Effective Utilization of Licensed and Unlicensed Spectrum in Large Scale Ad Hoc Networks

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***Abstract*—This paper studies the improvement in network throughput of an ad hoc network from using both licensed and unlicensed spectra compared to the case where only unlicensed spectrum is used. We address the problem of how the nodes of the network, or secondary users (SUs), should spread their transmissions on both licensed and unlicensed spectra to maximize network throughput, and characterize 'sharing gain' achievable in such spectrum sharing systems. The gain obtained can be significant and is increasing with the density of the SUs. The primary and secondary users are modeled as two independent Poisson point processes and their performance is evaluated using techniques from stochastic geometry. A co-operative case is considered where the channel selection strategy of the nodes is centrally controlled. Then, a non-cooperative *channel selection game* where the SUs selfishly select the channels is analyzed. A pricing scheme is proposed to drive the decisions of SUs to a favorable point. Specifically, by setting an 'appropriate' price, global optimal performance is attainable at equilibrium in some cases. Finally, the analysis is extended to the case where a network shares spectrum with a cellular network.**

***Index Terms*—Cognitive radio, game theory, stochastic geometry.**

I. Introduction

THE EMERGENCE of new network paradigms including Internet of Things (IoT) [1], Device to Device communications (D2D) [2] continue to raise the demand for scarce wireless spectrum. To maintain cost effectiveness, most of these networks operate on unlicensed spectrum, e.g., ISM band, which is reserved internationally. But, this spectrum band is limited and may not suffice to guarantee good network performance. On the other hand, most of the terrestrial wireless communication spectrum is licensed for proprietary usage. However, this static allocation is inefficient and lot of this spectrum is under-utilized [3], e.g., uplink channels in cellular networks [4]. In this work, we are interested in the performance (throughout) improvement that a large-scale wireless ad hoc network can achieve by simultaneously using both licensed spectrum (shared) and unlicensed spectrum for packet transmission.

Spectrum sharing is key to alleviate spectrum shortage and improve throughput in mobile ad hoc networks. One promising technology, which has been extensively explored, is the cognitive radio networks where unlicensed users (secondary) opportunistically transmit on a spectrum owned by licensed users (primary). The secondary users (SUs) either transmit on the unused portion of the licensed spectrum (overlay) or transmit on the same spectrum as the primary users (PUs) provided that their transmissions do not affect the performance of PUs (underlay). In underlay networks, the SUs continuously sense the primary spectrum to detect idle transmission slots, thus their performance depends on PUs' traffic pattern.

In applications such as the Internet of Things (IoT), a large number of heterogeneous devices need to continuously transmit critical data requiring seamless connectivity among them. If a licensed spectrum is available for spectrum sharing, some of these devices (SUs) can transmit on a licensed spectrum (provided that they do not degrade the performance of PUs) while the rest transmit on the unlicensed spectrum. Then the key question is what fraction of the SUs should transmit on each channel at any given instant. Clearly, more SUs transmitting on the licensed spectrum may degrade the quality of service (QoS) of the PUs. On the other hand, more SUs transmitting on the unlicensed spectrum can lead to a higher outage rate due to increased intra-channel interference. Therefore, we investigate how the secondary users/traffic should be split across licensed and unlicensed spectra so that the density of successful transmissions (throughput in short) for SUs increases without violating the QoS of PUs.

Many players in the telecom sector have bought spectrum for a huge amount for proprietor use which they may like to share with other users as long as it does not degrade performance of their PUs. For example, mobile ad hoc network (MANET) like FlashLinQ [5] may like to use some licensed spectrum that is available for sharing in addition to the unlicensed spectrum to get higher throughput. In LTE-U technology, cellular network operators propose to use unlicensed spectrum in addition to their licensed spectrum to boost coverage, whereas we address the reverse possibility of unlicensed spectrum users sharing a licensed spectrum to improve their throughput.

We refer to the licensed spectrum as the primary channel (PC) and the unlicensed spectrum as the secondary channel (SC). Most of the MANETs networks operate on SC and use

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congestion resolution protocol to achieve better throughput. If SUs can also use PC, they can aim to get better throughput by spreading their transmissions on both the channels. Our goal in this work is to characterize the gain in throughput of a secondary network by using both SC and PC simultaneously compared to the case they can only use SC. To characterize the gain, we introduce a metric called *Sharing Gain* defined as the ratio of highest achievable throughput using both PC and SC and that achievable using SC alone using contention resolution protocol like ALOHA.

We assume that the secondary network is a MANET operating on SC and the primary network can be another MANET or a cellular network operating on a PC. We model the PUs and SUs as homogeneous independent Poisson point processes and use techniques from stochastic geometry to derive performance metrics. Stochastic geometry has been widely used to analyze the performance of MANETs [6], cognitive radio networks [7], [8] and cellular networks [9], often leading to tractable analysis while giving performance comparable to that observed in reality [10]. In the case of heterogeneous Poisson point processes, even if analytical results are not easily available, simulations techniques can be used to evaluate some performance metrics [6]. Our goal is to obtain as far as we can closed-form expressions and therefore we restrict our problem to homogeneous case.

In our setting, the strategy of each SU is to select a channel (PC or SC) for transmission in each time slot. A network operator[1] interested in providing high connectivity in the secondary network would assign a channel selection strategy that maximizes the overall density of successful transmissions (network throughput) for SUs while maintaining QoS guarantee for PUs. However, due to decentralized nature of MANETs, the SUs may not follow the strategy assigned by the operator and selfishly select channels to maximize their throughput leading to a loss in performance of the secondary network and also degradation of QoS in the primary network. To mitigate the loss due to non-cooperation, we consider a pricing based 'de-incentivizing mechanism' where the network operator charges the SUs for transmissions on the PC. Such pricing can be easily enforced as SUs need authentication to transmit on the PC. We define a channel selection game among the SUs where the utility of each SU is defined in terms of the weighted difference of success probability and transmission costs and study *Symmetric Nash Equilibria* (SNE) of the game. Though the game involves an infinite number of players, our focus on SNE allows the game to be treated as a two-person game with closed-form expressions for equilibrium.[2] The price can be then used to control the fraction of the SUs transmitting on the PC.

The summary of our contributions and their discussions are as follows:

- We first model spectrum sharing between two MANETs, one operating on a PC and another on a SC. We introduce a metric called 'Sharing Gain' to quantify the gain. The metric allows us to compare the achievable throughput for SUs by using both PC and SC against a benchmark throughput achievable on SC alone using ALOHA.
- We show that when PUs QoS requirement of PUs is 'relaxed', the Sharing Gain can be significant (Theorem 1). Specifically, we show that as the QoS requirement of the PUs reduces SUs benefit more as they will have more flexibility to spread their transmission on primary and secondary channels. We also demonstrate that when the QoS for PUs is 'stringent,' the SUs would benefit from using only SC instead of both the channels.
- When the SUs are non-cooperative, we study the SNE of the game where SUs aim to maximize a utility defined as the weighted difference of success probability and transmission cost. We show that global optimal performance can be achieved at equilibrium (Theorem 2 & 3) in some cases through a pricing mechanism. Specifically, we show that a linear pricing mechanism where the SUs are charged in proportion to the amount of time they use PC induces co-operation among the selfish SUs.
- We show that SNE of the game is the same as Wardrop equilibrium [11] in an equivalent routing game (Theorem 4). This connection enables us to use replicator dynamics to compute the SNE. Further, using these dynamics we propose algorithms that achieve the equilibrium.
- Finally, we consider a secondary MANET share uplink of a cellular network and show that the observation about throughput and sharing gain are similar to the earlier case where they shared spectrum with another MANET.

A. Related Works

Several papers including [12]–[17] study the performance of co-existing heterogeneous networks under outage constraints. Below we discuss papers that consider underlay networks and non-cooperative users as we do. Survey articles [18]–[20] give details of other aspects of spectrum sharing.

In [15], the authors study transmission-capacity trade-off of a network where a MANET shares uplink of a cellular network; transmission capacity is defined as the highest density of PUs and SUs that can co-exist without violating the outage constraints. It is shown that the capacity region of co-existing network is triangular. In [17], the authors study co-existence of two MANETs and evaluate the transmission capacity of the secondary network under outage constraints. The authors in [13] study single hop transport capacity (STC) of two MANETs that co-exist, where STC involves both the transmission distance and transmission capacity. They consider different distribution on the distance between transmitter-receiver pair. In [12], the authors consider spectrum sharing between D2D devices and cellular networks. A D2D device either transmits directly to other devices or uses the cellular network. The authors derive rate expression and analyze achievable rates for D2D devices.

As discussed above, the papers on spectrum sharing analyze the highest density of SUs and PUs that can co-exist

[1]Network operator can be a regulator or a device manufacturer that sets the protocol of the SUs.

[2]It is well known that computing the games is, in general, a hard problem.

without violating outage constraints. Our work differs from these works as we study how SUs can spread their transmissions on both PC and SC to achieve higher throughput without degrading QoS of PUs and characterize the achievable gains. We stress that our goal in this work is to effectively utilize both PC and SC to achieve higher throughput rather the just co-exist with the PUs without degrading their QoS.

