Neural Network based Inverse Dynamics Identification and External Force Estimation on the da Vinci Research Kit

Nural Yilmaz, Jie Ying Wu, Peter Kazanzides, Ugur Tumerdem

***Abstract*— Most current surgical robotic systems lack the ability to sense tool/tissue interaction forces, which motivates research in methods to estimate these forces from other available measurements, primarily joint torques. These methods require the internal joint torques, due to the robot inverse dynamics, to be subtracted from the measured joint torques. This paper presents the use of neural networks to estimate the inverse dynamics of the da Vinci surgical robot, which enables estimation of the external environment forces. Experiments with motions in free space demonstrate that the neural networks can estimate the internal joint torques within 10% normalized root-mean-square error (NRMSE), which outperforms model-based approaches in the literature. Comparison with an external force sensor shows that the method is able to estimate environment forces within about 10% NRMSE.**

I. INTRODUCTION

Laparoscopic surgery brought the benefits of minimally invasive surgery to patients. At the same time, it created challenges for surgeons who had to operate with long, straight instruments through small incisions while viewing images of the internal anatomy captured by a laparoscope inserted through another small incision. Robotic assistance was introduced to solve both the loss of dexterity and poor hand/eye coordination encountered in laparoscopic surgery. For example, the da Vinci® Surgical System (Intuitive Surgical Inc., CA) provides wristed instruments on the Patient Side Manipulators (PSMs) that are teleoperated by the surgeon via the Master Tool Manipulators (MTMs), while viewing stereo images from a stereo laparoscopic camera. These advantages came at a cost, however, which was that the surgeon completely lost the sense of touch [1]. This had already been compromised by the shift from open surgery, where the surgeon's hands could directly palpate tissue, to laparoscopic surgery, where forces were transmitted to the surgeon's hands via the instruments. Experienced robotic surgeons learned to estimate forces through other cues, such as the tautness of suture or the discoloration of tissue being stretched, but it is widely believed that surgical performance would be improved by the addition of haptic feedback.

There are several challenges to achieving haptic feedback in a telesurgical system. First, it is difficult to integrate force sensors on the instrument tips, especially considering that the instrument must survive several cycles of cleaning and sterilization. Installing a force sensor in the non-sterile part of the robot (i.e., above the instrument) is also challenging because it would require good force estimation algorithms that account for the highly nonlinear cable-driven design of the instruments. There are also practical difficulties in implementing and testing solutions because commercial systems such as the da Vinci do not allow researchers to directly control the robot arms. Fortunately, the widespread adoption of open research platforms such as the da Vinci Research Kit (dVRK) [2], Fig.1, and the Raven II robot [3] enables researchers to implement and test different controllers, and ultimately to share working solutions with the community.

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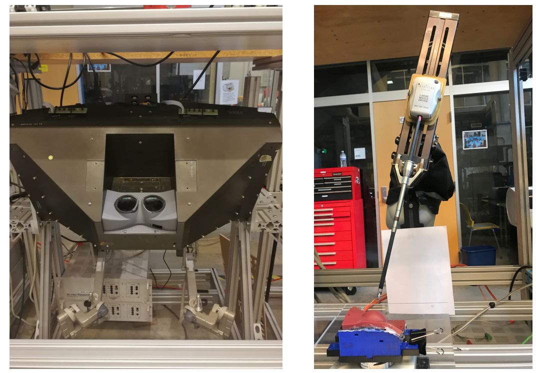


(a) (b)

Fig. 1: The da Vinci Research Kit: (a) Master Tool Manipulator (MTM) (b) Patient Side Manipulator (PSM)

This paper presents a neural network (NN) approach to estimate the inverse dynamics of the da Vinci PSM. The neural network can be used to control the PSM, for example to implement a computed torque controller, but the focus of this paper is to estimate the external torques/forces acting on the joints by subtracting the internal torques/forces (neural network outputs) from the measured torques. The method is implemented and tested on the dVRK.

II. RELATED WORK

The lack of haptic feedback in telesurgical systems has led to a significant amount of research. Some of this research has focused on developing miniaturized sensors that can be mounted on instrument tips. Researchers at DLR have developed a small 6-axis force sensor placed at the tip of the MIRO surgical robot [4]. Capacitive sensors that can be placed inside the forceps jaws of the Raven II surgical systems have been proposed in [5]. Strain gauges can also be printed on top of the da Vinci tool tip as described in [6].

