Exploring Quantum Reversibility with Young Learners

Diana Franklin*, Jen Palmer*, Woorin Jang*, Elizabeth M. Lehman*, Jasmine Marckwordt†,

Randall H. Landsberg*, Alexandria Muller†, Danielle Harlow†

*University of Chicago, Chicago, IL

† University of California, Santa Barbara, Santa Barbara, CA

{dmfranklin,jenpalmer,woorinjang,emlehman,rhlandsberg}@uchicago.edu;{jmarckwordt,almuller,dharlow}@ucsb.edu

ABSTRACT

Quantum computing is poised to revolutionize some critical intractable computing problems; but to fully take advantage of this computation, computer scientists will need to learn to program in a new way, with new constraints. The challenge in developing a quantum computing curriculum for younger learners is that two dominant approaches, teaching via the underlying quantum physical phenomenon or the mathematical operations that emerge from those phenomenon, require extensive technical knowledge. Our goal is to extract some of the essential insights in the principles of quantum computing and present them in contexts that a broad audience can understand.

In this study, we explore how to teach the concept of quantum reversibility. Our interdisciplinary science, science education, computer science education, and computer science team is co-creating quantum computing (QC) learning trajectories (LT), educational materials, and activities for young learners. We present a draft LT for reversibility, the materials that both influenced it and were influenced by it, as well as an analysis of student work and a revised LT. We find that for clear cases, many 8-9 year old students understand reversibility in ways that align with quantum computation. However, when there are less clear-cut cases, students show a level of sophistication in their argumentation that aligns with the rules of reversibility for quantum computing even when their decisions do not match. In particular, students did not utilize the idea of a closed system, analyzing the effects to every item in the system. This blurred the distinction between between reversing (undoing) an action, recycling to reproduce identical items with some of the same materials, or replacing used items with new ones. In addition, some students allowed for not restoring all aspects of the original items, just the ones critical to their core functionality. We then present a revised learning trajectory that incorporates these concepts.

KEYWORDS

quantum computing, learning trajectory, K-12 education

ICER '20, August 10–12, 2020, Virtual Event, New Zealand

ACM ISBN 978-1-4503-7092-9/20/08…$15.00

https://doi.org/10.1145/3372782.3406255

ACM Reference Format:

Diana Franklin, Jen Palmer, Woorin Jang, Elizabeth M. Lehman, Jasmine Marckwordt, Randall H. Landsberg, Alexandria Muller, Danielle Harlow. 2020. Exploring Quantum Reversibility with Young Learners. In *Proceedings of the 2020 International Computing Education Research Conference (ICER '20), August 10–12, 2020, Virtual Event, New Zealand.* ACM, New York, NY, USA, 11 pages. https://doi.org/10.1145/3372782.3406255

1 INTRODUCTION

Quantum computing has the potential to solve computational problems intractable using today's digital technology. By harnessing quantum properties, including superposition and entanglement, quantum computing defines a new set of states and operations that provide exponential computational power. Quantum computing is by no means a replacement for classical computing. However, the quantum algorithms that have been proposed are sufficiently compelling as to have resulted in billions of dollars in investment by governments across the globe, including the European Union, China, and the United States of America.

Some of the most compelling algorithms proposed are Shor's algorithm to factor prime numbers [22] (breaking modern cryptography), Grover's algorithm for search and optimization [9], quantum simulation [13], and quantum chemistry (e.g., to unlock the mysteries of nitrogen fixation to boost world food production) [15, 16].

For years quantum computers existed only in the realm of science fiction. Today, advances in the field mean that quantum supremacy, the time when a quantum computer can compute something that a classical computer can not feasibly compute, is on the horizon [11]. Much like Moore's Law predicted increases in the speed of classical computers, "Neven's Law" [12] predicts the potential computational power of quantum computers for years to come based on recent advances in the field. However, questions have arisen as to whether the rest of the system will be ready when hardware is large enough to perform useful computation becomes available. We see Neven's prediction coming to pass as Google, IBM, Intel, and Rigetti announced machines from 9 qubits to 72 qubits. While these machines are noisy with limited connectivity and far from what is needed for some of the most tantalizing applications, quantum supremacy is already claimed for a contrived problem [1] and is potentially on the near-term horizon for practical calculations (e.g., quantum chemistry).

