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Power Markets with Information-Aware Self-Scheduling Electric Vehicles

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Abstract We consider multi-period (24-hour day-ahead) multi-commodity (energy and regulation reserves) decentralized electricity Transmission and Distribution (T&D) market designs. Whereas conventional centralized generators with uniform price-quantity offers are scheduled by a Transmission System Operator, low voltage network connected Distributed Energy Resources (DERs) with complex preferences and requirements, such as Electric Vehicles (EVs), are allowed to self-schedule adapting to spatiotemporal marginal cost based prices. We model the salient characteristics of interconnected T&D networks, and we consider self-scheduling DER responses under alternative distribution network *information-Aware* or *information-Unaware* market designs. Moreover, we consider a single (EV load aggregator) network-information-aware scheduler market design. Our contribution is the characterization and comparative analysis — analytic as well as numerical — of equilibria, using game theoretical approaches to prove existence and uniqueness, and the investigation of the role of information on self-scheduling and EV aggregator coordinated EV scheduling. Finally, we derive conclusions on the impact to social welfare and distributional equity of information-Aware/Unaware self-scheduling as well as single EV aggregator scheduling, and implications that are relevant to market design and policy considerations.

Keywords Decentralized market design · Self-scheduling market participants · Network-connected Distributed Energy Resources · Network Information-Aware/Unaware Market Participants · multi-commodity/multi-period equilibrium

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1 INTRODUCTION

1.1 Motivation and Background

Distributed Energy Resources (DERs) are becoming significant in electricity networks. They are capable of flexibly scheduling their hourly consumption, generation, and storage levels, thereby providing valuable hourly and real-time balancing services to the grid. Incorporating flexible DERs is crucial in view of increasing penetration of environmentally clean, yet intermittent and volatile renewable resources. Should existing wholesale markets implemented for centralized generation scheduling be extended to incorporate distribution networks with DER participants, they could bring significant benefits. In this work, we focus on Electric Vehicle (EV) battery charging as a rapidly increasing DER [1]. EVs can be put to dual use and provide regulation service reserves, while optimally scheduling their hourly charging. We focus on a multi-commodity 24-hour day-ahead market, where energy consumption/generation and regulation reserve provision by distribution network connected EVs and Transmission network connected generators are co-optimized under various decentralized market designs.

Conventional centralized market-clearing algorithms used today to determine socially optimal hourly locational marginal prices (LMPs) and generation schedules in the wholesale market (i.e., at high voltage transmission network nodes) become computationally intractable in the presence of a large number of distribution network connected DERs and their complex inter-temporal dynamics. Decentralized market-clearing with self-scheduling DERs adapting to granular distribution network locational prices appears to be the only feasible approach [2],[3]. Self-scheduling EVs optimally decide their hourly energy consumption and reserve provision levels in response to distribution locational marginal prices (DLMPs). DLMPs are hourly distribution network locational prices that are derived from transmission level LMPs adjusted to incorporate the cost of distribution network marginal line losses.[1] Decentralized Transmission and Distribution (T&D) market-clearing through DER self-scheduling raises several questions: Are there conditions, e.g., distribution network information available to individual DERs and load aggregators, that allow self-scheduling with anticipation of its impact on prices? Can self-scheduling lead to market-clearing equilibria, and if so, might social welfare be compromised? This paper analyzes and compares decentralized market designs to provide answers to these questions.

The various decentralized market designs considered in this work differ in terms of the distribution network information that self-scheduling DERs and load aggregators have access to. Self-scheduling EVs that are aware of distribution network information (e.g., how marginal line losses are affected by power flowing through them) can predict how DLMPs are affected by their own

[1] In reality, DLMPs are a function of other marginal costs, for instance transformer loss of life [4], that are not modeled here for simplicity, but have qualitatively similar impacts to those of marginal network losses.

and neighboring DER actions for given wholesale market LMPs. As such, self-scheduling DERs are no longer pure price-takers compatible with a competitive market. Due to the dependency of individual EV decisions on neighboring EVs, EVs engage in a non-cooperative game and the resulting equilibrium can be analyzed in the Nash Equilibrium context [5].

A major objective of this paper is to investigate the impact of information-aware DER scheduling designs on *(i)* the uniqueness of aggregate DER responses to wholesale market-clearing prices (LMPs), and *(ii)* the ability of information-aware DERs to influence T&D markets at the expense of social welfare. To this end, we examine the existence, uniqueness and efficiency of the Nash equilibrium of self-scheduling EVs. Our investigation compares self-scheduling DER decentralized market designs to a centralized market design where a single system operator, the T&D System Operator (TDSO), clears the market to optimize social welfare with complete knowledge of the T&D network characteristics and DER preferences and capabilities. Our comparison involves both the optimality conditions, and, for small system instances where the centralized market design can be solved, numerical solutions as well. We also consider a tractable and scalable centralized market design variation where a Transmission System Operator (TSO) collaborates with multiple Distribution System Operators (DSOs) to clear the T&D market. The TSO receives information on distribution network losses from DSOs and schedules both centralized generators and decentralized DERs in a small number of iterations needed for DER schedules and marginal losses to stabilize. Given the large number of distribution networks associated with a real life transmission system, this centralized market design is more practical, since there is no single operator that has knowledge of both T&D network characteristics. Moreover, we show that this design admits a decomposition to distributed algorithms that renders it scalable to real size systems and, in fact, equivalent to an Information-Unaware decentralized design.

We consider two centralized and three decentralized market designs, as shown in Figure 1. Centralized designs consist of *(i)* a single all-knowing operator (TDSO) who solves a T&D Optimal Power Flow (OPF) to schedule centralized generation and DERs to maximize social welfare, and *(ii)* a single TSO who schedules centralized generation and DERs iteratively using total and marginal line loss relations provided by collaborating DSOs. This enables the parallelization of T&D network load flows that eases the computation and information communication burden at the expense of social welfare attainment. As such, the latter centralized design provides a second best social welfare. Decentralized designs involve EV self-scheduling under distribution network Information-Aware/Unaware conditions. information-Aware EVs can explicitly calculate DLMPs at their own distribution network location as a function of LMPs, their own decisions and the aggregate demand of all others connected to the same location. Hence, distribution network information-Aware EVs are no longer price-takers and can exploit this information to influence market-clearing to their favor and at the expense of social welfare. Although LMPs are calculated by the TSOs, and EVs do not know, nor can they learn,

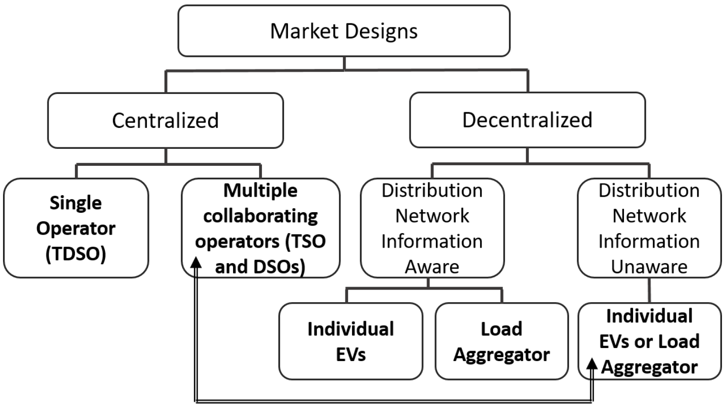

Fig. 1 Centralized and Decentralized Market Designs. Double arrow identifies designs with identical market-clearing outcome.

how to influence LMPs, distribution network information-aware EVs know how their actions impact marginal distribution network losses and through them DLMPs. When information-aware EVs rely on a load aggregator, who has access to their collective information and schedules them simultaneously, they can extract an even larger portion of social welfare for themselves. In contrast, distribution network information-Unaware EVs simply respond to DLMPs, and do not know, nor can they learn, how their own or others' actions affect DLMPs. Clearing of the decentralized markets under consideration may rely on distributed algorithm parallel computation. Interestingly, we show that the second best centralized design based on TSO-DSO collaboration is characterized by identical optimality conditions and reaches the same equilibrium outcome as the decentralized information-Unaware EV design. As such, the second best centralized design may also rely on parallel computation.

In conclusion, we analyze the aforementioned centralized and decentralized market designs to address key issues and questions that arise with self-scheduling participants. These issues center around the existence and uniqueness of equilibria, and the role that distribution network information awareness by self-scheduling DERs may play on social welfare and hence the fairness and efficiency of a market design. We finally note that under information-unaware conditions, the associated equilibrium of load aggregator assisted EV scheduling is identical to that of the individual EV self-scheduling market design.

1.2 Related Work and Contributions

Decentralized control of DERs has gained significant attention ([6]-[16]) due to scalability issues of load aggregation and direct utility control methods [17], [18]. Decentralized EV charging control with the objective of load flattening is studied in [12] and [13], without considering distribution network market with locational and hourly marginal prices. In [13], optimal EV charging schedules obtained from the decentralized and the centralized problems are identical, however, network properties, such as losses, are not considered. A similar result is shown in [16], however, even though distribution network costs are approximated and included in the objective function, network losses are not modeled explicitly. In general, decentralized control methods for EV charging are based on the assumption that EVs are price-takers [11], [16]. In this work, we study market designs where EVs or load aggregators use network information, hence anticipate and influence local distribution level prices.

The differences in individual versus load aggregator EV scheduling in the presence of a large number of EVs are studied in [10]. However, distribution networks are not modeled, their impact on the marginal cost that drives prices is omitted, and the eventual long-term adaptation of the network to increasing number of EVs is not considered either. The energy price is also simplified as a linear function of the total load. In this work, we present a closed-form of EV schedules enabled by a special case assuming an unlimited EV charging rate capacity similar to the assumption used in [10]. In addition, we construct the closed-form optimal market-clearing schedules under network conditions necessary for adaptation of a large number of EVs and study important qualitative differences between individual and load aggregator scheduling.

Two important issues arise in the decentralized market designs: *(i)* Is there a unique equilibrium that can be obtained for the market to clear, and moreover, *(ii)* is the equilibrium efficient? That is, does the decentralized self-scheduling equilibrium match the equilibrium obtained by centralized market clearing? Or is the opposite true, that is, can DERs self-schedule to increase their individual benefits at the expense of social welfare? Due to the hierarchical nature of decentralized designs, game theoretical methods have been widely adapted to answer such questions. For instance, hierarchical DER-system operator interactions are studied in [19]-[21], [24]. We adopt similar methods and develop novel conclusions on the second question, focusing on the interactions among self-scheduling DERs. We investigate equilibrium uniqueness under system operator calculated LMPs, but with distribution network information-Aware/Unaware conditions using in both cases the potential function approach [34] employed in the DER control literature, e.g., [26]-[28]. Competitive and Nash equilibria for price-taking and price-anticipating DERs are also studied in [27], however, salient network characteristics, such as line losses and their price ramifications, are considered only in this work.

In [7], [14], [15], equilibrium existence and uniqueness is investigated for asymptotically increasing EVs under the mean field game theory concept, where EVs respond to an aggregate population signal and the contribution of each individual EV to the aggregate signal is negligible. In such a case, it is shown in [7] that the EV best-response iterations converge to a unique Nash equilibrium. However, given the granularity of the distribution network, it is interesting to consider interactions among a finite number rather than an asymptotically increasing number of DERs. To this end, we study the uniqueness of Nash Equilibrium for finite number of EVs connected to the distribution network. Furthermore, for a simplified model with infinite charging rate capacity and identical EVs, we derive closed-form Nash equilibrium schedules for distribution network information-Aware/Unaware EVs.

Most game theoretical methods in DER control are applied to single-period, single-commodity markets [21]-[24], [28]. Equilibrium uniqueness is shown in [19] and [25] for a multi-period DER Game, but DERs minimize the same global objective. The hierarchical game between the price-determining utility company and EVs that also compete among them — since each EV charging strategy space depends on other EV strategies — is investigated in [20], however, EVs are price-takers and are not subject to inter-temporal constraints.

Our contributions in this paper are as follows:

- We model the salient details of decentralized multi-period multi-commodity (i.e., 24-hour energy and regulation reserve) market designs and consider distribution network information-Aware/Unaware EVs. We employ a high fidelity model internalizing key T&D network characteristics, such as losses.
- In decentralized market designs, we show Nash equilibrium existence and uniqueness for schedules of network information-Aware EVs, enabling them to anticipate local prices (DLMPs). We repeat for network information-Unaware, local price non-anticipating EVs, and we show that this equilibrium is identical to that of a centralized design, where the TSO (who schedules the DERs) receives DLMP information from collaborating DSOs.
- We show existence and uniqueness of a decentralized market equilibrium, where all EVs are scheduled by a network information-Aware aggregator to whom they entrust their individual preferences.
- Both numerically and analytically, we investigate differences between centralized and decentralized market design equilibria, and we evaluate impacts on social welfare and EV charging costs.

The remainder of this paper is organized as follows. Section 2 introduces a stylized T&D network and the market participants. Section 3 describes the clearing processes of centralized and decentralized market designs, and Section 4 compares their equilibria by contrasting the first order optimality conditions. Section 5 shows existence and uniqueness of equilibria for fixed wholesale prices (LMPs), and Section 6 derives closed-form expressions assuming similar EV preferences and relaxing the charging rate constraints. Section 7 presents numerical results on the full market design/EV preference and capability models illustrating social welfare and EV charging cost differences across market designs. Section 8 summarizes the main findings and their likely impact on policy and regulatory concerns, and identifies interesting directions for future research. For ease of exposition, the proofs are moved to Appendix A.

2 T&D Network, Participants, and Nodal Marginal Costs

We consider a T&D network with centralized generators connected to the high voltage low losses transmission network and distribution network consumers with both inelastic and flexible EV demand connected to the low voltage high losses distribution network. For simplicity, we assume there is a single transmission bus, as shown in Figure 2. This simplification does not affect the generality of our results; multiple transmission buses with limited capacity lines can be easily handled. Similar to [12], we consider N radial feeders. Notably, radial operation is common in distribution networks. Each feeder $n \in \mathcal{N}$ is connected to the transmission bus through a single line, with aggregated distribution feeder demand, both inelastic and flexible, located at the end of each line (see Figure 2). Each EV $j \in \mathcal{J}$ consumes $q^P_{j,t}$ amount of energy and provides $q^R_{j,t}$ amount of regulation reserves during hour t. We define T_j as the number of hours EV j is plugged in, and assume that T_j is deterministic. The subset $\mathcal{J}_{n,t}$ is the set of EVs that are connected to feeder n at hour t. The total demand at location n, hour t is given by:

$$d^P_{n,t} = d^f_{n,t} + \sum_{j \in \mathcal{J}_{n,t}} q^P_{j,t}, \tag{1}$$

where $d^f_{n,t}$ is the inelastic demand at feeder n. We explicitly model quadratic distribution line losses as a function of total load at feeder n. Quadratic and marginal losses on line n are given by:[2]

$$L_{n,t} = \frac{\gamma_n}{2}(d^P_{n,t})^2, \tag{2}$$

$$m_{n,t} = \frac{\partial L_{n,t}}{\partial d^P_{n,t}} = \gamma_n d^P_{n,t}, \tag{3}$$

where γ_n is the line loss factor and is equal to two times the ratio of the line resistance over the square of voltage at n.

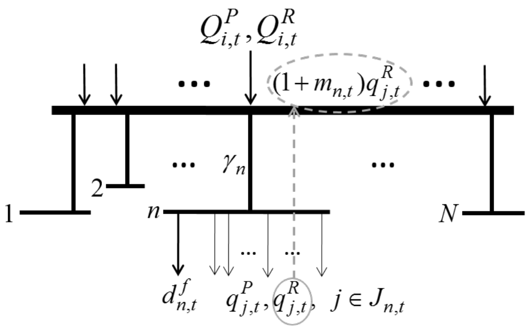

Fig. 2 T&D network, single transmission bus, multiple distribution networks $n = 1, ..., N$.

Each generator $i \in \mathcal{I}$ has a capacity and technical minimum of $\overline{Q}_i, \underline{Q}_i$, as well as marginal energy generation and reserve provision cost denoted by c^P_i

[2] Index n is interchangeably used for both the distribution feeder location and the line connecting this location to the transmission bus.

and c_i^R. The total energy generation and reserve provision cost of generator i during hour t is $C_i^P(Q_{i,t}^P) = c_i^P Q_{i,t}^P$, and $C_i^R(Q_{i,t}^R) = c_i^R Q_{i,t}^R$, respectively. The T&D market is simultaneously cleared with the objective of achieving the minimum energy generation plus reserve provision cost.

Lastly, we introduce the following vectors: $q_j^P = \{q_{j,t}^P, \forall t\}$, $q^P = \{q_j^P, \forall j\}$, $d^P = \{d_{n,t}^P, \forall n, t\}$, $L = \{L_{n,t}, \forall n, t\}$; vectors q_j^R, q^R, Q^P, Q^R are defined similarly. For clarity, we list the notation in Table 1.