MANETs are often decentralized and user in them make decisions on their own and can be selfish. several authors have studied non-cooperative behavior of SUs [21]. In [22], the authors consider a single PU and N SUs. Spectrum sharing problem is formulated as an Oligopoly market where the SUs fight for good share in the spectrum offered by the PU. In [23], multiple PUs trade spectrum with multiple SUs. The authors consider a repeated game model where SUs adopt to different quality and price set by the PUs. The behavior of SUs analyzed using the theory of evolution [24]. In [25], [26] the authors study a non-cooperative game among a finite set of primary networks that share part of their spare spectrum with the secondary users for a fee. In [27] the authors consider a population game framework composed of both PU and SU that determine selfishly their transmission rates on the PU channel. The quality is evaluated using an average delay measure and a pricing mechanism is used to minimize the price of anarchy of the system. Finally, transient analysis of the system is studied through the prism of the replicator dynamics. Its convergence is proved and yields to effective selection strategy for this overlay CR network. In this last reference, even if mathematical tools from game theory with a large number of players are also considered like in our section related to the noncooperative case, the problem under consideration is totally different. In fact, we consider the spatial distribution of users and coverage probabilities measure, they do not consider the geographical aspect at all, only the transmission rate and the average delay.

In this paper, we establish that under the stochastic geometric setting, the spectrum sharing problem with non-cooperative users can be modeled as a routing game. Though game theory has been applied extensively in the study of cognitive radio networks and spectrum sharing, to the best of our knowledge this connection is new and helps us leverage the tools developed for the routing games to study dynamics of the spectrum sharing systems.

Paper Organization: We begin with spectrum sharing problem between two MANETs in Section II and discuss the model and setup. In Section III, we consider a cooperative scenario where all the SUs follow a strategy assigned by a network operator. In Section IV, we consider selfish behavior of SUs and study Nash equilibria of the game. In Section V we establish that the game is equivalent to a routing game and give a method based on replicator dynamics to compute the Nash equilibrium of the game. In Section VI we extend the analysis to the case where SUs of a MANET share spectrum with a cellular network and give numerical results in Section VI-B. We end with concluding remarks in Section VII.

II. Spectrum Sharing Between MANETs

In this section, we consider two mobile ad hoc networks, one operating on a licensed spectrum and the other on an unlicensed spectrum. The extension to the case where multiple licensed spectra are available for sharing can also be treated similarly, but it brings in more combinatorial complexities. Using the terminology of cognitive radio networks, we refer to licensed spectrum as a primary channel (PC) and transmitters on it as primary users (PUs). Similarly, unlicensed spectrum is referred to as secondary channel (SC) and transmitters on it as secondary users (SUs). We assume that PC is of higher bandwidth than SC and is of better quality. The SUs can also transmit on the PC provided that they do not degrade PUs' quality of service (QoS). We assume that both SUs and PUs are saturated, i.e., always have a packet to transmit. This saturated traffic scenario is ideal to evaluate the performance of the system in a kind of worst-case scenario. This is mainly studied before deployment of solutions in practice.

Both PUs and SUs are spread in a common geographical area and we assume that they are distributed according to independent homogeneous Poisson point processes (P. P. P.) of intensity λ_I and λ_{II} respectively.

We consider the simplified mobile ad hoc network (MANET) model called the Poisson bipolar model proposed in [6] for the SUs and PUs. Each dipole of the MANET consists of a transmitter and an associated receiver. Let $\Phi^{II} := \{X_i^{II}\}_{i\geq 1}$ denote the locations of secondary transmitters that are scattered in the Euclidean plane according to a homogeneous Poisson point process of intensity λ_{II}. The set of secondary receivers $\{y_i^{II}\}_{i\geq i}$, where y_i^{II} denotes the receiver associated with transmitter X_i^{II}, are assumed to be distributed uniformly on a circle of radius r centered around its transmitter, i.e., $y_i^{II} = X_i^{II} + rR(\theta_i^{II})$, where θ_i^{II} is uniformly, independently and identically distributed on $[0\ 2\pi]$, and $R(\theta) = (\cos(\theta), \sin(\theta))$. Let $\Phi^I := \{X_i^I\}_{i\geq 1}$ denote the location of primary transmitters and, like SUs, the receiver of each PU is located uniformly on a circle of radius r_1 centered around its transmitter.[3]

Let $n = 0, 1, 2, \ldots,$ denote index of time slots with respect to which all nodes are synchronized.[4] We associate with each SU a multi dimensional mark that carries information about decision of which channel to use (PC or SC) and the fading condition at each time slot. Following the notation of [6, Ch. 17], let the sequence $M_i^{II}(n) = \{e_i(n), F_i^{II}(n), F_i^{II-I}(n)\}_{n\geq 0}$ denote the marks associated with SU i, and $M_i^I(n) = \{F_i^I(n), F_i^{I-II}(n)\}_{n\geq 0}$ denote the marks associated with PU i, where

- $e_i = \{e_i(n)\}_{n\geq 0}$ denotes the sequence of channel access decisions of SU i. $e_i(n)$ is an indicator function that takes value 1 if node i decides to transmit on PC in slot n, otherwise it takes value zero. The random variables $e_i(n)$ are assumed to be independently and identically distributed (i.i.d.) in i and n, and independent of everything else.

[3]Extension to include a random distance between Tx-Rx pairs is straightforward. But it provides little new insights.

[4]Analysis extends to non-synchronous using techniques in [28]

- $F_i^{II}(n) = \{F_{ij}^{II}(n) : j \geq 1\}$ denotes the sequence of channel conditions between the transmitter of SU *i* and all the secondary receivers (including its own receiver).
- $F_i^{II-I}(n) = \{F_{ij}^{II-I}(n) : j \geq 1\}$ denotes the channel condition between the transmitter of *i*th SU and all primary receivers.
- $F_i^{I}(n) = \{F_{ij}^{I}(n) : j \geq 1\}$ denotes the sequence of channel conditions between the transmitter of *i*th PU and all the receivers (including its own receiver).
- $F_i^{I-II}(n) = \{F_{ij}^{I-II}(n), j \geq 1\}$ denotes the sequence of channel conditions between the transmitter of *i*th PU and all secondary receivers.
- It is assumed that channel conditions are i.i.d. across the nodes and time slots, with a generic distribution on $\mathbb{R}^+$ denoted by *F* with mean $1/\mu$. The marks are assumed to be independent in space and time.

The probability that the *i*th SU transmits in time slot *n* on PC is $p_i := \Pr\{e_i(n) = 1\} = \mathbb{E}[e_i(n)]$. When all the SUs use the same p_i (symmetric scenario), we drop the subscript *i* and write it as *p*. If each SU use PC with probability *p* and its decision is independent of everything else, we obtain a pair of independent Poisson processes at each time slot *n*, one representing a set of SUs on PC $\Phi_1^{II}(n) = \{X_i^{II}, e_i(n) = 1\}$ and the other representing the rest of SUs on the SC $\Phi_0^{II}(n) = \{X_i^{II}, e_i(n) = 0\}$ with intensities $p\lambda_{II}$ and $(1-p)\lambda_{II}$ respectively. All the SUs transmit at a fixed power P_{II}, and the PUs at a fixed power level of P_I. For notational convenience we write $P = P_I/P_{II}$.

Let *l*(*x*, *y*) denote the attenuation function between any two given points $x, y \in \mathbb{R}^2$. We assume that this function depends on the distance between the points, i.e., $|x - y|$. We consider the following form for attenuation:

$$l(x, y) = |x - y|^{-\beta} \text{ with } \beta > 2. \tag{1}$$

A. Coverage Probability of a SU

A signal transmitted by *i*th SU is successfully received in time slot *n* on SC if the SINR at its receiver is larger than some threshold T_{II}, i.e.,

$$SINR_i^{II}(n) := \frac{P_{II} F_{ii}^{II}(n) l(r)}{I_{\Phi_0^{II}(n)}\left(y_i^{II}\right) + W(n)} > T_{II}, \tag{2}$$

where $W(n)$ is the thermal noise power at the receiver and

$$I_{\Phi_0^{II}(n)}\left(y_i^{II}\right) = \sum_{X_j^{II} \in \Phi_0^{II}(n) \setminus X_i^{II}} P_{II} F_{ij}^{II}(n) l\left(|X_j^{II} - y_i^{II}|\right)$$

denotes the *shot noise* of the P. P. P. $\Phi_0^{II}(n)$ in time slot *n*. We assume that the noise is an i.i.d. process. A signal transmitted by the SU at X_i^{II} is successfully received in time slot *n* on PC if the SINR at its receiver is larger than a threshold T_I, i.e.,

$$SINR_i^{II-I}(n) := \frac{P_{II} F_{ii}^{II}(n) l(r)}{I_{\Phi_1^{II}(n)}\left(y_i^{II}\right) + I_{\Phi^I(n)}\left(y_i^{II}\right) + W(n)} > T_I, \tag{3}$$