However, these approaches that require modifications to the instrument structure also pose limitations on the functionality and may not be applicable under all operational settings.

In order to obtain force measurements without additional sensors, force estimation methods can be used. Such approaches have been investigated starting with the first surgical robot prototypes like the Black Falcon [7]. One of the issues observed in the early attempts has been the transmission of robot coupling/internal dynamic forces to the operators in free motion. To solve this issue, in [8], a Coulomb friction compensator was proposed to improve the force estimation results on a customized da Vinci patient side manipulator. In [9], a cable tension estimator was developed to eliminate the effects of cable elasticity in the Raven II system. In [10], a sliding mode perturbation observer was developed for the estimation of grasping force on a customized da Vinci gripper.

To overcome problems with cable-tendon driven surgical manipulators, different actuator/transmission systems and force estimation schemes have also been developed, such as a pneumatic forceps mechanism and pressure based force estimation method [11], a rigid rod driven mechanism with strain gauges on the shafts for force estimation [12], and a rigid transmission system with load cell based estimation [13]. In [14][15], a disturbance observer and neural network based inverse dynamics was utilized to estimate external forces on a rigid link driven robotic forceps prototype.

With the development of the da Vinci Research Kit (dVRK), many research groups have started developing dynamic identification and external force estimation methods for the da Vinci systems. In [16], an explicit physics-based dynamic model of the dVRK PSM was developed and parameters of this model were identified together with the free motion torques to estimate external forces under quasi-static external loading. In [17], an LMI method was utilized for dynamic parameter and joint torque identification of both the MTMs and PSMs without external force estimation. In [18], a linearized model of the PSMs was obtained and the parameters were identified with least squares optimization. External forces/torques were also estimated by filtering out the free motion torques. In [19], an open source convex optimization based toolbox was proposed for dynamic model identification of the dVRK. All of these approaches assume an explicit dynamic model for the system and attempt to identify the parameters of the respective models with the robot following an automated optimal excitation trajectory.

In this work, we are following a similar approach with a key difference: the inverse joint space dynamics is identified by black box models in the form of neural networks for each joint, whose complexity can be increased and updated to adapt to various operating conditions. This approach can also help reduce the fitting errors in the explicit model-based approaches. A similar neural network based approach has been proposed in [20] for a 3-DOF Planar Twin-Pantograph haptic interface. However, unlike our approach, a single neural network has been trained for the robot, and training has been performed with a random persistent excitation trajectory.

In the proposed method, identification is performed with the operator in the loop: as the operator controls the slave in the workspace of the robot, dynamic identification is performed without a need for an automated excitation trajectory. This also helps reduce the discrepancy between the surgical workspace and the excitation trajectory. Furthermore, the method is flexible as it can also provide a basis for deep neural networks that can be trained with data from different operations/instruments/surgeons.

Yet another approach is the use of deep learning to estimate external forces by training a deep neural network using system state measurements from the robot such as current and position as the network inputs and external sensor measurements as the training data. Such a setup has been used in [21] using an external force sensor, to estimate the grasping forces on a custom da Vinci gripper. However, the proposed approach is fundamentally different from this approach because it does not require a force sensor for training. In [22], interaction forces are estimated using deep neural networks and external camera images as the inputs with a force sensor used to provide the ground truth. Our approach makes use of the fact that the fundamental relationship between external forces and joint currents/torques is well known and this can be exploited to obtain accurate external force estimates from joint torque and position measurements without the use of external sensors for ground truth.

III. Inverse Dynamics Identification and External Force Estimation

The force estimation method proposed in this paper is composed of two parts. First, identification of the inverse dynamic model of a dVRK PSM is performed by training a set of neural networks, as the robot is telemanipulated in free motion by an operator in the loop. Once the inverse dynamics is obtained, the identified dynamic torques are filtered out from the joint torque measurements and using the robot Jacobian, external forces exerted on the end effector of the robot can be estimated. First, the neural network based dynamic identification method is explained.

