Decision-Based Approach to Account for Uncertainty in Estimating the Overtopping Hazard to Manage Risk for Dams

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Abstract: This paper describes and demonstrates an approach to improve the management of risks from small-probability events that can lead to large consequences. It applies a decision-based theory to account for limited information in estimating frequencies for rare events to large rockfill dam in Norway that is being assessed for rehabilitation. Uncertainties are considered specifically in estimating the overtopping hazard for the existing dam and for an elevated dam crest. Uncertainty in the estimates of the overtopping hazard curve means that smaller costs of dam failure and/or larger costs of rehabilitation may be justified. From a practical perspective, a cost of rehabilitation in this case that is nearly ten times larger could be justified when the uncertainty in the estimate of the hazard curve is considered. The value of perfect information about the hazard curve increases as the amount of information available decreases and as the cost of failure relative to the cost of rehabilitation decreases. In this case, the value of perfect information about the hazard curve is about 25 percent of the cost to raise the dam crest.

Keywords: Decision Entropy theory; dam remediation; decision analysis; Bayes theory; risk; flood analysis.

1 Introduction

The motivation of this paper is to describe and demonstrate an approach to improve the management of risks from small-probability events that can lead to large consequences, such as estimating the overtopping hazard for an existing dam in order to decide whether or not to raise its crest. The challenge with these events is that there is relatively little information available to assess their probability. For example, a 1/10,000-year frequency will be estimated with generally less than 100 years of data.

The approach to account for limited information used here is a new theory being developed by the authors to frame uncertainty in the context of decisions that are affected by that uncertainty. The goal of the theory is to facilitate assessing small probability values for use in decision analysis. This theory is applied to a large rockfill dam in Norway being assessed for rehabilitation. Uncertainties in assessing the overtopping frequencies for the existing dam and for an elevated dam crest are considered. The theoretical basis for the theory is described and its implementation is demonstrated. Lastly, practical insight obtained from applying the theory to this problem is discussed.

2 Decision Analysis for Nesjen Dam

Nesjen is a regulated reservoir created by a main rockfill dam and three secondary dams in Kvinesdal municipality of Southern Norway. It is one of the input power sources of the Tonstad hydropower plant downstream. The Nesjen dams and the Tonstad hydropower plant are owned by the Sira-Kvina Power Company. The main Nesjen Dam was constructed in the 1960's. To be compliant with current regulations, the crest of the dam needs to be raised by 1 m, from H_{A1} = 716.6 m to H_{A2} = 717.6 m. The question is: Is the cost of raising the dam crest by 1 m worth the benefit in reducing the risk?

A decision tree framing this decision is shown in Fig. 1, where Pf_{A1} and Pf_{A2} are the probabilities the dam fails due to overtopping in a 100-year planning period and f_{A1} and f_{A2} are the annual frequencies of overtopping for crest heights H_{A1} and H_{A2}, respectively. Based on hydrologic analyses and expert elicitation, the frequencies of overtopping are estimated as f_{A1} = 1/1,000 1/year for the existing crest height and f_{A2} = 1/10,000 1/year for the raised crest height. If the dam is overtopped, there is assessed to be a 0.002 probability of dam failure (NGI, 2018), meaning the probability of dam failure in 100 years is $Pf_{Ai} = 0.002[1-(1-f_{Ai})^{100\ \text{years}}]$ for dam crest heights i = 1 or 2.

The consequence of raising the dam crest by 1 m is c_I, and the consequence of failure if the dam due to overtopping is c_F, where c_I, and c_F are negative and can either be expressed as an economic cost or a non-dimensional utility value. For context, the economic cost of raising the dam is on the order of $10,000,000 US. The preferred alternative has the maximum expected utility, u_{A1} versus u_{A2} obtained from:

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$$u_{A_1}(f_{A_1}, f_{A_2}) = 0.002\left[1-\left(1-f_{A_1}\right)^{100\text{ years}}\right]c_F \quad (1)$$

$$u_{A_2}(f_{A_1}, f_{A_2}) = 0.002\left[1-\left(1-f_{A_2}\right)^{100\text{ years}}\right]c_F + c_I \quad (2)$$