Table 1 Notation

	Abbreviations
TDSO	Transmission Distribution System Operator
TSO	Transmission System Operator
DSO	Distribution System Operator
DER	Distributed Energy Resource
EV	Electric Vehicle
	Indices
ind	Individual scheduling
agg	Aggregator scheduling
A	Information-Aware
Un	Information-Unaware
P	Energy
R	Reserves
	Sets
$\mathcal{J}$	Set of EVs, $\mathcal{J} = \{1, .., j, .., J\}$
$\mathcal{I}$	Set of generators, $\mathcal{I} = \{1, .., i, .., I\}$
$\mathcal{N}$	Set of distribution network nodes, $\mathcal{N} = \{1, .., n, .., N\}$
	Input & Decision Variables
$q_{j,t}^P, q_{j,t}^R$	Energy consumption and reserve provision of EV j during hour t
$Q_{i,t}^P, Q_{i,t}^R$	Energy generation and reserve provision of generator i during hour t
$\overline{q}_j$	Charging rate capacity of EV j
$\overline{Q}_i$, $\underline{Q}_i$	Generation capacity and technical minimum of generator i
$d_{n,t}^f, \overline{R}_t$	Inelastic demand at node n and system reserve requirement during hour t
$L_{n,t}, m_{n,t}$	Total quadratic and marginal losses at node n during hour t

3 Centralized and Decentralized T&D Market Designs

This section details the algorithms underlying the centralized (Subsection 3.1) and decentralized market designs (Subsection 3.2).

3.1 Centralized Market Designs

3.1.1 Centralized Market-Clearing with a Single TDSO ($\mathbf{TD_A}$)

In this centralized design, a single TDSO clears the market in a single step with access to full distribution network information including distribution feeder specific loss factor values, γ_n, central generator variable costs, c_i^P, c_i^R, as well as EV preferences and capabilities. The market equilibrium consists of a complete centralized generation and EV energy and reserve schedule across feeders and hours and is obtained by the following social cost minimization problem:

$$\min_{Q^P,Q^R,q^P,q^R} \sum_{i,t} \left(c_i^P Q_{i,t}^P + c_i^R Q_{i,t}^R \right) + \sum_{j,t} \delta (q_{j,t}^P)^2, \tag{4}$$

subject to

– **Energy balance and reserve requirement constraints:**

$$\sum_i Q_{i,t}^P = \sum_n [L_{n,t}((\Sigma q_{j,t}^P)^2) + d_{n,t}^P] \quad \forall t \rightarrow \lambda_t^P, \tag{5}$$

$$\sum_i Q_{i,t}^R + \sum_{n,j \in J_{n,t}} [1 + m_{n,t}(\Sigma q_{j,t}^P)] q_{j,t}^R \geq \overline{R}_t \quad \forall t \rightarrow \lambda_t^R, \tag{6}$$

– **Generator constraints:**

$$Q_{i,t}^P + Q_{i,t}^R \leq \overline{Q}_i, \quad Q_{i,t}^P - Q_{i,t}^R \geq \underline{Q}_i \quad \forall i,t, \tag{7}$$

– **EV constraints:**

$$\sum_t q_{j,t}^P \geq \underline{s}_j \ \forall j \rightarrow \underline{\zeta}_j, \tag{8}$$

$$\sum_t q_{j,t}^P \leq \overline{S}_j \ \forall j \rightarrow \bar{\zeta}_j, \tag{9}$$

$$q_{j,t}^R + q_{j,t}^P \leq \overline{q}_j \ \forall j,t \rightarrow \nu_{j,t}^1, \tag{10}$$

$$q_{j,t}^R - q_{j,t}^P \leq 0 \ \forall j,t \rightarrow \nu_{j,t}^2. \tag{11}$$

where $L_{n,t}((\Sigma q_{j,t}^P)^2)$ and $m_{n,t}(\Sigma q_{j,t}^P)$ are given by (2) and (3). Note that all decision variables are non-negative. We refer to the above problem as $\mathbf{TD_A}$, where **TD** denotes simultaneous clearing of the T&D networks, and **A** denotes the distribution feeder information-Aware TDSO, i.e., its knowledge of the detailed functional form of $m_{n,t}$, and loss factor values, γ_n, $\forall n$.

The objective function (4) models total generation and reserve provision cost, as well as aggregate EV costs. The term $\delta(q_{j,t}^P)^2$ represents the EV battery degradation cost, which penalizes fast charging [29]. The dual variables, λ_t^P and λ_t^R, of the energy balance and reserve requirements constraints in (5) and (6) represent the energy and reserve LMPs at the transmission bus (or distribution substation) over the 24-hour daily cycle. The energy balance (5) is a non-convex equality constraint due to the inclusion of quadratic distribution network losses that can be relaxed (convexified) to an inequality constraint:

$$\sum_i Q_{i,t}^P \geq \sum_n [L_{n,t}((\Sigma q_{j,t}^P)^2) + d_{n,t}^P] \quad \forall t \rightarrow \lambda_t^P,$$

which will in fact be binding in most cases, expect for extreme cases, e.g. when the reserve provision cost of EVs is very high.

Note that reserves offered at the end of a distribution feeder n provide a higher per unit amount at the transmission bus level due to line losses. Since the ratio of total reserve deployment from EVs to inelastic demand is fairly small, it is reasonable and practical to approximate incremental losses by marginal losses. Therefore, offering $q^R_{j,t}$ amount of reserves by EV j at the end of distribution feeder n is equivalent to offering $q^R_{j,t}(1+m_{n,t})$ of reserves at the transmission bus. The system reserve inequality constraint, 0which notably includes bilinear terms $q^P_{j,t}q^R_{j,t}$, requires a total reserve provision by generators and EVs that is equal to or exceeds system reserve requirements, $\overline{R}_t$.

Given the transmission bus wholesale LMPs, λ^P_t and λ^R_t, the marginal cost based prices at distribution feeder n (DLMPs), $\lambda^P_{n,t}$ and $\lambda^R_{n,t}$, are given by:[3]

$$\lambda^P_{n,t} = [1 + m_{n,t}(\Sigma q^P_{j,t})]\lambda^P_t, \tag{12}$$

$$\lambda^R_{n,t} = [1 + m_{n,t}(\Sigma q^P_{j,t})]\lambda^R_t. \tag{13}$$

We note again that the centralized TDSO market design requires complete knowledge of individual DER preferences and distribution network feeder information, an onerous task in itself. Moreover, the non-convexities identified above render its solution particularly hard and not scalable for real size systems. We solve for this market design equilibrium on a relatively small system for the purpose of comparing social welfare impacts from the adoption of decentralized market designs. Before proceeding with the decentralized designs, we explore next a second best (in terms of social welfare) centralized market design, which we show to admit a parallelizable clearing process, and is hence scalable to real size systems.

3.1.2 Centralized Market Design with TSO-DSO Collaboration ($\mathbf{TD_{Un}}$)

Centralized generators and DERs are cleared by the TSO, who does not have access to distribution feeder information, relying instead on DSOs that communicate the total value of losses and marginal losses on each distribution network feeder. All DERs, however, provide the TSO with their preferences and capabilities, namely the battery degradation cost and their individual constraints (8)-(11). The TSO proceeds to an iterative process interacting with DSOs to obtain the value of total and marginal losses ($L_{n,t}$ and $m_{n,t}$) associated with the most recent DER schedule set by the TSO. Note that the TSO does not need to know the values of distribution line loss factors, γ_n. We refer to this design as $\mathbf{TD_{Un}}$, since the TSO is Unaware of feeder information, but T&D markets are cleared simultaneously. The energy balance and reserve requirements constraints (5) and (6) in the TDSO centralized problem are replaced with the following:

$$\sum_i Q^P_{i,t} = \sum_n (L^{(k)}_{n,t} - m^{(k)}_{n,t} d^{P,(k)}_{n,t}) + \sum_n [(1 + m^{(k)}_{n,t}) d^P_{n,t}] \quad \forall t, \tag{14}$$

[3] The reader is forewarned that in the DLMP directed decentralized market designs, distribution feeder information-Aware EVs will have access to the exact functional form of DLMPs and price anticipation will be possible impacting the associated Nash Equilibrium.

$$\sum_i Q^R_{i,t} + \sum_{n,j \in J_{n,t}} [(1 + m^{(k)}_{n,t}) q^R_{j,t}] \geq \overline{R}_t \quad \forall t, \tag{15}$$

where a first order Taylor approximation is used for the quadratic losses to linearize the energy balance and reserve requirements constraints. The iterative market-clearing process for $\mathbf{TD_{Un}}$ is described in Algorithm 1.

Algorithm 1 : $\mathbf{TD_{Un}}$

Initialize $d_n^{P,(0)}$, calculate $m_n^{(0)}$, $L_n^{(0)}$ given $d_n^{P,(0)}$, and set $k := 0$
while $||m_n^{(k+1)} - m_n^{(k)}|| > tolerance$ **do**
  Step 1. Conditional upon $m_n^{(k)}$, $L_n^{(k)}$, $d_n^{P,(k)}$, the TSO optimizes (4) subject to (14), (15), (7)–(11) and simultaneously schedules $Q^{P,(k+1)}$, $Q^{R,(k+1)}$, $q^{P,(k+1)}$, $q^{R,(k+1)}$.
  Step 2. Given the updated $q^{P,(k+1)}$, the DSO calculates $d_n^{P,(k+1)}$, $m_n^{(k+1)}$, $L_n^{(k+1)}$ and submits them to the TSO.
  $k := k + 1$
end while
return q^P, q^R, Q^P, Q^R, λ^P, λ^R, λ^P_n, λ^R_n

3.2 Decentralized Market Designs

In the decentralized designs, the market clears to an equilibrium obtained by a converging iterative process. In each iteration, the TSO solves a simple (linear) OPF problem that schedules centralized generator energy and reserve provision for given DER energy and reserve schedules adjusted for the impact of losses over distribution feeders. This tentative OPF solution updates the LMPs and DLMPs. EVs then modify their energy and reserve schedules so as to minimize their individual costs against the updated DLMPs. To avoid or damp oscillations, the EV cost minimization problems are solved with Lagrangian Augmentation regularization terms added to their objective function [31], and the iterations continue until convergence. Hence, the iterative process that involves the TSO solving a Transmission System OPF given EV schedules to determine DLMPs, and the EVs that respond to DLMPs by optimally self-scheduling — solving well defined convex cost-minimization problems — falls into the broad category of proximal algorithms for which we can achieve descent (in the system cost) at each iteration [31]. We note that given existence and uniqueness of the equilibrium (which we show in Section 5), convergence of the iterative algorithms described in detail below is achieved by an appropriate selection of the regularization term coefficients.

EV self-scheduling is tantamount to parallelizing the market equilibrium process. In fact, the centralized $\mathbf{TD_{Un}}$ design clears to the same equilibrium as the information-Unaware decentralized design, hence amenable to parallelization.[4]

[4] Moreover due to privacy concerns, EV owners might not opt to communicate their preferences directly to the TSO.

We investigate next three decentralized market designs:

1. Feeder information-Unaware EVs individually self-scheduling: $\mathbf{EV}^{\mathbf{ind}}_{\mathbf{Un}}$;
2. Feeder information-Aware EVs individually self-scheduling: $\mathbf{EV}^{\mathbf{ind}}_{\mathbf{A}}$, and
3. EVs in the same feeder are scheduled by a feeder information-Aware Load Aggregator: $\mathbf{EV}^{\mathbf{agg}}_{\mathbf{A}}$.

The reason that we do not consider an information-unaware load aggregator market design is that it can be analytically shown to reach an identical market-clearing equilibrium as the $\mathbf{EV}^{\mathbf{ind}}_{\mathbf{Un}}$ design. In fact, load aggregation does not provide EVs with any advantage in the absence of access to distribution feeder information (i.e., the functional form of marginal losses $m_{n,t}$).

For simplicity of exposition and without loss of generality, we used the staircase supply function employed in today's wholesale power markets, which renders LMPs rather insensitive to EV schedule changes. This is a reasonable choice given that LMP determination is dominated by the mostly price inelastic conventional demand that will continue to make up the bulk of consumption even after EV battery charging accounts for over 10 percent of total electricity demand by 2030. Nevertheless, DLMPs in EV connected feeders remain quite sensitive to EV self-scheduling decisions.

3.2.1 Information-Unaware EV Self-Scheduling ($\mathbf{EV}^{\mathbf{ind}}_{\mathbf{Un}}$)

In the $\mathbf{EV}^{\mathbf{ind}}_{\mathbf{Un}}$ decentralized design, EVs do not have access to the functional form of marginal losses — see (12) and (13), and as such they are price-takers that schedule their battery charging to minimize their perceived charging costs.

The equilibrium is reached by an iterative process. At iteration k, upon receiving the tentative LMPs from the TSO and the value of marginal losses from the DSO, EV j minimizes its costs against DLMPs estimated according to (12) and (13) by solving:

$$\min_{q_j^P, q_j^R} \sum_t \left[\lambda_{n,t}^{P,(k)} q_{j,t}^P - \lambda_{n,t}^{R,(k)} q_{j,t}^R + \delta (q_{j,t}^P)^2 + \theta \| q_{j,t}^P - q_{j,t}^{P,(k)} \|^2 \right], \tag{16}$$

subject to (8)–(11). Note that the Lagrangian Augmentation regularization term, $\theta \| q_{j,t}^P - q_{j,t}^{P,(k)} \|^2$, approaches to zero upon convergence and assists in avoiding/damping oscillatory behavior during the iterative process [31]. The TSO then solves an OPF problem, which we refer to as $\mathbf{T_{Un}}$, since it is essentially the $\mathbf{TD_{Un}}$ market-clearing problem conditional upon (i.e., given) self-scheduled (fixed) EV energy and reserve quantities $q_{j,t}^{P,(k+1)}$, $q_{j,t}^{R,(k+1)}$:

$$\min_{Q^P, Q^R} \sum_{i,t} \left(c_i^P Q_{i,t}^P + c_i^R Q_{i,t}^R \right), \tag{17}$$

subject to (7) and

$$\sum_i Q_{i,t}^P = \sum_n \left(L_{n,t}^{(k+1)} + d_{n,t}^{P,(k+1)} \right) \;\forall t \rightarrow \lambda_t^{P,(k+1)}, \tag{18}$$

$$\sum_i Q^R_{i,t} + \sum_{n,j\in J_{n,t}} \left(1 + m^{(k+1)}_{n,t}\right) q^{R,(k+1)}_{j,t} \geq \overline{R}_t \ \forall t \rightarrow \lambda^{R,(k+1)}_t. \tag{19}$$

The iterative process, which describes the market-clearing of the $\mathbf{EV^{ind}_{Un}}$ decentralized market design, is summarized in distributed Algorithm 2.

Algorithm 2 : $\mathbf{EV^{ind}_{Un}}$ (Distributed Algorithm)

Initialize LMPs $\lambda^{P,(0)}, \lambda^{R,(0)}$, marginal losses $m^{(0)}_n$ and set $k := 0$
while $\sum_{j,t} ||q^{P,(k+1)}_{j,t} - q^{P,(k)}_{j,t}|| > tolerance$ **do**
  Step 1. $\forall j \in \mathcal{J}$, EV j synthesizes $\lambda^{P,(k)}_n, \lambda^{R,(k)}_n$ given $\lambda^{P,(k)}, \lambda^{R,(k)}$, and $m^{(k)}_n$ according to (12) and (13).
  Step 2. $\forall j \in \mathcal{J}$, EV j optimizes (16), subject to (8)–(11), given $\lambda^{P,(k)}, \lambda^{R,(k)}$, and updates $q^{P,(k+1)}_j$ and $q^{R,(k+1)}_j$, and submits them to the DSO.
  Step 3. Given $q^{P,(k+1)}$, the DSO updates $L^{(k+1)}_{n,t}$ and $m^{(k+1)}_{n,t}$ according to (2) and (3), and submits them to the TSO.
  Step 4. Given $q^{P,(k+1)}$, $q^{R,(k+1)}$, $m^{(k+1)}_{n,t}$, and $L^{(k+1}_{n,t}$, the TSO solves $\mathbf{T_{Un}}$, optimizing (17), subject to (7), (18), (19), and determines $\lambda^{P,(k+1)}, \lambda^{R,(k+1)}$.
  $k := k + 1$
end while
return q^P, q^R, Q^P, Q^R, λ^P, λ^R, λ^P_n, λ^R_n

3.2.2 Information-Aware EV Self-Scheduling ($\mathbf{EV^{ind}_{A}}$)

In the $\mathbf{EV^{ind}_{A}}$ decentralized design, EVs are feeder information-Aware, i.e., they know the functional form of marginal losses, $m_{n,t}$, and as such they are able to anticipate how their own and other EVs' actions influence the DLMPs at their own feeder, i.e., they know $m_{n,t}\lambda^P_t$ and $m_{n,t}\lambda^R_t$. EV j minimizes:

$$\min_{q^P_{j,t}, q^R_{j,t}} \sum_t \left\{ \begin{array}{r} \left[1 + m_{n,t}(q^P_{j,t}, q^P_{-j,t})\right] \left(\lambda^P_t q^P_{j,t} - \lambda^R_t q^R_{j,t}\right) \\ +\delta(q^P_{j,t})^2 + \theta ||q^P_{j,t} - q^{P,(k)}_{j,t}||^2 \end{array} \right\}, \tag{20}$$

subject to (8) – (11), where $q^P_{-j,t} = \sum_{j' \in J_{n,t}, j' \neq j} q^P_{j',t}$ is the complementary EV load to EV j, and $m_{n,t}(q^P_{j,t}, q^P_{-j,t}) = \gamma_n(q^P_{j,t} + q^P_{-j,t} + d^f_{n,t})$. Since the optimal self-schedule of EV j depends on its own and neighboring EV schedules, the equilibrium can be studied in the Nash Equilibrium context.