Even with gains in hardware, there are major challenges before quantum computing becomes viable. First, there are very few quantum algorithms [23]. Second, computer architects and systems engineers need to fill the gap between the perfect hardware that algorithms assume and the noisy, buggy hardware that is available today [4]. Finally, the lack of trained professionals to solve these problems becomes a major challenge in itself.

As useful quantum computation comes closer to reality, questions arise as to what elements to teach, how to teach it, and to what depth to adequately prepare quantum computer scientists. Quantum computer scientists will need to develop algorithms that use operations that are very different from classical operations, as well as compilers and architectures to bridge the gap between theoretical assumptions and device-level realities. Quantum computing courses are beginning to be offered at the undergraduate level, prior to in-depth learning on physics and computer science. As this subject gets taught to less and less technical audiences, it is important to understand how to present concepts in more accessible ways. Computer science educators will be key to making educative resources at all levels (K-12, undergraduate, graduate, and industry professionals) that are designed following established, research-based computer science education principles. In order to do so, we need to answer two fundamental questions. First, how much can we teach about each quantum computing principle before we must introduce the mathematics or physics behind the phenomenon? In other words, how far do analogies or related context go in teaching the core quantum concept? Second, what learning goals are appropriate for different age groups?

This paper strives to answer the following research question: *In what ways do 8-9-year-old children's conceptions of reversibility align with and diverge from reversibility within quantum information science contexts?*

In order to explore that question, this paper makes the following contributions:

- a draft quantum reversibility learning trajectory (LT), with early learning goals accessible to the general population and later learning goals requiring basic probability and linear algebra skills. This can be used as a starting point for research with various age groups.
- a zine and an activity exploring quantum reversibility aimed at broad non-technical audiences for use in museums, libraries, classrooms, and other learning environments.
- identification of the differences between everyday understandings of reversibility and quantum reversibility.
- implications of the study on the draft LT and instruction on quantum reversibility.

The rest of the paper is organized as follows. We next present the theoretical framework followed by background and related work in Section 4. Section 6 contains the methods, and Section 7 contains the results. We conclude in Section 10.

2 THEORETICAL FRAMING

In this section, we discuss learning theories and how they apply to creating learning trajectories. Learning Trajectories are hypothesized paths of knowledge building that students can move through on their learning journey[24]. They are often depicted as Directed Acyclic Graphs with nodes representing learning goals and arrows depicting potential orderings between learning goals. They begin with lower anchor points, which describe ideas that students are expected to have prior to instruction. Practically speaking, they are useful tools for building curriculum and have been used extensively in mathematics[5].

Learning trajectories are especially useful when taking a constructivist approach to curriculum development, which posits that students learn new material by building upon prior knowledge and with interpretations through the lens of existing knowledge[20]. Creating opportunities for students to construct their own knowledge to integrate that knowledge with existing knowledge leads to better understanding.

However, learning is often not linear, especially with student-constructed knowledge. In particular, when students are constructing knowledge through their own observations and discussions, it becomes clear that there are several somewhat independent "pieces of knowledge" necessary to understand a whole concept[10]. This has implications on the shape of the learning trajectory. When ordering the learning goals in our learning trajectories, we look for where learning goals can be learned in any order, resulting in less of a linear structure and more of a 2-d structure.

However, it is important to note that learning trajectories are not to be interpreted as the only, or even best, way for students to progress in their understandings. Initial learning trajectories, like ours, depend heavily on theories on the ways students build knowledge, as described above. These initial learning trajectories and the theories that influenced them not only inform initial activities, but revisions are influenced by them [3, 20].

Because a single learning trajectory does not encompass all possible paths, concepts may be ordered not because students *cannot* learn a later concept without an earlier one, but because students have been able to grasp the earlier concepts at younger ages than other ones or it is easier to build on that prior knowledge.