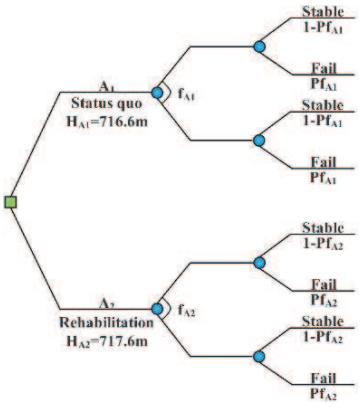


Figure 1. Decision tree comparing two alternatives (A_1 and A_2) of the Nesjen dam

3 Uncertainty in Overtopping Hazard

There is considerable uncertainty in estimating the frequencies of overtopping for the two dam heights, f_{A1} and f_{A2}. The dam has only been in operation for less than 60 years; changing climatic conditions may change hydrologic patterns in the future; the storage volume and release rates from the reservoir versus time are strongly dependent on daily economic decisions related to power generation and storage; and the frequencies of interest are very small (i.e., on the order of once per 1,000's to 10,000's of years).

If overtopping occurrences for each crest height are independent from year, then the likelihood of observing X_{A1} events where the existing dam crest is overtopped and X_{A2} events where the raised dam crest is overtopped (note that $X_{A1} \geq X_{A2}$) in a time period of t years is given as a function of f_{A_1} and f_{A_2} by a multinomial distribution:

$$P\left(X_{A_1}, X_{A_2} \middle| f_{A_1}, f_{A_2}\right) = k\left(1-f_{A_1}\right)^{t-X_{A_1}}\left(f_{A_1}-f_{A_2}\right)^{X_{A_1}-X_{A_2}} {f_{A_2}}^{X_{A_2}} \quad (3)$$

where k is constant independent of f_{A_1} and f_{A_2}. Since it is extremely unlikely that either of the dam heights will have been overtopped in only 60 years (in fact, neither have been exceeded), the estimates for f_{A1} and f_{A2} are based on significant extrapolation. If the time of observation is changed, then Eq. (3) can be generalized as follows:

$$P\left(X_{A_1}, X_{A_2} \middle| f_{A_1}, f_{A_2}\right) = k_N\left[\left(1-f_{A_1}\right)^{t-X_{A_1}}\left(f_{A_1}-f_{A_2}\right)^{X_{A_1}-X_{A_2}} {f_{A_2}}^{X_{A_2}}\right]^N \quad (4)$$

where N is the number of available time periods and k_N is a constant. Without loss of generality, N could be greater than one (a longer time period) or less than one (a shorter time period).

For this application, the available information will be represented as an equivalent time period of information equal to tN (say 60 years). The estimate that the overtopping frequency for the existing dam height is equal to 0.001 1/year will be represented by setting the equivalent number of overtopping occurrences to be equal to the expected number of occurrences in tN, $X_{A1}N = 0.001tN$; similarly, $X_{A2}N = 0.0001tN$. This likelihood function is illustrated in Fig. 2; note that the likelihood is sharper (more informative) as the equivalent time period of information increases.

4 Incorporating Information about Overtopping Hazard into Decision

In order to decide between maintaining or raising the dam crest (Fig. 1), the probability for different combinations of f_{A1} and f_{A2} is obtained from Bayes' theorem as follows:

$$P\left(f_{A_1}, f_{A_2} \middle| X_{A_1}, X_{A_2}\right) = \frac{P\left(X_{A_1}, X_{A_2} \middle| f_{A_1}, f_{A_2}\right) P\left(f_{A_1}, f_{A_2} \middle| Decision\right)}{\sum_{all\, f_{A_1}, f_{A_2}} P\left(X_{A_1}, X_{A_2} \middle| f_{A_1}, f_{A_2}\right) P\left(f_{A_1}, f_{A_2} \middle| Decision\right)} \quad (5)$$

where $P(f_{A_1}, f_{A_2}|X_{A_1}, X_{A_2})$ is the updated probability for f_{A1} and f_{A2} given the available information, $P(X_{A_1}, X_{A_2}|f_{A_1}, f_{A_2})$ is the likelihood function (Eq. 4)[1], and $P(f_{A_1}, f_{A_2}|Decision)$ is the prior (i.e., before information) probability for f_{A1} and f_{A2} given the decision between maintaining or raising the dam crest. This updated probability is incorporated into the decision analysis to obtain the expected utilities for each alternative i:

$$E[u_{A_i}(f_{A_1}, f_{A_2})] = \sum_{all\ f_{A_1}, f_{A_2}} u_{A_i}(f_{A_1}, f_{A_2}) P(f_{A_1}, f_{A_2}|X_{A_1}, X_{A_2}) \quad (6)$$