The iterative process, which describes the market-clearing of the $\mathbf{EV^{ind}_{A}}$ decentralized market design, is summarized in distributed Algorithm 3. It differs from Algorithm 2, since in Step 1, EV j knows the parameters and arguments of the feeder marginal losses $m_{n,t}(q^P_{j,t}, q^P_{-j,t})$. More precisely, EV j can infer the sum of total neighboring EV load and inelastic consumption, $q^P_{-j,t} + d^f_{n,t}$, when it is provided the value of $m_{n,t}$. EVj can therefore use this information when solving the $\mathbf{EV^{ind}_{A}}$ cost minimization problem assuming $q^P_{-j,t}$ will be equal to the previous iteration's value deduced from the most recent value of

$m_{n,t}$. As such, EVs engage in a best response iterative action moderated by the regularization term.

Algorithm 3 : $\mathbf{EV_A^{ind}}$ (Distributed Algorithm)

Initialize LMPs $\lambda^{P,(0)}, \lambda^{R,(0)}$, marginal losses $m_n^{(0)}$ and set $k := 0$
while $\sum_{j,t} ||q_{j,t}^{P,(k+1)} - q_{j,t}^{P,(k)}|| > tolerance$ **do**
  Step 1. $\forall j \in \mathcal{J}$, EV j calculates $d_n^f + q_{-j}^{P,(k)}$ given the functional form and value of $m_n^{(k)}$ according to (3).
  Step 2. $\forall j \in \mathcal{J}$, EV j optimizes (20), subject to (8)–(11), given $\lambda^{P,(k)}, \lambda^{R,(k)}$, $d_n^f + q_{-j}^{P,(k)}$, and updates $q_j^{P,(k+1)}$ and $q_j^{R,(k+1)}$, and submits them to the DSO;
  Steps 3-4. Repeat Steps 3-4 of Algorithm 2.
  $k := k + 1$
end while
return q^P, q^R, Q^P, Q^R, λ^P, λ^R, λ_n^P, λ_n^R

3.2.3 Information-Aware Load Aggregator Scheduling ($\mathbf{EV_A^{agg}}$)

In the $\mathbf{EV_A^{agg}}$ decentralized design, a Load Aggregator is assumed to schedule all EVs connected to a distribution feeder. Information-Aware load aggregators, similar to information-Aware self-scheduling EVs, can infer the value of the arguments of $m_{n,t}(\Sigma q_{j,t}^P)$ affecting energy and reserve DLMPs. Upon receiving DLMPs, the load aggregator at feeder n schedules EVs by solving the following minimization problem:

$$\min_{q^P, q^R} \sum_{j,t} \left\{ \left[1 + m_{n,t}(\Sigma q_{j,t}^P)\right] \left(\lambda_t^P q_{j,t}^P - \lambda_t^R q_{j,t}^R\right) + \delta(q_{j,t}^P)^2 \right\}, \qquad (21)$$

subject to (8)–(11) for all EVs. Note that the objective function couples all EVs that are located in the same feeder. The $\mathbf{EV_A^{agg}}$ decentralized market design is summarized in Algorithm 4. It is similar to the $\mathbf{EV_A^{ind}}$ design with the exception that in Step 1, load aggregator at feeder n only needs to infer d_n^f, and in Step 2, EVs at the same feeder are simultaneously scheduled by the load aggregator. Note that although we model a single aggregator at each feeder, our analysis extends to several aggregators and groups of EVs ([30]).

Algorithm 4 : $\mathbf{EV_A^{agg}}$ (Distributed Algorithm)

Initialize LMPs $\lambda^{P,(0)}, \lambda^{R,(0)}$, marginal losses $m_n^{(0)}$ and set $k := 0$
while $\sum_{j,t} ||q_{j,t}^{P,(k+1)} - q_{j,t}^{P,(k)}|| > tolerance$ **do**
  Step 1. Repeat Step 1 of Algorithm 3 and only calculate d_n^f.
  Step 2. Repeat Step 2 of Algorithm 3 but optimize (21).
  Steps 3-4. Repeat Steps 3-4 of Algorithm 2.
  $k := k + 1$
end while

4 Comparison of Equilibrium Conditions across Market Designs

In this section, we study the differences in the centralized and decentralized market designs by comparing the first order optimality conditions of the market-clearing optimization problems *w.r.t.* EV decisions $q^P_{j,t}$ and $q^R_{j,t}$. Since generators are scheduled by the TSO, the optimality conditions *w.r.t.* generator variables $Q^P_{i,t}$ and $Q^R_{i,t}$ are identical.

The optimality conditions of the $\mathbf{TD_A}$ centralized design are as follows:

$$\lambda^P_t(1+m_{n,t}) - \lambda^R_t\gamma_n \sum_{j\in J_{n,t}} q^R_{j,t} + A = 0, \tag{22}$$

$$-\lambda^R_{n,t} + \nu^1_{j,t} + \nu^2_{j,t} = 0, \tag{23}$$

where $A = 2\delta q^P_{j,t} - \underline{\zeta}_j + \overline{\zeta}_j + \nu^1_{j,t} - \nu^2_{j,t}$. Since the condition in (23) is identical across all market designs, we present optimality conditions *w.r.t.* $q^P_{j,t}$ only. For the $\mathbf{TD_{Un}}$ market design we have:

$$\lambda^P_{n,t} + A = 0. \tag{24}$$

For the distribution feeder information-Unaware decentralized market design $\mathbf{EV^{ind}_{Un}}$, we obtain exactly the same optimality conditions as above. This is not surprising, since we can observe by inspection that its clearing process is an exact decomposition of the centralized market design $\mathbf{TD_{Un}}$. The distribution feeder information-Aware decentralized design $\mathbf{EV^{ind}_{A}}$ yields:

$$\lambda^P_t(1+m_{n,t}) + \lambda^P_t\gamma_n q^P_{j,t} - \lambda^R_t\gamma_n q^R_{j,t} + A = 0. \tag{25}$$

Finally, the EV Load Aggregator with feeder information decentralized design, $\mathbf{EV^{agg}_{A}}$, yields:

$$\lambda^P_t(1+m_{n,t}) + \lambda^P_t\gamma_n \sum_j q^P_{j,t} - \lambda^R_t\gamma_n \sum_j q^R_{j,t} + A = 0. \tag{26}$$

Optimality conditions (22)-(26) reveal non-matching terms that imply differences in the market equilibria. The following proposition describes a major difference in the scheduling decisions across the three decentralized designs.

Proposition 1 (Key impact of decentralized designs on scheduling decisions) *Consider the decentralized market designs* $\mathbf{EV^{ind}_{Un}}$, $\mathbf{EV^{ind}_{A}}$, *and* $\mathbf{EV^{agg}_{A}}$. *At equilibrium, assume that the minimum daily energy charging constraint* (8) *is binding. Then, for given hours* t, t' *with* $q^P_{j,t} > 0$, *the marginal cost reduction associated with moving infinitesimal consumption away from hour* t *is equal to the increase in marginal cost associated with moving infinitesimal consumption into hour* t'. *Moreover, the marginal change in cost in hour* t *associated with moving consumption away from hour* t *is given by:*

- $\mathbf{EV^{ind}_{Un}}$: $\lambda^P_{n,t} + \lambda^R_{n,t} + 2\delta q^P_{j,t}$,
- $\mathbf{EV^{ind}_{A}}$: $(\lambda^P_t + \lambda^R_t)[\gamma_n q^P_{j,t} + 1 + m_{n,t}(q^P_{-j,t}, q^P_{j,t})] + 2\delta q^P_{j,t}$,

– $\mathbf{EV_A^{agg}}$: $(\lambda_t^P + \lambda_t^R)[\gamma_n \sum_j q_{j,t}^P + 1 + m_{n,t}(\Sigma q_{j,t}^P)] + 2\delta q_{j,t}^P$.

Proof The proof is available in Appendix A, Subsection A.1.

Proposition 1 shows the difference across decentralized market designs in the marginal costs perceived by individual EVs (or their aggregator) when they consider to transfer charging from hour t to hour t'. The differences depend on whether EVs (or their aggregator) do or do not have access to distribution feeder information. In the $\mathbf{EV_{Un}^{ind}}$ design, EVs have no information on how they can affect marginal losses, $m_{n,t}$, and the resulting DLMPs that determine their cost, whereas in the $\mathbf{EV_A^{ind}}$ design, EVs do have that information as they are aware of $m_{n,t}(q_{-j,t}^P, q_{j,t}^P)$. However, individual self-scheduling EVs can impact marginal losses $m_{n,t}$ and the DLMPs that determine their cost only by changing their own consumption $q_{j,t}^P$, whereas in the $\mathbf{EV_A^{agg}}$ design, the load aggregator has a greater leverage in impacting marginal losses $m_{n,t}$ by controlling the consumption of all EVs, $\sum_j q_{j,t}^P$. In summary, Proposition 1 shows how the market-clearing equilibria under each decentralized market design differ during hours when EV charging is strictly positive.

5 Existence and Uniqueness of EV Schedule Equilibrium in the Decentralized Market Designs

The cost function of self-scheduling EVs in the $\mathbf{EV_{Un}^{ind}}$ and $\mathbf{EV_A^{ind}}$ decentralized designs, characterized by (16) and (20), respectively, in addition to their own energy and reserve decisions, also depends on the total hourly energy consumption by other EVs connected to the same feeder. Hence, we study the associated market-clearing equilibrium as a non-cooperative game resulting in a Nash equilibrium [5]. To this end, we study existence and uniqueness of the EV schedules and DLMP Nash equilibria under decentralized EV information-Aware/Unaware market designs. Our goal is to show that spatiotemporal equilibria of EV energy and reserve schedules aggregated over identical groups of EVs exist and are unique. Although these results are shown for a single distribution feeder and EV group, their extension to mobile EVs visiting multiple feeders and constituting multiple identical groups is straightforward.

The fact that EVs co-optimize their Energy consumption and Reserve offers in the $\mathbf{EV_A^{ind}}$ and $\mathbf{EV_A^{agg}}$ decentralized designs renders the characterization and analysis of the multi-commodity market-clearing equilibrium a non-trivial task. Indeed, the associated objective functions (20) and (21) are non-convex due to bilinear $q_{j,t}^P q_{j,t}^R$ terms in the cost functions whose Hessian is indefinite. We overcome these difficulties by reducing the EV scheduling problem to a single-commodity problem involving explicitly only the energy decisions $q_{j,t}^P$. This is by no means a straightforward task, which we address as follows:

– We rely on the fact that energy and reserve decisions are coupled through a double constraint imposed by the fact that reserves are bidirectional (up and down) and can neither exceed the battery charging rate, $q_{j,t}^P$, nor the unused charging rate capacity, $\overline{q}_j - q_{j,t}^P$.

- We partition the input space to mutually exclusive and exhaustive subsets. Within each subset, we exploit the necessary optimality conditions (presented in Section 4) and duality, i.e., $\underline{\zeta}_j, \overline{\zeta}_j \geq 0$, $\nu^1_{j,t}, \nu^2_{j,t} \geq 0$, to express reserve decisions $q^R_{j,t}$ in terms of energy decisions $q^P_{j,t}$, and thus reduce the multi-commodity problem to a single-commodity one.
- We construct appropriate potential functions [35] that convert individual EV decisions to a centralized strictly convex problem within each partition.

In the $\mathbf{EV}^{\mathbf{agg}}_{\mathbf{A}}$ market design, where EVs are scheduled by an information-Aware load aggregator, there is no notion of a Nash Equilibrium among EVs in the same group.[5] Hence, we simply show that the $\mathbf{EV}^{\mathbf{agg}}_{\mathbf{A}}$ optimization problem is strictly convex within each input space partition.

Lemma 1 *Consider the* $\mathbf{EV}^{\mathbf{ind}}_{\mathbf{Un}}$, $\mathbf{EV}^{\mathbf{ind}}_{\mathbf{A}}$, *and* $\mathbf{EV}^{\mathbf{agg}}_{\mathbf{A}}$ *decentralized market designs. For the EV scheduling problems solved in Step 2 of Algorithms 2, 3, and 4, respectively, the optimal solution satisfies the following:*
(i) At least one of constraints (10) *or* (11) *is binding.*
(ii) If there exists t *such that* $q^P_{j,t} > 0$ *and* $\lambda^P_t - \lambda^R_t > 0$, *then constraint* (8) *is binding.*
(iii) If there exists t *such that* $q^R_{j,t} < q^P_{j,t}$, *then constraint* (8) *is binding.*
(iv) If constraint (8) *is not binding and there exists* t *such that constraint* (10) *is not binding, then constraint* (9) *is binding.*

Proof The proof is available in Appendix A, Subsection A.2.

Corollary 1 *If* $q^P_{j,t} < \frac{\overline{q}_j}{2}$, *then* $q^R_{j,t} = q^P_{j,t}$.

Corollary 2 *If* $q^P_{j,t} > \frac{\overline{q}_j}{2}$, *then* $q^R_{j,t} = \overline{q}_j - q^P_{j,t}$.

Note that when $q^P_{j,t} = \frac{\overline{q}_j}{2}$, Corollaries 1 and 2 are identical.

If the battery needs to charge during an hour when the energy price exceeds the reserve price, total charging is equal to minimum charging demand $\underline{s}_j$, since the total cost increases if the battery charges beyond this level. The reverse of Lemma 1(ii) is also intuitive and helpful; if the minimum charging demand constraint (8) is not binding, then one can show that the hours when the battery charges satisfy $\lambda^P_t - \lambda^R_t < 0$, i.e., the minimum charging demand is exceeded only in the case EV charges during hours when the reserve prices exceed energy prices. When constraint (8) is not binding, constraint (11) is binding $\forall t$, hence, $q^R_{j,t} = q^P_{j,t}, \ \forall t$. If reserve prices exceed energy prices, the battery will charge up to its maximum capacity, $\overline{S}_j$, if possible. The existence of a residual hour t where $q^P_{j,t} + q^R_{j,t} < \overline{q}_j$ suggests that reaching maximum total charging capacity is feasible.

Corollaries 1 and 2 imply that the relationship between reserves and energy consumption depends on whether energy consumption is smaller or greater

[5] Note, however, that if multiple EV groups that are internally homogeneous, as for example fleets of electrified UPS, FEDEX, Amazon delivery etc., are scheduled by competing load aggregators, the Nash equilibrium notion becomes relevant.

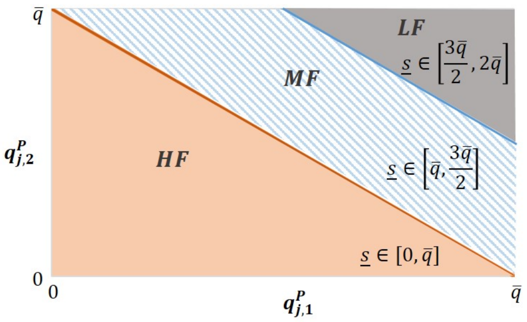

Fig. 3 Illustration of the charging flexibility levels for a two-hour problem. HF=High Flexibility, MF=Moderate Flexibility, LF=Low Flexibility.

than $\frac{\overline{q}_j}{2}$. In addition, whether EV j needs to charge more than $\frac{\overline{q}_j}{2}$ during an hour t depends on the relationship between the ratio $\frac{\underline{s}_j}{\overline{q}_j}$ and the number of hours that the EV is plugged in, T_j. This leads to the definition of the following input space partitions:

- *High Charging Flexibility*: The charging demand of EV j is small enough so that $q^P_{j,t} \leq \frac{\overline{q}_j}{2}$, $\forall t$ is a feasible solution, i.e.,

$$\underline{s}_j \leq T_j \frac{\overline{q}_j}{2}, \tag{27}$$

  implying that EV j does not need to charge more than $\frac{\overline{q}_j}{2}$ in any hour to meet its minimum charging demand. Therefore, Corollary 1 can hold $\forall t$. We later show that under a mild assumption this holds at equilibrium.
- *Moderate Charging Flexibility*: It is characterized by:

$$T_j \frac{\overline{q}_j}{2} < \underline{s}_j \leq (T_j - 1)\overline{q}_j + \frac{\overline{q}_j}{2}, \tag{28}$$

  implying that EV j needs to charge more than $\frac{\overline{q}_j}{2}$ during at least one hour.
- *Low Charging Flexibility*: EV j needs to charge more than $\frac{\overline{q}_j}{2}$ every hour to fulfill its charging demand, i.e., its charging demand $\underline{s}_j$ satisfies:

$$\underline{s}_j > (T_j - 1)\overline{q}_j + \frac{\overline{q}_j}{2}. \tag{29}$$

  Since $q^P_{j,t} > \frac{\overline{q}_j}{2}$ $\forall t$, Corollary 2 holds $\forall$ t as well. Therefore, $q^R_{j,t} = q^P_{j,t}, \forall t$.

The charging flexibility levels are illustrated in Figure 3 for a two-hour problem. In this case, High Flexibility is satisfied when the minimum charging demand, $\underline{s}_j$, does not exceed $\overline{q}_j$.