Finally, in order to identify partial understandings to create a learning trajectory, the debate about misconceptions and deficit thinking is relevant. Building on diSessa's framing of Knowledge in Pieces [7], Danielak recently argued that [6], focusing on misconceptions, or things students do *not* understand, makes it more difficult to identify what students *do* understand. In this work, when faced with apparent misconceptions, we took care to identify what was *correct* about the understandings. These partial understandings and pieces of knowledge became the learning goals, or nodes, within our learning trajectories.

3 PRIOR WORK

There have been recent efforts in computer science to create learning trajectories for fundamental computing concepts in order to inform assessments and curriculum development. These have come in very different formats. Early work in computer science was performed by Seiter and Foreman, who extracted learning trajectories from Scratch projects available on the Scratch website[21]. The CS K-12 Framework could be viewed as having many components of learning trajectories. More recently, learning trajectories have been published by Rich et. al. for sequence, iteration, conditionals[19], decomposition[17], and debugging[18]. There has also been work in extracting the depth of understanding of different concepts through artifact analysis[2, 14].

This effort has not yet begun in quantum computing. Instruction typically occurs at the graduate level from a mathematical

/ computational perspective. Principles are illustrated by demonstrating the mathematical results from specific quantum operations. Because our goal was to convey quantum computing concepts to an audience with less mathematical and computing background, we needed to identify the principles behind the examples and relate those to non-computing, non-mathematical contexts.

4 QUANTUM COMPUTING

We begin by introducing the basics of quantum computation, limited to the concepts necessary to give context and understand the learning trajectory and research study presented in this paper. Quantum computing harnesses quantum physical properties in order to define a new state and computations on that state.

4.1 Defining Quantum Computing

There are three similar areas related to quantum computing: quantum physics, quantum information science, and quantum computing. Quantum physics, also referred to as quantum mechanics, studies the quantum phenomena on which quantum computing is built. Quantum information science (QIS) utilizes these quantum phenomena to encode and operate on information stored in quantum states. Quantum computing is one of many applications of quantum information science - quantum sensors and quantum communications are two alternate areas to which QIS can be applied. For the purposes of this paper, the distinctions between quantum information science and quantum computing are not important. Both are concerned with how to harness quantum computation to implement quantum algorithms or protocols. However, the reasons for the physical phenomenon are not paramount; we want to understand how to utilize the constraints presented to us while abstracting away the quantum physics details.

4.2 Quantum State

Instead of definitively holding a 0 or 1 like classical bits, qubits can hold a superposition, or linear combination, of both 0 and 1. However, at any time, there is a probability of measuring a 0 or a 1. Quantum operations manipulate that probability. The exponential advantage in quantum computing compared to classical computing is the storage and manipulation of the superposition of many bits working together through entanglement. However, an understanding of superposition and entanglement are not required to understand quantum reversibility.

4.3 Reversibility

A physical limitation that greatly hinders quantum computation is the requirement that all computations must be reversible. In daily life, there are many actions that we consider reversible or not reversible (Figure 1). For quantum computing, the requirements of reversibility are stricter. Here, the definition of reversible is that you must always be able to perform an opposite operation that results in the original inputs, knowing only the outputs of an operation and the operation itself.

Consider logical operations. If you apply the *NOT* operation three times to a variable with a result of 0, can you reverse that operation and obtain the original input? Yes. Because *NOT* is its

Figure 1: Two panes of an 8-pane zine about Reversibility

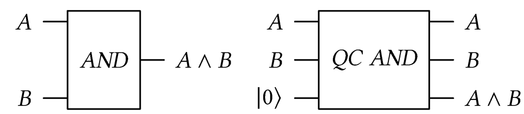

Figure 2: Making a reversible *AND* operation. The third input is an *ancilla bit*, which holds the result (allowing the extra information necessary to reverse the operation).

