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# Resilience improvement of a critical infrastructure via optimal replacement and reordering of critical components

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**ABSTRACT**
One of the most important obstacles in improving the resilience of critical infrastructures is the timely replacement of critical components. However, the shortage of spare parts often keeps practitioners from achieving this goal. Moreover, it is common that spare parts may deteriorate on the shelf. In this paper, we focus on a one-component deteriorating system carrying one deteriorating spare part. Both failure-switching and preventive-switching strategies are considered for component replacement during each operating cycle to minimize the long-run cost rates. A case study on gearbox replacements for an offshore wind farm is provided to illustrate the proposed joint component replacement and reordering policies in improving the resilience of critical infrastructures. Although the chance of failure of such a system may be reduced by the preventive-switching strategy, the failure-switching strategy may still result in better economic performance due to the on-shelf deterioration and salvage of spare parts.

**Abbreviations:** CDF: cumulativedistributionfunction; PDF: probabilitydensityfunction; PR: preventivereplacement; CE: cumulativeexposure

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**KEYWORDS**
Resilience; deteriorating spare part; maintenance; part reordering; optimization

## 1. Introduction

A critical infrastructure system is defined as a network of independent, mostly privately owned, man-made systems and processes that function collaboratively and synergistically to produce and distribute a continuous flow of essential goods and services (Ellis et al., 1997). Hence, critical infrastructures (CIs), such as electric power, water distribution, natural gas, transportation and telecommunications, are the backbone of modern societies depending on their continuous and proper functioning, availability and reliability (Almoghathawi et al., 2019). However, such CIs are subject to different types of disruptive events, including random failures, terrorist attacks, and natural disasters that could affect CIs' performance and have direct consequences on communities and daily lives of people (Almoghathawi et al., 2019; Morshedlou, 2018). It would be extremely important for CIs to be resilient against disruptive events.

The concept of resilience can be defined generally as the ability of a system or an organization to react and recover from unanticipated disturbances and events (Hollnagel et al., 2006). Resilience, and in particular CI resilience, has emerged in recent years due to the awareness of governments about the possible risks associated with CIs and the catastrophic impacts of various disruptive events affecting CIs (White House, 2013). For example, when Hurricane Sandy struck those populated regions on the east coast, it caused about $65 billion in damages and economic loss, disabling infrastructure networks including roads, public transit, electric power, and telecommunication. Moreover, about 8.5 million customers were left without power, and the commuting time increased substantially (Hurricane Sandy Rebuilding Task Force, 2013).

Furthermore, resilience of CIs can be seen, conceptually, as a reliability complement. Zio (2009) advanced the view of resilience as complementing reliability by stating '… systems should not only be made reliable, i.e., with acceptably low failure probability, but also resilient, i.e. with the ability to recover from disruptions of the nominal operating conditions'. Based on this view, one could think that two essential attributes of resilience are reliability and restoration, where the design strategies and operation actions employed to enhance system reliability and speed up restoration could be simultaneously carried out with the intention of improving resilience (Yodo & Wang, 2018). Moreover, traditional means of design for reliability, such as component or subsystem redundancy, can aid in improving system resilience (Yodo & Wang, 2016). Furthermore, an extra step for improving resilience of

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CIs could be improving both reliability and restoration time by optimizing spare parts reordering policies. The goals are to minimize the operational costs, speed up recovery actions by increasing spare parts availability, and improve reliability of the system by replacing a deteriorated component or subsystem at the right time.

Many studies have tried to define, quantify and model the resilience of CIs, especially in the area of network optimization. Yodo and Wang (2016) and Hosseini et al. (2016) provided detailed reviews of resilience quantification measures. So far, only a few resilience measures have explicitly included reliability in the related quantification models. One of such measures is the one suggested by Youn et al. (2011) who considered both mitigation and contingency strategies to define a resilience metric as the sum of a passive survival rate focusing on reliability and a proactive survival rate emphasizing restoration following a disruption. Another study on resilience modeling was done by Sharma et al. (2018). They linked resilience with reliability by considering system instantaneous reliability as the basis in planning all recovery activities. Gasser et al. (2019) reviewed relevant literature and provided an in-depth analysis of resilience assessment and quantification particularly for energy systems. It is worth pointing out that the idea of linking CI resilience to the role of spare parts provides an additional way to address CI resilience. However, a novel approach considering the availability and functionality of spare parts in improving the CI resilience has not been explored yet.

Spare parts are stored to replace deteriorating or failed components in a system to keep the system up and running. Ideally, spare parts can be stored indefinitely to meet the future maintenance demand. However, this is often not the case in many real-world situations, especially when the systems and spare parts are used in harsh environments. To achieve the required system reliability and improve system resilience at the minimum cost, maintenance and spare parts inventory control must be well coordinated considering the nature and cost of spare parts as well as component reliability during operation.

### 1.1. Motivation & background

In a recent report by the European Commission's science and knowledge service, the Joint Research Centre (JRC) has addressed challenges in power grid recovery after natural hazards (Karagiannis et al., 2017). In this study, the impact of different natural hazards on equipment is highlighted, and equipment failures associated with specific natural hazards are described. For example, failures of generators and transformers are the expected consequences of earthquakes, and failures of transmission towers are associated with floods. Moreover, it is found that for many components used to operate CIs, the type of damage as a result of disasters is the major determinant of recovery time. If spare parts are unavailable, the resilience of such CIs will be significantly reduced. In addition, the availability of spare parts of capital-intensive components, such as large power transformers, can decrease the repairing time by a few days to even a whole year.

Karagiannis et al. (2017) concluded their study with a list of eight recommendations that can improve the resilience of power grids. One of these recommendations is to 'stockpile spare items to expedite the repair or replacement of key assets and equipment'. It is found that the availability of spares used to replace damaged equipment or parts can reduce recovery time not only theoretically but also practically. For example, when the Kocaeli earthquake happened in Turkey on 17 August 1999, the transmission system at that time was undergoing a major expansion such that new equipment and materials were available which had sped up the recovery time (Karagiannis et al., 2017). Spare parts availability comes at a cost from buying to storing and even to paying insurance premiums. Therefore, when designing an inventory policy for the spare parts of critical components, reducing the costs while improving the availability should be the first priority.

Another important aspect of spare parts is reliability. Some products may deteriorate on the shelf and eventually become nonoperational, and others may encounter failures on demand caused by deterioration. Whenever that happens, the risk of elongating repair times increases, which in turn makes CI recovery time increase as well. A good example is sealed lead acid battery; without being recharged, it may be unrecoverable due to sulfation after approximately 12 months if stored at $20^{o}C$ or be permanently damaged after approximately 4 months if stored at $40^{o}C$. Such batteries are used, for example, as backup energy storage for wind pitch control systems used to protect wind turbines in wind farm substations. Many historical events emphasize the reliability of spare parts and backup solutions in operating different CIs (Karagiannis et al., 2017; Schroeder, 2015).

Nuclear Regulatory Commission (NRC) has reported different occasions when the U.S. nuclear power plants were disconnected from their offsite power grids. The term used to describe such events by NRC is Loss of Offsite Power (LOOP) (Lochbaum, 2015; Schroeder, 2015). During LOOP, backup generators are considered as the main backup system to operate the power plant, and backup batteries are the third source of electricity designed to supply sufficient electricity to a minimal subset of emergency equipment needed to cool the reactor core for a few hours before restoring power from one of the other sources (Lochbaum, 2015). On 20 March 1990, a LOOP at the Vogtle nuclear power plant in Georgia occurred, and one of the two emergency diesel generators was out of service for maintenance. Although the other emergency diesel generator automatically started, a sensor on its cooling system malfunctioned causing the generator to stop (Lochbaum, 2015; Mrowca, 2011). It is reported that the same sensor had malfunctioned 69 times since 1985 but had never been fixed or replaced. Moreover, an emergency diesel generator during a LOOP on 14 June 2004, at the Palo Verde nuclear power plant in Arizona, did not work due to a mis-positioned switch along with the failure of two electrical circuit breakers, which made the restoration time of the plant increase significantly (Lochbaum, 2015). Other events of backup generators caught on fire in Arkansas Nuclear One (Arkansas), Calvert Cliffs (Maryland), North Anna (Virginia) and others were also reported (Lochbaum, 2015; Mrowca, 2011; Schroeder, 2015). Furthermore, many battery problems in nuclear power plants were reported (Lochbaum, 2015; Schroeder, 2015). One interesting event was that when workers at Waterford (Louisiana) found that the capacity of a station battery was 86.25% of the manufacturer's rating during a test on 16 May 2008 which was below the average value of 103.7% recorded during prior tests (Lochbaum, 2015). Workers conducted a follow-up test on 22 May 2008 and found that the capacity of the battery had dropped to 71.67% with no clear causes. Another event happened at San Onofre Unit 2 (California) when workers conducted a weekly surveillance test and identified an apparent low-voltage condition on one of the four banks of station batteries on 25 March 2008. The problem was attributed to loose bolts on the connection of the charging cable (Lochbaum, 2015). Such events and others show that reliability of spare parts and backup components is an important feature to consider when designing spare parts reordering policies, especially for critical components in CIs.

In addition to natural characteristics of spare parts, it is not always possible to store spare parts and backup subsystems in a safe and well-controlled environment. For example, lead–acid batteries, which are widely used as pitch control systems backup energy storage in wind farms, are vulnerable to varying ambient temperature conditions. They are being replaced multiple times during the lifetime of each turbine, which caused European wind farms to trend toward ultracapacitor-based backup energy storage solutions to save battery replacement and downtime costs (Heick, 2018). From an economic viewpoint, to avoid high investments on warehouses, users may store spare parts in places where temperature, humidity and dust are not under control (Van Volkenburg et al., 2014). Essentially, once the part reaches the end of shelf life, it cannot be used for replacement. On the other hand, if the part has deteriorated to an intermediate state when used for replacement, its potential operational life will be shorter compared to a new part. In other words, such on-shelf deterioration shortens the useful life of spare parts.

Another important concern in operations management is that holding many deteriorating spare parts not only incurs high holding costs but also increases the amount of waste, which is unacceptable to CI operations. Economically, this may not be a wise choice for maintaining the needed high availability of spare parts. Indeed, in many cases, users can only afford to carry one unit of critical spare part and reorder one when needed. This policy is considered reasonable and quite common for different CI facilities, stations and sites, where system operations heavily rely on capital-intensive components. To maximize the usage of all parts while reducing the shortage, holding and spoilage costs, it is necessary to develop component replacement and spare parts reordering policies considering spare part deterioration.

It is worth pointing out the difference between spare part obsolescence and spare part on-shelf deterioration. Essentially, spare part obsolescence is attributed to changes in technology, designs, equipment, or processes, which make the spare parts useless and thus must be eventually discarded or recycled.