To proceed with the proof of equilibrium existence and uniqueness for decentralized information-Unaware and Aware EV market designs, the input space partition approach is strengthened by the following reasonable input assumptions that hold true under extensive tests on actual LMP data:

Assumption 1 $\lambda^P_{t'} + \lambda^R_{t'} > \lambda^P_t - \lambda^R_t, \quad \forall t, t'|t' \neq t.$

Assumption 1 relies on the anticipated increase in reserve prices due to renewable energy penetration. Since hourly marginal losses on distribution network lines average 10%, Assumption 1 should hold for distribution feeder DLMPs as well, i.e.,

$$\lambda^P_{n,t'} + \lambda^R_{n,t'} > \lambda^P_{n,t} - \lambda^R_{n,t}, \quad \forall t, t'|t' \neq t, \tag{30}$$

implying that charging more than $\frac{\overline{q}_j}{2}$ is not optimal unless it is required for feasibility. The marginal cost of charging beyond $\frac{\overline{q}_j}{2}$ at hour t is equal to $\lambda^P_{n,t}+\lambda^R_{n,t}$, since an additional unit of energy consumption requires one less unit of reserve provision due to constraint (10). On the other hand, the marginal cost of charging when $q^P_{j,t} < \frac{\overline{q}_j}{2}$ equals to $\lambda^P_{n,t} - \lambda^R_{n,t}$. Therefore, as long as the marginal cost of charging when $q^P_{j,t} > \frac{\overline{q}_j}{2}$ is greater than when $q^P_{j,t} < \frac{\overline{q}_j}{2}$, $\forall t, t'$ pairs, it is not optimal to charge more than $\frac{\overline{q}_j}{2}$ at any hour.[6] Analysis of PJM energy and reserve LMP data reveals that Assumption 1 holds almost certainly [32].

Assumption 2 $\lambda^P_t > \lambda^R_t, \ \forall t.$

We assume energy LMPs are higher than reserve LMPs. This assumption is required only for some charging flexibility input space partitions. In the analysis that follows, we assume that EVs may belong to the same charging flexibility category, but their actual minimum charging demand values may differ. Assumption 2 is used in employing the potential function approach required to prove Nash equilibrium uniqueness.

The following two theorems are restatements of known results on the existence and uniqueness of a Nash equilibrium that are employed in our analysis.

Theorem 1 (Nash Equilibrium Existence) *Given a game with a set of players* $\mathcal{J} = \{1, .., J\}$ *where player* $j \in \mathcal{J}$ *has a strategy* $q_j \in S_j$ *and a cost function* $f_j(q^P_j, q^P_{-j}) : S_1 \times S_2 \times \ldots \times S_J \to \mathbb{R}$*, a Nash equilibrium exists if* $f_j(q_j, q_{-j})$ *is strictly convex in* q^P_j*, continuous in both* q^P_j *and* q^P_{-j}*, and* $S_j \subset \mathbb{R}^{m_j}$ *is convex, closed and bounded* $\forall j$*, where* m_j *is the dimension of player j's strategy.*

Proof Theorem 1 is a restatement of Theorem 1 in [33].

Theorem 2 (Nash Equilibrium Uniqueness) *For the game described by Theorem 1, given a strictly convex function* $\overline{P}(q^P_1, \ldots, q^P_j, \ldots, q^P_J)$ *that satisfies* $\underset{q^P_j \in S_j}{\text{argmin}}\, f_j(q^P_j, q^P_{-j}) = \underset{q^P_j \in S_j}{\text{argmin}}\, \overline{P}(q^P_1, \ldots, q^P_j, \ldots, q^P_J)$*, any Nash Equilibrium is a minimum point of* $\overline{P}$ *if* $f_j(q^P_j, q^P_{-j})$ *is bounded* $\forall j$ *and* $\overline{P}$ *is smooth on* $S_1 \times \ldots \times S_J$*. Moreover, strict convexity of* $\overline{P}$ *implies a unique Nash Equilibrium.*

[6] If (30) does not hold, an EV might decide to provide no reserve or strictly smaller reserve than $q^P_{j,t}$. For instance, for a two-hour case where $\{\lambda^P_{n,1}, \lambda^R_{n,1}\} = \{20, 15\}$, $\{\lambda^P_{n,2}, \lambda^R_{n,2}\} = \{50, 10\}, \overline{q}_j = 3, \underline{s}_j = 3$, the optimal solution is $q^P_{j,1} = 3, q^R_{j,1} = q^P_{j,2} = q^R_{j,2} = 0$.

Proof Theorem 2 is a restatement of known results in [34]–[37]. A short discussion and proof can be found in Appendix A, Subsection A.3.

5.1 Existence and Uniqueness of Equilibrium under High Charging Flexibility

In the following Lemma, we first show the coupling between energy and reserves under High Charging Flexibility, and transform the multi-commodity to a single-commodity problem.

Lemma 2 (Single-Commodity, High Charging Flexibility) *Consider the* $\mathbf{EV^{ind}_{Un}}$*,* $\mathbf{EV^{ind}_{A}}$*, and* $\mathbf{EV^{agg}_{A}}$ *decentralized designs. For the EV scheduling problems solved in Step 2 of Algorithms 2, 3, and 4, respectively, given Assumptions 1 and 2, under High Charging Flexibility, the optimal solution satisfies* $q^P_{j,t} \leq \frac{\overline{q}_j}{2}, \forall t$.

Proof The proof is available in Appendix A, Subsection A.4.

Having shown $q^P_{j,t} \leq \frac{\overline{q}_j}{2}$, we can also conclude $q^R_{j,t} = q^P_{j,t}$, $\forall t$, by Corollary 2. For the $\mathbf{EV^{ind}_{A}}$ design, the self-scheduling problem solved by EV j (Step 2 of Algorithm 3) can then be written as a function of $q^P_{j,t}$ only:

$$\min_{q^P_j \in S^{HF}_j} f_j^{\mathbf{EV^{ind}_{A}}}(q^P_j, q^P_{-j}) = \sum_t \left\{ \Delta\lambda_t \left[1 + m_{n,t}(q^P_{j,t}, q^P_{-j,t})\right] q^P_{j,t} + \delta (q^P_{j,t})^2 \right\}, \tag{31}$$

where $S^{HF}_j = \{q^P_{j,t} | q^P_{j,t} \in [0, \frac{\overline{q}_j}{2}],\ \sum_t q^P_{j,t} \geq \underline{s}_j\}$ and $\Delta\lambda_t = \lambda^P_t - \lambda^R_t$. We refer to $\Delta\lambda_t$ as the effective price (LMP) of hour t. Note that since $\underline{s}_j \leq \overline{S}_j$ and $\sum_t q^P_{j,t} = \underline{s}_j$, we can omit the maximum total charging capacity constraint (9). Similarly, for the $\mathbf{EV^{ind}_{Un}}$ design, the EV self-scheduling problem (Step 2 of Algorithm 2) can be written as:

$$\min_{q^P_j \in S^{HF}_j} f_j^{\mathbf{EV^{ind}_{Un}}}(q^P_j) = \sum_t \left\{ \Delta\lambda_{n,t} q^P_{j,t} + \delta (q^P_{j,t})^2 \right\}, \tag{32}$$

where $\Delta\lambda_{n,t} = \Delta\lambda_t(1 + m_{n,t})$. Lastly, for the $\mathbf{EV^{agg}_{A}}$ design, the problem solved by the load aggregator at location n (Step 2 of Algorithm 4) is written as:

$$\min_{q^P \in S^{HF}} f^{\mathbf{EV^{agg}_{A}}}(q^P) = \sum_{j,t} \left\{ \Delta\lambda_t \left[1 + m_{n,t}(\Sigma q^P_{j,t})\right] q^P_{j,t} + \delta (q^P_{j,t})^2 \right\}. \tag{33}$$

Proposition 2 (Existence and Uniqueness, High Charging Flexibility) *Under High Charging Flexibility, given Assumptions 1 and 2, a unique Nash Equilibrium exists for the* $\mathbf{EV^{ind}_{A}}$ *and* $\mathbf{EV^{ind}_{Un}}$ *decentralized market designs, described by Algorithms 2 and 3. For the* $\mathbf{EV^{agg}_{A}}$ *decentralized market design, described by Algorithm 4, the aggregate EV response is also unique.*

Proof The proof is available in Appendix A, Subsection A.5.

We remind the reader that in this paper, we do not consider the game among multiple load aggregators in a single feeder. This could also be investigated in the Nash Equilibrium context, where a few but big players are market participants. We refer the reader to our previous work [30], where we provide results with respect to quantitative differences among market design outcomes when multiple load aggregators in a single feeder engage in a game.

5.2 Existence and uniqueness of Equilibrium under Moderate and Low Charging Flexibility

Since the definition of Moderate Charging Flexibility in (28) requires that there exists t s.t. $q^P_{j,t} > \frac{\overline{q}_j}{2}$, the equilibrium satisfies $\sum_t q^P_{j,t} = \underline{s}_j$ based on Lemma 1(iii) regardless of the sign of $\Delta\lambda_t$. Therefore, in the Lemma below, Assumption 2 is not required.

The one to one relationship between energy and reserves is shown in the following lemma.

Lemma 3 (Single-Commodity, Moderate/Low Charging Flexibility) *Consider the* $\mathbf{EV^{ind}_{Un}}$, $\mathbf{EV^{ind}_{A}}$, *and* $\mathbf{EV^{agg}_{A}}$ *decentralized designs. For the EV scheduling problems solved in Step 2 of Algorithms 2, 3, and 4, respectively, given Assumption 1 and under Moderate Charging Flexibility, or under Low Charging Flexibility, the optimal solution satisfies* $q^P_{j,t} \geq \frac{\overline{q}_j}{2}, \forall t$.

Proof The proof is available in Appendix A, Subsection A.6.

By Corollary 1 and Lemma 3, we can write $q^R_{j,t} = \overline{q}_j - q^P_{j,t}$. Therefore, for the $\mathbf{EV^{ind}_{A}}$ design, the individual information-aware EV scheduling problem (Step 2 of Algorithm 3), under Moderate or Low Charging Flexibility, can be written only in terms of $q^P_{j,t}$ as follows:

$$\min_{q^P_j \in S^{MLF}_j} \; f_j^{\mathbf{EV^{ind}_{A}}}(q^P_{j,t}, q^P_{-j,t}) = \sum_t \left\{ \left[1 + m_{n,t}(q^P_{j,t}, q^P_{-j,t})\right] \left[\lambda^P_t q^P_{j,t} - \lambda^R_t(\overline{q}_j - q^P_{j,t})\right] + \delta(q^P_{j,t})^2 \right\}, \tag{34}$$

where $S^{MLF}_j = \{q^P_{j,t} | q^P_{j,t} \in [\overline{q}_j/2, \overline{q}_j],\ \sum_t q^P_{j,t} \geq \underline{s}_j\}$. Due to positivity of λ^P_t and λ^R_t, the above objective function in (34) is strictly convex regardless of the sign of $\Delta\lambda_t$. Similarly, for the $\mathbf{EV^{ind}_{U}n}$ and $\mathbf{EV^{agg}_{A}}$ designs, the objective functions of the individual information-unaware and load aggregator scheduling problems (Step 2 of Algorithms 3 and 4) are given by:

$$f_j^{\mathbf{EV^{ind}_{Un}}}(q^P_j) = \sum_t \left[\lambda^P_{n,t} q^P_{j,t} - \lambda^R_{n,t}(\overline{q}_j - q^P_{j,t}) + \delta(q^P_{j,t})^2\right], \tag{35}$$

$$f^{\mathbf{EV^{agg}_{A}}}(q^P) = \sum_{j,t} \left\{ \left[1 + m_{n,t}(\Sigma q^P_{j,t})\right] \left[\lambda^P_t q^P_{j,t} - \lambda^R_t(\overline{q}_j - q^P_{j,t})\right] + \delta(q^P_{j,t})^2 \right\}. \tag{36}$$

Proposition 3 (Existence and Uniqueness, Moderate/Low Charging Flexibility) *Under Moderate Charging Flexibility and given Assumption 1 or under Low Charging Flexibility, a unique Nash Equilibrium exists for the* $\mathbf{EV_A^{ind}}$ *and* $\mathbf{EV_{Un}^{ind}}$ *decentralized market designs, described by Algorithms 2 and 3. For the* $\mathbf{EV_A^{agg}}$ *decentralized market design, described by Algorithm 4, the aggregate EV response is also unique.*

Proof The proof is available in Appendix A, Subsection A.7.

Having shown existence and uniqueness, we proceed in the next two sections to a qualitative and quantitative exploration of the role of information in decentralized market-clearing. Section 6 relies on analytic closed-form expressions of decentralized market equilibria that are possible under simplified EV models, whereas Section 7 uses the full EV model and provides quantitative analysis based on numerical solutions of equilibria instances.

6 Closed-Form Characterization of EV Decisions in Decentralized Markets

In this section, we assume that EVs have practically unlimited charging capacity, which enables us to derive closed-form equilibrium expressions across decentralized market designs. Since the total EV load is a small percentage of total system demand in a transmission network, transmission bus LMPs are fairly insensitive to the changes in the total EV load. We may therefore focus on the role of EV-decision-sensitive DLMPs on market-clearing. Individual EVs or assisting EV load aggregators determine optimal EV schedules in response to marginal loss, $m_{n,t}$, dependent DLMPs. A major objective is to use the analytic characterization of the various equilibria to understand the role of information.

We first consider (in Subsection 6.1) a two-hour model of the $\mathbf{EV_{Un}^{ind}}$, $\mathbf{EV_A^{ind}}$, and $\mathbf{EV_A^{agg}}$ market designs, for which we can obtain simple closed-form equilibrium expressions. We then generalize our model to 24 hours (in Subsection 6.2), and we analyze asymptotic behaviour (in Subsection 6.3) as the number of EVs connected to each feeder increases while distribution feeder losses adapt to increasing load. Since the charging rate capacity is assumed unlimited,[7] the High Charging Flexibility input partition holds always and minimum total charging demand, $\underline{s}_j$, satisfies (27) $\forall j$. Lemma 2 and Corollary 2 then imply that the equilibrium will satisfy $q^P_{j,t} = q^R_{j,t}$.

6.1 Closed-Form Equilibria in the Two-Hour Model

If EVs are connected for two hours, the only decision variable for EV j is energy consumption during hour 1 ($q^P_{j,1}$), since, by Lemma 1(ii), $q^P_{j,1}+q^P_{j,2} = \underline{s}_j$. Hence

[7] This will be the case when fast chargers, with say a 240kW capacity, are widely available.

in the explicit two-hour model, the feasible decision set of EV j is given by $S_j^{2hr} = \{q_{j,1}^P | q_{j,1}^P \in [0, s_j]\}$.

For the $\mathbf{EV_{Un}^{ind}}$ market design, assuming identical EVs and writing $\underline{s}_j = \underline{s}$, the equilibrium schedule of EV j is given by[8]:

$$q_{j,1}^{P,\mathbf{EV_{Un}^{ind}}} = \frac{\Delta\lambda_2(1+\gamma d_2^f) - \Delta\lambda_1(1+\gamma d_1^f) + 2\delta\underline{s} + \Delta\lambda_2\gamma J\underline{s}}{4\delta + (\Delta\lambda_2 + \Delta\lambda_1)\gamma J}, \qquad (37)$$

when $q_{j,1}^{P,\mathbf{EV_{Un}^{ind}}} \in (0, \underline{s})$. We note that $\Delta\lambda_{n,t} = \Delta\lambda_t(1+m_{n,t})$ and that information-Unaware EVs do not know how their decisions influence $\Delta\lambda_{n,t}$.

For the $\mathbf{EV_A^{ind}}$ market design, using the EV information-Aware optimality conditions, we characterize $q_{j,1}^P(q_{-j,1}^P)$, which depends on aggregate consumption of other EVs given the effective LMPs $(\Delta\lambda_1, \Delta\lambda_2)$:

$$q_{j,1}^{P,\mathbf{EV_A^{ind}}}(q_{-j,1}^P) = \frac{\Delta\lambda_2(1+\gamma d_2^f) - \Delta\lambda_1(1+\gamma d_1^f) + \gamma\Delta\lambda_2(J+1)\underline{s} + 2\delta\underline{s} - \gamma(\Delta\lambda_1 + \Delta\lambda_2)q_{-j,1}^P}{4\delta + 2\gamma(\Delta\lambda_1 + \Delta\lambda_2)} \qquad (38)$$

when $q_{j,1}^{P,\mathbf{EV_A^{ind}}} \in (0, \underline{s})$. The above can be viewed as the best response function of EV j to given LMPs and other EVs' aggregate consumption, $q_{-j,t}^P$. Unsurprisingly, the optimal consumption during hour 1 is decreasing in $q_{-j,t}^P$. The best response function of EV j also implies that if the effective price difference across the hours, $|\Delta\lambda_2 - \Delta\lambda_1|$, is large, EVs will charge their batteries only during one of the hours. Similarly, as the inelastic demand difference between the hours, $|d_2^f - d_1^f|$, increases, the difference in marginal losses and effective DLMPs increases too.