A. Neural Network-based Dynamic Identification of the dVRK PSM

The dynamic model of a dVRK PSM can be described by the joint space equation:

$$M(q)\ddot{q} + C(q,\dot{q}) + G(q) + F(\dot{q}) + \tau_{int} = \tau \qquad (1)$$

where q, $\dot{q}$ and $\ddot{q}$ represent the joint position, velocity and acceleration vectors, M, C and G denote mass/inertia matrix, Coriolis and centrifugal force/torque and gravity vectors, F represents the friction force/torque vector, τ_{int} is the internal force/torque vector representing the uncertain internal forces in the robot and τ denotes the actuator force/torque vector,

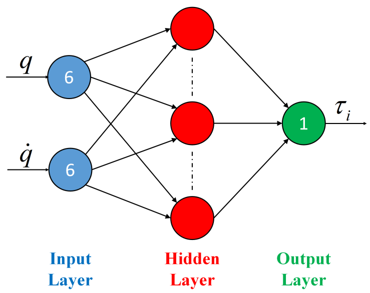


Fig. 2: Neural network for inverse dynamics identification

respectively. Since the dVRK utilizes tendon-driven mechanisms for both motion and force transmission, it is difficult to identify the dynamic model accurately due to the uncertain system parameters, friction, elongation and elasticity. As a result, the dynamic terms in (1) are not exactly known, however a lumped model can be defined to provide the sum of these effects:

$$\hat{\tau}_{dyn} = H(q, \dot{q}) \tag{2}$$

where q and $\dot{q}$ are the model inputs, H is the lumped internal dynamics model of the robot and $\hat{\tau}_{dyn}$ is the inverse dynamics torque estimate.

The crux of the proposal in this paper is to obtain H, and to identify the lumped robot dynamics, with a set of neural networks. A neural network can be used as a black box model to approximate the nonlinear relationship between robot joint states (position, velocity) and the joint torques without the need for an explicit robot model. In this paper, we are trying to approximate this function without acceleration measurements, as these measurements can be quite noisy, however they can also be used if good measurements are available. Also, in this paper, a separate neural network is utilized for each joint, with a total of 6 neural networks for the combined manipulator (excluding the gripper axis), and each neural network (see Fig. 2) includes:

- One input layer with 12 neurons for the position and velocity measurements of each joint
- One hidden layer with 100 neurons
- One output layer with 1 neuron representing the individual predicted actuator torque

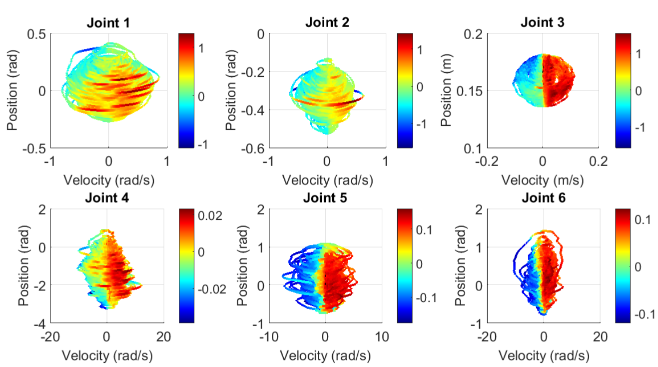


Fig. 3: The relation between joint states (x and y axes) and actuator force/torque (color scale) during training operation

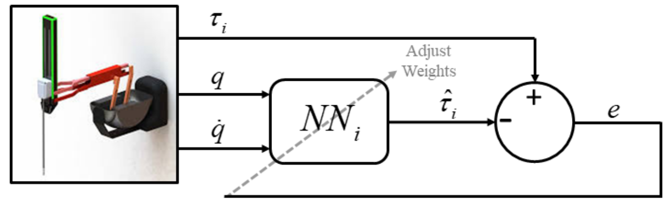


Fig. 4: Offline training of each neural network

Here, each neural network has input measurements from all the robot joints, but the output is the torque/force estimate for the respective joint. Thus each joint's identification error is used to train the respective neural network, and this provides better performance than a single neural network with multiple outputs where different scales of the measurements becomes an issue. The NN learning process is achieved by back-propagation and Bayesian Regularization. This approach has been selected for training as it can provide good generalization for difficult and noisy datasets [23]. In the multilayer network structure, the tan-sigmoid transfer function (tansig) and linear transfer function (purelin) have been used for the hidden and output layers, respectively. Initial weight and bias values were selected randomly and then updated according to adaptive weight minimization (regularization) with the chosen algorithm.