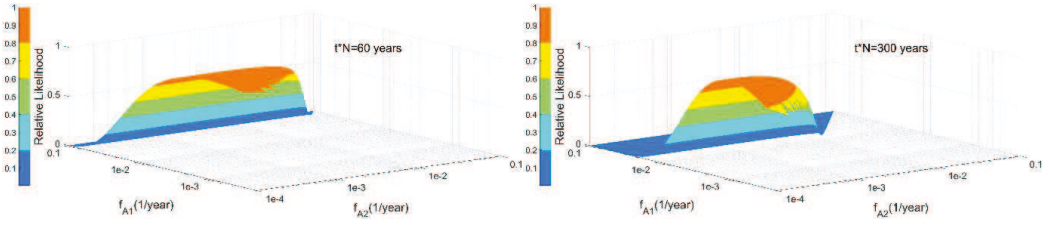


Figure 2. Likelihood function of combinations of f_{A1} and f_{A2}.

5 Non-Informative Prior Probability Based on Decision Entropy Theory

The Theory of Decision Entropy (Gilbert et al. 2012, 2016; Mostofi 2018) is being developed to establish non-informative prior probabilities in the context of making a decision, $P(f_{A_1}, f_{A_2}|Decision)$. This theory is derived from three principles:

1. If no information is available about the probabilities of f_{A1} and f_{A2}, then a selected alternative is equally probable to be or not to be the preferred alternative.
2. If no information is available about the probabilities of f_{A1} and f_{A2}, then the possible differences in preference between a selected alternative and the preferred alternative are equally probable.
3. If no information is available about the probabilities of f_{A1} and f_{A2}, then the possibilities of learning with new information about the selected alternative compared to the preferred alternative are equally probable.

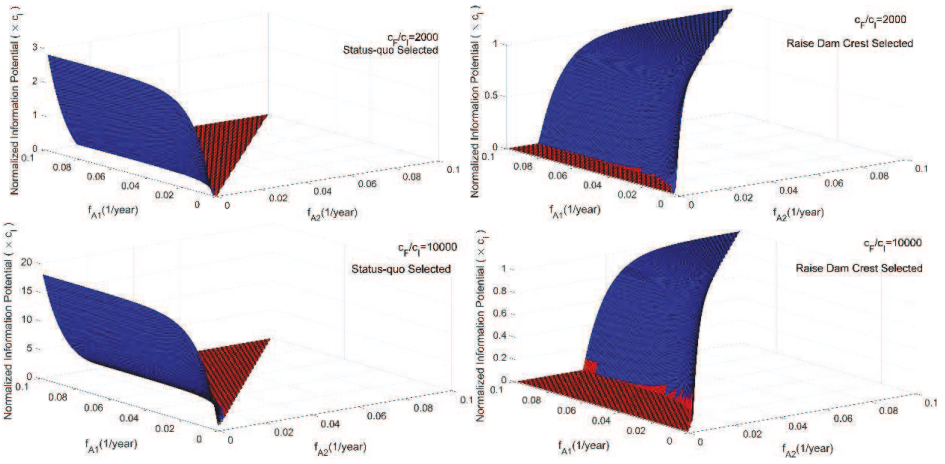
The theory is implemented mathematically using the Theory of Information Entropy (Shannon 1948). The prior probability for a selected decision alternative is obtained by maximizing the entropy of the information potential:

$$\Delta u_{A_j}(f_{A_1}, f_{A_2}) = u_{A_j}(f_{A_1}, f_{A_2}) - max[u_{A_1}(f_{A_1}, f_{A_2}), u_{A_2}(f_{A_1}, f_{A_2})] \quad (7)$$

where $\Delta u_{A_j}(f_{A_1}, f_{A_2})$ is the information potential if either the dam is not raised (A_1) or the dam is raised (A_2). The information potential is less than or equal to zero: it equals 0 if A_j is the preferred alternative and it is less than zero if A_j is not the preferred alternative. Accounting for uncertainty in f_{A1} and f_{A2}, the preferred alternative will have the maximum expected value of the information potential. For generality, the information potential will be normalized here by the cost to raise the dam, c_I. The information potential depends both on the selected alternative and on the ratio of c_F/c_I (Fig. 3).