### 1.2. Related literature

In general, engineering systems are designed to last for several years, and more attention towards sustainability and resilience of engineering systems requires such systems to have a longer lifetime and high reliability (Jia et al., 2017). To achieve a high level of reliability, modeling component deterioration has become an essential tool for predicting the reliability and remaining useful life of such a system. Quite a few studies have addressed different deterioration mechanisms of engineering systems. These studies were mainly focused on the concept of life-cycle analysis (LCA) where the performance of a system, over its entire life-cycle, is studied in terms of a performance measure, such as time-dependent reliability, and of the cost and benefit analysis (Kumar & Gardoni, 2014). Furthermore, most infrastructure systems deteriorate as a result of both sudden extreme events (i.e., shocks) and continuous degradation caused mainly by aging and environmental factors (Sanchez-Silva et al., 2011). Sanchez-Silva et al. (2011) proposed a model for structural deterioration resulting from the combined action of progressive degradation (e.g., corrosion, fatigue) and sudden events (e.g., earthquakes). Kumar and Gardoni (2014) proposed a novel LCA of deteriorating systems based on a renewal theory-based life-cycle analysis (RTLCA) that is capable of estimating the expected values and variances of availability, age, benefit, and costs of operation and failures of the system for a finite time horizon. Jia et al. (2017) proposed a general stochastic model for LCA of deteriorating engineering systems named stochastic life-cycle analysis (SLCA) that includes the mathematical modeling of multiple deterioration and recovery processes along with a probabilistic resilience analysis. However, all such studies and others focus on the deterioration of an operating component, subsystem and system without considering possible deterioration of spare parts which can affect system reliability, maintenance plans and spare parts reordering policies especially when the spare parts are capital-intensive.

In the literature, joint maintenance and spare parts reordering policies have been studied. Armstrong and Atkins (1996) studied a joint optimal parts replacement and reordering policy for a one-component system with only one spare part. The objective is to minimize the total cost considering the costs for replacement, shortage, holding, and breakage. The convexity properties were obtained to show the feasibility of the cost minimization problem. Brezavscek and Hudoklin (2003) considered a similar joint optimization problem where the preventive replacement interval and maximal inventory level are determined to minimize the expected total cost of maintenance per unit time. Rausch and Liao (2010) developed a production and spare part inventory control model where maintenance is initiated based on the current state of a deteriorating component. Huang et al. (2008) proposed a generalized joint optimal policy under block replacement and a periodic review inventory model with a random lead time. Louit et al. (2011) optimized spare parts selection for both repairable and non-repairable slow-moving spare parts. Recently, Wang and Syntetos (2011) presented a joint maintenance and spare part control model considering a failure delay time concept.

In reality, some spare parts, if not used, will remain in the inventory and eventually become obsolete. Indeed, the loss of obsolete inventory is a critical problem especially for capital-intensive units (Cho & Parlar, 1991). Kim et al. (1996) included the costs of obsolescence into the holding cost of spare parts in a multi-echelon system. Cobbaert and Van Oudheusden (1996) studied fast moving spare parts and incorporated the 'sudden death' obsolescence risk into an economic order quantity (EOQ) model. In a recent study, Nguyen et al. (2013) considered the obsolescence of spare parts inventory due to technology evolution. They studied the impact of spare parts inventory on equipment maintenance and replacement decisions under technological change via a Markov decision process formulation. It is worth pointing out that these studies considered the loss of obsolete spare parts as part of inventory costs, but none of them studied the effect of on-shelf deterioration on the operational lifetime of spare parts. In reality, on-shelf deterioration is quite important to users who need to balance the cost of downtime due to the stockout of spare parts, the inventory holding cost and the loss due to on-shelf part deterioration and failure.

On-shelf deterioration and on-equipment deterioration are the two stochastic processes a unit may experience, and the degradation state of a spare part prior to installation determines its remaining operational lifetime. A useful model that describes such cumulative effects is the Cumulative Exposure (CE) model (Nelson, 2009). Finkelstein (1999) introduced two models for components subject to wear-out under different environments considering stress changing points and CE. Moreover, Finkelstein (2007) used the virtual age of a system based on a model reported by the same author (Finkelstein, 1999). In this paper, we will use the CE model to connect on-shelf deterioration and on-equipment deterioration of a spare part. To the best of our knowledge, research about spare parts availability in CI applications is scarce except for one study conducted by Ferdinand et al. (2018) who developed the optimal spare parts inventory control strategy for offshore wind farm substations based on failure mode and effects analysis.

The operation of a system with one operating component and one deteriorating spare part is similar to a warm-standby system (Elsayed, 2012) with an active component and a warm-standby unit. Yun and Cha (2010) considered a two-unit warm-standby system and formulated an optimization problem to determine the optimal time for switching the units by maximizing the expected system lifetime. Chen and Sapra (2013) developed a decision-making model targeting on the optimal long-run profit. Both first-in, first-out and last-in, first-out scenarios were considered for a product with a two-period lifetime. Sung et al. (2013) studied a two-unit system subject to shocks and failure rate interaction. A long-run cost rate criterion was used to determine the optimal replacement policy. However, these optimization models do not consider joint component replacement and reordering policies for deteriorating inventories.

### 1.3. Overview

In this paper, we study a system with an operating (primary) component and a deteriorating spare part. We develop joint component replacement and reordering policies considering the related costs and different types of replacement within each operation cycle. The objective is to minimize the long-run cost rate for the system under each of these policies. It is worth pointing out that the focus of this paper is different from most work reported in the literature. First, unlike a system with a cold- or warm-standby component, the spare part considered in our model can be used to preventively replace the primary component in the system to maintain high reliability of the system. Second, the spare part is deteriorating and may fail before the replacement of primary component or be salvaged without being used till the end of each operation cycle. More importantly, the policies of our interest focus on both replacement and reordering of components, which are more realistic and complex than those policies handling component replacement and reordering separately.

The remainder of this paper is organized as follows. In Section 2, the long-run cost rate for the system is derived based on the related costs and a CE model for switching a spare part from its storage condition to the operation mode. In Section 3, we formulate two optimization models for the failure-switching and preventive-switching strategies, respectively, and provide managerial insights into the resulting optimal policies. Section 4 presents a case study on gearbox replacements for an offshore wind farm operation to illustrate the use of the proposed joint component replacement and reordering policies. Finally, Section 5 concludes the paper.

## 2. Methodology & model development

### 2.1. Problem description

The primary component can be replaced preventively or upon failure. Regardless of the types of replacement, such actions are assumed to be instantaneous (Wang, 2002). For the spare parts reordering process, a batch of two units will be ordered periodically in order to maintain the high reliability levels for both primary and spare components. The lead time $\tau$ is fixed. In some cases, emergency orders can be placed without following this regular reordering schedule. Such cases will be addressed later. A new cycle begins upon the arrival of the two brand new units most recently ordered. One of the units will be used as the primary component, and the other will be stocked as a spare part. A holding cost must be paid for the spare part until it fails or is used to replace the primary component. When the new cycle begins, mandatory preventive replacement (PR) of the primary component in the previous cycle will be performed, and the spare part not used in the previous cycle will be salvaged. Because each replacement action is instantaneous if a spare part is available, the system downtime is defined as the time between a system failure and the order arrival time. Such downtime will incur a production loss.

Without loss of generality, we denote the primary unit as unit $A$ and the spare part as unit $B$ in the beginning of each cycle. Unit $A$ deteriorates as its usage and age increase. Unit $B$ also deteriorates on the shelf but with a slower rate and can be used to replace unit $A$ upon a request if it is not failed on the shelf. Figure 1 shows all possible cases for one operation cycle. Let $t_{i-1}$ be the time when the $(i-1)$th order is received, $t_p$ be the interval for making a regular order, and $t_{i-1} + t_p - \tau$ be the scheduled time to place the next (i.e., the $i$th) order. It is worth pointing out that Figure 1 provides a general picture of how the system works, where letter 'R' can represent either corrective or preventive replacement of unit $A$.

Cases (a)–(e) all have a regular order received at $t_i$ and the same cycle length of $t_p$. Clearly, the length of $t_i - t_{i-1}$ in these cases is the time between two consecutive mandatory PR actions and is the exact time between the arrivals of two consecutive regular orders. In particular, in cases (a)–(c) the system is still up and running at the end of cycle ($t_i$). In cases (d) and (e), the system fails before a regular cycle ends. It is worth pointing that unit $B$ is failed on the shelf in cases (a) and (d), unit $A$ is replaced by unit $B$ in cases (b) and (e), and in case (c) unit $A$ is not replaced and unit $B$ is salvaged at the end of the cycle. In cases (f) and (g) where both the primary and spare units are failed prior to the scheduled order-placing time, an emergency order will be placed upon the system failure resulting in an operation cycle shorter than $t_p$. This situation is depicted in cases (f) and (g). Note that in cases (d) and (e), both units fail before the regular cycle ends, but a regular order has been made so there is no incentive to do emergency order while waiting for the parts already ordered.

For an actual operation, unit $B$ can be switched to its operation mode by replacing unit $A$ either correctively or preventively (see cases (b)(e)(g) in Figure 1). We use the word 'switching' throughout this paper to differentiate such replacement actions from mandatory PR performed in the beginning of each cycle. In particular, the following two switching strategies are considered:

- **Failure-switching strategy** – Spare unit $B$ will be installed only when unit $A$ fails. If it fails before unit $A$ fails, it will be removed from the inventory.