For training, the position, velocity and torque data, q, $\dot{q}$ and τ, are recorded as the dVRK is unilaterally teleoperated by an operator providing different poses and velocities via the MTM. The training dataset contains about 415,000 samples. In Fig. 3, the x and y axes present velocity and position measurements, respectively, and the color scale shows the measured force/torque acting on the actuators. As the robot moves in free motion, the external force exerted on the robot is known to be zero, which means that the measured joint torques are purely due to inverse dynamics. The error variable to be minimized by the networks is the difference between the neural network estimate and the measured torques in free motion so that the neural network can be trained to estimate the joint torque due to inverse dynamics in free motion. The optimal weights to minimize the error are found through back propagation, as shown in Fig. 4. In this paper, training was performed offline, but it could be performed online with adaptive neural networks [24].

B. External Force Estimation

When there is an external force/torque applied to the end effector of a surgical robot, the generalized dynamic equation in joint space is:

$$M(q)\ddot{q} + C(q, \dot{q}) + G(q) + F(\dot{q}) + \tau_{int} + \tau_{ext} = \tau \tag{3}$$

where τ_{ext} is the external force/torque vector acting on each joint. The external force/torque can be calculated by subtracting the inverse dynamics torque estimated by the trained neural network, $\hat{\tau}_{dyn}$ defined in (2), from the measured actuator force/torque τ:

$$\hat{\tau}_{ext} = \tau - \hat{\tau}_{dyn} \tag{4}$$

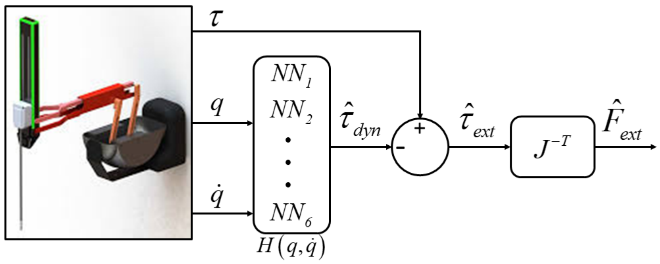


Fig. 5: External force estimation with the trained network

Utilizing the Jacobian matrix of the robot (J) and the external joint torques/forces, the external force acting on the tool-tip in Cartesian space (see Fig. 5), can be estimated as:

$$\hat{F}_{ext} = J^{-T}\hat{\tau}_{ext} \tag{5}$$

This estimate can then be compared with an external force sensor for validation, as described in the next section.

IV. EXPERIMENTS AND RESULTS

This section describes the experiments conducted to validate the inverse dynamics identification and external force estimation. Also, a set of palpation experiments, performed on different phantom surfaces, for stiffness differentiation are provided as a case study. In the experiments, the da Vinci PSM was unilaterally teleoperated by a human operator with an MTM and all the identification/estimation results were performed offline with post-processing. In the experiments, the dVRK communicates with the computer through FireWire and the data is captured at 1 kHz. Figure 6 shows a block diagram of the dVRK controller hardware; a more detailed description of the system can be found in [2]. The updated velocity estimation algorithm from [25] was used. The torque measurement τ is obtained by the multiplication of the measured current values with motor torque/force constants. The data is published as ROS topics and is recorded as rosbags for offline processing on Matlab Simulink. To evaluate the performance of the estimation method, the normalized root mean square errors (NRMSE) between the actual and estimated forces/torques can be found using the formula given in [18]:

$$NRMSE_i^* = \frac{\sqrt{\frac{1}{N}\sum_{n=1}^{N}[\hat{y}(n) - y(n)]_i^2}}{(y_{max} - y_{min})_i} \tag{6}$$

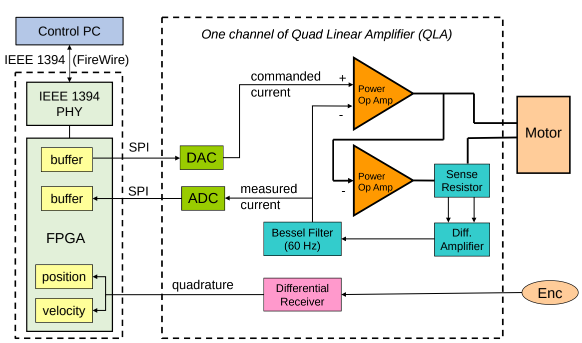


Fig. 6: dVRK controller hardware for one channel

TABLE I: Error values (NRMSE* or RelE$^+$) of joint force (f) or torque (τ) in free-motion for proposed method (PM) compared to other reported methods

Method	τ_1	τ_2	f_3	τ_4	τ_5	τ_6
PM^*	4.37	3.51	4.99	4.70	6.28	6.80
[18]*	5.92	5.78	18.84	10.41	16.84	22.96
SNN^*	13.88	10.87	14.64	13.57	19.92	20.50
PM^+	18.40	11.19	13.95	12.10	21.23	22.39
[17]$^+$	22.07	31.55	29.55	11.93	35.10	45.30
[19]$^+$	9.30	17.80	19.10	13.40	23.90	21.30

Here, N is the number of samples in the time series data from the experiments, y is the vector of reference force/torque, $\hat{y}$ is the vector of estimated force/torque of y, and $y(n)$ and $\hat{y}(n)$ are the n^{th} samples of y and $\hat{y}$, respectively.