Maximizing the entropy of the information potential according the three principles of the Theory of Decision Entropy produces non-informative probability distributions for the information potential that have a probability mass of 0.5 when information potential is 0 and a uniform probability density over the range of information potential values less than 0 (Fig. 4). The non-informative probability distributions for f_{A1} and f_{A2} are obtained by mapping the probability distributions for the information potential (Fig. 4) onto relationship between information potential and f_{A1} and f_{A2}, in which the third principle of the Theory of Decision Entropy is approximately satisfied by making all combinations of f_{A1} and f_{A2} that give the same information potential equally probable. Example non-informative probability distributions for f_{A1} and f_{A2} are shown in Fig. 5.

[1] Note that the constant k_N does not need to be explicitly evaluated since it is in both the denominator and the numerator of Eq. (5).



(a) Status-quo selected (b) Rehabilitation selected

Figure 3. Information potential for alternative dam crests versus annual frequencies of overtopping.

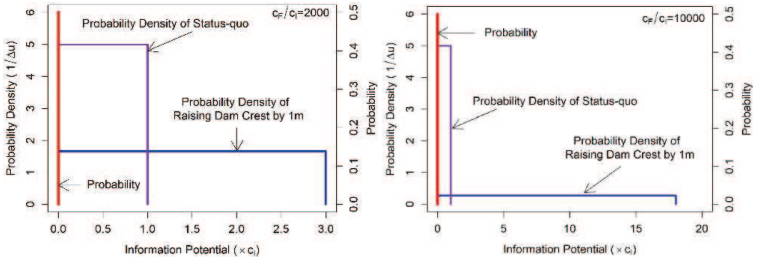
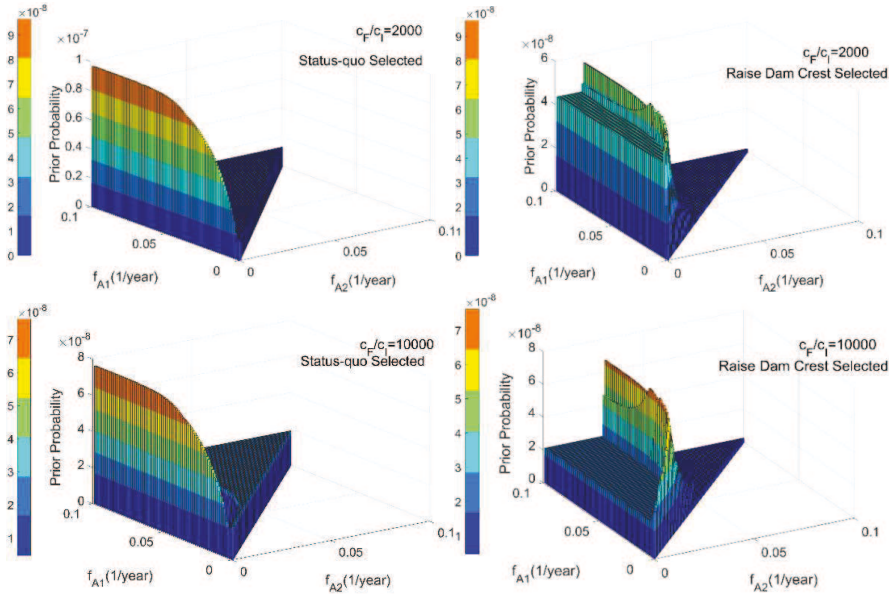


Figure 4. Non-informative probability distributions of information potential for each alternative.