where

$$I_{\Phi_1^{II}(n)}\left(y_i^{II}\right) = \sum_{X_j^{II} \in \Phi_1^{II}(n) \setminus X_i^{II}} P_{II} F_{ij}^{II}(n) l\left(|X_j^{II} - y_i^{II}|\right) \tag{4}$$

is the shot noise from SUs on the PC and

$$I_{\Phi^I(n)}\left(y_i^{II}\right) = \sum_{X_j^I \in \Phi^I(n)} P_I F_{ji}^{I-II}(n) l\left(\left|X_j^I - y_i^{II}\right|\right)$$

denote the shot noise from PUs, at y_i^{II} in time *n*. Since bandwidth of PC is assumed to be much larger than that of SC we set $T_{II} > T_I$. Consider a typical SU at the origin with mark $M_i^{II}(0) = (e_i(0), F_i^{II}(0), F_i^{II-I}(0))$ at $n = 0$. The typical node is said to be covered in slot $n = 0$ on SC if (2) holds given that it selects to operate on SC. Then the coverage probability of the typical node is

$$\mathbf{P}_0^{II}\left\{ SINR_i^{II}(0) > T_{II} \;\middle|\; e_i(0) = 0 \right\},$$

where $\mathbf{P}_0^{II}$ denotes the Palm distribution [29, Ch. I] of the stationary marked P. P. P. Φ_0^{II}. Note that due to time-homogeneity, this conditional probability does not depend on *n*. The coverage probability of a typical node when all other nodes use the same channel access decision is evaluated in [30]. Continuing the notation used in [30] we denote this coverage probability (non-outage probability) as $p_c^{II} := p_c^{II}(r, (1-p)\lambda_{II}, T_{II})$. A tagged node is said to be covered in slot $n = 0$ on PC if (3) holds given that it selects to operate on PC. We denote this coverage probability as $p_c^{II-I} := p_c^{II-I}(r, \hat{\lambda}_I, T_I)$ and is given by

$$\mathbf{P}_0^{II-I}\left\{ SINR_i^{II-I}(0) > T_I \;\middle|\; e_i(0) = 1 \right\},$$

where $\mathbf{P}_0^{II-I}$ denote the Palm distribution of the stationary marked P. P. P. $\tilde{\Phi}^I = \Phi_1^{II} + \Phi^I$ with density $\hat{\lambda}_I := \lambda_I + p\lambda_{II}$.

Proposition 1: [6, Proposition 16.2.2] Let the fading process be Rayleigh distributed and each SU select PC with probability *p* in each slot. Then, coverage probability of a typical SU on SC and PC, respectively, is

$$\begin{aligned} p_c^{II} &= \mathcal{L}_{I_{\Phi_0^{II}}}(\mu T_{II}/P_{II} l(r)) \mathcal{L}_W(\mu T_{II}/P_{II} l(r)) \\ p_c^{II-I} &= \mathcal{L}_{I_{\Phi_1^{II}}}(\mu T_I/P_{II} l(r)) \mathcal{L}_{I_{\Phi^I}}(\mu T_I/P_{II} l(r)) \\ &\quad \times \mathcal{L}_W(\mu T_I/P_{II} l(r)), \end{aligned}$$

where $\mathcal{L}_X(s) = \mathbb{E}[e^{-sX}]$ denotes the Laplace transform of random variable *x* evaluated at *s*.

Corollary 1: Let the fading process be Rayleigh distributed and $W \equiv 0$. For the path loss model in (1), we have

$$\begin{aligned} p_c^{II} &= \exp\{-(1-p)\lambda_{II} C_{II}\} \\ p_c^{II-I} &= \exp\{-p\lambda_{II} C_I\} \exp\left\{-\lambda_I P^{2/\beta} C_I\right\}, \end{aligned}$$

where $C_{II} = r^2 T_{II}^{2/\beta} K(\beta)$, $C_I = r^2 T_I^{2/\beta} K(\beta)$ and $K(\beta) = 2\pi^2/(\beta \sin(2\pi/\beta))$.

B. Coverage Probability of a PU

The ith PU is said to be covered if SINR at its receiver is larger than threshold T_I. We denote the coverage probability of a typical PU as $p_c^I := p_c^I(r_1, \hat{\lambda}_I, T_I)$. Following the steps used for typical PU, it can be evaluated as follows:

Proposition 2: [6, Proposition 16.2.2] Let each SU transmit on PC with probability p. For Rayleigh fading, the success probability of a typical PU is given as

$$p_c^I := \mathcal{L}_{I_{\Phi_1^{II}}}(\mu T_I/P_I l(R))\mathcal{L}_{I_{\Phi^I}}(\mu T_I/P_I l(r_1)) \times \mathcal{L}_W(\mu T_I/P_I l(r_1)).$$

Corollary 2: Let the fading process be Rayleigh distributed and $W \equiv 0$. For the path loss model in (1), the coverage probability of a PU is

$$p_c^I = \exp\left\{-p\lambda_{II} P^{-2/\beta} \bar{C}_I\right\} \exp\{-\lambda_I \bar{C}_I\},$$

where $\bar{C}_I = r_1^2 T_I^{2/\beta} K(\beta)$.

C. Density of Successful Transmission of SUs

Let $d_s^{II}(p)$ denote the spatial density of successful transmissions of the SUs on SC. Since the SUs form a P. P. P. of intensity $(1-p)\lambda_{II}$ in each slot on SC, we obtain $d_s^{II}(p) = (1-p)\lambda_{II} p_c^{II}(r, (1-p)\lambda_{II}, T_{II})$. Similarly, let $d_s^I(p)$ denote the spatial density of the successful transmissions of the SUs on PC. Since the SUs form a P. P. P. of intensity $p\lambda_{II}$ in each time slot on PC, we obtain $d_s^I(p) = p\lambda_{II} p_c^{II-I}(r, \hat{\lambda}, T_I)$. Then, if each SU decides to transmit on PC with probability p independent of others, the total density of successful transmissions of the SUs, denoted as $d_s(p)$, is

$$d_s(p) = d_s^{II}(p) + d_s^I(p).$$

In the following we first consider the case where all the SUs co-operate and aim to maximize their spatial density of success without degrading the QoS of the PUs. We then consider that the SUs selfishly select the channels and propose a mechanism to improve equilibrium performance.

III. Co-Operative Case

In this section we assume that all the SUs belong to a network operator and access PC with a probability assigned by the operator. A fraction of SUs are allowed to transmit on the PC provided that their transmissions do not degrade the QoS for the PUs on PC.

A. Quality of Service Guarantee for the PUs

A natural way to guarantee a certain QoS to the PUs is to limit the amount of interference by the SUs on PC, or, alternatively, to maintain a minimum coverage probability of the PUs in presence of SUs. Specifically, we consider that each PU is covered with probability at least $1-\delta$, where δ determines the predefined QoS level. The objective of the network operator is to maximize the density of successful transmissions of the SUs without degrading the QoS requirement of the PUs, i.e.,

$$\begin{aligned} \text{maximize}_{p\in[0\ 1]} \quad & d_s(p) \\ \text{subject to} \quad & p_c^I \geq 1-\delta \end{aligned} \tag{5}$$

Let p_o^* denote the global maximum of the optimization problem in (5).

Sharing Gain: If SUs can transmit only on the SC, then contention resolution protocols like ALOHA, CSMA can be used to improve the success density. In ALOHA/CSMA, only a fraction of the SUs transmits in each time slot, while the others remain silent to reduce interference. If SUs can also transmit on the PC in addition to SC, some of the SUs can transmit on PC (provided that the QoS constraints are met) and thus increase the number of their concurrent transmissions without increasing intra-channel interference on SC. We measure this gain in terms of the success density of the SUs by comparing maximum success density achievable using both PC and SC to that achievable using SC alone. Specifically, we define SUs success density gain by $G := G(\lambda_{II}, \lambda_I, T_{II}, T_I)$, as:

$$G = \frac{d_s(p_o^*)}{\max_{p\in[0\ 1]} d_s^{II}(p)}. \tag{6}$$

B. Performance Evaluation

For analytic tractability, we focus on the case of Rayleigh fading and neglect channel noise[5] ($W \equiv 0$). From Corollaries 1 and 2, the optimization problem in (5) for the Rayleigh fading is given by:

$$\begin{aligned} \text{maximize}_{p\in[0\ 1]} \quad & (1-p)\lambda_{II} \exp\{-(1-p)\lambda_{II} C_{II}\} \\ & + p\lambda_{II} \exp\left\{-p\lambda_{II} C_I - \lambda_I P^{\frac{2}{\beta}} C_I\right\} \\ \text{subjected to} \quad & \exp\{-p\lambda_{II} P^{\frac{-2}{\beta}} \bar{C}_I - \lambda_I \bar{C}_I\} \geq 1-\delta. \end{aligned} \tag{7}$$

The following simple observation states in which parameters configuration it is possible for both the SUs and PUs to share the PC.