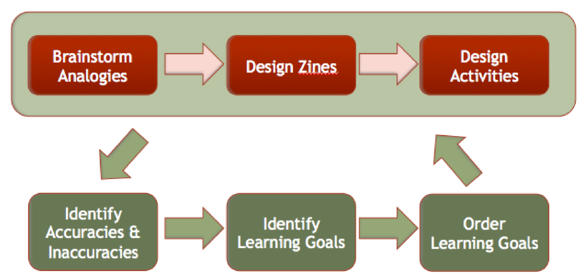

Figure 3: Materials Development Process

own inverse, you would apply *NOT* three times to reverse the three original *NOT*s and obtain 1.

Now consider the logical *AND* operation. There are four possible input combinations but only two possible output values. If the output is 0, there are three possible input combinations: $(0, 0)$, $(0, 1)$, $(1, 0)$. This means that if you know the operation (AND) and the output (0), you do not have enough information to determine the inputs because there are three valid combinations. To make the *AND* operation reversible (Figure 2), we would need to add an output bit to distinguish between the combinations of inputs that have the same output. Because there are three possibilities, we must add two bits. Thus, there are three output bits: one bit that contains the *AND* calculation and two bits that make it reversible. The two inputs are passed through, and an extra bit is necessary for the result. This extra bit is called an ancilla bit. Note that this adds only what is necessary for reversibility - quantum computing operations have other constraints that we do not address in this paper.

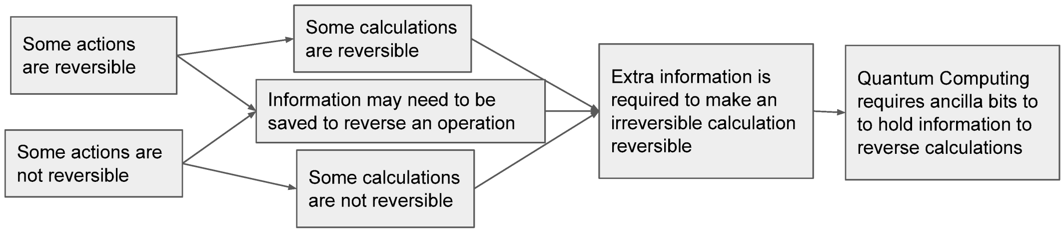

Figure 4: Reversibility Learning Trajectory

5 MATERIALS DEVELOPMENT

Our goal is to develop learning trajectories for reversibility and other concepts in quantum computing, as well as activities that help learners meet the goals within the learning trajectories. The challenge in this endeavor is that, unlike previous efforts to create learning trajectories, we are attempting to create learning trajectories before materials exist, for an audience with breadth in both age and technical background. Existing textbooks take a mathematical approach, showing how quantum computing works through the application of quantum gates on quantum state. Our goal is to extract larger principles that can be separated from quantum computing itself so they can be conveyed in other contexts. This allows us to take advantage of the fonts of knowledge that a broader audience has to build intuition that supports quantum concepts before introducing the mathematics.

Our interdisciplinary group consisted of faculty members, graduate students, and staff with expertise in quantum computing, computer science education, and/or science education. Through weekly discussions, this group collaboratively designed informal activities for wide age ranges and created zines, cartoon-ish introductions of each concept (see Figure 1 for an example). As team members contributed ideas for activities for the everyday analogies included in the zines, these ideas were dissected as to the ways in which they *did* and *did not* accurately reflect the quantum computing concept involved. This process of identifying possible misconceptions or misunderstandings, then pivoting to identifying partial understandings, allowed us to identify concepts that could be taught apart from the traditional math-based or physics-based approaches. These partial understandings became nodes in the learning trajectory. The nodes were ordered utilizing learning theories with respect to learning trajectories. In particular, using constructivist principles, partial understandings most directly connected to everyday knowledge were ordered first. Aligned with a "knowledge in pieces" approach, different nodes without strong dependence were not connected.

The Learning Trajectory (LT), zine, and informal activity development followed a cyclic process, whereby the zines and activities were initially used to draft the LT's. The LT's were then used to refine the zines to more smoothly present the concept as well as identify gaps in activity coverage and so that brainstorming could occur to fill that gap (Figure 3). We now present the learning trajectory, zine, and activity.