(a) Status Quo selected (b) Rehabilitation selected

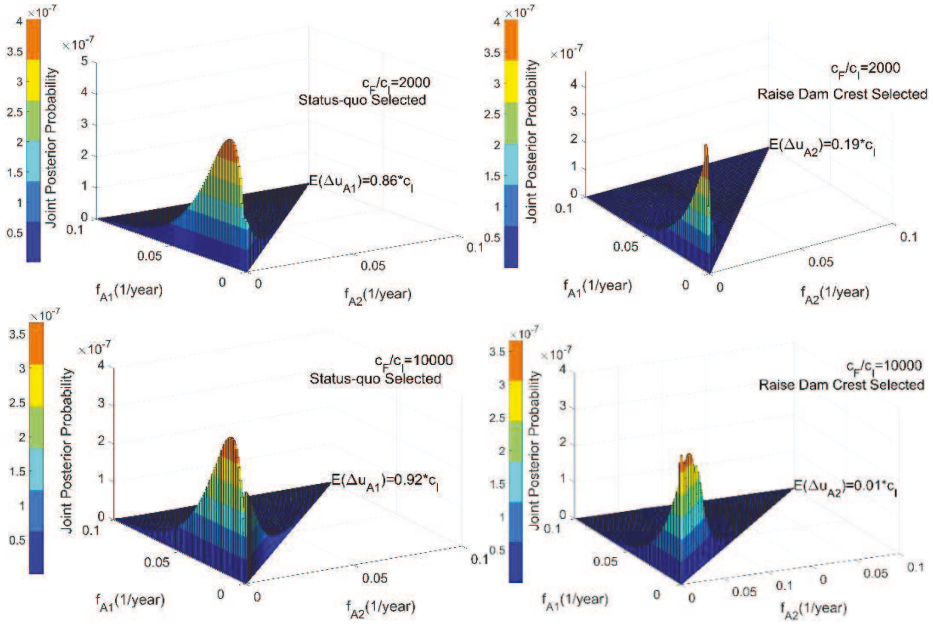
Figure 5. Example non-informative prior probability distributions for f_{A1} and f_{A2}.

6 Results

Example updated probability distributions for the overtopping frequencies, f_{A1} and f_{A2}, from Bayes' Theorem (Eq. 5) are shown in Fig. 6. The preferred alternative has the maximum expected information potential.

$$E\left[\Delta u_{A_j}(f_{A_1}, f_{A_2})\right] = \sum_{all\ f_{A_1}, f_{A_2}} \Delta u_{A_j}(f_{A_1}, f_{A_2}) P(f_{A_1}, f_{A_2} | X_{A_1}, X_{A_2}) \quad (8)$$

In addition, the value of perfect information about the overtopping frequencies is equal to the negative of the expected information potential.



(a) Status quo selected (b) Raising dam crest selected

Figure 6. Posterior joint probability distributions for f_{A1} and f_{A2}.

The expected information potential for the preferred alternative is shown in Fig. 7 versus the normalized cost of failure, c_F/c_I, for different amounts of information (i.e., the equivalent years of experience used to estimate the overtopping frequencies, tN). For a given amount of information, the alternative of raising the dam crest becomes preferred over maintaining the status quo as the cost of failure increases relative to the cost of raising the dam crest. For the case of no information ($tN = 0$, which gives a flat likelihood function), raising the dam crest is preferred for $c_F/c_I > 1000$ (Fig. 7). For the case of perfect information ($tN = \infty$, which gives a likelihood of 1.0 for the estimated values of $f_{A1} = 0.001$ 1/year and $f_{A2} = 0.0001$ 1/year and a likelihood of zero for all other combinations), raising the dam crest is preferred for $c_F/c_I > 6{,}000$ (Fig. 7). For amounts of information between nothing and everything, the threshold value of c_F/c_I ranges between the two extremes. The uncertainty in estimates of the hazard curve (f_{A1} and f_{A2}) captured by the non-informative prior probability distribution means that rehabilitation may be justified with smaller costs of dam failure and/or larger costs of rehabilitation.

From a practical perspective, the actual amount of information is closer to 100 years than 10,000 years. However, in many risk assessments for a dam, the hazard curve would assume to be known (i.e., $f_{A1} = 0.001$ 1/year and $f_{A2} = 0.0001$ 1/year). The difference between these two cases is that a cost of rehabilitation that is nearly ten times larger could be justified when the uncertainty in the estimate of the hazard curve is considered.