Lemma 1: Given $\delta > 0$, SUs can transmit on the PC if and only if

$$\lambda_I \leq -\log(1-\delta)/\bar{C}_I.$$

Proof: A fraction p of the SU's are permissible to transmit on PC if and only if they do not degrade the QoS of the PU's, i.e., when

$$\exp\left\{-p\lambda_{II} P^{-2/\beta} \bar{C}_I - \lambda_I \bar{C}_I\right\} \geq 1-\delta.$$

Rearranging the above, we obtain

$$p \leq \frac{-\log(1-\delta) - \lambda_I \bar{C}_I}{\lambda_{II} P^{-2/\beta} \bar{C}_I}.$$

Now it is clear that for $p \geq 0$, λ_I should satisfy

$$\lambda_I \leq -\log(1-\delta)/\bar{C}_I.$$

■

[5]For the case with non-negligible noise the same analysis holds up to a constant scaling factor that depends on the Laplacian of noise random variable.

Scaling Properties: For notational convenience write $\bar{\lambda}_I := -\log(1-\delta)/\bar{C}_I$ and $\bar{p} := -(\log(1-\delta)+\lambda_I \bar{C}_I)/\lambda_{II} P^{\frac{-2}{\beta}} \bar{C}_I$. If $\lambda_I < \bar{\lambda}_I$, then some SUs are permitted to use PC. On the other hand if $\lambda_I \geq \bar{\lambda}_I$, SUs are prohibited to use PC. Notice that $\bar{\lambda}_I$ is inversely proportional to T_I (through $\bar{C}$), and if T_1 decreases, more SUs can be accomodated on the PC without degrading QoS of PUs. Specifically, if T_I decreases by a factor $a \in (0\ 1]$, then the density of both PUs and SUs can increase by a factor $a^{-2/\beta}$ on PC without affecting the QoS for the SUs. To see this, note that $\bar{p}$ depends on the products $\lambda_I T_I^{2/\beta}$ and $\lambda_{II} T_I^{2/\beta}$ (through $\lambda_I \bar{C}$ and $\lambda_{II} \bar{C}$) and their value does not change if T_I decreases by a factor $a \in [0\ 1]$ and the values of λ_I and λ_{II} increases by a factor $a^{-2/\beta}$.

Let us first consider unconstrained optimization problem ignoring the constraint in (7). It is easy to note that both $d_s^{II}(p)$ and $d_s^I(p)$ are quasi-concave in p and each attains a unique maximum. But, their sum is not always quasi-concave and the maximum value of $d_s(p)$ may not be unique. Some of its properties are listed below.

Proposition 3: Let p^* denote a maximum of $d_s(p)$. Assume $\lambda_{II} C_I \leq 1$, then p^* is unique. Further,

- If $\lambda_{II} C_{II} \leq 1$, then $d_s(p)$ is concave in p.
- If $\lambda_{II} C_{II} > 1$, then $p^* \geq 1 - 1/\lambda_{II} C_{II}$.

Proof: Differentiating each term in $d_s(p)$ w.r.t p we obtain

$$\begin{aligned} \mathrm{d}d_s^{II}(p)/\mathrm{d}p &= \lambda_{II} \exp\{-(1-p)\lambda_{II} C_{II}\} \\ &\quad \times (\lambda_{II} C_{II}(1-p) - 1), \qquad (8) \\ \mathrm{d}d_s^{I}(p)/\mathrm{d}p &= \lambda_{II} \exp\{-p\lambda_{II} C_I\}(1 - p\lambda_{II} C_I) \\ &\quad \times \exp\Big\{-\lambda_I P^{2/\beta} C_I\Big\}. \qquad (9) \end{aligned}$$

First consider the case $\lambda_{II} C_{II} \leq 1$. Differentiating (8) and (9) again, we see that they take negative values for $p \in [0, 1]$. Hence $d_s(p)$ is concave in p. Now consider the case $\lambda_{II} C_{II} > 1$. In this case $d_s(p)$ is monotonically increasing for all $0 \leq p \leq 1 - 1/\lambda_{II} C_{II}$ as both (8) and (9) are positive in this interval. Hence it is clear that $p^* \geq 1 - 1/\lambda_{II} C_{II}$.

Uniqueness of p^* is clear in the case $\lambda_{II} C_{II} \leq 1$ due to concavity. To see the same holds for the case $\lambda_{II} c_{II} \geq 1$, consider the following first order optimality conditions obtained by setting $\mathrm{d}d_s(p)/\mathrm{d}p = 0$. Simplifying the expression and we obtain

$$\begin{aligned} &\exp\Big\{-\lambda_I P^{2/\beta} C_I\Big\}(1 - p\lambda_{II} C_I)\exp\{-p\lambda_{II} C_I\} \\ &\quad = \exp\{-(1-p)\lambda_{II} C_{II}\}(1 - (1-p)\lambda_{II} C_{II}). \qquad (10) \end{aligned}$$

Since we assume that $\lambda_{II} C_I \leq 1$, the left hand side is positive and decreasing, taking value $\exp\{-\lambda_I P^{-2/\beta} \bar{C}_I\}$ $(1 - p\lambda_{II} C_I)\exp\{-p\lambda_{II} C_I\}$ at $p = 1$. Whereas, the right side is positive and increasing only for $p \geq 1 - 1/\lambda_{II} C_{II}$ (otherwise it is negative), taking values 0 and 1 at $p = 1 - 1/\lambda_{II} C_{II}$ and $p = 1$, respectively. Hence, the condition $\mathrm{d}d_s(p)/\mathrm{d}p = 0$ (Eqn. (10)) holds for a unique p. ∎

Based on this result, the following remarks can be done.

Remark 1: Let p^* denote a maximum of $d_s(p)$. If $\lambda_{II} C_I > 1$, then p^* may not be unique. Further, when $\lambda_{II} C_{II} > 1$, any optimum p^* is such that $1/\lambda_{II} C_I \leq p^* \leq 1 - 1/\lambda_{II} C_{II}$.

The proofs of this remark come directly and are similar to that of Proposition 3. Recall that $C_{II} > C_I$ implies $T_{II} > T_I$, and smaller the values of T_I and T_{II}, better the channel quality (with higher success rate). For a given λ_{II}, if $\lambda_{II} C_{II} \leq 1$, d_s^{II} is maximized at $p = 0$, i.e., there is no need for SUs to transmit on PC, hence G= 1. If $\lambda_{II} C_{II} > 1$, $d_s^{II}(p)$ is maximized at $p = 1 - 1/\lambda_{II} C_{II}$ and the operator benefits by allowing a fraction $1 - 1/\lambda_{II} C_{II}$ of the SUs to transmit on PC. Further, if $\lambda_{II} C_I \leq 1$, PC quality is better than that of SC, hence the operator benefits more by allowing more than $1 - 1/\lambda_{II} C_{II}$ fraction of the SUs to transmit on PC as noted in the second part of Proposition (3). On the other hand, if $\lambda_{II} C_I \geq 1$, PC quality is not significantly better than that of SC, and the operator may gain only by allowing a smaller than $1 - 1/\lambda_{II} C_{II}$ fraction of the SUs to transmit on PC (Proposition 1). Thus we focus on the scenario where $\lambda_{II} C_{II} > 1$ and $\lambda_{II} C_I \leq 1$, i.e., the quality of PC is better compared to that of SC where the operator prefers to place more SUs on PC but is constrained by the QoS requirements for the PUs.

Now we return to the constrained optimization in (7). Note the objective function is concave in the regime of interest and constraint is a convex set. Hence solution of (7) is unique. Its properties are listed below.

Proposition 4: Let $\lambda_{II} C_{II} > 1$ and $\lambda_{II} C_I \leq 1$. For a given $\delta > 0$ and $\bar{p}$ the global optimum d_o^* of (5) satisfies:

- If $\bar{p} \leq 1 - 1/\lambda_{II} C_{II}$, then $p_o^* = \bar{p}$.
- If $\bar{p} > 1 - 1/\lambda_{II} C_{II}$, then $p_o^* \geq 1 - 1/\lambda_{II} C_{II}$.

Proof: From Proposition (3), the objective function in (7) is monotonically increasing for all $p \leq 1 - 1/\lambda_{II} C_{II}$. Hence the optimum is achieved at $p_c^* = \bar{p}$. Further, $d_s(p)$ achieves the maximum at a point $p \geq 1 - 1/\lambda_{II} C_{II}$. Hence if $\bar{p} > 1 - 1/\lambda_{II} C_{II}$, then the optimum is such that $p_o^* > 1 - 1/\lambda_{II} C_{II}$. ∎

As noted earlier, the network operator benefits if at least $1 - 1/\lambda_{II} C_{II}$ fraction of the SUs operate on PC. However, if doing so violates QoS guarantee for PUs, then the network operator can place at most $\bar{p}$ fraction of SUs on PC. The following Theorem characterizes the gain in different regimes.