5.1 Learning Trajectory

Figure 4 depicts the reversibility learning trajectory. It begins with the accessible concepts of actions being reversible and not reversible *actions*, and continues to reversible and not reversible *calculations*. The reversibility of calculations was chosen because for young children, they learn addition and subtraction (1st and 2nd grade, ages 6-8), as well as multiplication and division (3rd grade, ages 8-9)[8]. The trajectory follows with how to convert an operation that is not reversible into something that is reversible (by storing extra information, which leads to the inclusion of ancilla bits). This is not unlike making a non-invertible function invertible, which is in the Common Core State Standards for high school[8]. Therefore, reasoning about whether something is reversible is simpler than converting something from not being reversible to being reversible.

The actions−>calculations−>quantum gates progression was a specific attempt based on constructivism to be accessible to the broadest population at the beginning and narrow as we got closer to quantum computing. We begin with analogies like navigation on city streets and then move on to simple calculations with addition.

5.2 Reversibility Zine

The progression in the learning trajectory is depicted in our zine. As shown in Figure 1, we begin with the learning goal *some actions are reversible*, followed by *some actions are not reversible*. Then, we modify the definition of reversible to require getting back the same inputs and apply this to math in panel 3 (Figure 5). This covers both *some calculations are reversible* and *some calculators are not reversible*. In panel 4, we then address learning goals related to information: *information may need to be saved to reverse an operation* and *extra information is required to make an irreversible calculation reversible* by showing how to make addition reversible. Finally, we tie these concepts to quantum computing in the final panes, Figure 6.

5.3 Activity

Our iterative process led to the design of several reversibility activities. In this paper, we focus on one activity that we piloted in a classroom, "Is It Reversible?" (inspired by a life science activity, "Is It Living?"). At its heart, it is a categorization activity. Students categorize examples of actions, ranging from clear to ambiguous with respect to reversibility, with the goal of getting them to reason about ambiguous cases and articulate their thinking as to how they categorize things. The activity consists of three phases - categorization, argumentation, and assessment.

We had two goals in designing our activity. Our first goal in designing this idea was to provide students with minimal direct instruction and instead provide them opportunities to think about, discuss, and articulate to use their thoughts about reversibility. As in constructivist thinking, we wanted to understand how they

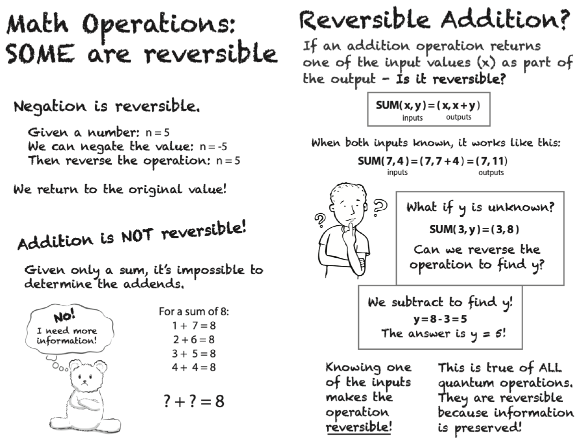

Figure 5: Reversible calculations

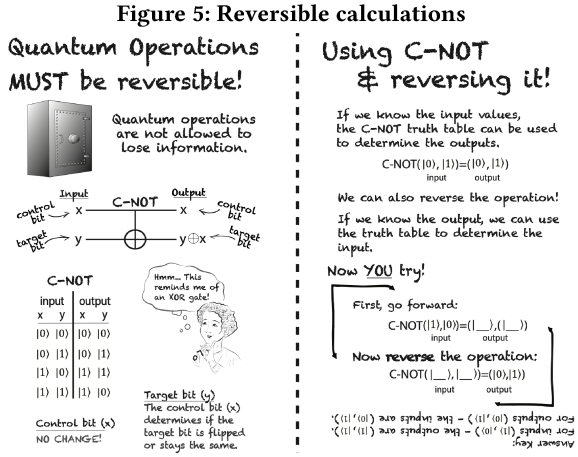

Figure 6: Tying reversibility to quantum computing.