However, in [17] and [19], the identification performances were evaluated by calculating relative prediction error by the following formula:

$$RelE_i^+ = \frac{\|y_i - \hat{y}_i\|_2}{\|y_i\|_2} = \sqrt{\frac{\sum_{n=1}^{N}[\hat{y}(n) - y(n)]_i^2}{\sum_{n=1}^{N}[y(n)]_i^2}} \tag{7}$$

In this paper, both formulas have been used to make comparisons with the results in the mentioned papers. In this paper, errors with superscript $^+$ are computed using the relative prediction error (7), and those with the superscript * are calculated using the NRMSE (6).

A. Validation of Dynamic Identification

In the first experiment, the neural network outputs and the measured inverse dynamics forces/torques from the actuators in free motion are compared. For this experiment, a test data set was collected separately from the training set with the operator unilaterally controlling the robot in free motion for both datasets. It can be seen in Fig. 7 that the measured force/torque and estimated force/torque by the neural network are very close and dynamic identification is realized accurately on each joint with NRMSE of less than 10%, as shown in Table I. Table I also shows the comparison of the errors with the proposed method (PM) to results in various papers using the previously mentioned metrics. Also, to serve as a reference, a single neural network (SNN), similar to [20], has been trained using the Levenberg-Marquardt method, with 12 inputs (joint variables) and 6 outputs (torques) and 100 hidden neurons. It can be seen that the results obtained with the proposed method are generally better than the prior results in the literature, and the use of neural networks for each joint provides an improvement over the SNN.

B. Validation of External Force Estimation

The second experiment was conducted to validate the force estimation in the Cartesian X, Y and Z axes using a force sensor. A Gamma F/T Sensor (ATI Industrial Automation, Apex, NC, USA) was fixed to a platform so as to have the same frame orientation as the dVRK base. Also, a 3D printed apparatus with square holes was mounted to the top of the