Theorem 1: Let $\lambda_{II} C_{II} > 1$ and $\lambda_{II} C_I \leq 1$. Let $a := \exp\{-\lambda_I P^{2/\beta} C_I\}$, we have

- If $\bar{p} \geq 1 - 1/\lambda_{II} C_{II}$, then

$$\begin{aligned} &1 + ea\lambda_{II} C_{II} \exp\{-\lambda_{II} C_I\} \geq G \\ &\quad \geq 1 + ea(\lambda_{II} C_{II} - 1)\exp\{-(\lambda_{II} C_{II} - 1)C_I/C_{II}\} \qquad (11) \end{aligned}$$

- If $\bar{p} < 1 - 1/\lambda_{II} C_{II}$, then

$$G \leq 1 + ea(\lambda_{II} C_{II} - 1)\exp\{-(\lambda_{II} C_{II} - 1)C_I/C_{II}\}$$

Proof: 1 First consider the case $\bar{p} \geq 1 - 1/\lambda_{II} C_{II}$. It is easy to note that maximum of $d_s^{II}(p)$ is attained at $p = 1 - 1/\lambda_{II} C_{II}$ and $\max_{p\in[0,1]} d_s^{II}(p) = 1/eC_{II}$. Also, $d_s^I(p)$ is monotonically increasing and maximized at $p = 1$. Hence, $d_s(p)$ is trivially upper bounded and then we obtain:

$$G \leq \frac{\max_{p\in[0,1]} d_s^I(p) + \max_{p\in[0,1]} d_s^{II}(p)}{\max_{p\in[0,1]} d_s^I(p)},$$

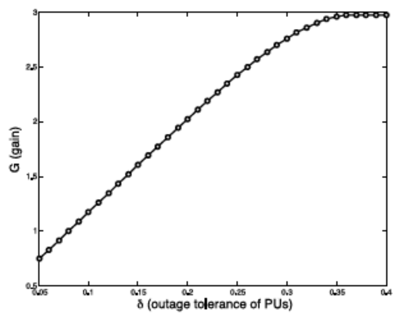

Fig. 1. Gain vs. outage . We set $P_{II} = P_I$, $\lambda_{II} C_{II} = 2$, $\lambda_{II} C_I = 0.2$, $\lambda_{II} \bar{C}_I = 0.5$, $\lambda_I \bar{C}_I = 0.05$. The gain reaches a limit value as the outage tolerance grows. However, the centralized scheme may degrade the network performance when the outage tolerance is very low ($G <1$ when $\delta < 0.08$.).

$$\leq 1 + eC_{II} \max_{p\in[0,1]} d_s^I(p),$$
$$= 1 + ea\lambda_{II} C_{II} \exp\{-\lambda_{II} C_I\}.$$

From Propositions 3 and 4 note that $p_o^* \geq 1 - 1/\lambda_{II} C_{II}$ and $d_s(p)$ is increasing for all $p \leq 1 - 1/\lambda_{II} C_{II}$. Evaluating $d_s^{II}(p)$ and $d_s^I(p)$ at $p = 1 - 1/\lambda_{II} C_{II}$ and simplifying, we obtain

$$d_s(p_o^*) \geq \frac{1}{eC_{II}} + a\left(\lambda_{II} - \frac{1}{C_{II}}\right) \exp\left\{-\left(\lambda_{II} - \frac{1}{C_{II}}\right) C_I\right\}.$$

Finally, using the definition of G and simplifying the expression, we get the lower bound:

$$G \geq 1 + ae(\lambda_{II} C_{II} - 1) \exp\left\{-(\lambda_{II} C_{II} - 1)\frac{C_I}{C_{II}}\right\}.$$

When $\bar{p} < 1-1/\lambda_{II} C_{II}$, from Proposition 4 we have $p_o^* = \bar{p}$. Also, from Proposition 3, $d_s(p)$ is monotonically increasing for all $p \leq 1-1/\lambda_{II} C_{II}$. Hence $d_s(p_o^*) \leq d_s(1-1/\lambda_{II} C_{II})$. Then using the same steps used previously, we obtain the required upper bound. ■

Both the upper and lower bounds in (11) are increasing in the ratio T_{II}/T_I (through C_{II}/C_I) for a fixed $\lambda_{II} C_{II}$. Thus, the gain G is higher if PC quality improves compared to that of SC quality. We note that for the case $\bar{p} \geq 1-1/\lambda_{II} C_{II}$ the lower bound is strictly larger than 1, thus the network operator always gains using PC. However, if QoS requirement for the SUs is 'stringent' such that $\bar{p} < 1 - 1/\lambda_{II} C_{II}$, then it may be possible that $G < 1$ and the operator may not gain by using PC. In Figure 1 we plot G as a function of the outage tolerance δ. As seen, for small values of δ, $G < 1$. In this regime, the network operator should avoid using PC and aim to increase the success density on SC alone using contention resolution protocols. In Figure 1, the minimum outage tolerance such that the use of PC enhances SUs successful transmissions is given by $\delta = 0.08$. Above this outage level, the gain G is higher than 1. Therefore, transmitting on both SC and PC is not always profitable at the global system level.

IV. Non-Cooperative Case

In this section, we consider that the nodes selfishly select the channel to transmit on. The goal of each SU is to maximize its good-put. We refer this non-cooperative game among SUs as a *channel selection* game.

With abuse of terminology let *PC* and *SC* also denote actions of each SU. In this context, we consider that the SUs do not know a priori their neighborhood and the level of interference they will see on either channel and hence play the same mixed strategy in each time slot that is decided at the beginning of the game. Our interest is in considering Symmetric Nash Equilibrium (SNE) of the game, in which all the SUs play same (mixed) strategy at equilibrium. SNE is often considered to evaluate the performance of systems involving a large population of homogeneous players. The game considered is static; i.e., the environments of the players such as CSI and geographic positions are invariant. It provides a baseline game-theoretical analysis as in many networking problems (see [27], [31]) for obtaining closed-form expression of equilibrium solutions and performances.

Each SU determines one channel to transmit on as a random process. Let p_i denote the probability with which i-th user selects the PC. We refer to p_i as the mixed strategy (or simply strategy) of user i. Consider a tagged SU i and let $U_i(p_i, p)$ denote its reward/utility when it plays a strategy p_i while the rest of SUs play strategy p. We define the utility of the tagged SU as its average good-put, given as[6]

$$U_i(p_i, p) = p_i p_c^I(r, \lambda_I + p\lambda_{II}, T_I) + (1 - p_i)p_c^{II}(r, (1 - p)\lambda_{II}, T_{II}). \quad (12)$$

Definition 1: A probability $p_g^* \in [0\ 1]$ is a SNE if

$$U_i(p_i, p_g^*) \leq U_i(p_g^*, p_g^*),$$

holds for all i and $p_i \in [0\ 1]$.

p_g^* indicates that no SU has an incentive to unilaterally deviate from the strategy p_g^* if all other SUs are playing it. p_g^* also gives the fraction of SUs using PC at equilibrium.

Proposition 5: Let the fading be Rayleigh distributed and $W \equiv 0$. Then we have

$$p_g^* = \frac{(T_{II}/T_I) - (\lambda_I/\lambda_{II})P^{\frac{2}{\beta}}}{T_{II}/T_I + 1}. \quad (13)$$

Proof: The proof idea is that when all the players play mixed strategy (13), $U(p_i, p)$ becomes independent of p_i, i.e., a tagged node becomes indifferent to its own strategy. Let SU i play mixed strategy p_i and all other SUs play mixed strategy p. Then for Rayleigh fading and $W \equiv 0$, we have

$$\begin{aligned} &U_i(p_i, p) \\ &= p_i a \exp\{-p\lambda_{II} C_I\} + (1 - p_i) \exp\{-(1 - p)\lambda_{II} C_{II}\} \\ &= p_i(a \exp\{-p\lambda_{II} C_I\} - \exp\{-(1 - p)\lambda_{II} C_{II}\}) \\ &\quad + \exp\{-(1 - p)\lambda_{II} C_{II}\}, \end{aligned}$$

where $a := \exp\{-\lambda_I P^{\frac{2}{\beta}} C_I\}$. Suppose all the SUs other than i-th SU play a strategy $\tilde{p}$ is such that

$$a \exp\{-\tilde{p}\lambda_{II} C_I\} = \exp\{-(1 - \tilde{p})\lambda_{II} C_{II}\}, \quad (14)$$

then $U_i(p_i, \tilde{p})$ does not depends on p_i and the i-th SU becomes indifferent to its own strategy. Further, if every SU

[6]Though there are infinitely many players in the game, we can compactly write utility U_i as if there are two players in the game- one is the tagged player and the other representing rest of the players that use the same strategy. We do so as our interest in the SNE of the game.

TABLE I
SUMMARY OF NOTATIONS

Notation	Meaning
p^*	unconstrained optimum of $d_s(p)$ in (7)
p_o^*	constrained optimum of $d_s(p)$ in (7)
$\bar{p}$	QoS constraint bound
p_g^*	SNE without cost
$p_g^*(\rho)$	SNE with cost $\rho \geq 0$

play a mixed strategy $\tilde{p}$ that satisfies (14), then unilateral deviation by a SU do not increase its reward. Hence, solution of (14) as given in (13) is the SNE of the game. ■

Notice that p_g^* is increasing with the ratio T_{II}/T_I and decreasing with λ_I/λ_{II}, thus

- if PC quality improves or SC quality degrades, then SUs will transmit more frequently on PC at equilibrium,
- if PU density decreases or SU density increases then SUs will transmit more frequently on PC at equilibrium.