Figure 4 illustrates the best response functions with two EVs and zero inelastic demand, for various values of $\Delta\lambda_2 - \Delta\lambda_1$. The unique equilibrium can be recovered from the intersection of best response functions. When the best response function is piecewise linear, the optimal consumption of EV j given by (38) is within the $[0, \underline{s}]$ range for lower values of $q_{-j,1}^P$. However, as $q_{-j,1}^P$ increases, EV j consumes energy only during the hour with the lower effective LMP. When $\Delta\lambda_1 - \Delta\lambda_2$ is larger, the best response of $q_{j,1}^P$ falls outside the $[0, \underline{s}]$ range for all possible levels of $q_{-j,1}^P$. In other words, there exists a dominantly cheap hour and EV j's best response is not affected by the other EV's consumption. Comparing the $\mathbf{EV_A^{ind}}$ equilibrium in Figure 4 to $\mathbf{EV_{Un}^{ind}}$ and $\mathbf{EV_A^{agg}}$ for $\Delta\lambda_1 - \Delta\lambda_2 = 0.3$, we observe that $q_{j,1}^{P,\mathbf{EV_{Un}^{ind}}} = 4.38$ kW, $q_{j,1}^{P,\mathbf{EV_A^{ind}}} = 4.90$ kW, and $q_{j,1}^{P,\mathbf{EV_A^{agg}}} = 5.17$ kW.

Proposition 4 (Stability of N.E. under Information-Awareness) *Consider the $\mathbf{EV_A^{ind}}$ market design. In the two-hour model, the equilibrium of the information-Aware self-scheduling EVs is stable for $J \leq 3$.*

[8] For simplicity, we omit location index n from loss factor γ_n and inelastic demand $d_{n,t}^f$.

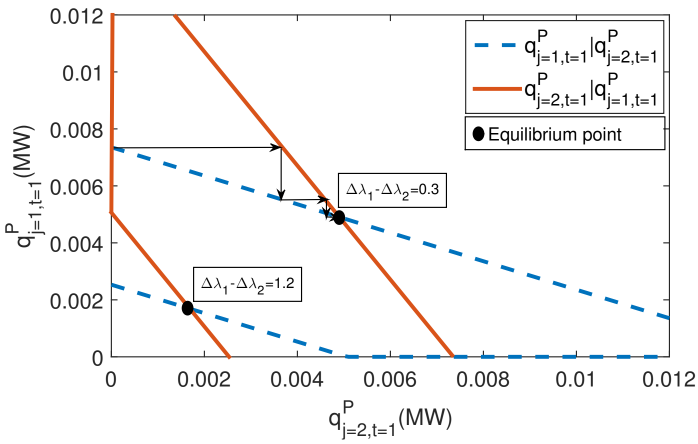

Fig. 4 EV best responses; $\mathbf{EV^{ind}_A}$ design; 2 EVs, $\varDelta\lambda_1 - \varDelta\lambda_2 = \{0.3, 1.2\}$; $\underline{s}_1 = \underline{s}_2 = 12$kW.

Proof The proof is available in Appendix A, Subsection A.8.

Proposition 4 suggests that the unique Nash equilibrium is guaranteed to be reached when EV j iteratively plays the best response $q^{P,\mathbf{EV^{ind}_A}}_{j,1}(q^P_{-j,1})$. This refers to Step 2 of Algorithm 3. The best response iteration path with two EVs is illustrated in Figure 4, for $\varDelta\lambda_1 - \varDelta\lambda_2 = 0.3$. For identical EVs, the solution to the system of the best response functions in (38) results, when $q^{P,\mathbf{EV^{ind}_A}}_{j,1} \in (0, \underline{s})$, to the equilibrium solution:

$$q^{P,\mathbf{EV^{ind}_A}}_{j,1} = \frac{\varDelta\lambda_2(1+\gamma d^f_2) - \varDelta\lambda_1(1+\gamma d^f_1) + (J+1)\varDelta\lambda_2\gamma\underline{s} + 2\delta\underline{s}}{4\delta + (J+1)\gamma(\varDelta\lambda_1 + \varDelta\lambda_2)}. \tag{39}$$

Note that $q^P_{-j,t} = (J-1)q^P_{j,t}$. Similarly to the information-unaware self-scheduling equilibrium given in (37), as $|\varDelta\lambda_1 - \varDelta\lambda_2|$ increases, the equilibrium solution falls outside the $(0, \underline{s})$ range.

We now write the optimal consumption of EV j, $q^{P,\mathbf{EV^{agg}_A}}_{j,1}(q^P_{-j,1})$, obtained from the first order optimality condition of the $\mathbf{EV^{agg}_A}$ market design:

$$q^{P,\mathbf{EV^{agg}_A}}_{j,1}(q^P_{-j,1}) = \frac{\varDelta\lambda_2(1+\gamma d^f_2) - \varDelta\lambda_1(1+\gamma d^f_1) + 2\varDelta\lambda_2 J\gamma\underline{s} + 2\delta\underline{s} - 2\gamma(\varDelta\lambda_1+\varDelta\lambda_2)q^P_{-j,1}}{4\delta + 2\gamma(\varDelta\lambda_1 + \varDelta\lambda_2)}. \tag{40}$$

The difference between the expression given in (40) and the best response in the $\mathbf{EV^{ind}_A}$ market design given by (38) is the relative slope of EV j to q^P_{-j}. For $q^{P,\mathbf{EV^{agg}_A}}_{j,t} \in (0, \underline{s})$, the equilibrium of EV j in the $\mathbf{EV^{agg}_A}$ market design becomes:

$$q^{P,\mathbf{EV^{agg}_A}}_{j,1} = \frac{\varDelta\lambda_2(1+\gamma d^f_2) - \varDelta\lambda_1(1+\gamma d^f_1) + 2J\varDelta\lambda_2\gamma\underline{s} + 2\delta\underline{s}}{4\delta + 2J\gamma(\varDelta\lambda_1 + \varDelta\lambda_2)}. \tag{41}$$

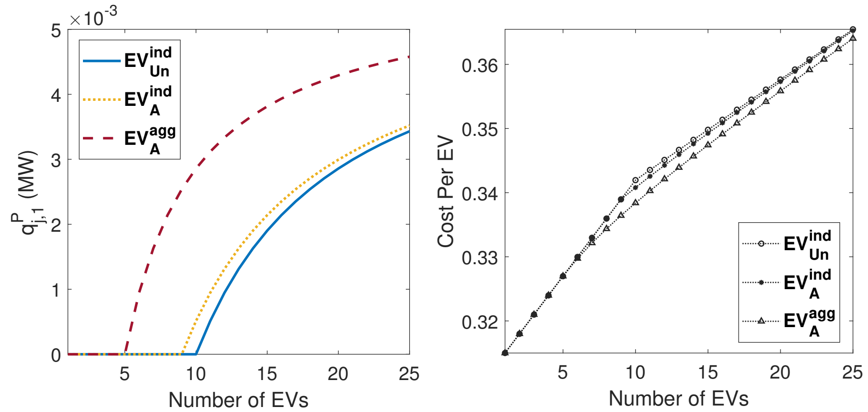

Fig. 5 Hour 1 Consumption level ($q^P_{j,1}$) and Cost per EV for the $\mathbf{EV^{ind}_{Un}}$, $\mathbf{EV^{ind}_{A}}$ and $\mathbf{EV^{agg}_{A}}$ market designs for fixed effective LMPs, *w.r.t.* the number of EVs. Loss factor γ is fixed and equal to 0.8, $\Delta\lambda_1 - \Delta\lambda_2 = 2.5\$/MWh$, $\underline{s} = 12$kW.

Proposition 5 (Closed form expressions for differences in equilibria across market designs) *Assuming $q^P_{j,t} \in (0, \underline{s})$, $t = 1, 2$ and disregarding the battery degradation term δ, the magnitude of the difference in optimal EV consumption at $t = 1$, $q^P_{j,1}$, across the decentralized market design equilibria is as follows:* $\left|q^{P,\mathbf{EV^{ind}_{Un}}}_{j,1} - q^{P,\mathbf{EV^{ind}_{A}}}_{j,1}\right| = \frac{1}{J(J+1)}M$, $\left|q^{P,\mathbf{EV^{ind}_{A}}}_{j,1} - q^{P,\mathbf{EV^{agg}_{A}}}_{j,1}\right| = \frac{(J-1)}{2J(J+1)}M$, $\left|q^{P,\mathbf{EV^{ind}_{Un}}}_{j,1} - q^{P,\mathbf{EV^{agg}_{A}}}_{j,1}\right| = \frac{1}{2J}M$, *where* $M = \left|\frac{\Delta\lambda_2(1+\gamma d^f_2) - \Delta\lambda_1(1+\gamma d^f_1)}{(\Delta\lambda_1+\Delta\lambda_2)\gamma}\right|$.

Proof Proposition 5 follows from (37), (39), and (41).

Proposition 5 implies that the magnitude of the difference in consumption between the load aggregator market design ($\mathbf{EV^{agg}_{A}}$) and the individual self-scheduling ($\mathbf{EV^{ind}_{Un}}$, $\mathbf{EV^{ind}_{A}}$) equilibria, whether information-Aware or Unaware, $q^{P,\mathbf{EV^{ind}_{Un}}}_{j,1}$ or $q^{P,\mathbf{EV^{ind}_{A}}}_{j,1}$, is greater than the magnitude of the difference between the self-scheduling $\mathbf{EV^{ind}_{Un}}$ and $\mathbf{EV^{ind}_{A}}$ equilibria for a given $J \geq 3$ with $q^P_{j,t} \in (0, \underline{s})$ in all equilibria. This is because $\frac{(J-1)}{2J(J+1)} \geq \frac{1}{J(J+1)}$ and, $\frac{1}{2J} > \frac{1}{J(J+1)}$for $J \geq 3$. This result implies that there is a greater leverage of distribution feeder information-aware EVs, when they entrust their scheduling to an aggregator, or, in other words, when they collude. Moreover, Proposition 5 implies further that, as the number of EVs increases, the difference in energy consumption between the self-scheduling $\mathbf{EV^{ind}_{Un}}$ and $\mathbf{EV^{ind}_{A}}$ market designs decreases. This makes intuitive sense, since a single EV's influence on the marginal losses, and hence the DLMPs, diminishes as the "other" loads affecting feeder marginal losses become increasingly larger relative to an individual EV's battery charging load.

Proposition 5 is illustrated further in Figure 5, which shows the equilibrium consumption level of a single EV during hour 1 and the total cost per EV in $\mathbf{EV^{ind}_{Un}}$, $\mathbf{EV^{ind}_{A}}$, and $\mathbf{EV^{agg}_{A}}$ market designs and plots them against the number of EVs that are connected to the distribution feeder. We note that the cost per EV is the lowest when EVs are scheduled collectively by a distribution feeder information-aware EV aggregator. As expected, and demonstrated analytically in Proposition 5, the difference in the first hour consumption level and the total cost per EV is higher between the $\mathbf{EV_A^{ind}}$ and $\mathbf{EV_A^{agg}}$ market designs than between the $\mathbf{EV_{Un}^{ind}}$ and $\mathbf{EV_A^{ind}}$ designs. Figure 5 also shows that for smaller number of EVs, optimal consumption level is identical across the $\mathbf{EV_{Un}^{ind}}$, $\mathbf{EV_A^{ind}}$, and $\mathbf{EV_A^{agg}}$ equilibria, because the equilibrium obtained from all the market designs fall outside the $[0, \underline{s}]$ range and EVs charge only during hour 2. However, as the number of EVs increases, the effective DLMP difference across the hours becomes small enough for EVs to reduce their cost by splitting their charging across both hours. The number of EVs beyond which EV consumption occurs during both hours is the smallest in the load aggregator market design.

6.2 Closed-Form Equilibria in the Multi-Hour Model

In what follows, we provide the 24-hour simplified EV model closed-form equilibrium relations.

Proposition 6 (Closed form equilibria expressions for simplified 24-hour problem) *Consider the* $\mathbf{EV_{Un}^{ind}}$, $\mathbf{EV_A^{ind}}$, *and* $\mathbf{EV_A^{agg}}$ *market designs. For the multi-hour model, assuming identical EVs with unlimited charging rate capacity, the closed-form equilibria are given by:*
$q_{j,t}^{P,\mathbf{EV_{Un}^{ind}}} = [G(J,t)]^+$, $q_{j,t}^{P,\mathbf{EV_A^{ind}}} = [G(J+1,t)]^+$, *and* $q_{j,t}^{P,\mathbf{EV_A^{agg}}} = [G(2J,t)]^+$, *where* $[x]^+ = max\{x, 0\}$, $t' = \{t | q_{j,t}^P > 0\}$, $g(t) = \Delta\lambda_t(1 + \gamma d_t^f)$, $h(J,t) = J\gamma\Delta\lambda_t + 2\delta$, *and* $G(J,t) = \frac{\sum_{t'} \frac{g(t')}{h(J,t')} + \underline{s}}{\sum_{t'} \frac{h(J,t)}{h(J,t')}} - \frac{g(t)}{h(J,t)}$.

Proof The proof is available in Appendix A, Subsection A.9.

Proposition 6 has an interesting interpretation: If the small battery degradation term δ is disregarded, it is clear that the EVs scheduled by the EV aggregator consume, during the hours with the higher effective LMPs, more than they would if they were to self schedule whether information Aware or Unaware. This is easier to conclude in the two-hour model; the difference $q_{j,1}^{P,\mathbf{EV_A^{agg}}} - q_{j,1}^{P,\mathbf{EV_A^{ind}}}$ is positive for $J \geq 2$ when $\Delta\lambda_1 > \Delta\lambda_2$.[9] As shown in Proposition 1, this is because the load aggregator is able to estimate the marginal change in cost during an hour associated with shifting the consumption of all EVs.

[9] Assuming that inelastic demand levels in a single feeder are not large enough to affect this inequality.

6.3 Asymptotic Results for an Unlimited Number of EVs

We conclude the simplified EV model based closed-form equilibrium characterization analysis by investigating the limiting behavior of EV schedules as the number of EVs grows substantially at the distribution feeder level. Although in real distribution networks increasing EV numbers will be compensated by increasing feeder capacity as well as feeder numbers, and, as such, granularity will persist, we adopt a simpler model for the purpose of looking at some interesting qualitative results. We assume that feeder line loss factors will decrease in the long run linearly in the number of EVs connected to the distribution network. Specifically, for J EVs, we define $\gamma_n = \hat{\gamma}_n/J$. The closed-form EV schedule equilibria depend now on J.[10] For the two-hour model selected for simplicity, we have:

$$q_{j,1}^{P,\mathbf{EV}_{\mathbf{Un}}^{\mathbf{ind}}} = \left[\frac{A + \Delta\lambda_2\hat{\gamma}\underline{s}}{4\delta + (\Delta\lambda_1 + \Delta\lambda_2)\hat{\gamma}}\right]^+, \tag{42}$$

$$q_{j,1}^{P,\mathbf{EV}_{\mathbf{A}}^{\mathbf{ind}}} = \left[\frac{A + \frac{(J+1)}{J}\Delta\lambda_2\hat{\gamma}\underline{s}}{4\delta + \frac{(J+1)}{J}(\Delta\lambda_1 + \Delta\lambda_2)\hat{\gamma}}\right]^+, \tag{43}$$

$$q_{j,1}^{P,\mathbf{EV}_{\mathbf{A}}^{\mathbf{agg}}} = \left[\frac{A + 2\Delta\lambda_2\hat{\gamma}\underline{s}}{4\delta + 2(\Delta\lambda_1 + \Delta\lambda_2)\hat{\gamma}}\right]^+, \tag{44}$$

where $A = \Delta\lambda_2(1 + \frac{\hat{\gamma}}{J}d_2^f) - \Delta\lambda_1(1 + \frac{\hat{\gamma}}{J}d_1^f) + 2\delta\underline{s}$.

Decentralized market design equilibria differences persist with increasing EVs. More specifically, $\mathbf{EV}_{\mathbf{Un}}^{\mathbf{ind}}$ and $\mathbf{EV}_{\mathbf{A}}^{\mathbf{agg}}$ market design equilibria are only implicitly dependent on the number of EVs through the effective loss factor, and become completely independent as the inelastic demand becomes relatively small.[11] This makes sense, since the marginal hourly losses are constant in the short run *w.r.t.* the number of EVs. Another intuitive result from the closed-form equilibrium relations is that as $J \to \infty$, the impact of inelastic demand on EV schedules approaches zero, since the total EV load dominates inelastic demand. Figure 6 demonstrates clearly the preceding discussion.

Proposition 7 (Asymptotic coincidence of individual self scheduling information Aware and Unaware EVs) *Given the postulated long term dependence of the feeder loss factor on the number of EVs, $\frac{\hat{\gamma}}{J}$, information-unaware and aware self-scheduling $\mathbf{EV}_{\mathbf{Un}}^{\mathbf{ind}}$ and $\mathbf{EV}_{\mathbf{A}}^{\mathbf{ind}}$ equilibria become asymptotically identical as $J \to \infty$.*

Proof The proof is listed in Appendix A, Subsection A.10.

[10] Under a large number of EVs, we also assume the generation capacity of generator i is adjusted as $\overline{Q}_i = \overline{Q}_i + J\underline{s}$, so that wholesale prices are not significantly affected by the increased demand from EV charging.

[11] Replace $d_t^f = 0$ to see this clearly.