The negative or absolute value of the expected information potential is equal to the value of perfect information about the overtopping hazard curve ($f_{A1} = 0.001$ 1/year and f_{A2}). For example, if the cost of a dam failure is 1,000 times the cost of raising the dam crest and an equivalent of 100 years of experience are used to estimate the overtopping hazard curve, then the value of perfect information about the hazard curve is about 25 percent of the cost to raise the dam crest (Fig. 7). This value of perfect information decreases as the amount of information available increases (i.e., tN increases) because more is known at the starting point. Also, this value of perfect information for a given tN value reaches a maximum when the cost of failure relative to the cost of rehabilitation (c_F/c_I) is such that the decision maker is most indifferent between the two alternatives, and

decreases as c_F/c_I decreases or increases from this point because one alternative becomes more strongly preferred (e.g., the alternative of raising the dam becomes strongly preferred regardless of uncertainty in the overtopping hazard curve for relatively large c_F/c_I values.

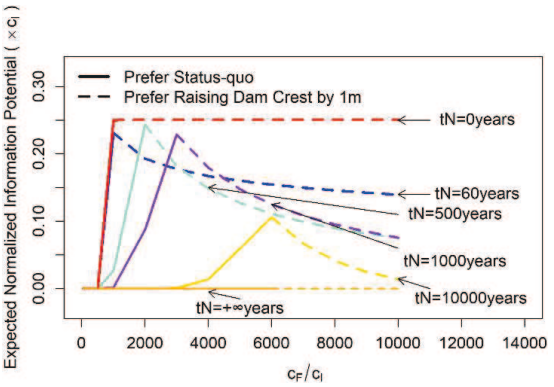


Figure 7. Expected normalized information potential for different normalized failure cost.

7 Conclusion

This paper describes and demonstrates an approach to improve the management of risks from small-probability events that can lead to large consequences. It applies a decision-based theory to account for limited information in estimating frequencies for rare events to large rockfill dam in Norway that is being assessed for rehabilitation. Uncertainties are considered specifically in estimating the overtopping frequencies for the existing dam and for an elevated dam crest (i.e., points on the overtopping hazard curve).

Uncertainty in the estimates of the overtopping hazard curve means that smaller costs of dam failure may justify rehabilitation and/or larger costs of rehabilitation may be justified. From a practical perspective, a cost of rehabilitation in this case that is nearly ten times larger could be justified when the uncertainty in the estimate of the hazard curve is considered. The value of perfect information about the hazard curve increases as the amount of information available decreases, and it increases as the cost of failure relative to the cost of rehabilitation decreases. In this case, the value of perfect information about the hazard curve is about 25 percent of the cost to raise the dam crest.

Acknowledgments

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References

Gilbert, R.B., Habbi, M., and Min, N. (2012). Achieving reliability in the force of extreme uncertainty. *Proc., Asian Pacific Sumposium on Structural Reliability and Its Applications*, Singapore.

Gilbert R.B. (2016). Important role of uncertainty in forensic geotechnical engineering. *Forensic Geotechnical Engineering. Developments in Geotechnical Engineering*, Rao V., Sivakumar Babu G. (eds.), Springer, New Delhi.

Habbi, M., Gilbert, R.B., Lake, L.W., and Mclntosh, P. (2014). Assessing value of test walls in developing an unconventional play-decision entropy theory. *Proc., SPE Hydrocarbon Economics and Evaluation Symposium*, Houston, Texas.

Mostofi, A. (2018). *Development of a New Decision-Based Framework for Risk Assessment and Management of Landslides*, PhD Dissertation, The University of Texas at Austin.

Norwegian Geotechnical Institute (2018). *Damsikkerhet i et helhetlig perspektiv – Risikovurdering for Nesjendammene*. NGI Document No. 20150624-04-R, 27 April 2018 (in Norwegian).

Shannon, C.E. (1948). A mathematical theory of communication. *The Bell System Technical Journal*, 27(3), 379-423.

Taleb, N.N. (2007). *The Black Swan: The Impact of the Highly Improbable*, Random House, Inc., New York.