Though the bias of the SUs to operate on PC increases with increasing quality of PC or decreasing density of the PUs, this is not always desired as it can violate the QoS requirement for the PUs. To avoid this, the SUs has to be de-incentivized from aggressively using PC. We next propose a pricing based de-incentivizing scheme.

A. Pricing Based Mechanism

To prevent the SUs from aggressively using PC, we introduce a price on SUs for transmitting on the PC. We assume that the network operator charges a fixed price per transmission from SUs that transmit on the PC, and this price is determined in collaboration with the owner of the PC knowing its QoS constraints. The network operator controls this price form the SUs and can regulate its value. Let $\rho > 0$ denote the price charged form SUs each time they transmit on PC. Taking into account this cost, we redefine utility of each SU as a weighted difference of performance (average good-put) and a dis-utility (expected transmission cost) as

$$\bar{U}_i(p_i, p) = U_i(p_i, p) - \alpha w(p_i) \quad \forall\, i \qquad (15)$$

where α is a trade-off factor indicating the importance each SU assigns to the performance compared to costs. For each SU, the transmission cost depends only on its strategy and for ith SU it is given as $w(p_i) = \rho p_i$. The other possible utility function is the average good-put per unit cost, i.e., $U_i(p_i, p)/\alpha w(p_i)$. But this utility leads to a game with degenerate SNE of $p_g^* = 0$,where SUs have no incentive to use PC. Henceforth, we focus on the game with utility (15) where we denote the resulting SNE as $p_g^*(\rho)$. For notational convenience, we write the product $\alpha\rho$ simply as ρ and refer to it as price. A list of notations summarized in Table I for easy reference.

Theorem 2: Assume $\lambda_{II} C_I \leq 1$ and $\lambda_{II} C_{II} > 1$. For Rayleigh fading and $W \equiv 0$, the game where SU's utility defined in (15) has a unique SNE. Further, the SNE $p_g^*(\cdot)$ is such that

- $p_g^*(\rho) = 0$ if $\rho \geq \exp\{-\lambda_I P^{\frac{2}{\beta}} C_I\} - \exp\{-\lambda_{II} C_{II}\}$
- $p_g^*(\rho)$ is strictly decreasing for all
$$0 \leq \rho \leq \exp\left\{-\lambda_I P^{\frac{2}{\beta}} C_I\right\} - \exp\{-\lambda_{II} C_{II}\}.$$

Proof: Rearranging the utility of a tagged SU, we have

$$\begin{aligned}\bar{U}_i(p_i, p) &= p_i(a\exp\{-p\lambda_{II} C_I\} - \rho - \exp\{-(1-p)\lambda_{II} C_{II}\}) \\ &\quad + \exp\{-(1-p)\lambda_{II} C_{II}\}, \qquad (16)\end{aligned}$$

where $a := \exp\{-\lambda_I P^{\frac{2}{\beta}} C_I\}$. Let $a - \exp\{-\lambda_{II} C_{II}\} \leq \rho$. Note that the coefficient multiplying p_i, i.e., $a\exp\{-p\lambda_{II} C_I\} - \rho - \exp\{-(1-p)\lambda_{II} C_{II}\}$ is a decreasing function in p, and is negative valued for all $p \in [0\ 1]$. Then $p_i = 0$ is optimal for SU i and is also a dominant strategy. Hence $p_g^* = 0$ is the SNE.

To prove the second part, recall from the proof of Proposition (5) that SNE of the game is given as the solution of

$$a\exp\{-p\lambda_{II} C_I\} - \exp\{-(1-p)\lambda_{II} C_{II}\} = \rho. \qquad (17)$$

The left hand side is continuous and strictly decreasing in parameter p. Hence, the value of solution is decreasing in ρ. ■

The quantity $\exp\{-\lambda_I P^{\frac{2}{\beta}} C_I\} - \exp\{-\lambda_{II} C_{II}\}$ denotes the marginal gain in success probability of a SU from transmitting on PC compared to that on SC when rest of the SUs transmit on the SC. If this marginal gain is smaller than the transmission cost on PC, then the SU has no incentive to use PC. Hence we obtain $p_g^* = 0$ in the first part of the theorem. On the other hand, when the marginal gain is larger than the transmission cost, the SUs benefit from transmitting on PC. However, with higher costs, the benefit is diminishing and SUs use PC less frequently as noted in the second part of the theorem. Further, by setting an appropriate price the operator can achieve any SNE in the interval $[0\ p_g^*(0)]$. See the proof for details.

Optimal Price: The operator would like to set a price such that the success density of the SUs is maximized without violating the QoS requirement for the PUs. If $p_o^* \in [0\ p_g^*(0)]$, then the operator can find a price $\rho^* \geq 0$ such that $p_g^*(\rho^*) = p_o^*$ and achieve global optimal performance at equilibrium. However the condition $p_o^* \in [0\ p_g^*(0)]$ need not always hold. The following Theorem characterizes the achievability the global optimal performance at equilibrium.

Theorem 3: Let the conditions in theorem (2) hold. Global optimal performance is achievable at equilibrium if and only if $\bar{p} \leq p_g^*(0)$.

Proof: A straightforward evaluation of derivatives in (8) and (9) at $p = p_g^*(0)$, we see that $\mathrm{d}d_s^{II}(p_g^*(0))/\mathrm{d}p > 0$. Hence, from Proposition (3) the unconstrained optima p^* of $d_s(\cdot)$ satisfies $p_g^*(0) < p^*$. Also, from Proposition (4) we have $\bar{p} = po^*$. Let $\bar{p} \leq p_g^*(0)$. The second part of Theorem (2) guarantees existence of a ρ^*>0 such that $\bar{p} = p_g^*(\rho^*)$. This proves the sufficient condition. Let $\bar{p} > p_g^*(0)$. From Proposition (3) and (4) we have that $p_o^* = p^*$ if $\bar{p} > p^*$ or $p_o^* = \bar{p}$ if $\bar{p} \leq p^*$. Since $p_g^*(0) \leq p^*$, it must be the case that $p_g^*(0) < p_o^*$. Now, by for any $\rho \geq 0$, we only achieve a SNE smaller than $p_g^*(0)$. Hence, global optimal is not possible for any $\rho \geq 0$. This concludes the proof. ■

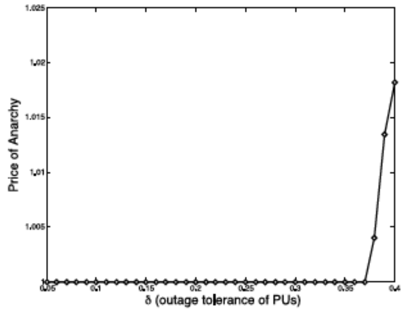

Fig. 2. PoA vs. outage tolerance. We set $P_I = P_{II}, \lambda_{II} C_{II} = 2, \lambda_{II} C_I = 0.2, \lambda_{II} \bar{C}_I = .5, \lambda_I \bar{C}_I = 0.05, \lambda_I C_I = 0.02$..

As given in the proof, for any given ρ, the corresponding SNE $\tilde{p} := p_g^*(\rho)$ satisfies

$$a \exp\{-\tilde{p}\lambda_{II} C_I\} - \exp\{-(1-\tilde{p})\lambda_{II} C_{II}\} = \rho.$$

Alternatively, to achieve any SNE p_g^* we can set a value of ρ for which the above relation holds with $\tilde{p} = p_g^*$. Specifically, by setting $\tilde{p} = p_o^*$, we can find the corresponding ρ that results in global optimal performance at equilibrium.

When $\bar{p} > p_g^*(0)$, the pricing strategy is not effective and there is degradation in the social performance due to selfish nature of SUs. This degradation can be measured using price of anarchy (PoA) [32] defined in our context as follows:

$$PoA = \min_{\rho \geq 0, p_g^*(\rho) \leq \bar{p}} \frac{d_s(p_o^*)}{\lambda_{II} U(p_g^*(\rho))}, \tag{18}$$

where $\lambda_{II} U(p_g^*(\rho))$ is the SUs' spatial density of successful transmissions at SNE $p_g^*(\rho)$. PoA as a function of δ is shown in Figure 2. When δ is small, through pricing selfish nature of the SUs can be directed to achieve global optimal performance at equilibrium (PoA $= 1$). For large values of δ, there exists no non-negative price to control the selfish behavior and hence the PoA is high. However, in this regime, if the operator pays the SUs instead of charging them (negative ρ), global optimal performance can be still achieved.

V. Dynamics of the Game

In this last section, we study the dynamics of decision for this channel selection game. We first establish that the game is equivalent to a routing game with a continuum of users.[7] Specifically, we show that the SNE of the game is the same as the Wardrop Equilibrium (WE) [11] of an equivalent routing game. Though the equivalence seems natural, we have not come across this equivalence anywhere and present it here for completeness. The advantage of this equivalence is that we can readily use replicator dynamics to compute SNE of the channel selection game.