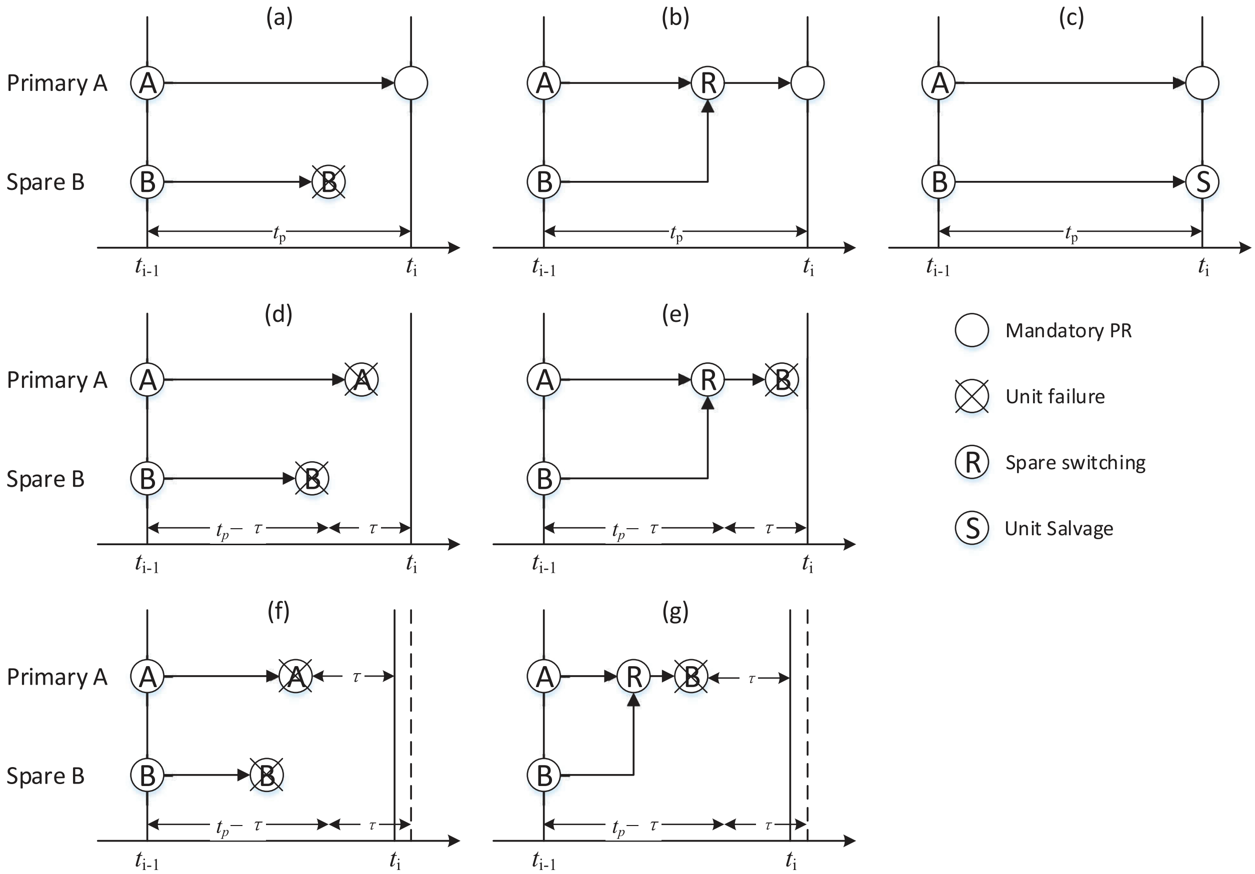


**Figure 1.** Illustration of all scenarios of reorder cycle.

- **Preventive-switching strategy** – Spare unit $B$ will be used to replace unit $A$ at time $t_s$ ($< t_p$) or upon failure of unit $A$, whichever comes first.

It is important to note that all the units considered in this paper will not be used twice. In other words, even though the primary unit being preventively replaced is still functional, it will not be reused. This is reasonable because in many cases, such units after being replaced will be either recycled or sent to a repair shop making the units unavailable to the users. This is also in line with the assumption that an unused spare part, if still operational, will be salvaged at the end of each cycle. We summarize our model assumptions and the limitations as follows.• The replacement time, compared to the lifetime of each unit, can be ignored (i.e., replacement is instantaneous). This assumption is made to ease the formulation of the related optimization problem and mathematical calculations. If not, we expect the downtime cost to increase, and an easy workaround to account for it in our model is to increase the lead time by including the replacement time.

- The lead time $\tau$ is assumed to be fixed. This assumption is quite common in the literature of maintenance decision-making for deteriorating systems (Wang, 2002). If the assumption is not made, we expect earlier reordering of spare parts given that the holding costs are generally significantly lower than the downtime costs, especially in CI applications. Nonetheless, the limitation can be overcome by simulating different instances of the model using varying lead times and choosing, on average, the optimal solution (Sarker & Haque, 2000).

- A new cycle begins upon the arrival of the two brand new units most recently ordered and a holding cost must be paid for the spare part until it fails or is used to replace the primary component.

- The preventive switching is assumed to incur a similar cost as a mandatory PR since they are considered as similar actions. If not, we expect the estimated cost of preventive-switching strategy to increase, and thus, the switching-strategy might be less attractive in terms of the all cost.

### 2.2. Modeling a component switching from storage to operation mode

Regardless of the types of switching, the environment to which a spare part is exposed will be changed, so the lifetime distribution of the part will change accordingly (Cha et al., 2008). In this paper, we model the reliability of spare unit $B$ switching from its storage condition to the operation mode using the CE concept (Nelson, 2009). The basic idea is that the probability that a unit will fail at time $u$ under a certain stress level equals the probability that the unit would fail after accumulating equivalent time $\omega$ under a different stress level.

Let $X_B$ be the random lifetime of unit $B$ under the storage condition, and $F_B(t)$ and $f_B(t)$ be the corresponding cumulative distribution function (CDF) and probability density function (PDF), respectively. On the other hand, let $X_A$ be the random lifetime of an identical component in the operation mode with CDF $F_A(t)$ and PDF $f_A(t)$, respectively. Under the CE model, the relationship between $F_A(t)$ and $F_B(t)$ can be expressed as (Yun & Cha, 2010)

$$F_A(t) = F_B(\rho t), \qquad t \geq 0, \tag{1}$$

where $\rho \geq 1$ is the acceleration factor. A more general model can be expressed as (Yun & Cha, 2010)

$$F_A(t) = F_B(\rho(t)), \qquad t \geq 0, \tag{2}$$

where $\rho(t)$ depends on the operating environment that is often harsher than the storage condition. Similar to (1), it is usually assumed that $\rho(t) \geq t$ and $\rho(0) = 0$. The models given in (1) and (2) imply that unit $B$ on the shelf is more reliable than primary unit $A$ at the same age

$$R_B(t) > R_A(t), \qquad t \geq 0, \tag{3}$$

where $R_j(t) = 1 - F_j(t)$.

Suppose that unit $B$ survives during $[0, u)$ in stock and gets installed at time $u$. We denote its corresponding effective age in the operation mode as $\omega(u)$. According to the CE model (Nelson, 2009), $\omega(u)$ satisfies

$$F_B(u) = F_A(\omega(u)). \tag{4}$$

Applying the inverse operator $F_A^{-1}$ to both sides of (4) yields

$$\omega(u) = F_A^{-1}(F_B(u)) = \rho^{-1}(u), \qquad t \geq 0. \tag{5}$$

To clearly describe the connection between the two failure processes under different conditions, we use the following piece-wise reliability function $R_B(t|u)$ for spare unit $B$ being switched at time $u$

$$R_B(t|u) = \begin{cases} R_B(t), & t < u, \\ R_A(t - u + \omega(u)), & t \geq u, \end{cases} \tag{6}$$

where $R_A(\omega(u)) = R_B(u)$ when $t = u$. To show the effect of switching time on the spare unit's reliability function, Figure 2 provides the reliability functions of unit $B$ for no switching and at three different switching times.

## 2.3. Modeling of long-run cost rate

We formulate the long-run cost rate to evaluate the effectiveness of a component replacement and reordering policy. We assume that preventive switching and mandatory PR incur the same cost $c_{pr}$ as they essentially belong to the same type of action, any failure

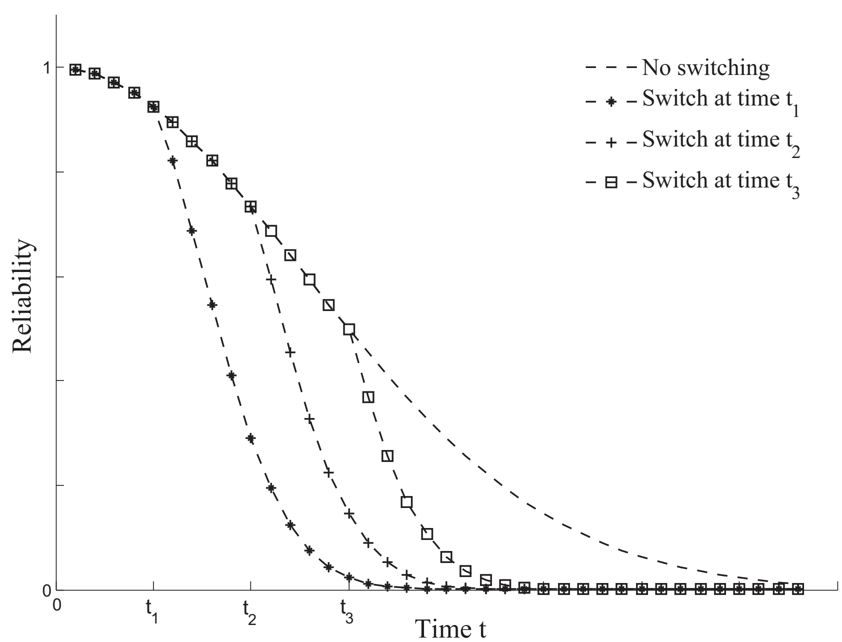


Figure 2. Reliability of spare unit $B$ being switched at different times.

replacement costs $c_f$, and the system's downtime cost is $\pi$ per unit time. Moreover, in each cycle, the holding cost will be charged on the spare part at rate $c_h$ per unit time, and the unused part will be salvaged with a value of $c_s(t_p)$ that could be a decreasing function of $t_p$.

Let $TT^i$ be the system's uptime in the $i$th cycle. Figure 3 shows two typical cases regarding the timing between $TT^i$ and the reordering time. In a cycle with a regular reordering where the system fails after $t_p - \tau$ (i.e., after a regular order is made), the corresponding cycle length $L^i(t_p)$ is $t_p$, and the system downtime is $[t_p - TT^i]^+$. In a cycle where the system fails before $t_p - \tau$, an emergency order is immediately placed at $TT^i$ and is received at $TT^i + \tau$. The corresponding cycle length will be $TT^i + \tau$ that is less than $t_p$, and the system downtime is $\tau$. By considering these cases, the system's downtime can be expressed as $[\min(t_p - TT^i, \tau)]^+$, and the cycle length $L^i(t_p)$ is $\min(TT^i + \tau, t_p)$.

In the following, we will develop the long-run cost rates for the two types of part switching strategies.