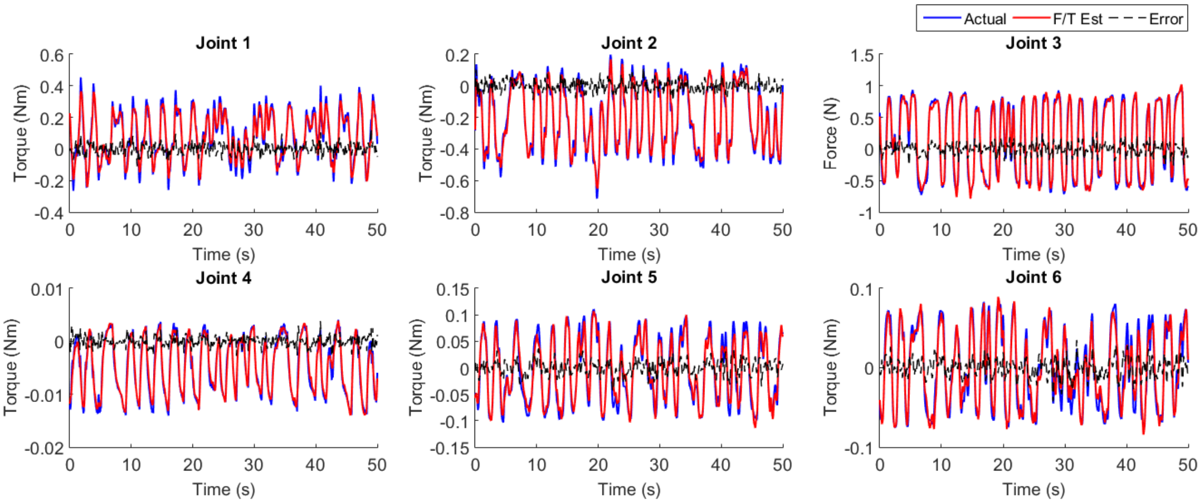


Fig. 7: Joint torque identification results

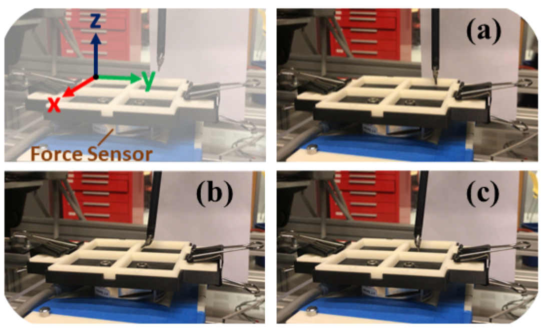


Fig. 8: Test setup used in validation experiments. Contact in (a) X-axis, (b) Y-axis, (c) Z-axis

sensor for the purpose of touching in each axis separately, as illustrated in Fig. 8. Figure 9 shows the comparison of measured forces and estimated forces when the end effector is in contact with the sensor, with results summarized in Table II. When there is contact with the sensor, it can be observed that estimation results of each axis are in agreement with the force sensor outputs with less than 10% error in each axis. However, the fact that the robot was not always in direct contact with the sensor, but rather with an apparatus that was mounted on top of the sensor, and had contact at locations other than the tip, may account for some of the errors observed. Table II also compares the force estimation errors obtained in this experiment with the results provided in [18], which is the only cited paper that uses a similar dVRK setup and has performed dynamic contact with a force sensor for validation of force estimates. While the exact experiment setup cannot be replicated, it can be seen that the performance of the proposed approach is comparable.

TABLE II: Normalized RMS error values (NRMSE) of Cartesian force (F) in contact and free-motion

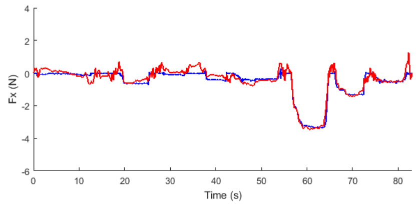
Method	F_x	F_y	F_z
PM^*	6.80	9.86	4.45
[18]*	8.26	5.96	6.10

C. Stiffness Identification

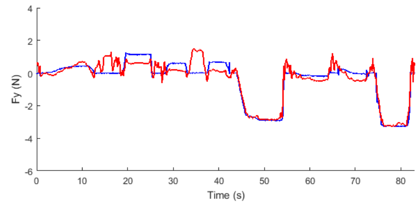
To demonstrate the feasibility of the proposed method for clinical purposes, a palpation case study was also performed. The goal was to differentiate the relative stiffness values of three different phantoms by touching 7 random points on each phantom, as shown in Fig. 10. The phantom in Fig. 10(a) has the lowest stiffness and the phantom in Fig. 10(c) has the highest stiffness. During the experiments, force and position data were recorded, external forces were estimated with the proposed algorithm and these were plotted with respect to changes in tool position, as shown in Fig. 11. Stiffness can be determined from the slope of these plots. As the exact stiffness values of the phantoms were not available, the same calculation was performed with the measurements from the force sensor placed under the phantoms. Table III shows consistent estimates of the average stiffness for each phantom by the proposed method (PM) and force sensor (FT). This result shows that the proposed method can be useful in applications such as tissue differentiation which could be of practical use to surgeons.

D. Discussion

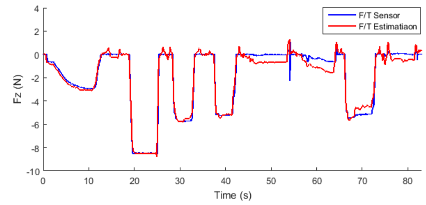
When compared with existing identification and estimation results in the literature on the dVRK, the method in its current form has comparable or better error rates. However, the main advantage of the proposal is the learning nature of the identification method and it can be improved with more datasets and training. It is also versatile when compared with other identification/estimation methods. In the current implementation, training, identification and estimation have been performed off-line, but this is purely an implementation issue. Even if the neural network is trained offline, it can be used in a real-time application. Also, it should be possible to implement an adaptive neural network to enable real-time training. This would mean that training can be performed by the surgeon/operator as the method does not require an optimal excitation trajectory, given that sufficient excitation is