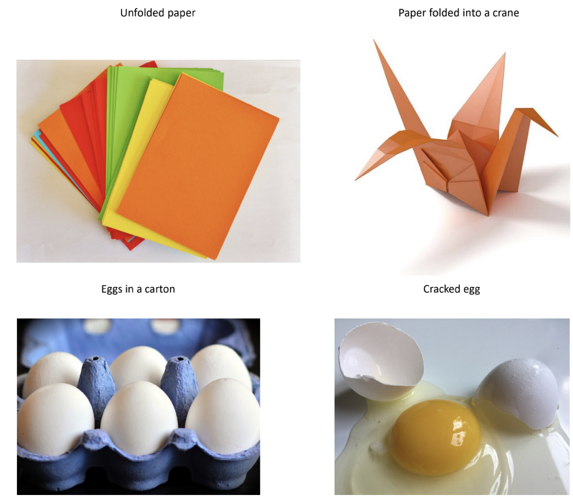

Figure 7: Reversibility cards

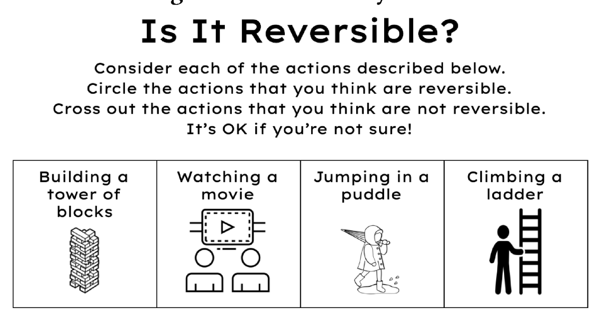

Figure 8: Reversibility assessment

connected reversibility to their everyday thoughts so that we could identify the ways in which their everday understandings did or did not match the way reversibility is used in quantum information science. Therefore, in each phase, students are given a task in which there is an outcome that they decide and articulate their reasons for that decision. Our second goal was to create an activity that satisfies learning goals outside of quantum information science for this age group. The second phase satisfies this goal - we created an argumentation activity that exercises students' logical thinking and articulation of an argument.

The first phase of the activity, categorization, begins with an introductory discussion on the concept of reversibility. The facilitator bring up every day actions, such as zipping up a jacket and cutting a piece of paper, and discuss whether the action can be undone to bring everything back to its exact original state. Then, groups of participants are given a set of "reversibility cards" and asked to categorize each card as "reversible", "not reversible", and "unsure". Each reversibility card has before and after pictures and text describing an action on an object. There are a total of 17 cards; two examples are shown in Figure 7: folding paper into a crane and cracking eggs in a carton. Once participants have finished categorizing the cards, the whole group discusses how each small group categorized the cards and why. During this part of the activity, we encourage participants to think about the detailed differences or similarities between the original item and the "reversed" item.

The next activity phase is argumentation. Each participant completes a "Reversible or Not?" worksheet. On the worksheet, they describe one action or change. Then, they indicate whether they think that action/change is reversible or not reversible, and write down their reasoning for this categorization. Once all the participants complete this portion of the worksheet, they participate in a gallery walk to view what others wrote. The bottom half of the worksheet allows participants to agree or disagree with the original author's claim and explain their own reasoning.

Finally, the last phase of the activity is the assessment portion, which is completed by each participant individually. The assessment worksheet, "Is it Reversible?", consists of 16 actions which the participant is asked to categorize as reversible or not. Again, these actions range from clear to ambiguous with respect to reversibility. Four of the actions are shown in Figure 8. Then, participants briefly describe the "rule" they used to determine if the actions are reversible or not.