### 2.3.1. Failure-switching strategy

Under the failure-switching strategy, unit $B$ becomes the primary unit only if it survives in stock when unit $A$ fails. One extreme case is that both units survive till the end of the cycle (e.g., for $t_p$ in case (c)), so that there is no switching and both units will be discarded. Let $T_A^i$ and $T_B^i$ be the lifetimes of the two units under operation and storage conditions, respectively, without considering any switching in the $i$th cycle. Clearly, the inventory holding time in the $i$th cycle can be expressed as $\min(T_A^i, T_B^i, t_p)$. As a result, the operation cost, i.e., the sum of spare holding cost and system downtime cost, incurred in the $i$th cycle can be expressed as

$$G^i(t_p) = c_h \min(T_A^i, T_B^i, t_p) + \pi[\min(t_p - TT^i, \tau)]^+. \tag{7}$$

To address the possibilities for unit $B$ to be either switched to operation or salvaged at the end of a cycle, we use an indicator variable $I_{T_B^i \geq T_A^i, T_A^i < t_p}$ to indicate whether (= 1) or not (= 0) switching is performed and another variable $I_{T_A^i \geq t_p, T_B^i \geq t_p}$ to indicate whether (= 1) or not (= 0) unit $B$ is salvaged

$$I_{T_B^i \geq T_A^i, T_A^i < t_p} = \begin{cases} 1, & T_B^i \geq T_A^i,\ T_A^i < t_p, \\ 0, & \text{otherwise.} \end{cases} \tag{8}$$

$$I_{T_A^i \geq t_p, T_B^i \geq t_p} = \begin{cases} 1, & T_A^i \geq t_p,\ T_B^i \geq t_p, \\ 0, & \text{otherwise.} \end{cases} \tag{9}$$

In addition, we use $I_{TT^i \geq t_p}$ to indicate either mandatory PR (= 1) (i.e., the system is still working) or failure replacement (= 0) will be performed upon the arrival of ordered parts (i.e., the beginning of next cycle)

$$I^F_{TT^i \geq t_p} = \begin{cases} 1, & TT^i \geq t_p, \\ 0, & \text{otherwise.} \end{cases} \tag{10}$$

Considering the replacement cost and salvage value of unit $B$, the total maintenance cost can be expressed as $M^i(t_p) = c_f(I_{T_B^i \geq T_A^i, T_A^i < t_p} + 1 - I^F_{TT^i \geq t_p}) + c_{pr} I^F_{TT^i \geq t_p} - c_s(t_p) I_{T_A^i \geq t_p, T_B^i \geq t_p}$.

Combining all the operation and maintenance costs, the long-run cost rate for $n$ cycles is

$$c(n, t_p) = \frac{\sum_{i=1}^{n} \{G^i(t_p) + M^i(t_p)\}}{\sum_{i=1}^{n} L^i(t_p)}, \tag{11}$$

and the long-run cost rate for an infinite time horizon can be expressed as (Ross, 2014)

$$c(\infty, t_p) = \frac{\mathbb{E}\{G(t_p) + M(t_p)\}}{\mathbb{E}[L(t_p)]}, \tag{12}$$

where $\mathbb{E}\{G(t_p) + M(t_p)\}$ is the expected cost in a cycle and $\mathbb{E}[L(t_p)]$ is the expected cycle length. Note that the

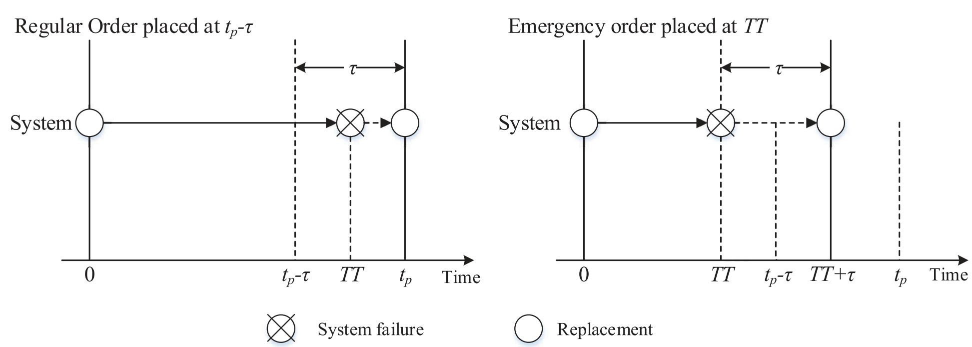


**Figure 3.** Illustration of two scenarios in ordering policy.

infinite time horizon assumption holds properly for CIs that are expected to operate indefinitely. This is usually the case in practice. Nonetheless, if the time horizon is finite, the model will operate mainly based on (11) with finite cycles, and the rest of related equations will be slightly modified by mainly indexing the cost and time within each cycle.

### 2.3.2. Preventive-switching strategy

Under the preventive-switching strategy, a preventive switching action is performed if both primary unit $A$ and spare unit $B$ survive at the preventive-switching time $t_s$ (assumed to be $< t_p - \tau$); if unit $B$ is not available for a preventive switching at $t_s$, unit $A$ will be kept on the system until failure or the next mandatory PR action; if unit $A$ fails before $t_s$ while unit $B$ survives, failure replacement is performed immediately; if unit $B$ is not available for failure replacement before time $t_s$, the system will stop operation until next mandatory PR action. Clearly, unit $B$ is removed from the inventory either because of on-shelf failure or failure/preventive replacement, whichever occurs first. Note that after unit $A$ is preventively replaced by unit $B$ it is still possible for unit $B$ to fail prior to the arrival of new order. In this paper, we assume that unit $A$ will not be reused due to the reasons mentioned previously.

Similar to the failure-switching strategy, the inventory holding time within a cycle is $\min(T_A^i, T_B^i, t_s)$, the system downtime is $[\min(t_p - TT^i, \tau)]^+$, and the cycle length is $L^i(t_s, t_p) = \min(TT^i + \tau, t_p)$. As a result, the operation cost $G^i(t_s, t_p)$ can be expressed as

$$G^i(t_s, t_p) = c_h \min(T_A^i, T_B^i, t_s) + \pi[\min(t_p - TT^i, \tau)]^+. \tag{13}$$

To describe possible preventive switching in each cycle, we define an indicator variable $I_{T_A^i \geq t_s, T_B^i \geq t_s}$

$$I_{T_A^i \geq t_s, T_B^i \geq t_s} = \begin{cases} 1, & T_A^i \geq t_s, T_B^i \geq t_s, \\ 0, & \text{otherwise.} \end{cases} \tag{14}$$

Another indicator variable $I_{T_A^i < t_s, T_B^i \geq T_A}$ is defined for possible corrective replacement before $t_s$

$$I_{T_A^i < t_s, T_B^i \geq T_A} = \begin{cases} 1, & T_A^i < t_s, T_B^i \geq T_A, \\ 0, & \text{otherwise.} \end{cases} \tag{15}$$

Similar to $I^F_{TT^i \geq t_p}$, we use $I^P_{TT^i \geq t_p}$ to indicate whether or not a mandatory PR action is performed at the end of the cycle

$$I^P_{TT^i \geq t_p} = \begin{cases} 1, & TT^i \geq t_p, \\ 0, & \text{otherwise.} \end{cases} \tag{16}$$

Then, the maintenance cost in a cycle can be expressed as $M^i(t_s, t_p) = c_{pr}(I_{T_A^i \geq t_s, T_B^i \geq t_s} + I^P_{TT^i \geq t_p}) + c_f(I_{T_A^i < t_s, T_B^i \geq T_A} + 1 - I^P_{TT^i \geq t_p})$.

Considering the operation and maintenance costs, the long-run cost rate under the preventive-switching strategy is given by,

$$c(\infty, t_s, t_p) = \frac{\mathbb{E}\{G(t_s, t_p) + M(t_s, t_p)\}}{\mathbb{E}[L(t_s, t_p)]}. \tag{17}$$

Unlike the failure-switching strategy, the expected cycle cost $\mathbb{E}\{G(t_p) + M(t_p)\}$ and the expected cycle length $\mathbb{E}[L(t_p)]$ in the preventive-switching strategy depend on both $t_p$ and $t_s$.

## 3. Optimal policies for different switching strategies

### 3.1. Failure-switching strategy

For the failure-switching strategy, we formulate an optimization problem with the objective of minimizing the long-run cost rate

$$t_p^* = \arg \min_{t_p \in (\tau, \infty)} c(\infty, t_p) = \frac{\mathbb{E}\{G(t_p) + M(t_p)\}}{\mathbb{E}[L(t_p)]}, \tag{18}$$

where $t_p$ is the only decision variable.

We consider the following two cases in a cycle in which the system survives at time $t$ (Elsayed, 2012):

- Case 1: Unit $A$ is still working at time $t$.
- Case 2: Unit $A$ fails before $t$ and unit $B$ survives the remaining time.

Let $F_1(t), f_1(t)$ and $R_1(t)$ be the CDF, PDF and reliability function of a unit in the operating environment, and $F_2(t), f_2(t)$ and $R_2(t)$ be the corresponding functions of a unit in the storage condition. Considering the two exclusive cases, system reliability $R_s(t)$ (i.e., probability that there is at least one working unit) can be expressed as (Elsayed, 2012)

$$R_s(t) = R_1(t) + \int_0^t R_2(u) f_1(u) \frac{R_1(t - u + \omega(u))}{R_1(\omega(u))} du. \tag{19}$$

Based on (4), we can rewrite $R_s(t)$ as

$$R_s(t) = R_1(t) + \int_0^t f_1(u) R_1(t - u + \omega(u)) du. \tag{20}$$

With the system reliability function and the lifetime distributions of units under the two different environments, the following results can be obtained for the failure-switching strategy (see Appendices for the proof).

**Remark 1.** The expected cycle length is $\mathbb{E}[L(t_p)] = t_p R_s(t_p - \tau) + \int_0^{t_p-\tau} (\tau + t) f_s(t) dt$; the expected system downtime is $\mathbb{E}[(\min(t_p - TT, \tau))^+] = \int_{t_p-\tau}^{t_p} (t_p - t) f_s(t) dt + \tau F_s(t_p - \tau)$; and the expected spare holding time is $\mathbb{E}[\min(T_A, T_B, t_p)] = \int_0^{t_p} u(f_1(u)R_2(u) + f_2(u)R_1(u)) du + t_p R_1(t_p) R_2(t_p)$.

Moreover, it is easy to see that the probability to perform failure switching is $\mathbb{E}[I_{T_B \geq T_A, T_A < t_p}] = \int_0^{t_p} f_1(u) R_2(u) du$, the probability of salvaging unit $B$ is $\mathbb{E}[I_{T_A \geq t_p, T_B \geq t_p}] = R_1(t_p)R_2(t_p)$, the probability to perform mandatory PR is $\mathbb{E}[I_{TT \geq t_p}] = R_s(t_p)$, and the probability to perform system-level failure replacement is $F_s(t_p)$. Then, the expected cycle cost can be expressed as

$$\mathbb{E}\{G(t_p) + M(t_p)\} =$$

$$\left\{ \begin{array}{l} \pi \times \left[ t_p F_s(t_p) - (t_p - \tau) F_s(t_p - \tau) - \int_{t_p-\tau}^{t_p} t f_s(t) dt \right] \\ + c_h \times \left[ t_p R_1(t_p) R_2(t_p) + \int_0^{t_p} u(f_1(u)R_2(u) + f_2(u)R_1(u)) du \right] \\ + c_{pr} R_s(t_p) + c_f \left( \int_0^{t_p} f_1(u) R_2(u) du + F_s(t_p) \right) - c_s R_1(t_p) R_2(t_p) \end{array} \right\} \tag{21}$$

Based on (12), the long-run cost rate $c(\infty, t_p)$ can be obtained by dividing the above expression by the expected cycle length $\mathbb{E}[L(t_p)]$. The following theorem gives a property of the long-run cost rate when the failure-switching strategy is adopted (see Appendices for the proof).

