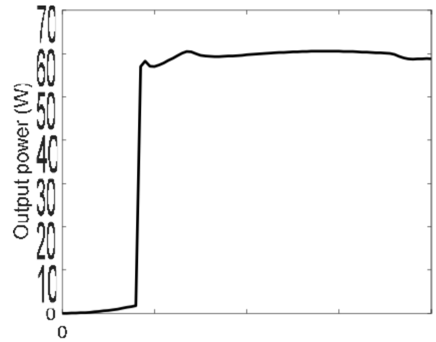


Fig. 6. Simulated output power versus coupling factor for the optimized two nonlinear resonance WPT circuit, compared to the optimized one nonlinear circuit.

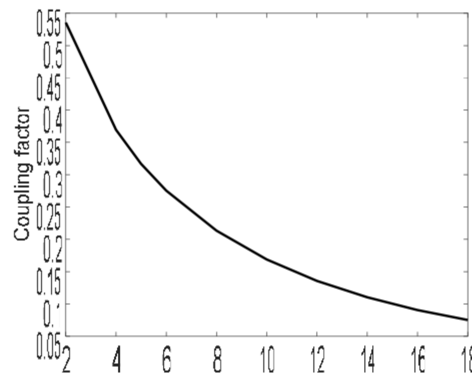


Fig. 7. Relationship between coupling factor and coil distance.

III. WPT Circuit Employing Two Nonlinear Coupled Resonators and Experimental Results

A WPT system employing two nonlinear resonators at the receiver and transmitter is designed and built. A 60 W circuit prototype, with nonlinear components on both transmitter and receiver side is fabricated. A linear WPT circuit of the same output power level and loaded coupling factor is also fabricated for comparison purposes. Both circuits use the same transmit and receive coils with inner and outer radii of 13.5 cm and 15 cm respectively. The coils are made from AWG 12 wires and each has 4 turns. A half-bridge GaN inverter that converts 120 V DC input voltage into a square wave at 1.5 MHz and excites the transmit resonant circuit.

Given the physical parameters of the transmitter and the receiver coil, the relationship between coil distances and coupling factor can be determined. Fig. 7 shows the results from a two-coil Ansys electromagnetic field simulation. The coupling factor in simulation results are converted into transmit and receive coils distance and compared with the measured results. The coupling factor variation in simulation can also be translated into lateral or angular misalignment.

As mentioned, both linear and nonlinear 60 W WPT circuits are designed at the critical coupling factor corresponding to 15 cm. For the nonlinear system, a constant 60 W output power can be maintained over the transmission distance from 6 cm to

15 cm as shown in Fig. 8. At distances larger than 15 cm, the output power drops, which is a characteristic of the Duffing type resonators. In contrast, the output power from the linear WPT circuit decreases when the coil distance changes in both directions away from the critical coupling point. The output power drops by 85% at 6 cm, while the nonlinear circuit power increases slightly as expected due to the smaller distance between the coils. Clearly there is a significant improvement in nonlinear WPT circuit performance in the over coupled region in comparison with the linear WPT circuit.

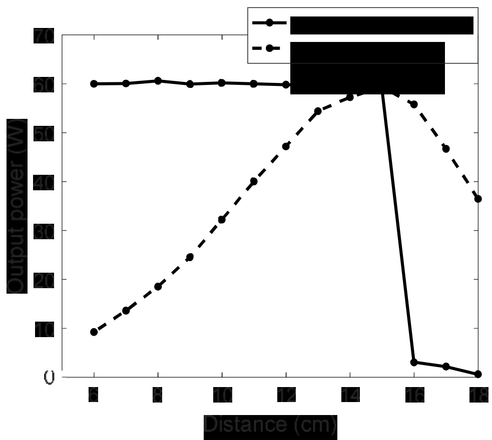


Fig. 8. Measured transmitted po[redacted]ance variation for a nonlinear and linear WPT circuit.

IV. Conclusion

This paper presents a new coupling factor insensitive WPT circuit design using two nonlinear devices in both the transmitter and receiver sides. The behaviour of such a nonlinear WPT circuit based on anti-series connected varactor pairs is analysed and simulated. The behaviour of single and coupled nonlinear resonators are modelled in the frequency domain and solved numerically. To verify the theoretical analysis, a 60 W series-series resonant type WPT circuit working at 1.5 MHz is designed and fabricated. It maintains a near constant output power over the transmission distance ranging between 6 cm to 15 cm.

Acknowledgment

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