A. Equivalent Routing Game

We first recall the Wardrop setting. In this setting, certain traffic is to be shipped across a set of routes. Total traffic to be shipped constitute the set of non-atomic players, and the cost of using each route depends on the total traffic on that route. To map channel selection game to a Wardrop setting, we can view each SU as a non-atomic player and transmissions of SUs as the total traffic (load) to be transported across two routes that are represented by PC and SC. The set of transmitters constitute a common *source* (S) and the set of receivers a common *destination* (D). In the channel selection game, the success probability depends on the total density of SUs using that channel. We can thus assign the cost of a route in Wardrop setting as the success probability (with price) on that route/channel. The routing model is such that congestion costs denoted $R_1(\cdot)$ and $R_2(\cdot)$ are

$$R_1(p) = \exp\left\{-p\lambda_I C_I P^{\frac{2}{\beta}}\right\} \exp\{-p\lambda_{II} C_I\} - \rho$$
$$R_2(p) = \exp\{-(1-p)\lambda_{II} C_{II}\}.$$

We refer to this game model as routing equivalent of channel selection game where the notion of equilibrium is WE which satisfies the following principles [11]. *"The journey times on all the routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused route"* Let $(p_w^*, (1-p_w^*))$ denote the WE of the equivalent routing game.

Theorem 4: The SNE of the channel selection is the same as the WE of its equivalent routing game, i.e., $p_w^* = p_g^*(\rho)$.

Proof: Applying the principles of Wardrop equilibrium, we can find a η such that

$$R_1(p_w^*) \geq \eta$$
$$R_2((1-p_w^*)) \geq \eta$$
$$p_w(R_1(p_w^*) - \eta) = 0$$
$$(1-p_w^*)(R_1(1-p_w^*) - \eta) = 0.$$

Let $0 < p_w^* \leq 1$, above conditions then imply that $R_1(p_w^*) = R_2(1-p_w^*)$ which is the same as the condition of SNE (see (14)). Now consider $p_w^* = 0$. Applying the above conditions again, we obtain $R_1(0) = \eta \leq R_2(1)$, which implies $\exp\{-p\lambda_I C_I P^{\frac{2}{\beta}}\} - \exp\{-\lambda_{II} C_{II}\} \leq \rho$. Thus $(p_w^*, (1-p_w^*))$ satisfy properties of SNE in Theorem 2 and hence $p_w^* = p_g^*(\rho)$. ■

B. Replicator Dynamics

Given that the channel selection problem is equivalent to a routing game (or a populations game), we can apply replicator dynamics [24] to study its behavior. Let $p(t, P)$ denote the fraction of SUs that take action P at time t. It is straightforward to compute the solutions to the replicator equation for $p(t, P)$, which is given as follows:

$$\mathrm{d}p(t,P)/\mathrm{d}p$$
$$= p(t,P)(1-p(t,P))\{R_1(p(t,P)) - R_2(p(t,P))\}. \tag{19}$$

The stationary points of the above dynamics are clearly either 0, 1 or a p such that $R_1(p) = R_2(p)$, which is in agreement with our earlier observations. We note that replicator dynamics indicates that $p = 1$ could be an SNE, but under the regime $\lambda_{II} C_{II} > 1$ and $\lambda_{II} C_I \leq 1$ this equilibrium is not realizable. For the sake of completeness, we discuss the derivation of the replication dynamics for our setting next.

[7] Note that in the channel selection game the number of players is countable, whereas it is uncountable in the routing game.

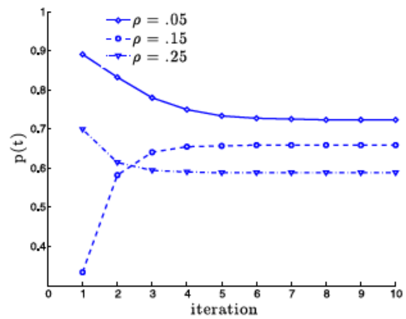

Fig. 3. Convergence of DRD as ρ varies.

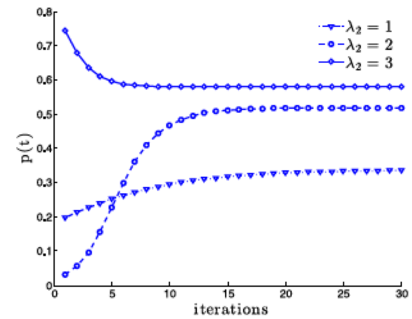

Fig. 4. Convergence of DRD as λ_2 varies.

Derivation of Replicator Dynamics

To use population game terminology, let $F : \{P, S\} \times [0\ 1] \to \mathbb{R}$ denote fitness function of each SU, where $F(A, p')$ is fitness of a SU when it plays action $A \in \{P, S\}$ while the rest of the SUs play mixed strategy p'. Fitness function indicates the benefit of taking an action. Then fitness for taking action P is $F(P, p') = R_1(p')$ and fitness for taking action S is $F(S, p') = R_2(1-p')$. The utility of a SU that plays mixed strategy p when rest play p' (see (15)) is related to its fitness function as $U(p, p') = pF(P, p') + (1-p)F(S, p')$. Let $p(t, p)$ denote the fraction of SUs that take action P at time t. $p(t, p)$ evolves according to replicator dynamics as [24]:

$$\mathrm{d}p(t, P)/\mathrm{d}p = p(t, P)\{F(P, p(t, P)) - \bar{F}(t)\} \qquad (20)$$

where $\bar{F}(t)$ denotes the mean fitness of the populations given by $\bar{F}(t) = p(t, P)F(P, p(t, P)) + (1-p(t, P))F(S, p(t, P))$. Substitute the values of the fitness function and we arrive at

$$\begin{aligned}
&\mathrm{d}p(t, P)/\mathrm{d}p \\
&= p(t, P)(1 - p(t, P))\{F(P, p(t, P)) - F(S, p(t, P))\} \\
&= p(t, P)(1 - p(t, P))\{R_1(p(t, P)) - R_2(1 - p(t, P))\}.
\end{aligned}$$

■

Discrete Version: Considering a Euler discretization of the replicator equation (13), we obtain an equivalent discrete-time version of the replicator dynamics:

$$p_{t+1} = p_t \frac{R_1(p_t)}{(1 - p_t)R_2(p_t) + p_t R_1(p_t)}.$$

From the folk theorem of evolutionary game theory, the Nash equilibria are rest points of the above dynamics [33]. In Fig. (3)–(4), convergence of discrete replicator dynamics is shown for various parameters starting from random initial points and following parameters: $\lambda_1 = .1$, $\lambda_2 = 5$, $T_1 = .1$, $T_2 = 1$, $P_1 = 4$, $P_2 = 1$, $r = .3, r_1 = .5$. In Figure 3, we plot updates of discrete replicator dynamics as the value of ρ varies. We repeat this experiments by varying λ_2 in Figure (4). As seen, the dynamics converges in a few iterations.

C. Decentralized Protocol

The replicator dynamic equations (19) are the limit of protocols and learning algorithms that can be practically implemented in real networks. For example, protocols based on imitation rules (described in [34] in a cognitive radio network) or utility estimation (as proposed in [35]) played repeatedly converge to the replicator equations. Such algorithms are easily implementable because only local information on goodput is needed. Therefore, many works nowadays combine machine learning techniques to enhance performance of future wireless systems and it motivates the need of AI for wireless technologies. We describe next a pseudo-code of the imitation protocol that can be implemented in each SU.

1) Initially, randomly choose between primary and secondary channel to transmit. Then, store the goodput obtained.
2) While at each iteration do
    a) Select randomly a channel and compare the new goodput to the previous one.
    b) If the new goodput is higher, then **Migrate** to this channel with probability proportional to the difference of goodput.

This algorithm is based on particular information for each mobile, which is his goodput depending on the channel used to transmit on. Many other protocols based on imitation rules are proposed in [36, Ch. 3]. In this chapter, the authors propose an algorithm in which SUs exchange estimated goodput and then choose the best one to transmit. Their proposed imitation-based protocol can achieve efficient spectrum utilization meanwhile provide good fairness across SUs. In the case of imperfect information on the goodput, one can resort to recent machine learning techniques for games as in [37].

VI. Sharing Spectrum With Cellular Network

A MANET co-existing with a cellular network is considered. The users of MANET can transmit on an unlicensed spectrum and uplink channel of the cellular network. As earlier, we refer to users of the MANET as SUs and that of the cellular network as PUs. The discussion in this section is brief; once the success probability of a PU and an SU on the uplink channel is derived, the rest of the analysis is similar to that of the previous sections.

A. Model and Setup

For SUs, we use the same bipolar MANET model of Section II. We follow the setup in [9] to model the uplink of a cellular network with power control. The base stations (BSs) and PUs are modeled as two independent homogeneous P. P. P. of intensity λ_B and λ_M respectively. Let $\Phi^C = \{X_i^C\}_{i\geq 1}$ denote locations of the PUs. Each PU attaches to the nearest

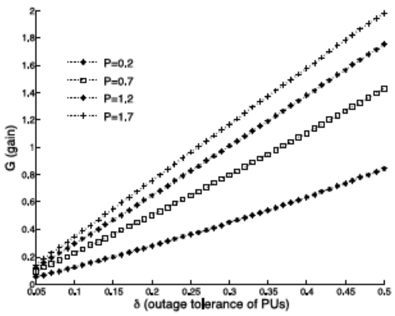

Fig. 5. Gain vs Outage for different primary and secondary transmit power ratios.