**Theorem 1.** For the failure-switching strategy, as $t_p$ increases, the long-run cost rate will become stable and approach a constant, and the expected cycle length will approach $\tau + T_M$, where $T_M = \int_0^{\infty} R_s(t) dt$ is the system's mean-time-to-failure.

### 3.2. Preventive-switching strategy

In this section, we study the case where preventive switching is performed. In this case, the optimal long-run cost per unit time depends on two decision variables $t_s$ and $t_p$ as

$$(t_s^*, t_p^*) = \arg \min_{\substack{t_p \geq \tau \\ 0 < t_s \leq t_p - \tau}} \frac{\mathbb{E}\{G(t_s, t_p) + M(t_s, t_p)\}}{\mathbb{E}[L(t_s, t_p)]}. \tag{22}$$

We begin by formulating the system reliability at time $t$ for this case. According to the switching strategy, if the system survives at time $t$ $(< t_s)$, only when unit $A$ fails unit $B$ will be installed. The system reliability function is given by

$$R_s(t) = R_1(t) + \int_0^t f_1(u) R_1(t - u + \omega(u)) du, \quad t < t_s. \tag{23}$$

If the system survives at time $t$ $(> t_s)$, we have the following three exclusive cases during one cycle (see Figure 4):

- Case 1: Unit $B$ fails before time $t_s$, and unit $A$ is still working at time $t$.
- Case 2: Unit $A$ works up to time $t_s$, and unit $B$ replaces unit $A$ preventively and survives the remaining time.
- Case 3: Unit $B$ gets installed upon failure of unit $A$ before $t_s$ and survives the remaining time.

Combining the above three exclusive cases, we can obtain the system reliability $R_s(t)$ when $t > t_s$ as

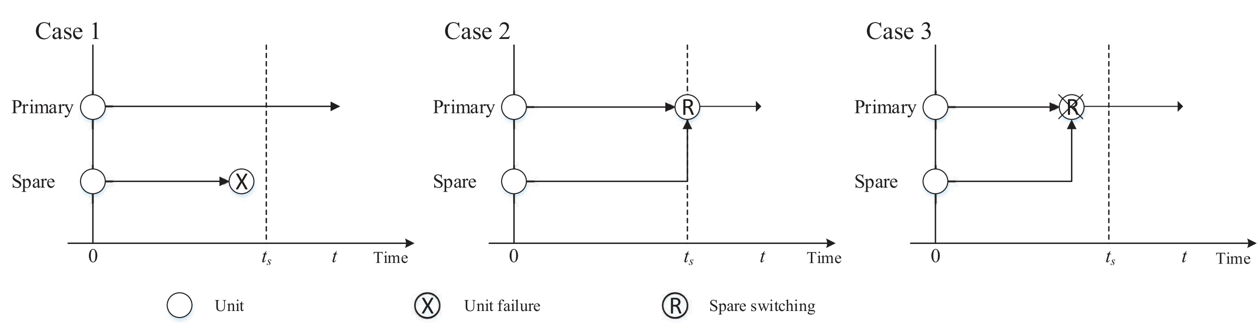


**Figure 4.** Three exclusive cases when system survives at $t$ $(> t_s)$.

$$R_s(t) = R_1(t)F_2(t_s) + \int_0^{t_s} f_1(u)R_1(t-u+\omega(u))\,du + R_1(t_s)R_1(t-t_s+\omega(t_s)), \qquad t > t_s. \tag{24}$$

Note that when $t = t_s$, (23) and (24) are the same. Therefore, the system reliability has a piece-wise form

$$R_s(t) = \begin{cases} R_1(t) + \int_0^{t} f_1(u)R_1(t-u+\omega(u))\,du, & t \le t_s, \\ R_1(t)F_2(t_s) + \int_0^{t_s} f_1(u)R_1(t-u+\omega(u))\,du \\ \quad + R_1(t_s)R_1(t-t_s+\omega(t_s)), & t > t_s. \end{cases} \tag{25}$$

From (25), it is not difficult to show that for two different preventive-switching times $t_s' > t_s$ $R_s(t|t_s') > R_s(t|t_s)$ when $t \ge t_s$.

Considering all the cases, the following results can be obtained for the preventive-switching strategy (see Appendices for the proof).

**Remark 2**. The expected cycle length is $\mathbb{E}[L(t_s, t_p)] = t_p R_s(t_p - \tau) + \int_0^{t_p-\tau} (\tau + t) f_s(t) dt$; the expected system downtime is $\mathbb{E}[(\min(t_p - TT, \tau))^+] = \int_{t_p-\tau}^{t_p} (t_p - t) f_s(t) dt + \tau F_s(t_p - \tau)$; and the expected spare holding time is $\mathbb{E}[\min(T_A, T_B, t_s)] = t_s R_1(t_p) R_2(t_s) + \int_0^{t_s} u(f_1(u) R_2(u) + f_2(u) R_1(u)) du$.

Moreover, it is easy to see that the probability to perform a preventive switching at $t_s$ in one cycle is $\mathbb{E}[I_{T_A^i \ge t_s, T_B^i \ge t_s}] = R_1(t_s)R_2(t_s)$, the probability to perform a failure replacement is $\mathbb{E}[I_{T_A^i < t_s, T_B^i \ge T_A}] = \int_0^{t_s} f_1(t) R_2(t) dt$, the probability to perform mandatory PR at $t_p$ is $\mathbb{E}[I_{TT^i \ge t_p}] = R_s(t_p)$, and the probability to perform a system-level failure replacement at $t_p$ is $F_s(t_p)$. Therefore, the expected cycle cost is given by

$$\mathbb{E}[G(t_s, t_p) + M(t_s, t_p)] = \left\{ \begin{array}{l} \pi \times \left[ t_p F_s(t_p) - (t_p - \tau) F_s(t_p - \tau) - \int_{t_p-\tau}^{t_p} t f_s(t)\, dt \right] \\ + c_h \times \left[ t_s R_1(t_s) R_2(t_s) + \int_0^{t_s} u(f_1(u) R_2(u) + f_2(u) R_1(u))\, du \right] \\ + c_{pr}\left[R_1(t_s)R_2(t_s) + R_s(t_p)\right] + c_f \left[ \int_0^{t_s} f_1(t) R_2(t)\, dt + F_s(t_p) \right] \end{array} \right\} \tag{26}$$

For the long-run cost rate $\mathbb{E}[G(t_s, t_p) + M(t_s, t_p)] / \mathbb{E}[L(t_s, t_p)]$, the following theorem addresses its property (see Appendices for the proof).

**Theorem 2**. For the preventive-switching strategy with a fixed preventive-switching time $t_s$, as $t_p$ increases, the long-run cost rate will become stable and approach a constant, and the expected cycle length will approach $\tau + T_M$ regardless of $t_s$, where $T_M = \int_0^{\infty} R_s(t) dt$ is the system's mean-time-to-failure.

## 4. Case study

The use of offshore wind energy is expected to play a significant role in future energy supply (Tracht et al., 2013). Operations and maintenance management for such a wind power generation system aims at reducing the overall maintenance cost, improving the availability and resilience of the system (Tian et al., 2011). As a result, ensuring a reliable and cost-effective supply of spare parts is of great importance. In this context, planning and scheduling extensive maintenance activities, such as changing some critical components (e.g., gearbox and main bearing) can be considered as one of the most difficult tasks in maintaining offshore wind turbines (Tracht et al., 2013). Gearboxes are expensive and have long lead times. In addition, they cause high inventory costs which makes reducing the number of spare parts without significantly affecting system availability a strategic goal. Furthermore, different studies show that the failure rates of wind turbine gearboxes, especially in an offshore application, are underestimated (Deb et al., 2016). Indeed, the reported failure rates for onshore and offshore applications vary between .05 and 0.5 failures per year (Crabtree et al., 2015; Deb et al., 2016). In fact, a recent report of NoordzeeWind regarding Egmond aan Zee, the first Dutch offshore wind farm, showed that gearbox failures have caused the business venture to initiate a replacement program for gearboxes in their first 2 years (NoordzeeWind, 2010). This incident supports the use of lifetime distributions with increased failure rates, such as Weibull with a shape parameter greater than 1 (Tian et al., 2011).

In this section, we illustrate how a gearbox's long-run cost varies under different mandatory PR and switching policies. The lifetimes of a gearbox in the operation mode and a unit on the shelf follow different Weibull distributions. In particular, a unit in the operation mode has a CDF $F_1(t) = 1 - e^{-(t/\eta_1)^{\beta_1}}$, and an identical unit on the shelf has a CDF $F_2(t) = 1 - e^{-(t/\eta_2)^{\beta_2}}$ and a lower failure rate. The scale parameter and shape parameter of the unit under the operating environment are $\eta_1 = 2$ years (NoordzeeWind, 2010; Tian et al., 2011) and $\beta_1 = 3$ (Guo et al., 2009; Tian et al., 2011), respectively. The scale parameter $\eta_2$ of the on-shelf unit varies between 3 and 6 years in different scenarios, and the shape parameter is $\beta_2 = 2$. Note that more advanced stochastic process models for deterioration, such as the ones in (Jia et al., 2017; Kumar & Gardoni, 2014), can be plugged into $F_B(u)$ with a few modifications. Substituting the CDF's into (5), the effective age $\omega(t)$ of a spare unit can be obtained as $\omega(t) = \eta_1 (t/\eta_2)^{\beta_2/\beta_1}$. The values of replacement cost, downtime cost, salvage and others are adopted from (Carroll et al., 2016; Tian et al., 2011; Tracht et al., 2013). Table 1 gives the summary of these parameters used in this case study. The algorithm used to solve the optimization problems is the golden section search and parabolic interpolation algorithm.

### 4.1. Case for the failure-switching strategy

In this case, we consider the failure-switching strategy. Figure 5 shows the system reliability for different values of scale parameter $\eta_2$ $(= 3, 4, 5, 6)$ reflecting different stocking conditions. It is intuitively true that a lower on-shelf deterioration rate results in higher system reliability.