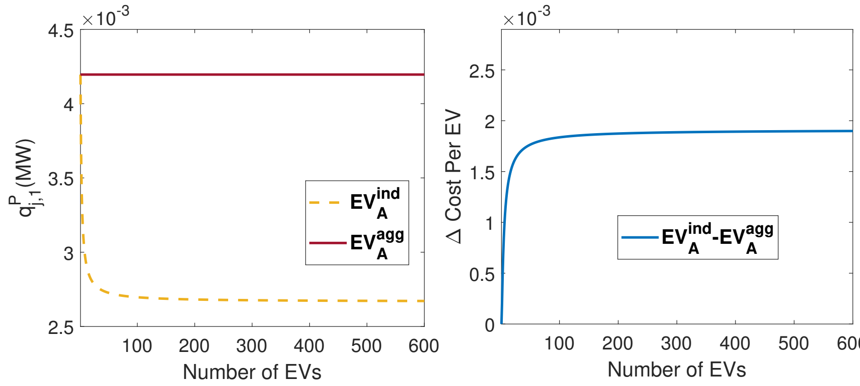

Fig. 6 Hour 1 consumption levels ($q^P_{j,1}$) and cost difference per EV between $\mathbf{EV^{ind}_A}$ and $\mathbf{EV^{agg}_A}$ designs, for fixed effective LMPs ($\Delta\lambda_t$), *w.r.t.* large number of EVs. Loss factor $\gamma = \frac{\hat{\gamma}}{J}$, where $\hat{\gamma} = 15$; $d^f_t = 0, \forall t$; $\Delta\lambda_1 - \Delta\lambda_2 = 2.5\$/\mathrm{MWh}$; $\underline{s} = 12\mathrm{kW}$.

Proposition 7 is consistent with and reinforces a conclusion we can draw from Proposition 5, which shows the analytical differences in equilibrium under finite number of EVs connected to a feeder. Proposition 5 implies that the difference between the equilibrium of information-aware/unaware $\mathbf{EV^{ind}_A}$ and $\mathbf{EV^{ind}_U}$**n** market designs approach zero as the number of EVs J increases. In Proposition 7, we show that these two market designs become identical for a large number of EVs even when the network loss factor adjusts to accommodate an unlimited increase in the number of connected EVs.

Proposition 8 (Individually scheduled information Aware and Unaware EVs are asymptotically indistinguishable relative to aggregator scheduled information Aware EVs) *Given the postulated long term dependence of the feeder loss factor on the number of EVs, $\frac{\hat{\gamma}}{J}$, and disregarding battery degradation term δ, the difference in consumption level between the two-hour self-scheduling and the load aggreator $\mathbf{EV^{agg}_A}$ market designs as $J \to \infty$ is given by:*

$$\left| q^{P,\mathbf{EV^{agg}_A}}_{j,1} - q^{P,\mathbf{EV^{ind}_A}}_{j,1} \right| = \left| q^{P,\mathbf{EV^{agg}_A}}_{j,1} - q^{P,\mathbf{EV^{ind}_{Un}}}_{j,1} \right| = \left| \frac{\Delta\lambda_1 - \Delta\lambda_2}{2(\Delta\lambda_1 + \Delta\lambda_2)\hat{\gamma}} \right| . \quad (45)$$

Proof The results are obtained directly from (42), (43), and (44).

Figure 6 shows the optimal consumption level in the information-aware $\mathbf{EV^{ind}_A}$ and $\mathbf{EV^{agg}_A}$ market designs for the two-hour model for a wide range of EV numbers. The difference in cost per EV stabilizes as $\frac{J}{J+1}$ approaches 1.

7 Numerical Results

This section illustrates differences among the equilibria of centralized and decentralized market designs under a realistic EV model. Since the emphasis is on the distribution network connected EVs, we use a single transmission bus[12] with N distribution network feeders. Note that since the EV schedule of the information-unaware decentralized market design ($\mathbf{EV^{ind}_{Un}}$) is identical to the centralized scheduling with TSO-DSO collaboration ($\mathbf{TD_{Un}}$), we only report results on the former for the sake of brevity. We assume that EVs are connected to only one feeder, and the remainder of the feeders carry only inelastic demand. EVs have a battery capacity of 24 kWh, a charging rate capacity of 15 kW, and they are are connected for 24 hours, i.e., they have High Charging Flexibility, and we use the respective reduced single-commodity formulation with energy consumption $q^P_{j,t}$ being the only decision variable. In the multi-hour numerical experiments, we add another regularization term in the battery dynamics representing the dependency of the charging efficiency on the charging rate. The minimum charging demand constraint (8) becomes: $\sum_t q^P_{j,t}(1 - \epsilon \frac{q^P_{j,t}}{\bar{q}_j}) \geq \underline{s}_j$, where we select $\epsilon = 0.1$, i.e., if the EV is charging at the maximum charging rate, charging efficiency drops to 90%.

The total hourly system inelastic demand levels and resulting effective LMPs ($\Delta\lambda_t$) are shown in Figure 7. As expected, we observe that across all market designs, EVs charge during the hours with the lowest system inelastic demand (hours 1-5, Figure 8).

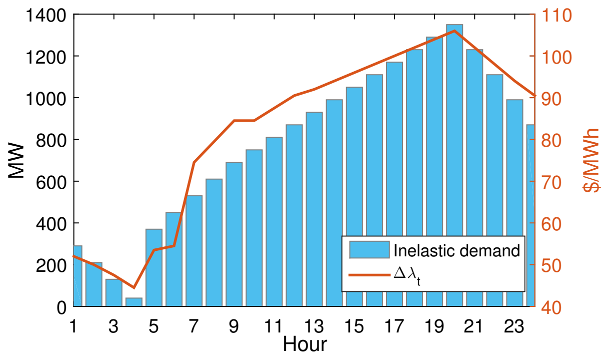

Fig. 7 Hourly total system inelastic demand and resulting effective LMPs ($\Delta\lambda_t$).

Figure 8 compares the hourly EV consumption and marginal losses with 15 EVs across the decentralized designs. As discussed in Proposition 6, EVs scheduled by the EV aggregator consume more during the hours with higher effective LMPs (hours 1, 2, 5) than they do when they self-schedule, whether with or without feeder information. If we include only the hours when $q^P_{j,t} > 0$, i.e., hours 2,3,4, the difference in total hourly EV load reaches 19% between $\mathbf{EV^{ind}_{A}}$ and $\mathbf{EV^{agg}_{A}}$, and 2.2% between $\mathbf{EV^{ind}_{Un}}$ and $\mathbf{EV^{ind}_{A}}$.

[12] Or equivalently a low losses transmission network with no line congestion, a choice that does not compromise the qualitative generality of our results.

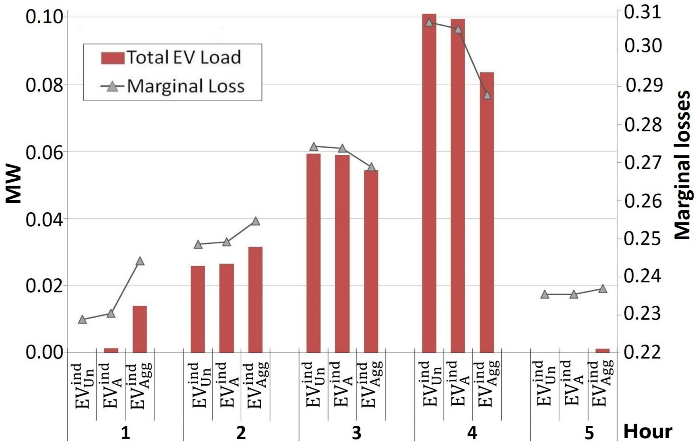

Fig. 8 Total EV consumption and marginal losses with 15 EVs. Comparison of decentralized $\mathbf{EV}_{\mathbf{Un}}^{\mathbf{ind}}$, $\mathbf{EV}_{\mathbf{A}}^{\mathbf{ind}}$, and $\mathbf{EV}_{\mathbf{A}}^{\mathbf{agg}}$ market designs, with $\gamma = 1.1$.

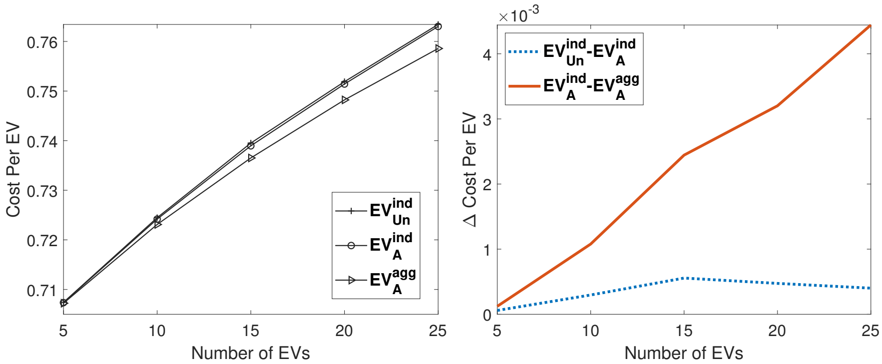

Fig. 9 Cost per EV and difference in Cost per EV among decentralized $\mathbf{EV}_{\mathbf{Un}}^{\mathbf{ind}}$, $\mathbf{EV}_{\mathbf{A}}^{\mathbf{ind}}$, and $\mathbf{EV}_{\mathbf{A}}^{\mathbf{agg}}$ market designs *w.r.t.* the number of EVs.

We compare the cost per EV differences among decentralized market designs *w.r.t.* the number of EVs. First we consider a small range of EV numbers without adjusting the line loss factor (Figure 9), and then for higher EV adoption rates we use an EV number sensitive loss factor γ set equal to $\frac{\hat{\gamma}}{J}$ (Figure 10). Total EV demand even in the highest EV penetration case is a relatively small percentage of total system demand, LMPs $(\lambda_t^P, \lambda_t^R)$ are still considered relatively insensitive to EV demand. More specifically, at a 250 EV penetration, total hourly EV demand reaches 2% of total system demand in a particular hour. Hourly DLMPs, however, are quite sensitive to hourly EV consumption, and hence the game context is still relevant.

The difference in cost per EV and hourly consumption between the EV load aggregator $\mathbf{EV}_{\mathbf{A}}^{\mathbf{agg}}$ and EV self-scheduling market designs is higher than the difference between EV self-scheduling information-Aware and Unaware mar-

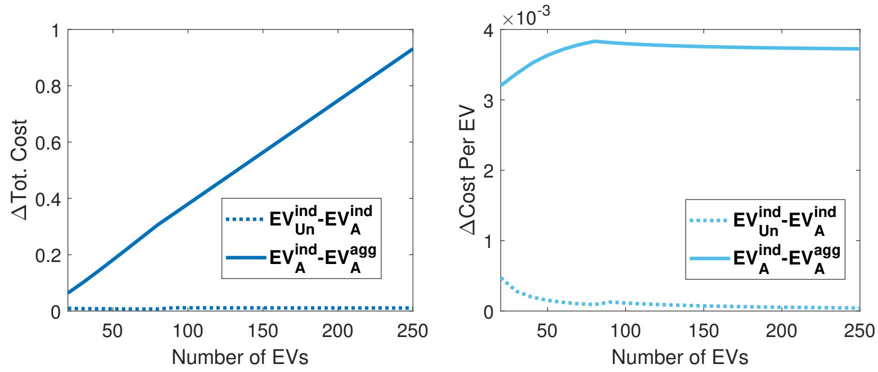

Fig. 10 Differences in total EV cost and cost per EV *w.r.t.* the number of EVs. Comparison among decentralized $\mathbf{EV^{ind}_{Un}}$, $\mathbf{EV^{ind}_{A}}$, and $\mathbf{EV^{agg}_{A}}$ market designs; $\gamma = \frac{\hat{\gamma}}{J}$, with $\hat{\gamma} = 22$.

ket designs (Figure 9 and 11). In fact, the cost per EV difference between the individual scheduling information Unaware $\mathbf{EV^{ind}_{Un}}$ and individual scheduling information Aware $\mathbf{EV^{ind}_{A}}$ market designs starts to decrease beyond 15 EVs (Figure 9). On the other hand, the difference in cost per EV between the individual and the EV aggregator $\mathbf{EV^{agg}_{A}}$ market designs stabilizes asymptotically as the number of EVs increases, resulting in a persistent difference in total EV cost (Figure 10).

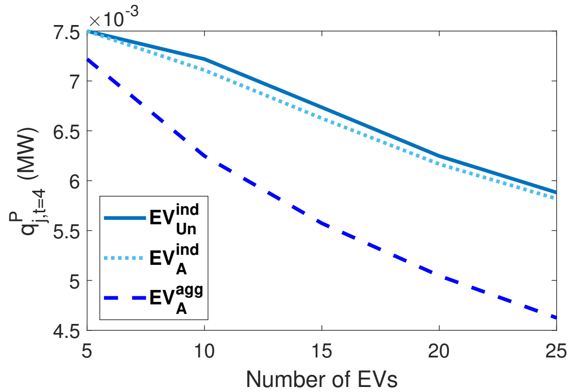

Fig. 11 Consumption during hour 4 per EV *w.r.t.* the number of EVs. Comparison of decentralized $\mathbf{EV^{ind}_{Un}}$, $\mathbf{EV^{ind}_{A}}$, and $\mathbf{EV^{agg}_{A}}$ market designs.

Table 2 summarizes the aggregate EV net costs and its components for small and high penetration levels of EVs, as well as the total marginal cost based charges assessed to inelastic demand located at the same feeder n' as EVs. Inelastic demand marginal cost based charges are equal to $\lambda^P_t(1 + m_{n',t})d^f_{n',t}$. Numerical results indicate that EVs achieve lower net costs when they self-schedule compared to the centralized $\mathbf{TD_A}$ market design equilibrium. EVs are able to enjoy lower costs relative to the centralized market design equilibrium, and achieve the lowest cost when scheduled by an information aware load aggregator under the $\mathbf{EV_A^{agg}}$ market design. They interestingly decrease their battery charging costs while also decreasing, albeit at a lower rate, their reserve revenue. As expected, social costs are lowest under the single operator centralized market design, $\mathbf{TD_A}$.[13] Notably, inelastic demand cost is higher under the decentralized market design equilibria, suggesting that self-scheduling EVs lower their cost at the expense of both social welfare and inelastic demand customers.

Table 2 Summarized EV and inelastic demand cost with Low and High EV Penetration

Number of EVs=15	$\mathbf{TD_A}$	$\mathbf{EV_{Un}^{ind}}$	$\mathbf{EV_A^{ind}}$	$\mathbf{EV_A^{agg}}$
Cost Per EV	0.744	0.740	0.739	0.736
Total EV cost	11.157	11.093	11.084	11.047
Energy cost	17.238	17.097	17.075	16.933
Reserve Revenue	6.082	6.004	5.990	5.886
Max. marg. losses	0.320	0.307	0.305	0.288
Social cost	1,712,928.66	1,712,928.68	1,712,928.69	1,712,928.76
Inelastic dem. cost	840.01	840.03	840.04	840.08
Number of EVs=250	$\mathbf{TD_A}$	$\mathbf{EV_{Un}^{ind}}$	$\mathbf{EV_A^{ind}}$	$\mathbf{EV_A^{agg}}$
Cost Per EV	0.648	0.638	0.638	0.635
Total EV cost	162.117	159.631	159.621	158.690
Energy cost	249.988	244.749	244.723	241.399
Reserve Revenue	87.881	85.118	85.102	82.714
Max. marg. losses	0.166	0.138	0.138	0.112
Social cost	1,712,979.79	1,712,980.25	1,712,980.25	1,712,981.87
Inelastic dem. cost	665.106	665.179	665.179	665.296

With increasing EV numbers the impact of a single EV's consumption on the marginal losses becomes insignificant, a fact shown analytically by Proposition 7. Numerical results confirm that $\mathbf{EV_{Un}^{ind}}$ and $\mathbf{EV_A^{ind}}$ equilibria become almost identical (Table 2) and differences in cost per EV approaches zero with increasing number of EVs (Figure 10). With 250 EVs, the difference in total EV cost between the information-aware decentralized $\mathbf{EV_A^{agg}}$ and $\mathbf{EV_A^{ind}}$ equilibria is about 0.6%, while the difference is about 2.1% relative to the centralized social welfare maximizing $\mathbf{TD_A}$ market design equilibrium.

In summary, numerically computed equilibria under accurate EV models are fully compatible with the analytical results of Section 6 derived under simplified EV models.

8 Conclusions, Policy Recommendations and Future Work

We considered multi-commodity and multi-period electricity markets in the anticipated massive presence of DERs, primary among them EVs, which have

[13] A large portion of the social cost is associated with fixed/inelastic demand.

complex inter-temporally coupled preferences. As such, EVs can practically participate in network enabled electricity markets through self-scheduling in response to granular and dynamically varying spatiotemporal marginal network cost based prices, or, through group scheduling assisted by load aggregators responding to similar marginal network cost rates. Notably, however, self or aggregator-assisted EV scheduling can benefit EVs with access to network cost information which enables them to anticipate how their scheduling decisions affect marginal costs. To this end, decentralized market designs enabling self or group EV scheduling with network information-Aware or Unaware decision makers were analyzed. Existence and uniqueness of decentralized market equilibrium were proven and compared both analytically and numerically. A novel and unique contribution of this work was the modeling and internalization of the salient network characteristics and their impact on the multi-commodity (energy and regulation reserves) and multi-period (24-hour day ahead) markets where inter-temporally coupled EV charging must be scheduled. Under network information-Aware scheduling, convexity conditions do not hold and proving equilibrium existence and uniqueness is nontrivial. Using a pioneering input state partition approach we were able to exploit the inequality constraints coupling battery charging and reserve provision decisions to map the multi-commodity to a single-commodity market. This mapping enabled us to construct and use potential functions to not only show equilibrium uniqueness and existence, but to also compare market designs and reach the following conclusions:

1. As expected, social Welfare is maximized under a centrally cleared market operator ($\mathbf{TD_A}$) who has full access to network information (especially line loss factors) as well as preferences of centralized generation, EVs, and inelastic demand. A second best social welfare is obtained when DERs self-schedule adapting to prices without being aware of how their decisions affect line losses.
2. Self-scheduling EVs achieve a lower cost at the expense of social welfare when they are fully Aware of network marginal losses ($\mathbf{EV_A^{ind}}$) and thus may (i) learn inelastic demand and the behavior of other EVs, and (ii) anticipate how their own scheduling decisions impact marginal cost based rates at their node.
3. Social welfare is even lower and EVs achieve an even lower cost if EVs are scheduled as a group by a distribution network information-aware EV aggregator ($\mathbf{EV_A^{agg}}$).
4. In asymptotic results where high and low voltage networks increase their capacity in proportion to the number of EVs, information-aware EV self-scheduling ($\mathbf{EV_A^{ind}}$) converges to information-Unaware scheduling ($\mathbf{EV_{Un}^{ind}}$) as the number of EVs approaches infinity. However, this is not a realistic comparison, since actual distribution networks will respond to accommodate a larger number of EVs by a proliferation of feeder lines rather than strengthening a single line to drive its loss factor down in proportion to the number of EVs connected to it.