6 METHODS

6.1 Experimental Design

Students in this IRB-approved study were recruited from one 3rd grade (8-9 year olds) classroom in a large, urban, private school in the Midwestern United States. The activity was facilitated by a researcher in quantum computing and computer science education. In addition, three computer science education and/or quantum computing researchers observed the activities. Students participated in three activities over three weeks, which focused on measurement disturbing state, reversibility, and superposition. Of the 23 students in the class, 15 students and their parents gave consent for data collection. Here we report on the reversibility activity to explore 8-9 year olds' thoughts and understandings of an early learning goal in reversibility.

6.2 Data Collected

Data were collected in the form of student work: the "Reversible or Not?" worksheet from the argumentation phase of the activity and the "Is It Reversible?" worksheet from the assessment phase of the activity. Researchers also recorded observation notes.

6.3 Data Analysis

First, each "Reversible or Not?" worksheet was analyzed. It had space for 4 student answers: the original's student's reasoning on whether the action they chose was reversible or not, another student's reason for agreeing, another student's reason for disagreeing, and another student's reason for being unsure. We analyzed students' understanding of reversibility as shown in these short answers by categorizing each answer qualitatively. In order to develop the categories, one researcher read the answers and identified emerging themes in how students reasoned about reversibility. The themes were refined and clarified over several discussions between the two researchers performing the coding. The final coding scheme is shown in Table 1. Prior to discussing the coding scheme, there was a 30% (14/47) agreement between the two researcher. Of the 70% of answers on which they disagreed, 26% (12/47) were determined to belong in multiple categories and subsequently labeled as thus. The remaining 44% (21/47) of answers were discussed and placed into one category after reaching agreement.

Second, the "Is It Reversible?" assessment worksheet was analyzed. The binary answer (reversible or not reversible) was recorded for each item for each student. Observation notes were consulted to identify reasons students cited that were counter to the majority of answers. Finally, we analyzed the final question asking how they determined whether something was reversible or not reversible using the same categorization scheme as for the "Reversible or Not?" worksheet.

7 RESULTS

We present two sets of results, focusing on our questions of how students think about reversibility when applied to real-life everyday actions and how student reasoning aligns with the definition of reversibility utilized in quantum computing.

7.1 Argumentation Activity

For the argumentation phase, we first present the categories that emerged from qualitative coding, then analyze the results.

Table 1: Three Themes for Categorization

Theme	Code	Categorization
1. Components of Object	R	RV if components of object can be replaced NR if components of object cannot be replaced
	O	RV if original components of object can be maintained NR if original components cannot be maintained
2. Function of Object	F	RV if object retains its function despite changes in appearance NR if function of object changes
	X	RV if object retains both function and appearance NR if function and/or appearance is modified
3. Specific Action	A	RV if action can be undone but might modify object NR if action cannot be undone
	S	RV if action can be undone without modifying object NR if undoing the action changes the object
	C	RV if action can be repeated continuously NR if action can only be done once

RV = Reversible, NR = Not reversible.

7.1.1 *Argumentation Categories.*

Three themes emerged from qualitative coding of student written arguments and discussions during the activity.

Ability to replace components. Some students reasoned not based on reversing the action but through recycling or replacing components. For example, a used tissue could be recycled by shredding it and recycling it into new tissue paper. One student stated in the worksheet that they were unsure whether writing with a pencil was a reversible action, saying, "If you had a mechanical pencil, you could put more lead in it." Here, the student is not analyzing the reversibility of the action of writing itself, but rather thinking about replacing a part of the pencil that has been lost because of the action of writing.

Within this framework of thinking, we developed two specific categories. Student answers that included statements about whether components could be replaced were categorized into code R. The mechanical pencil answer above is such an example.

Student answers that included statements about whether the replaced component was the exact same, original component and not a new one were categorized into code O. For example, one student stated that taking Tic Tacs out of the box and eating them was not a reversible action because "you can't find the exact same tic-tacs. You need to put the exact same tic-tacs." Here, the student still thinks about reversibility in terms of replacing components rather than reversing the action. However, the student recognizes that replacing original components with similar components still alters the object in some way.