(a) Cartesian X-axis Force

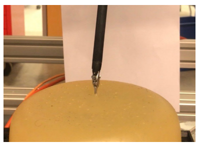


(b) Y-axis Force

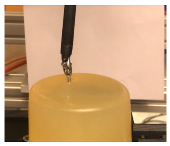


(c) Z-axis Force

Fig. 9: Validation of external force estimation



(a)



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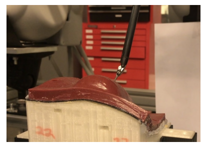
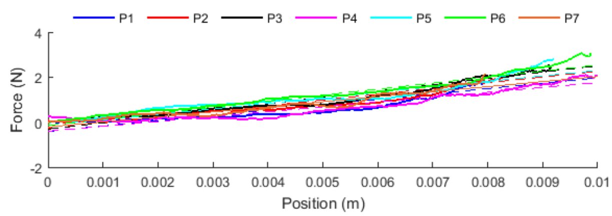
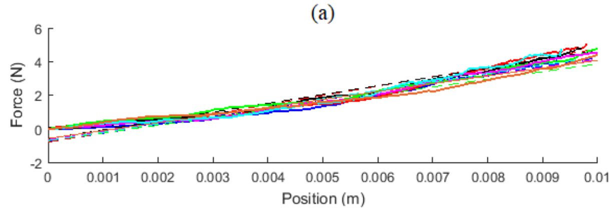


(c)

Fig. 10: Experiment setups utilized in stiffness determination: (a) Phantom #1, (b) Phantom #2, (c) Phantom #3



(a)



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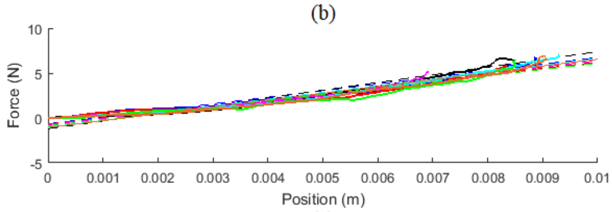

(c)

Fig. 11: Stiffness differentiation results of: (a) Phantom #1, (b) Phantom #2, (c) Phantom #3

provided by the human operator. The presented experiments were performed on a single PSM with a single instrument, so it is possible that real-time training updates may be required when applying the neural network to different PSMs and instruments. Also, during a surgical operation, robot dynamics is subject to changes as the robot is coupled with the environment at various contact points, including the trocar. Therefore, a lumped black box approach that can achieve real-time identification would be more robust compared with explicit model based approaches which could suffer from model uncertainties due to interaction with the environment. One possible application of the proposed method would be to enable surgeons to perform identification before/during

TABLE III: Estimated stiffness values for each phantom using proposed method (PM) and force/torque sensor (FT)

Method	Phantom #1		Phantom #2		Phantom #3	
	PM (N/m)	FT (N/m)	PM (N/m)	FT (N/m)	PM (N/m)	FT (N/m)
P1	237.74	238.84	499.23	388.64	738.11	799.08
P2	255.22	225.13	556.89	530.37	690.36	818.21
P3	281.45	249.96	525.62	517.47	857.74	806.83
P4	216.06	260.33	456.74	489.32	714.23	856.99
P5	243.78	252.18	479.41	533.21	759.9	806.04
P6	268.47	219.14	499.17	501.38	697.89	856.78
P7	224.03	245.26	465.45	522.67	761.48	807.65
Avg(K)	**246.68**	**241.55**	**497.50**	**497.58**	**745.67**	**821.65**

the surgical operation for identification and elimination of trocar interaction forces. Another limitation of the current implementation is that the training workspace did not cover the whole robot workspace, but this can be corrected by combining data from multiple training sets. Furthermore, the method can be augmented by the use of deep learning as extensive amounts of data are being gathered from different dVRK and da Vinci setups around the world. Finally, we have provided force estimates from the tip on three axes, but this was due to limitations in the experiment setup. A mounting apparatus for the force/torque sensor and a grip force sensor is required, but the proposed method is capable of handling estimation in these axes as well.

V. CONCLUSIONS

We proposed a neural network based inverse dynamics identification method for the da Vinci patient side manipulators and, using this identification method, we obtain a simple and robust way to filter out the dynamic components from the joint torque measurements for the estimation of the external forces. The method does not require a ground truth sensor and is easy to implement. With experiments, we have demonstrated that both the identification and external force estimation methods have results comparable to, or better than, similar methods used on the dVRK. The method can be improved with more training and real-time implementation. Different neural network architectures can also be implemented for better performance.

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