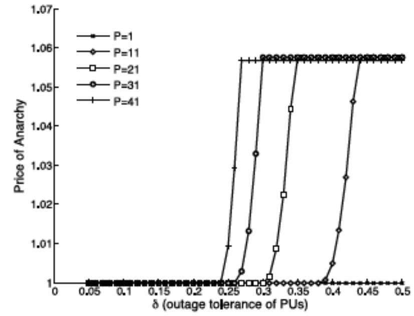

Fig. 6. PoA vs Outage for different primary and secondary transmit power ratios.

BS, thus the cellular network forms a Poisson-Voronoi tesselation of two-dimensional plane. Each BS serves a single PU, that is randomly selected from the Voronoi cells, on a time-frequency resource. Let $M_i^C = \{F_i^C(n), F_i^{C-II}(n)\}_{n\geq 0}$ denote the sequence of marks associated with PU i, where $F_i^C(n) = \{F_{ij}^C(n) : j \geq 1\}$ denote the sequence of channel conditions between the PU i and the BSs, and $F_i^{C-II}(n) := \{F_{ij}^{C-II}(n) : j \geq 1\}$ denote sequence of channel conditions between PU i and the SUs.

Consider a typical SU at origin and let it connect to a tagged BS located at B_0. Let R denote distance between the typical SU and B_0. It is easy to note that R is Rayleigh distributed, given as $f_R(r) = 2r\pi\lambda_B \exp\{-\lambda_B \pi r^2\}$. Further, let $D_i = |X_i - B_0|$ denote distance between an interfering SU located at X_i and B_0, and R_i denote interferer's distance to its serving BS. We denote the set of active users obtained from thinning of Φ^C as $\tilde{\Phi}^C$. The SINR of the typical SU in slot n is

$$SINR^C(n) = \frac{P_M R^{\beta\epsilon} F_{i0}^C(n) R^{-\beta}}{I_{\tilde{\Phi}^C(n)}(B_0) + I_{\Phi_1^{II}(n)}(B_0) + W(n)} \quad (21)$$

where P_M denotes the transmit power of PUs, $\epsilon \in [0, 1]$ is the power control factor, $I_{\tilde{\Phi}^C} := \sum_{X_i \in \tilde{\Phi}^C} P_M F_{i0}^C(n) R_i^{\epsilon\beta} D_i^{-\beta}$ denotes the shot noise filed of PUs and $I_{\Phi_1^{II}(n)}(\cdot)$ is the shot noise of SUs defined in (4). A typical SU is said to be covered when $SINR^C(n)$ is larger than T_C.

It is noted in [9] that $\{R_i\}s$, though identical, need not be independent, and the P. P. P. $\tilde{\Phi}^C$ is not tractable. Further, for analysis, the following reasonable approximation are made in [9]. (1) $\{R_i\}$s are independent and their marginal distributions are identical distributed given as $f_{R_i}(r|D_i) = 2\lambda_B \pi r \exp\{-\lambda_B \pi r^2\}/(1 - \exp\{-\pi\lambda D_i^2\})$ (2) $\tilde{\Phi}^C$ is non-homogeneous with distance dependent intensity function given as $\tilde{\lambda}_M(d) = \lambda_B(1 - \exp\{-\lambda_B \pi d^2\})$. We continue to make these assumptions to derive the coverage of the typical PU and SU on the uplink, denoted as $p_c^C := p_c^C(\lambda_C, T_C, \epsilon)$ and $p_c^{II-C} := p_c^{II-C}(r, \lambda_C, T_C, \epsilon)$, respectively. where $\lambda_C = \lambda_M + p\lambda_{II}$.

Proposition 6 ([9, Th. 2]): Assume $W \equiv 0$. Let the fading process be Rayleigh distributed with mean 1 and each SU selects PC with probability p in each slot. Then, coverage probability of a typical PU and SU on PC is, respectively,

$$p_c^C = \mathbb{E}_R\Big[\mathcal{L}_{I_{\tilde{\Phi}^C}}\Big(T_C/P_M R^{\beta(\epsilon-1)}\Big) \times \mathcal{L}_{I_{\Phi_1^{II}}}\Big(T_C/P_M R^{\beta(\epsilon-1)}\Big)\Big]$$

$$p_c^{II-C} = \mathcal{L}_{I_{\tilde{\Phi}^C}}(T_C/P_{II} l(r)) \mathcal{L}_{I_{\Phi_1^{II}}}(T_C/P_{II} l(r)),$$

where $\mathbb{E}_R[\cdot]$ denotes expectation with respect to rv R.

For the case of perfect power control, i.e., $\epsilon = 1$, the coverage probabilities of a typical PU and SU can be computed explicitly using

$$\mathcal{L}_{I_{\Phi_1^{II}}}\left(\frac{T_C}{P_{II} l(r)}\right) = \exp\Big\{-p\lambda_{II} r^2 T_C^{2/\beta} K(\beta)\Big\}$$

$$\mathcal{L}_{I_{\Phi_1^{II}}}\left(\frac{T_C}{P_M}\right) = \exp\Big\{-p\lambda_{II} (P_{II}/P_M)^{2/\beta} T_C^{2/\beta} K(\beta)\Big\}$$

$$\mathcal{L}_{I_{\tilde{\Phi}^C}}\left(\frac{T_C}{P_M}\right) = \exp\Bigg\{-2\pi\lambda_B \int_0^\infty \int_0^{x^2} \times \frac{\pi\lambda_B e^{-\pi\lambda_B u} \mathrm{d}u x \mathrm{d}x}{1 + T_C^{-1} u^{-\beta/2} x^\beta}\Bigg\}.$$

Notice the similarity of SU coverage probabilities on the cellular network with that in the previous section where PC is another MANET. Then, the density of successful transmissions of SUs as a function of p has similar properties in both the cases. By setting $P_I = P_M$, $\lambda_B = \lambda_I$ in Sections III and IV we can obtain the sharing gain and the equilibrium behavior (upto constant factors) for the case with cellular networks. Hence we skip the details.

B. Numerical Results

We compare the performance gain and price of anarchy of spectrum sharing systems when the primary system consists of a MANET or a cellular network. In Figure 5, we plot gain of SUs as a function of QoS parameter δ for different values of primary and secondary transmit power ratio $P = P_I/P_{II}$. As expected, the gain increases with relaxing the outage constraint. Note that for a fixed δ, the gain is increasing in p. To understand this, we first fix a P_{II} and note that $\bar{p}$ and observe that the maximum fraction of allowed SUs on PC increases with P (or P_I). Thus, more SUs can transmit on PC and increase their throughput. Now fix P_I, by lowering P_{II}, we observe that more SUs can operate on PC and thus exploit the benefit of nonconcurrent transmissions on both the channels. To understand the effect of transmission powers when the nodes are non-cooperative, we plot PoA as a function of δ for different value of p in Figure 6. For each p, increasing δ increases the loss due to non-cooperative behavior of the SUs. Also, when outage constraints are relaxed (increased), SUs have to be incentivized to transmit on PC. For a given δ, decreasing p decreases PoA, i.e., by decreasing (increasing) primary (secondary) transmit power, network throughput increases.

VII. Conclusion

In this work, we considered the scenario where the users of a MANET (secondary) operating on an unlicensed also transmit on a licensed spectrum to improve their throughput. We introduced the metric 'Sharing Gain' that measures the gain in throughput in a MANET using both the licensed and unlicensed channels compared to the case where they can only use the unlicensed channel. We characterized the sharing gain of the secondary network and showed that it can be significant for large-scale networks. specifically, we showed that if the QoS on the primary channel is stringent, one may be better off by not sharing the spectrum with the primary users on the licensed channel, but use only the unlicensed spectrum with some contention resolution protocol. We also considered non-cooperative behavior and studied the channel selection games where the SUs selfishly select a channel for transmission. For this case, we proposed a pricing mechanism that achieves global optimal performance at equilibrium in some cases. Finally, we established an equivalence between the channel selection game and a routing game and developed algorithms to study the dynamics of the system using replicator dynamics.

Decentralized access channel techniques are crucial for the deployment of Cognitive Radio technologies in the context of IoT and 5G networks. Our study evaluates theoretically the gain of such methodology and also opens interesting questions: is there an alternative framework (semi-distributed) that can be easily implemented into sensors/actuators, how the control has to be reactive with the current decision processes, how to manage the interoperability among the devices in the revision protocols, and so on. Finally, combining the decision process of secondary users with existing tools in cognitive radio networks such as "detect-and-avoid" may lead to interesting hierarchical game model.

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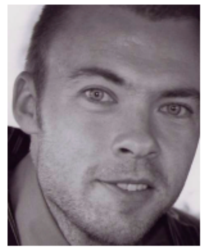

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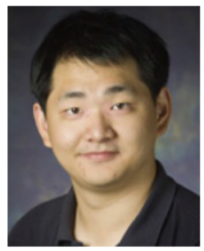

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