5. Interestingly, however, even under the simplistic assumption that feeder capacity increases in proportion to the connected number of EVs, information Aware EV load aggregators may still achieve a leveling but not diminishing cost advantage.
6. All the aforementioned conclusions are shown on numerically estimated realistic cases, but also on simplified EV models that allow analytic representation of the information-Aware and Unaware market design equilibria.
7. The computationally tractable market design with the highest social welfare is the information-Unaware self-scheduling described as the second best under item 1. Interestingly, information-Unaware group scheduling is identical to self-scheduling under the Information-Unaware market design. This provides a very significant regulatory policy recommendation that distribution network operators who are fully aware of the network information, should remain regulated and not allowed to engage in profit maximizing EV group schedulers. It is important to note that this is a conclusion that is true even in the asymptotic case where the number of EVs goes to infinity as discussed under item 5.

In terms of future work, the results obtained here by a simple (but not too simple as to prevent us to discriminate between salient characteristics of real distribution networks) model of the electricity network should be extended and implemented to higher fidelity network and DER models including EV utility functions. For example, the simple notion of marginal losses increasing in the level of charging during a given hour, is likely to be magnified significantly if the marginal cost of transformer life degradation is modeled [39], [40]. The computational issues, as for example algorithmic convergence to the Nash Equilibrium, are magnified by hard to deal with non-convexities when the distribution network is represented with a higher fidelity model which considers both real and reactive power flows (e.g., the branch flow model introduced in [41] and recently revisited by [42]). Our brief discussion of convergence under Section 3 should and most probably can be extended to hold under the non-convexities introduced by higher fidelity network models.

A APPENDIX

A.1 Proof of Proposition 1 (Key impact of decentralized designs on scheduling decisions)

For the purposes of the proof, we reduce the multi-commodity (energy and reserves) decisions to a single-commodity decision. To this end, we note that reserve provision $q^R_{j,t}$ either increases with energy consumption $q^P_{j,t}$ if constraint (10) is not binding, or decreases if constraint (11) is not binding. If the former is true, then *(i)* $\nu^1_{j,t} = 0$ and, *(ii)* $\frac{\partial q^R_{j,t}}{\partial q^P_{j,t}} = 1$. In that case, rearranging the optimality conditions *w.r.t.* $q^P_{j,t}$ and $q^P_{j,t'}$ yields:

- $\mathbf{EV^{ind}_{Un}}$: $\lambda^P_{n,t} - \lambda^R_{n,t} + 2\delta q^P_{j,t} = \lambda^P_{n,t'} - \lambda^R_{n,t'} + 2\delta q^P_{j,t'}$,
- $\mathbf{EV^{ind}_{A}}$: $(\lambda^P_t - \lambda^R_t)[\gamma_n q^P_{j,t} + 1 + m_{n,t}(q^P_{-j,t}, q^P_{j,t})] + 2\delta q^P_{j,t} =$
  $= (\lambda^P_{t'} - \lambda^R_{t'})[\gamma_n q^P_{j,t'} + 1 + m_{n,t'}(q^P_{-j,t'}, q^P_{j,t'})] + 2\delta q^P_{j,t'}$,

– $\mathbf{EV^{agg}_{A}}$: $(\lambda^P_t - \lambda^R_t)\left[\gamma_n \sum_j q^P_{j,t} + 1 + m_{n,t}(\Sigma q^P_{j,t})\right] + 2\delta q^P_{j,t} =$
$= (\lambda^P_{t'} - \lambda^R_{t'})\left[\gamma_n \sum_j q^P_{j,t'} + 1 + m_{n,t'}(\Sigma q^P_{j,t'})\right] + 2\delta q^P_{j,t'}$.

If, on the other hand, reserve provision decreases with energy consumption, then *(i)* $\nu^2_{j,t} = 0$ and *(ii)* $\frac{\partial q^R_{j,t}}{\partial q^P_{j,t}} = -1$. Rearranging the optimality conditions *w.r.t.* $q^P_{j,t}$ and $q^P_{j,t'}$ yields:

– $\mathbf{EV^{ind}_{Un}}$: $\lambda^P_{n,t} + \lambda^R_{n,t} + 2\delta q^P_{j,t} = \lambda^P_{n,t'} + \lambda^R_{n,t'} + 2\delta q^P_{j,t'}$,
– $\mathbf{EV^{ind}_{A}}$: $(\lambda^P_t + \lambda^R_t)\left[\gamma_n q^P_{j,t} + 1 + m_{n,t}(q^P_{-j,t}, q^P_{j,t})\right] + 2\delta q^P_{j,t} =$
$= (\lambda^P_{t'} + \lambda^R_{t'})\left[\gamma_n q^P_{j,t'} + 1 + m_{n,t'}(q^P_{-j,t'}, q^P_{j,t'})\right] + 2\delta q^P_{j,t'}$,
– $\mathbf{EV^{agg}_{A}}$: $(\lambda^P_t + \lambda^R_t)\left[\gamma_n \sum_j q^P_{j,t} + 1 + m_{n,t}(\Sigma q^P_{j,t})\right] + 2\delta q^P_{j,t} =$
$= (\lambda^P_{t'} + \lambda^R_{t'})\left[\gamma_n \sum_j q^P_{j,t'} + 1 + m_{n,t'}(\Sigma q^P_{j,t'})\right] + 2\delta q^P_{j,t'}$.

The equality of marginal cost reduction in hour t to the marginal cost increase in hour t' is thus shown and quantified by expressing $q^R_{j,t}$ in terms of $q^P_{j,t}$.

A.2 Proof of Lemma 1

(i) The result follows directly from the optimality conditions (23). Since the equality is not satisfied when $\nu^1_{j,t} = \nu^2_{j,t} = 0$, at least one of these dual variables must be nonzero, assuming that $\lambda^R_t > 0, \forall t$. Note that we do not consider the unlikely event that $\lambda^R_t = 0$.
(ii) The result follows from (24) for $\mathbf{EV^{ind}_{Un}}$, from (25) for $\mathbf{EV^{ind}_{A}}$ and from (26) for $\mathbf{EV^{agg}_{A}}$. Note that $\underline{\zeta}_j$ is positive when $\Delta\lambda_t = \lambda^P_t - \lambda^R_t > 0$. Therefore, the minimum charging demand constraint in (8) is binding. That is, $\sum_j q^P_{j,t} = \underline{s}_j$.
(iii) For $\mathbf{EV^{ind}_{A}}$, the result follows from (23) and (25). Note that when $q^R_{j,t} < q^P_{j,t}$, constraint (11) is not binding and $\nu^2_{j,t} = 0$. Then, combining (23) and (25), we obtain: $\underline{\zeta}_j = \lambda^P_t(1 + m_{n,t}) + \lambda^R_t(1 + m_{n,t}) + \gamma_n(\lambda^P q^P_{j,t} - \lambda^R_t q^R_{j,t}) + 2\delta q^P_{j,t} + \overline{\zeta}_j$. Therefore, $\underline{\zeta}_j > 0$ and constraint (8) is binding. The positivity of $\underline{\zeta}_j$ in the $\mathbf{EV^{ind}_{Un}}$ and $\mathbf{EV^{agg}_{A}}$ market designs is shown similarly.
(iv) For $\mathbf{EV^{ind}_{A}}$, the result follows from (23) and (25). When $\underline{\zeta}_j = \nu^2_{j,t} = 0$, we obtain $\overline{\zeta}_j = -\Delta\lambda_t + \gamma_n(\lambda^R_t q^R_{j,t} - \lambda^P_t q^P_{j,t}) - 2\delta q^P_{j,t}$. Since a non-binding constraint (8) implies $\lambda^R_t - \lambda^P_t > 0$, we conclude that $\overline{\zeta}_j > 0$ and constraint (9) is binding. The positivity of $\overline{\zeta}_j$ in the scheduling problems solved in Step 2 of Algorithms 2 ($\mathbf{EV^{ind}_{Un}}$) and 4 ($\mathbf{EV^{agg}_{A}}$) can be shown similarly.

A.3 Proof of Theorem 2 (Nash Equilibrium Uniqueness)

Theorem 2 employs the potential function approach to show uniqueness of an equilibrium. A game in strategic form is called a potential game if the change in the payoff of a player due to change in strategy is equal to the change in the global potential function due to unilateral change of strategy of the player. That is, a game where each player i has payoff function $f_i(q_i, q_{-i})$, is a potential game if for every $q_i \in S$, $f_i(q_i, q_{-i}) - f_i(q'_i, q_{-i}) = \overline{P}(q_i) - \overline{P}(q'_i)$, $\forall i, q_i$. The function $\overline{P}$ that couples all players is called the *potential function*. Constructing it is key to invoke Theorem 2. The relation between the Nash equilibrium condition and the optimization problem with objective function replaced by $\overline{P}$ is based on [35, Definition 1]. The conditions under which the Nash equilibrium is unique are given by [37, Theorem 2] and restated in [36]. In fact (see [36]), the diagonal strict convexity condition for equilibrium uniqueness given by [33, Theorem 2] is satisfied when the potential function is strictly convex. Lastly, it is stated in [34] that there is no clear economic interpretation of the potential function, in other words, the function that players jointly maximize.

A.4 Proof of Lemma 2 (Single-Commodity, High Charging Flexibility)

We prove the result by contradiction for the $\mathbf{EV}^{\mathbf{ind}}_{\mathbf{Un}}$, $\mathbf{EV}^{\mathbf{ind}}_{\mathbf{A}}$ and, $\mathbf{EV}^{\mathbf{agg}}_{\mathbf{A}}$ market designs and illustrate it for $\mathbf{EV}^{\mathbf{ind}}_{\mathbf{A}}$. Assume the opposite is true, i.e., $\exists t$ s.t. $q^P_{j,t} > \frac{\overline{q}_j}{2}$. By Assumption 2 and Lemma 1(ii), $\sum_t q^P_{j,t} = \underline{s}_j$. Then, given (27), there should exist some hour t' s.t. $q^P_{j,t'} < \frac{\overline{q}_j}{2}$. Comparing the expressions of $\underline{\zeta}_j$ obtained from the first order optimality conditions associated with $q^P_{j,t}$ and $q^P_{j,t'}$, we get: $\underline{\zeta}_j = (\lambda^P_t + \lambda^R_t)(1 + m_{n,t}) + \gamma_n(\lambda^P_t q^P_{j,t} - \lambda^R_t q^R_{j,t}) + 2\delta q^P_{j,t} + \overline{\zeta}_j$, and $\underline{\zeta}_j = \Delta\lambda_{t'}(1+m_{n,t'}) + \gamma_n(\lambda^P_{t'} q^P_{j,t'} - \lambda^R_{t'} q^R_{j,t'}) + 2\delta q^P_{j,t'} + \overline{\zeta}_j$. Note that the terms $\gamma_n(\lambda^P_t q^P_{j,t} - \lambda^R_t q^R_{j,t})$ include energy and reserve decisions of EV j only. Hence, these quantities, including the degradation term, are small enough, implying that the two expressions for $\underline{\zeta}_j$ cannot be identical since $\lambda^P_{n,t} + \lambda^R_{n,t} > \Delta\lambda_{n,t'}$ from Assumption 1. Therefore, the optimal solution of the EV scheduling problem satisfies $q^P_{j,t} \leq \frac{\overline{q}_j}{2}$, $\forall t$.

A.5 Proof of Proposition 2 (Existence and Uniqueness, High Charging Flexibility)

Existence of equilibrium for the $\mathbf{EV}^{\mathbf{ind}}_{\mathbf{A}}$ and $\mathbf{EV}^{\mathbf{ind}}_{\mathbf{Un}}$ market designs follows from Lemma A1 and Theorem 1.

Lemma A1 *Consider the* $\mathbf{EV}^{\mathbf{ind}}_{\mathbf{Un}}$ *and* $\mathbf{EV}^{\mathbf{ind}}_{\mathbf{A}}$ *market designs. The feasible set* S^{HF}_j *of EV* j *in the EV self-scheduling problems, under High Charging Flexibility, is closed, bounded, and convex. In addition, given Assumption 2, the cost functions* $f^{\mathbf{EV}^{\mathbf{ind}}_{\mathbf{A}}}_j$ *and* $f^{\mathbf{EV}^{\mathbf{ind}}_{\mathbf{Un}}}_j$ *of EV* j *are both strictly convex and continuous in* q^P_j.

Proof Due to the positivity of $\Delta\lambda_t$ from Assumption 2, $f^{\mathbf{EV}^{\mathbf{ind}}_{\mathbf{A}}}_j(q^P_j, q^P_{-j})$ and $f^{\mathbf{EV}^{\mathbf{ind}}_{\mathbf{Un}}}_j(q^P_j)$ given in (31) and (32) are strictly convex in q^P_j and continuous in both q^P_j and q^P_{-j}. In addition, the feasible set S^{HF}_j of EV j is closed and bounded. Without the inter-temporal minimum charging demand constraint (8), the feasible set of EV j is the box constraint $q^P_{j,t} \in [0, \frac{\overline{q}_j}{2}]$. Therefore, the feasible set of EV j without constraint (8), denoted by $\overline{S}_j$, is given by $\overline{S}_j = \overline{S}^1_j \times \ldots \times \overline{S}^t_j \times \ldots \times \overline{S}^T_j$ where $\overline{S}^t_j = [0, \frac{\overline{q}_j}{2}]$ and T is the number of periods EVs are plugged in. Therefore, $\overline{S}_j$ is closed and bounded. The feasible set in the presence of constraint (8), S^{HF}_j, is then a closed subset of $\overline{S}_j$, hence also bounded. S^{HF}_j is convex as well since the constraints are linear.

Consider the $\mathbf{EV}^{\mathbf{ind}}_{\mathbf{A}}$ market design. The potential function $\overline{P}^{\mathbf{EV}^{\mathbf{ind}}_{\mathbf{A}}}$ for the game with information-aware EVs, where EV j has the cost function $f^{\mathbf{EV}^{\mathbf{ind}}_{\mathbf{A}}}_j$ in (31) and strategy set S^{HF}_j, is given by:

$$\overline{P}^{\mathbf{EV}^{\mathbf{ind}}_{\mathbf{A}}} = \sum_{j,t} \Delta\lambda_t \left[1 + m_{n,t}(\Sigma q^P_{j,t})\right] q^P_{j,t} + \delta(q^P_{j,t})^2 - \sum_{j,j'|j'>j,t} \Delta\lambda_t\gamma_n q^P_{j,t} q^P_{j',t}. \quad \text{(A.1)}$$

In addition, $\overline{P}^{\mathbf{EV}^{\mathbf{ind}}_{\mathbf{A}}}$ is strictly convex in $q^P_{j,t}, \forall j$. The $J \times J$ Hessian matrix of the potential function $\overline{P}^{\mathbf{EV}^{\mathbf{ind}}_{\mathbf{A}}}(q^P_t)$ in (A.1) for a given t has the following form:

$$H_t = \begin{bmatrix} 2\Delta\lambda_t\gamma_n + 2\delta & \Delta\lambda_t\gamma_n & \Delta\lambda_t\gamma_n & \ldots & \Delta\lambda_t\gamma_n \\ \Delta\lambda_t\gamma_n & 2\Delta\lambda_t\gamma_n + 2\delta & \Delta\lambda_t\gamma_n & \ldots & \Delta\lambda_t\gamma_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \Delta\lambda_t\gamma_n & \Delta\lambda_t\gamma_n & \Delta\lambda_t\gamma_n & \ldots & 2\Delta\lambda_t\gamma_n + 2\delta \end{bmatrix},$$

which can be written as the sum of a positive semi-definite matrix and the identity matrix multiplied by a positive scalar. Since the identity matrix is positive definite, and the sum of a positive semi-definite and positive definite matrix is also positive definite, H_t is positive definite, and hence, the potential function (A.1) is strictly convex, which implies uniqueness. We then proceed to show that the equilibrium conditions are identical to the optimality conditions of the equilibrium recovery problem (Nash equilibrium is recovered by minimizing the potential function $\overline{P}^{\mathbf{EV}^{\mathbf{ind}}_{\mathbf{A}}}$ subject to $S_j^{HF}, \forall j$). The first order optimality condition of the EV self-scheduling problem (Step 2 of Algorithm 3) *w.r.t.* $q^P_{j,t}$ is written as:

$$\Delta\lambda_t(1+m_{n,t}) + \Delta\lambda_t\gamma_n q^P_{j,t} + 2\delta q^P_{j,t} - \underline{\zeta}_j + \nu^1_{j,t} = 0. \tag{A.2}$$

The optimality condition *w.r.t.* $q^P_{j,t}$ of the problem with the objective function replaced by the potential function $\overline{P}^{\mathbf{EV}^{\mathbf{ind}}_{\mathbf{A}}}$ given in (A.1) is:

$$\Delta\lambda_t(1+m_{n,t}) + \Delta\lambda_t\gamma q^P_{j,t} + \Delta\lambda_t\gamma q^P_{-j,t} + 2\delta q^P_{j,t} - \Delta\lambda_t\gamma q^P_{-j,t} - \underline{\zeta}_j + \nu^1_{j,t} = 0. \tag{A.3}$$

By inspection, optimality conditions (A.2) and (A.3) are identical. Hence, by Theorem 2, the Nash equilibrium of the $\mathbf{EV}^{\mathbf{ind}}_{\mathbf{A}}$ market design, under High Charging Flexibility, is unique.