**Table 1.** Operation and maintenance-related parameters.

| $\tau$ | $\pi$ | $c_h$ | $c_{pr}$ | $c_f$ | $c_s$ |
|---|---|---|---|---|---|
| 6 months | \$180k | \$10k/year | \$150k | \$250k | \$50k |

Table 2 shows the minimal long-run cost rates, the expected cycle lengths as well as the expected lengths of system downtime and spare holding time for different values of $\eta_2$. One can see that as $\eta_2$ (i.e., the on-shelf characteristic life) increases reflecting the improvement of storage condition, the optimal mandatory PR interval, probability of performing failure replacement, expected system downtime and expected spare holding time will be increasing. Notice how the downtime decreases from 0.0239 years ( $\approx$ 9 days) when $\eta_2 = 3$ to 0.0104 years ( $\approx$ 3.8 days) when $\eta_2 = 6$. Moreover, the optimal long-run cost rate, cycle length and probabilities of performing mandatory PR and salvaging spare will be decreasing. This is favorable in terms of the CI's economic performance.

Moreover, Figure 6 shows the long-run cost rates as $t_p$ increases for different values of $\eta_2$. One can see that in each case, as $t_p$ increases, the long-run cost rate decreases first, reaches the minimum value, increases again, and eventually becomes stable (see Theorem 1). The optimum value tends to be lower as $\eta_2$ increases. This indicates a direction for the operator of CI to reduce the operation and maintenance cost by storing spare parts in a better stocking condition.

### 4.2. Case for the preventive-switching strategy

In this section, we consider the preventive-switching strategy where switching time $t_s$ and mandatory PR time $t_p$ are the decision variables. Figure 7 shows that a higher value of $t_s$ will result in higher system reliability.

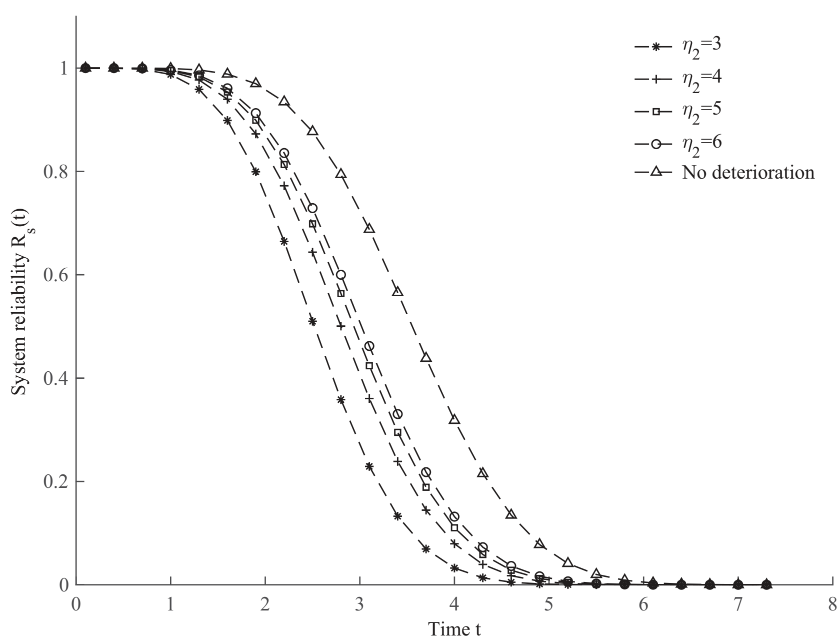


**Figure 5.** System reliability functions for different values of $\eta_2$.

**Table 2.** Results for different values of scale parameter $\eta_2$.

| $\eta_2 = 3$ | $t_p^* = 1.1867$ years | $\mathbb{E}[L(t_p^*)] = 1.1864$ years |
|---|---|---|
| | $c^*(\infty, t_p^*) = \$146.5541$k | $\mathbb{E}[I_{T_A^i \geq t_p, T_B^i \geq t_p}] = 0.6939$ |
| | $\mathbb{E}[\min(T_A, T_B, t_p^*)] = 1.0748$ years | $\mathbb{E}[I_{T_B^i \geq T_A^i, T_A^i < t_p}] = 0.1722$ |
| | $\mathbb{E}[\min\,(t_p^* - TT, \tau)^+] = 0.0239$ years | $\mathbb{E}[I^F_{TT^i \geq t_p}] = 0.9646$ |
| $\eta_2 = 4$ | $t_p^* = 1.1848$ years | $\mathbb{E}[L(t_p^*)] = 1.1847$ years |
| | $c^*(\infty, t_p^*) = \$144.8741$k | $\mathbb{E}[I_{T_A^i \geq t_p, T_B^i \geq t_p}] = 0.7441$ |
| | $\mathbb{E}[\min(T_A, T_B, t_p^*)] = 1.0962$ years | $\mathbb{E}[I_{T_B^i \geq T_A^i, T_A^i < t_p}] = 0.1784$ |
| | $\mathbb{E}[\min\,(t_p^* - TT, \tau)^+] = 0.0164$ years | $\mathbb{E}[I^F_{TT^i \geq t_p}] = 0.9756$ |
| $\eta_2 = 5$ | $t_p^* = 1.1863$ years | $\mathbb{E}[L(t_p^*)] = 1.1862$ years |
| | $c^*(\infty, t_p^*) = \$143.9872$k | $\mathbb{E}[I_{T_A^i \geq t_p, T_B^i \geq t_p}] = 0.7672$ |
| | $\mathbb{E}[\min(T_A, T_B, t_p^*)] = 1.1082$ years | $\mathbb{E}[I_{T_B^i \geq T_A^i, T_A^i < t_p}] = 0.1823$ |
| | $\mathbb{E}[\min\,(t_p^* - TT, \tau)^+] = 0.0126$ years | $\mathbb{E}[I^F_{TT^i \geq t_p}] = 0.9813$ |
| $\eta_2 = 6$ | $t_p^* = 1.1882$ years | $\mathbb{E}[L(t_p^*)] = 1.1881$ years |
| | $c^*(\infty, t_p^*) = \$143.4573$k | $\mathbb{E}[I_{T_A^i \geq t_p, T_B^i \geq t_p}] = 0.7796$ |
| | $\mathbb{E}[\min(T_A, T_B, t_p^*)] = 1.1156$ years | $\mathbb{E}[I_{T_B^i \geq T_A^i, T_A^i < t_p}] = 0.1849$ |
| | $\mathbb{E}[\min\,(t_p^* - TT, \tau)^+] = 0.0104$ years | $\mathbb{E}[I^F_{TT^i \geq t_p}] = 0.9847$ |

Table 3 presents the optimum solutions $(t_s^*, t_p^*)$, the corresponding minimal long-run cost rates, expected cycle length, and other related quantities for different values of $\eta_2$. Unlike the failure-switching strategy, as $\eta_2$ increases, the expected system downtime and spare holding time are decreasing while the expected cycle length is increasing. Notice that the downtime for the preventive-switching strategy when $\eta_2 = 6$ is about 0.2016 years ( $\approx$ 73.5 days) as opposed to 0.0104 years ( $\approx$ 3.8 days) under the failure-switching strategy. This significant change clearly reduces the availability of the system under the preventive-switching strategy. In addition, the system reliability under the failure-switching strategy is also, overall, higher indicating that this policy results in a more reliable and more resilient system. In addition, no spare parts will be salvaged due to preventive switching scheduled in each cycle (i.e., at $t_s$). Figure 8 provides four surface graphs for the long-run cost rate where the infeasible regions violate the constraint of $t_s < t_p - \tau$. One can see that for a fixed $t_s$, the long-run cost value levels off as $t_p$ increases (see Theorem 2).

It is worth pointing out that the long-run cost rate of the preventive-switching strategy is higher than the corresponding value of the failure-switching alternative in the previous example. This is counterintuitive in the sense that preventive maintenance actions may not always reduce the operating and maintenance costs. This is mainly caused by on-shelf deterioration and salvage of spare parts. Clearly, for the operation of CIs, such as wind farms, the desired economic performance and system resilience need to be balanced.

## 5. Conclusions & future work

In this paper, we studied joint component replacement and spare parts reordering policies for a one-unit system carrying a deteriorating spare part. Because of on-shelf deterioration, the spare part is not as good as new when it is used to replace the unit on the system. Moreover, the spare part may fail before installation on the system. Optimization models were developed for failure-switching and preventive-switching strategies. In the failure-switching case, the mandatory PR time is determined to minimize the long-run cost rate. For the preventive-switching case, both the mandatory PR time and spare switching time are determined considering the same

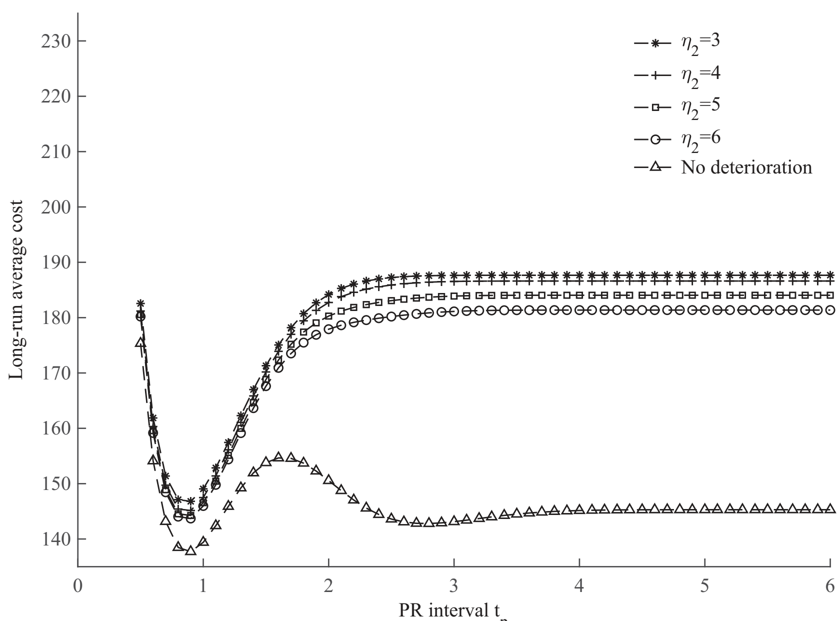


**Figure 6.** Long-run cost rates for different values of $\eta_2$.

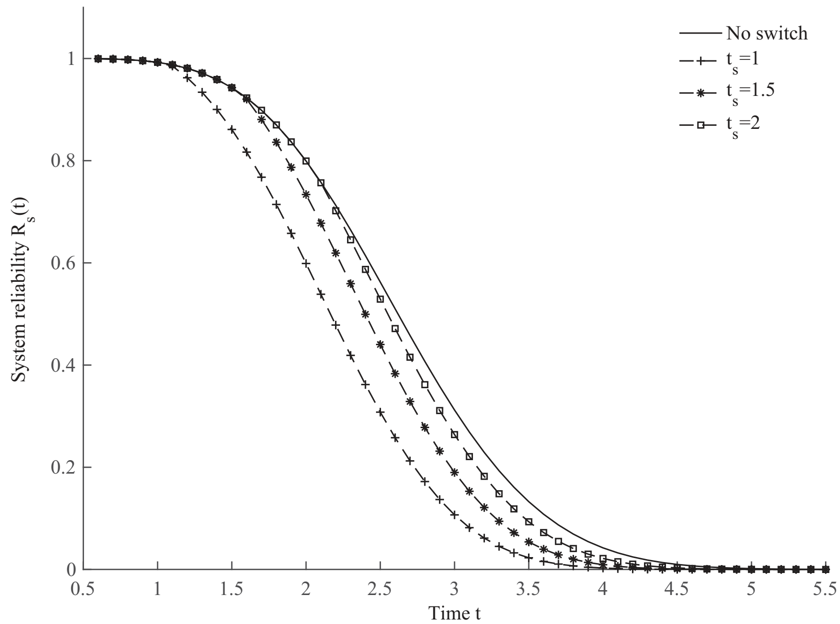


**Figure 7.** System reliability functions for different preventive-switching times (when $\eta_2 = 4$).

**Table 3.** Results for different values of scale parameter $\eta_2$.

| $\eta_2 = 3$ | $t_s^* = 2.6946$, $t_p^* = 3.1946$ years | $\mathbb{E}[L(t_s^*, t_p^*)] = 2.4173$ years |
|---|---|---|
| | $c(\infty, t_s^*, t_p^*) = \$193.2285$ k | $\mathbb{E}[I_{T_A^i \ge t_s, T_B^i \ge t_s}] = 0.0387$ |
| | $\mathbb{E}[\min(T_A, T_B, t_s^*)] = 1.5293$ years | $\mathbb{E}[I_{T_B^i \ge T_A^i, T_A^i < t_s}] = 0.6603$ |
| | $\mathbb{E}[\min(t_p^* - TT, \tau)^+] = 0.2253$ years | $\mathbb{E}[I^P_{TT^i \ge t_p}] = 0.0693$ |
| $\eta_2 = 4$ | $t_s^* = 2.8354$, $t_p^* = 3.3354$ years | $\mathbb{E}[L(t_s^*, t_p^*)] = 2.5387$ years |
| | $c(\infty, t_s^*, t_p^*) = \$193.2788$ k | $\mathbb{E}[I_{T_A^i \ge t_s, T_B^i \ge t_s}] = 0.0350$ |
| | $\mathbb{E}[\min(T_A, T_B, t_s^*)] = 1.6273$ years | $\mathbb{E}[I_{T_B^i \ge T_A^i, T_A^i < t_s}] = 0.7753$ |
| | $\mathbb{E}[\min(t_p^* - TT, \tau)^+] = 0.2220$ years | $\mathbb{E}[I^P_{TT^i \ge t_p}] = 0.0818$ |
| $\eta_2 = 5$ | $t_s^* = 2.8454$, $t_p^* = 3.3454$ years | $\mathbb{E}[L(t_s^*, t_p^*)] = 2.6165$ years |
| | $c(\infty, t_s^*, t_p^*) = \$193.2853$ k | $\mathbb{E}[I_{T_A^i \ge t_s, T_B^i \ge t_s}] = 0.0406$ |
| | $\mathbb{E}[\min(T_A, T_B, t_s^*)] = 1.6758$ years | $\mathbb{E}[I_{T_B^i \ge T_A^i, T_A^i < t_s}] = 0.8315$ |
| | $\mathbb{E}[\min(t_p^* - TT, \tau)^+] = 0.2132$ years | $\mathbb{E}[I^P_{TT^i \ge t_p}] = 0.1118$ |
| $\eta_2 = 6$ | $t_s^* = 2.5043$, $t_p^* = 3.2648$ years | $\mathbb{E}[L(t_s^*, t_p^*)] = 2.6328$ years |
| | $c(\infty, t_s^*, t_p^*) = \$193.8032$ k | $\mathbb{E}[I_{T_A^i \ge t_s, T_B^i \ge t_s}] = 0.1180$ |
| | $\mathbb{E}[\min(T_A, T_B, t_s^*)] = 1.6772$ years | $\mathbb{E}[I_{T_B^i \ge T_A^i, T_A^i < t_s}] = 0.7946$ |
| | $\mathbb{E}[\min(t_p^* - TT, \tau)^+] = 0.2016$ years | $\mathbb{E}[I^P_{TT^i \ge t_p}] = 0.1505$ |

optimization criterion. The numerical examples demonstrate the negative effect of bad storage condition on the operation and maintenance cost. For the cases with Weibull shelf life distributions with a fixed shape parameter, lower characteristic life, i.e., $\eta_2$, as a result of worse storage condition results in a higher long-run cost rate. Moreover, compared to a failure-replacement strategy, preventively replacing a deteriorating component using a spare part subject to on-shelf deteriorating may not reduce the operating and maintenance cost. This is quite important when practitioners are facing such joint component replacement and reordering problems.

The proposed policies in this paper focus on a one-component deteriorating system carrying one deteriorating spare part. The numerical results show that optimizing component replacement and spare parts reordering policies is a major factor in improving the resilience of CIs. By improving the reliability of the equipment and the availability of spare parts, it is possible to shorten the recovery time. Moreover, it is necessary to consider spare part deterioration (Ruiz et al., 2020) so that failures that elongate the recovery time can be handled more effectively.

Such policies could potentially scale up to a multi-component system carrying several spare parts. Indeed, the expected savings for multiple multi-component systems would be more significant. However, to model such a system and determine the optimal spare-switching and

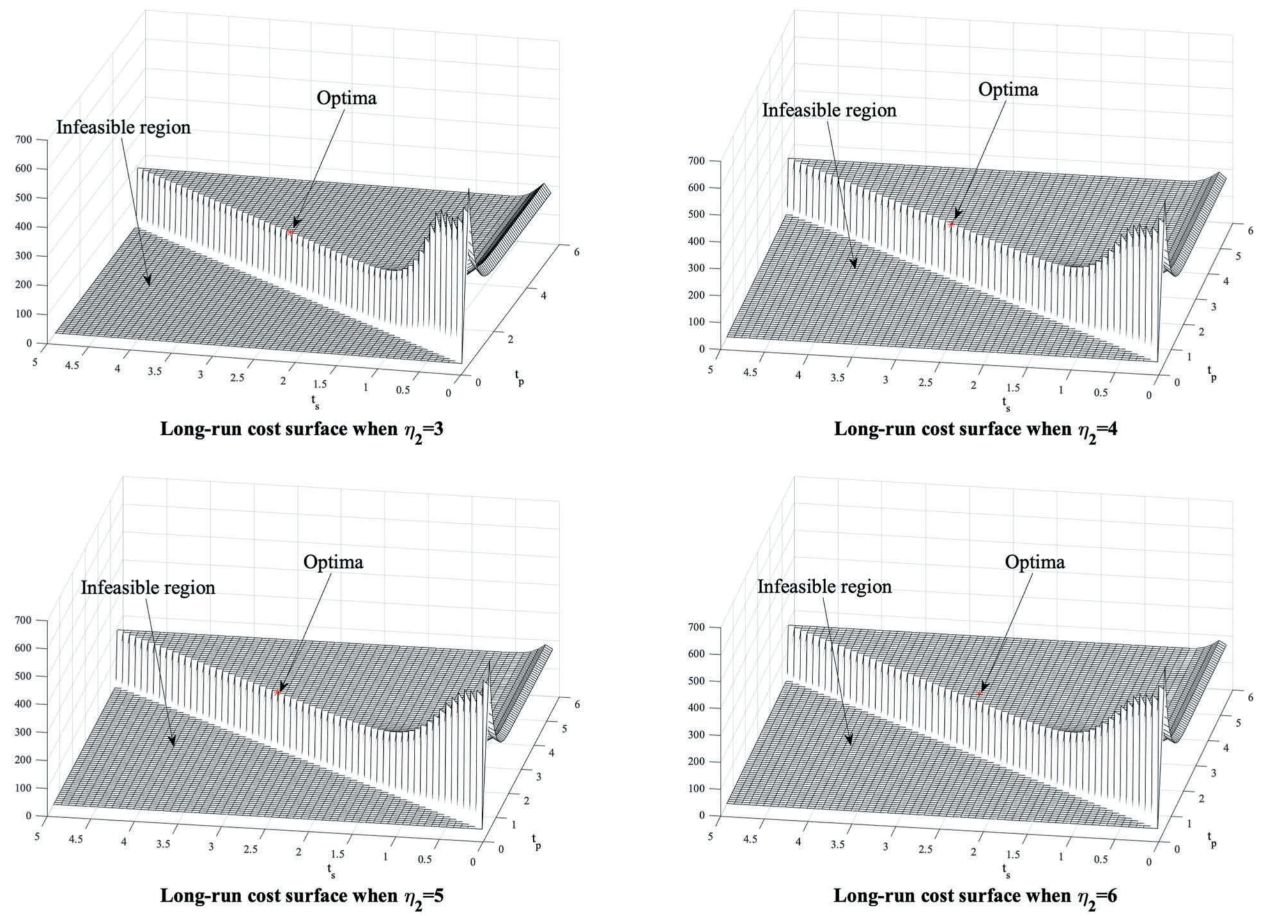


**Figure 8.** Surface of the long-run cost rate for different values of $\eta_2$.

reordering policy, a more complex stochastic model must be developed to take into account different ages of spare parts before being installed. In addition, considerations of the potential economic dependency among maintenance actions on different components add another layer of complexity in model development for improving the economic performance and resilience of the entire system. Other possible extensions to this research could be to model different types of spare parts along with their reordering policies for a certain facility and to provide a joint policy for multiple facilities sharing the same set of spare parts. To this end, different stochastic process models and joint optimization models need to be developed to describe the reliability of multiple deteriorating units with different deterioration rates and quantify the economic performance and resilience of each facility or multiple facilities as a whole.