Consider the $\mathbf{EV}^{\mathbf{ind}}_{\mathbf{Un}}$ market design. The potential function $\overline{P}^{\mathbf{EV}^{\mathbf{ind}}_{\mathbf{Un}}}$ for the game with information-unaware EVs, where EV j has the cost function $f_j^{\mathbf{EV}^{\mathbf{ind}}_{\mathbf{Un}}}$ in (32) and strategy set S_j^{HF}, is given by:

$$\overline{P}^{\mathbf{EV}^{\mathbf{ind}}_{\mathbf{Un}}} =$$

$$\sum_{j,t}\Delta\lambda_t\left[1+m_{n,t}(\Sigma q^P_{j,t})\right]q^P_{j,t} + \delta(q^P_{j,t})^2 - \sum_{j,j'|j'>j,t}\Delta\lambda_t\gamma_n q^P_{j,t}q^P_{j',t} - \frac{1}{2}\sum_{j,t}\Delta\lambda_t\gamma_n(q^P_{j,t})^2. \tag{A.4}$$

In addition, $\overline{P}^{\mathbf{EV}^{\mathbf{ind}}_{\mathbf{Un}}}$ is strictly convex in $q^P_{j,t}, \forall j$, and the Hessian of a single hour t of $\overline{P}^{\mathbf{EV}^{\mathbf{ind}}_{\mathbf{Un}}}$ in (A.4) is given by:

$$H_t = \begin{bmatrix} \Delta\lambda_t\gamma_n+2\delta & \Delta\lambda_t\gamma_n & \Delta\lambda_t\gamma_n & \dots & \Delta\lambda_t\gamma_n \\ \Delta\lambda_t\gamma_n & \Delta\lambda_t\gamma_n+2\delta & \Delta\lambda_t\gamma_n & \dots & \Delta\lambda_t\gamma_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \Delta\lambda_t\gamma_n & \Delta\lambda_t\gamma_n & \Delta\lambda_t\gamma_n & \dots & \Delta\lambda_t\gamma_n+2\delta \end{bmatrix},$$

where $\Delta\lambda_t = \lambda^P_t - \lambda^t_R$. Given Assumption 2, the Hessian is positive definite. We can also show that the equilibrium conditions and optimality conditions of the equilibrium recovery problem where $\overline{P}^{\mathbf{EV}^{\mathbf{ind}}_{\mathbf{Un}}}$ is minimized over S_j^{HF}, $\forall j$ are identical. Hence, by Theorem 2, the Nash equilibrium of the $\mathbf{EV}^{\mathbf{ind}}_{\mathbf{Un}}$ market design, under High Charging Flexibility, is unique.

For the $\mathbf{EV}^{\mathbf{agg}}_{\mathbf{A}}$ market design, we note that the objective function of the load aggregator, $f^{\mathbf{EV}^{\mathbf{agg}}_{\mathbf{A}}}(q^P_t)$, given by (33), under High Charging Flexibility, is strictly convex subject to linear constraints. Therefore, the optimal solution is unique.

We note that diagonal strict convexity of EV cost functions $f_j^{\mathbf{EV}^{\mathbf{ind}}_{\mathbf{A}}}$ and $f_j^{\mathbf{EV}^{\mathbf{ind}}_{\mathbf{Un}}}$ can also be shown both for information Aware and Unaware market designs, which is an equilibrium uniqueness condition provided by [33, Theorem 2].

A.6 Proof of Lemma 3 (Single-Commodity, Moderate/Low Charging Flexibility)

We illustrate the proof for the $\mathbf{EV}^{\mathbf{ind}}_{\mathbf{A}}$ market design; the proofs for $\mathbf{EV}^{\mathbf{ind}}_{\mathbf{Un}}$ and $\mathbf{EV}^{\mathbf{agg}}_{\mathbf{A}}$ are similar. Assume the opposite is true, i.e., $\exists t$ s.t. $q^P_{j,t} < \frac{\overline{q}_j}{2}$. By the definition of Moderate

Charging Flexibility, there should exist another hour t' s.t. $q^P_{j,t'} > \frac{\overline{q}_j}{2}$. We then use the same contradiction argument as in Subsection A.4 in the opposite direction, and we show that the two expressions of $\underline{\zeta}_j$ cannot be identical. Therefore, $q^P_{j,t} \geq \frac{\overline{q}_j}{2}$, $\forall t$. Under Low Charging Flexibility, due to (29), $q^P_{j,t} > \frac{\overline{q}_j}{2}$, $\forall t$, regardless of Assumption 1 or 2.

A.7 Proof of Proposition 3 (Existence and Uniqueness, Moderate/Low Charging Flexibility)

Equilibrium existence for the $\mathbf{EV^{ind}_{A}}$ and $\mathbf{EV^{ind}_{Un}}$ market designs follows from Lemma A2 and Theorem 1.

Lemma A2 *Consider the* $\mathbf{EV^{ind}_{Un}}$ *and* $\mathbf{EV^{ind}_{A}}$ *market designs. The feasible set* S^{MLF}_j *of EV* j *in the EV self-scheduling problems, under Moderate or Low Charging Flexibility, is closed, bounded, and convex. In addition, the cost functions* $f^{\mathbf{EV^{ind}_{A}}}_j$ *and* $f^{\mathbf{EV^{ind}_{Un}}}_j$ *of EV* j *are both strictly convex and continuous in* q^P_j.

Proof Since $\lambda^P_t + \lambda^R_t > 0$, $f^{\mathbf{EV^{ind}_{A}}}_j$ and $f^{\mathbf{EV^{ind}_{Un}}}_j$ are both convex. The convexity, closedness and boundedness of S^{MLF}_j is shown similarly to the proof of Lemma A1. In this case, we define $\overline{S}^t_j$ as $\overline{S}^t_j = [\frac{\overline{q}_j}{2}, \overline{q}_j]$.

Consider the $\mathbf{EV^{ind}_{A}}$ market design. The potential function $\overline{P}^{\mathbf{EV^{ind}_{A}}}$ for the game with information-aware EVs, where EV j solves (34) subject to S^{MLF}_j is:

$$\begin{aligned}\overline{P}^{\mathbf{EV^{ind}_{A}}} &= \sum_{j,t}(1 + m_{n,t}(\Sigma q^P_{j,t}))(\lambda_t{}^P q^P_{j,t} - \lambda^R_t(\overline{q}_j - q^P_{j,t}) + \delta(q^P_{j,t})^2, \\ &- \sum_{j,j'|j'>j,t}(\lambda^P_t + \lambda^R_t)\gamma q^P_{j,t} q^P_{j',t} + \sum_{j,j'|j'\neq j,t}\lambda^R_t \gamma q^P_{j,t}\overline{q}_{j'}.\end{aligned} \tag{A.5}$$

In addition, $\overline{P}$ is strictly convex in $q^P_{j,t}, \forall j$. The Hessian of the potential function in (A.5) has the following form:

$$H_t = \begin{bmatrix} 2\Sigma\lambda_t\gamma_n + 2\delta & \Sigma\lambda_t\gamma_n & \Sigma\lambda_t\gamma_n & \dots & \Sigma\lambda_t\gamma_n \\ \Sigma\lambda_t\gamma_n & 2\Sigma\lambda_t\gamma_n + 2\delta & \Sigma\lambda_t\gamma_n & \dots & \Sigma\lambda_t\gamma_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \Sigma\lambda_t\gamma_n & \Sigma\lambda_t\gamma_n & \Sigma\lambda_t\gamma_n & \dots & 2\Sigma\lambda_t\gamma_n + 2\delta \end{bmatrix},$$

where $\Sigma\lambda_t = \lambda^P_t + \lambda^R_t$. Due to positivity of $\Sigma\lambda_t$, the above Hessian is positive definite and the objective function is strictly convex. Uniqueness of Nash Equilibrium is then shown by identical optimality conditions. The optimality condition of the EV self-scheduling problem (Step 2 of Algorithm 3) *w.r.t.* $q^P_{j,t}$ is given by:

$$(\lambda^P_t + \lambda^R_t)(1 + m_{n,t} + \gamma q^P_{j,t}) - \gamma\lambda^R_t\overline{q}_j + 2\delta q^P_{j,t} - \underline{\zeta}_j + \nu^1_{j,t} = 0. \tag{A.6}$$

One can show that the optimality condition of the potential function $\overline{P}$ in (A.5) *w.r.t.* $q^P_{j,t}$ is identical to (A.6).

Consider the $\mathbf{EV^{ind}_{Un}}$ market design. The potential function $\overline{P}^{\mathbf{EV^{ind}_{Un}}}$ for the game with information-unaware EVs, where EV j minimizes the cost function in (35) is given by:

$$\begin{aligned}\overline{P}^{\mathbf{EV^{ind}_{Un}}} &= \sum_{j,t}(1 + m_{n,t})(\lambda_t{}^P q^P_{j,t} - \lambda^R_t(\overline{q}_j - q^P_{j,t}) + \delta(q^P_{j,t})^2 - \\ &\sum_{j,j'|j'>j,t}(\lambda^P_t + \lambda^R_t)\gamma q^P_{j,t} q^P_{j',t} + \sum_{j,j',t}\lambda^R_t\gamma q^P_{j,t}\overline{q}_{j'} - \frac{1}{2}\sum_{j,t}(\lambda^P_t + \lambda^R_t)\gamma(q^P_{j,t})^2.\end{aligned} \tag{A.7}$$

and strictly convex. The Hessian of the potential function in (A.7) has the following form:

$$H_t = \begin{bmatrix} \Sigma\lambda_t\gamma_n + 2\delta & \Sigma\lambda_t\gamma_n & \Sigma\lambda_t\gamma_n & \dots & \Sigma\lambda_t\gamma_n \\ \Sigma\lambda_t\gamma_n & \Sigma\lambda_t\gamma_n + 2\delta & \Sigma\lambda_t\gamma_n & \dots & \Sigma\lambda_t\gamma_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \Sigma\lambda_t\gamma_n & \Sigma\lambda_t\gamma_n & \Sigma\lambda_t\gamma_n & \dots & \Sigma\lambda_t\gamma_n + 2\delta \end{bmatrix},$$

where $\Sigma\lambda_t = \lambda_t^P + \lambda_R^t$, and is positive definite. In addition, it is straightforward to show that the optimality conditions of the EV self-scheduling problem (Step 2 of Algorithm 2) and the optimality conditions of the problem where $\overline{P}^{\mathbf{EV}^{\mathbf{ind}}_{\mathbf{Un}}}$ is minimized over S_j^{MLF} are identical. Hence, uniqueness follows from Theorem 2.

For the $\mathbf{EV}^{\mathbf{agg}}_{\mathbf{A}}$ market design, we note that the objective function of the load aggregator, $f^{\mathbf{EV}^{\mathbf{agg}}_{\mathbf{A}}}(q_t^P)$, given by (36), under Moderate or Low Charging Flexibility, is strictly convex subject to linear constraints. Therefore, the optimal solution is unique.

A.8 Proof of Proposition 4 (Stability of N.E. under Information-Awareness)

In the two-hour model, the cost function of information-aware EV j, $f_j^{\mathbf{EV}^{\mathbf{ind}}_{\mathbf{A}}}(q_{j,1}^P)$, given in (31), satisfies the following condition for $J \leq 3$:

$$\sum_{j|j\neq j'} \left| \frac{\partial^2 f_{j'}^{\mathbf{EV}^{\mathbf{ind}}_{\mathbf{A}}}}{\partial q_{j',1}^P \partial q_{j,1}^P}(q^P) \right| < \left| \frac{\partial^2 f_{j'}^{\mathbf{EV}^{\mathbf{ind}}_{\mathbf{A}}}}{\partial^2 q_{j',1}^P}(q^P) \right|, \quad \forall j', q_{j',1}^P \in [0, \underline{s}_j]. \tag{A.8}$$

Therefore, by [38, Theorem 4], the Nash equilibrium in the $\mathbf{EV}^{\mathbf{ind}}_{\mathbf{A}}$ market design is stable.

A.9 Proof of Proposition 6 (Closed form equilibria expressions for simplified 24-hour problem)

We provide the proofs for $\mathbf{EV}^{\mathbf{ind}}_{\mathbf{A}}$ and $\mathbf{EV}^{\mathbf{agg}}_{\mathbf{A}}$ market designs; the proof is similar for $\mathbf{EV}^{\mathbf{ind}}_{\mathbf{Un}}$. The first order optimality conditions of $\mathbf{EV}^{\mathbf{agg}}_{\mathbf{A}}$ market design in (33) subject to only the minimum charging demand constraint (8) with J identical EVs can be written as:

$$q_{j,t}^{P,\mathbf{EV}^{\mathbf{agg}}_{\mathbf{A}}} = \max\left\{0, \frac{\underline{\zeta} - \Delta\lambda_t(1+\gamma d_t^f)}{2J\gamma\Delta\lambda_t + 2\delta}\right\}, \tag{A.9}$$

$$(-\sum_t q_{j,t}^P + \underline{s})\underline{\zeta} = 0. \tag{A.10}$$

Since EVs are identical, $\underline{\zeta}_j = \underline{\zeta}$, $\underline{s}_j = \underline{s}$, and $q_{j,t}^P = q_t^P$, $\forall j$. Combining (A.9) and (A.10), we can write: $\underline{\zeta} = \left[\sum_{t'} \frac{\Delta\lambda_{t'}(1+\gamma d_t^f)}{2J\gamma\Delta\lambda_{t'}+2\delta} + \underline{s}\right] / \left(\sum_{t'} \frac{1}{2J\gamma\Delta\lambda_{t'}+2\delta}\right)$, where $t' = \{t | q_{j,t}^P > 0\}$, and substituting in (A.9), we obtain:

$$q_{j,t}^{P,\mathbf{EV}^{\mathbf{agg}}_{\mathbf{A}}} = \frac{\sum_{t'} \frac{\Delta\lambda_{t'}(1+\gamma d_{t'}^f)}{2J\gamma\Delta\lambda_{t'}+2\delta} + \underline{s}}{\sum_{t'} \frac{2J\gamma\Delta\lambda_t+2\delta}{2J\gamma\Delta\lambda_{t'}+2\delta}} - \frac{\Delta\lambda_t(1+\gamma d_t^f)}{2J\gamma\Delta\lambda_t + 2\delta}. \tag{A.11}$$

Note that (A.11) matches (41) if EVs are connected only for two hours.

The first order optimality conditions of $\mathbf{EV}_{\mathbf{A}}^{\mathbf{ind}}$ in (31) subject to only (8) are: $q_{j,t}^{P,\mathbf{EV}_{\mathbf{A}}^{\mathbf{ind}}} = \max\left\{0, \frac{\underline{\zeta}-\Delta\lambda_t(1+\gamma d_{t'}^f)}{(J+1)\gamma\Delta\lambda_t+2\delta}\right\}$, and $(-\sum_t q_{j,t}^P + \underline{s})\underline{\zeta} = 0$, yielding:

$$q_{j,t}^{P,\mathbf{EV}_{\mathbf{A}}^{\mathbf{ind}}} = \frac{\sum_{t'} \frac{\Delta\lambda_{t'}(1+\gamma d_{t'}^f)}{(J+1)\gamma\Delta\lambda_{t'}+2\delta} + \underline{s}}{\sum_{t'} \frac{(J+1)\gamma\Delta\lambda_t+2\delta}{(J+1)\gamma\Delta\lambda_{t'}+2\delta}} - \frac{\Delta\lambda_t(1+\gamma d_t^f)}{(J+1)\gamma\Delta\lambda_t+2\delta}.$$

A.10 Proof of Proposition 7 (Asymptotic coincidence of individual self scheduling information Aware and Unaware EVs)

We illustrate this on the two-hour equilibria given by (42) and (43). As $J \to \infty$, $\frac{J+1}{J} \to 1$ rendering $q_{j,1}^{P,\mathbf{EV}_{\mathbf{Un}}^{\mathbf{ind}}} = q_{j,1}^{P,\mathbf{EV}_{\mathbf{A}}^{\mathbf{ind}}}$.

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