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## Nomenclature

| | |
|---|---|
| $F_j(t)$ | CDF of the lifetime of a unit under operation (j = 1) or storage condition (j = 2) |
| $f_j(t)$ | PDF of the lifetime of a unit under operation (j = 1) or storage condition (j = 2) |
| $R_j(t)$ | reliability function of a unit under operation (j = 1) or storage condition (j = 2) |
| $F_s(t)$ | CDF of the lifetime of the system |
| $f_s(t)$ | PDF of the lifetime of the system |
| $R_s(t)$ | system reliability function |
| $\omega(t)$ | effective age of the spare part when switching ti end of the ith cycle |
| $t_p$ | length of a regular reorder cycle, i.e., mandatory PR interval |
| $t_s$ | preventive-switching time |
| $\tau$ | order lead time |
| $c_h$ | holding cost for a spare part per unit time |
| $\pi$ | system downtime cost per unit time |
| $c_s$ | salvage value of an unused spare part |
| $c_{pr}$ | cost for mandatory PR or preventive switching |
| $c_f$ | failure replacement cost |
| $T_A$ | potential lifetime of primary unit A |
| $T_B$ | potential lifetime of spare unit B |
| $TT$ | uptime of the system |
| $x^+$ | max(X; 0) |
| $G(t_p)$ | random system operation cost in one cycle given tp |
| $M(t_p)$ | random system maintenance cost in one cycle given tp |
| $L(t_p)$ | random system cycle length given tp |

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## Appendix A. Derivations and proofs

***A.1*. Derivation of Remark 1.**
From (20) for the system reliability, the PDF of the latent system lifetime can be obtained as

$$f_s(t) = -\frac{dR_s(t)}{dt} = f_1(t) + \int_0^t f_1(u) f_1(t-u+w(u))\,du - f_1(t)R_1(w(t)), \quad t \geq 0.$$

The cycle length $L(t_p) = \min(TT+\tau, t_p)$ can be expressed by a piece-wise function is

$$L(t_p) = \begin{cases} t_p, & TT \geq t_p - \tau, \\ TT+\tau, & TT < t_p - \tau. \end{cases}$$

With the above PDF, the expected cycle length can be obtained as $\mathbb{E}[L(t_p)] = t_p R_s(t_p-\tau) + \tau F_s(t_p-\tau) + \int_0^{t_p-\tau} t f_s(t)\,dt$. In addition, the spare part holding time $\min(T_A, T_B, t_p)$ can be rewritten as

$$\min(T_A, T_B, t_p) = \begin{cases} t_p, & \min(T_A, T_B) \geq t_p, \\ \min(T_A, T_B), & \min(T_A, T_B) < t_p. \end{cases}$$

With the lifetime distributions for both the operating and on-shelf conditions, we first obtain the CDF $F_{min}(u)$ of $\min(T_A, T_B)$ as $F_{\min}(u) = \Pr(\min(T_A, T_B) \leq u) = 1 - R_1(u)$
$R_2(u)$, and its corresponding PDF $f_{\min}(u)$

$$f_{\min}(u) = \frac{dF_{\min}(u)}{du} = f_1(u)R_2(u) + f_2(u)R_1(u).$$

Similar to the calculation of expected cycle length, the expected holding time in one cycle is

$$\mathbb{E}[\min(T_A, T_B, t_p)] \quad = t_p\left(1 - F_{\min}(t_p)\right) + \int_0^{t_p} u f_{\min}(u)\,du$$

The system downtime $[\min(t_p - TT, \tau)]^+$ is also in a piece-wise form as

$$[\min(t_p - TT, \tau)]^+ = \begin{cases} 0, & TT \geq t_p, \\ t_p - TT, & t_p - \tau < TT < t_p, \\ \tau, & TT \leq t_p - \tau. \end{cases}$$

Then, its expectation can be obtained as

$$\mathbb{E}[(\min(t_p - TT, \tau))^+] = \int_{t_p-\tau}^{t_p} (t_p - t) f_s(t)\,dt + \tau F_s(t_p - \tau).$$

***A.2*. Proof of Theorem 1.**
Let $C(t_p) = G(t_p) + M(t_p)$, $p(t_p) = d\mathbb{E}[C(t_p)]/dt_p$, and $l(t_p) = d\mathbb{E}[L(t_p)]/dt_p$. Taking the first-order derivative of the long-run cost rate with respect to $t_p$ yields

$$\left(\frac{\mathbb{E}[C(t_p)]}{\mathbb{E}[L(t_p)]}\right)' = \frac{\mathbb{E}[L(t_p)]p(t_p) - \mathbb{E}[C(t_p)]l(t_p)}{\mathbb{E}[L(t_p)]^2}.$$

Using the results given in Remark 1, we have

$$p(t_p) = \begin{Bmatrix} c_h R_1(t_p)R_2(t_p) + \pi(R_s(t_p - \tau) - R_s(t_p)) - c_{pr} f_s(t_p) \\ +c_s\left(f_1(t_p)R_2(t_p) + f_2(t_p)R_1(t_p)\right) + c_f\left(f_s(t_p) + f_1(t_p)R_2(t_p)\right) \end{Bmatrix}$$

and

$$l(t_p) = R_s(t_p - \tau),$$

respectively. Clearly, when $t_p \to \infty$, $\{\mathbb{E}[L(t_p)]p(t_p) - \mathbb{E}[C(t_p)]$
$l(t_p)\} \to 0$. Moreover, the expected cycle cost $\mathbb{E}[C(t_p)]$ satisfies

$$\lim_{t_p \to \infty} \mathbb{E}[C(t_p)] = \pi\tau + c_h \int_0^\infty u(f_1(u)R_2(u) + f_2(u)R_1(u))\,du + c_f \int_0^\infty f_1(u)R_2(u)\,du + c_f,$$

which is a constant. The expected cycle length $\mathbb{E}[L(t_p)]$ satisfies

$$\lim_{t_p\to\infty}\mathbb{E}[L(t_p)] = \lim_{t_p\to\infty}\left\{t_pR_s(t_p-\tau)+\tau F_s(t_p-\tau)+\int_0^{t_p-\tau} tf_s(t)\,dt\right\} = \tau+T_M,$$

where $T_M = \int_0^{\infty} tf_s(t)\,dt = \int_0^{\infty} R_s(t)\,dt$ is the system's mean-time-to-failure.

***A.3.* Derivation of Remark 2.**
From (25), the PDF of system lifetime can be expressed as

$$f_s(t) = \begin{cases} f_1(t) + \int_0^t f_1(u)f_1(t-u+w(u))\,du - f_1(t)R_1(w(t)), & t_s \geq t \geq 0 \\ f_1(t)F_2(t_s) + \int_0^{t_s} f_1(u)f_1(t-u+\omega(u))\,du & \\ \quad +R_1(t_s)f_1(t-t_s+\omega(t_s)), & t \geq t_s \end{cases}$$

The cycle length is $L(t_s,t_p) = \min(TT+\tau, t_p)$ having the following piece-wise form

$$L(t_s,t_p) = \begin{cases} t_p, & TT \geq t_p-\tau, \\ TT+\tau, & TT < t_p-\tau. \end{cases}$$

Its expectation is $\mathbb{E}[L(t_s,t_p)] = t_pR_s(t_p-\tau)+\int_0^{t_p-\tau}(\tau+t)f_s(t)\,dt$. Moreover, the spare holding time in one cycle is $\min(T_A,T_B,t_s)$, which has the following form

$$\min(T_A,T_B,t_s) = \begin{cases} t_s, & \min(T_A,T_B) \geq t_s, \\ \min(T_A,T_B), & \min(T_A,T_B) < t_s. \end{cases}$$

From the CDF $F_{min}(u)$ of $\min(T_A,T_B)$ given in Remark 1, one can see that

$$\mathbb{E}[\min(T_A,T_B,t_s)] = t_sR_1(t_s)R_2(t_s) + \int_0^{t_s} u(f_1(u)R_2(u)+f_2(u)R_1(u))\,du.$$

The system downtime has the following form

$$[\min(t_p-TT,\tau)]^+ = \begin{cases} 0, & TT \geq t_p, \\ t_p-TT, & t_p-\tau < TT < t_p, \\ \tau, & TT \leq t_p-\tau. \end{cases}$$

Then, its expectation can be expressed as

$$\mathbb{E}[(\min(t_p-TT,\tau))^+] = \int_{t_p-\tau}^{t_p}(t_p-t)f_s(t)\,dt + \tau F_s(t_p-\tau).$$

***A.4.* Proof of Theorem 2.**
Let $C(t_s,t_p) = G(t_s,t_p)+M(t_s,t_p)$, $p(t_s,t_p) = d\mathbb{E}[C(t_s,t_p)]/dt_p$, and $l(t_s,t_p) = d\mathbb{E}[L(t_s,t_p)]/dt_p$. Similar to the proof of Theorem 1, we have

$$\left(\frac{\mathbb{E}[C(t_s,t_p)]}{\mathbb{E}[L(t_s,t_p)]}\right)' \propto \{\mathbb{E}[L(t_s,t_p)]p(t_s,t_p) - \mathbb{E}[C(t_s,t_p)]l(t_s,t_p)\}.$$

Using the results given in Remark 2, we have

$$p(t_s,t_p) = \pi \times \left[F_s(t_p)-F_s(t_p-\tau)\right] + c_{pr}(-f_s(t_p)) + c_f(f_s(t_p))$$

and

$$l(t_s,t_p) = R_s(t_p-\tau) - t_pf_s(t_p-\tau) + t_pf_s(t_p-\tau) = R_s(t_p-\tau).$$

As $t_p\to\infty$, $\mathbb{E}[L(t_s,t_p)]p(t_s,t_p) - \mathbb{E}[C(t_s,t_p)]l(t_s,t_p) \to 0$. In other words, the long-run cost rate will approach a constant. Moreover, the expected cycle cost $\mathbb{E}[C(t_s,t_p)]$ satisfies

$$\begin{aligned}\lim_{t_p\to\infty}\mathbb{E}[C(t_s,t_p)] = \quad & \pi\tau + c_h\left(t_sR_1(t_s)R_s(t_s) + \int_0^{t_s} u(f_1(u)R_2(u) + f_2(u)R_1(u))\,du\right)\\ & +c_{pr}(R_1(t_s)R_2(t_s)) + c_f\int_0^{t_s} f_1(u)R_2(u)\,du + c_f,\end{aligned}$$

which is a function of $t_s$. The expected cycle length $\mathbb{E}[L(t_s,t_p)]$ satisfies

$$\lim_{t_p\to\infty}\mathbb{E}[L(t_s,t_p)] = \lim_{t_p\to\infty}\left\{t_pR_s(t_p-\tau) + \int_0^{t_p-\tau}(\tau+t)f_s(t)\,dt\right\} = \tau + T_M,$$

where $T_M = \int_0^{\infty} tf_s(t)\,dt = \int_0^{\infty} R_s(t)\,dt$ is the system's mean-time-to-failure.