Functionality of object. When reasoning about reversibility, some students focused on whether the functionality of the object remained intact. For example, one reversibility card shows scissors cutting a piece of paper. Instead of reasoning about the action of cutting the paper, some students looked at the functionality of the scissors instead. They stated that the action was reversible because the scissors would still be able to cut more paper after the action was complete. Here, students viewed an action as reversible if the functionality of the object involved was left intact.

Once again, we developed two categories within this theme. If the student's reasoning focused on the functionality of the object after a specific action regardless of cosmetic loss, we categorized it into code F. For example, one student reasoned that the action of erasing was reversible because "if it breaks the eraser will still work." Here, the student argues for reversibility based on the functionality of the eraser, but does not take into account that the appearance of the eraser changed because of the action.

If the student's reasoning focused on both the functionality and appearance of the object, we categorized it into code X. For example, one of the reversibility cards show the action of stretching a rubber band. One student stated that this action must be not reversible because while the rubber band can still hold things, it will be larger because it has been stretched out. Once again, the student focuses on the function of the rubber band to hold things, but this time takes into consideration the changes to the appearance of the rubber band.

Action Type. Finally, many students reasoned about the reversibility of an action by considering the action itself as opposed to focusing solely on the characteristics of the objects involved. For example, when reasoning about the action of tying shoelaces, one student responded that the action was reversible because, "You can untie your shoes." Here, the student is thinking about how the actual action can be reversed.

Three categories were developed within this theme. First, some students reasoned that an action is reversible if it can be reversed, regardless of whether the related object might be modified or have an altered appearance. These answers were categorized into code A. For example, one reversibility card shows the coloring of a coloring page. Some students argued that applying white-out would reverse the action of coloring, even though the markings would be obvious. We also categorized student answers as A if the answer didn't consider whether reversing the action would cause such modifications to the related objects. For example, one student stated that the action of filling a water bottle was reversible because, "You can take the water back out." Here, the student focuses on simply reversing the action and does not consider whether the wet state of the water bottle from the action impacts the reversibility of the action.

If a student answer did consider how reversing the action could modify the related object, it was categorized into code S. For example, when reasoning about the reversibility of tying a shoelace, one student said that the action was not reversible because, "it will be wrinkled later." Here, the student not only analyzes the reversibility of the action itself, but also considers any possible modifications to the object from its previous state before the action.

Some students looked at the action, but rather than thinking about reversing the action, thought about whether the action could be repeated or not. These answers were categorized into code C. For example, when analyzing the action of sharpening a pencil, one student responded saying the action was not reversible because, "You can't sharpen it too many times or it will be too short to sharpen." The student's reasoning does look at the action, but instead of thinking about how to reverse the sharpening, it thinks about whether the sharpening can be repeated.

Relationship to Quantum Reversibility. Analysis that matches quantum reversibility would take into consideration all objects in the system (the scissors and the paper, for example) and only allow the action to reverse, not any components to be replaced or recycled. From this perspective, an action that is reversible must adhere to qualitative codes O (all components of the system must be maintained), X (the system retains both function and appearance), and S (can be done without modifying the components in the system). Qualitative code C indicates a very useful way of analyzing whether changes occur but is not an additional requirement for reversibility.

7.1.2 Categorization Results. The categorization results are shown in Table 1. Answers that were missing were categorized as M, and answers that were difficult to understand due to grammatical or logical errors were categorized as U. With 15 students and 4 possible student answers for each worksheet, there were 60 possible answers. Of these, 13 were missing (M), leaving 47 analyzed answers.

Figure 9 shows the number of answers categorized into each theme. The graph shows the total number of counts for all the answers (blue), the answers supporting reversibility of an action (green), the answers supporting non-reversibility of an action (red), and the answers that were uncertain (yellow). Note that the number of categorizations do not add up to 47 because 12 of the answers were categorized with two different codes. When student reasoning was coded into multiple categories, the most common combination was A and R.

As seen in Figure 9, most student arguments for the reversibility of an action analyzed the actual action itself (Theme 3). This was followed by analysis of the components of the object affected by the action (Theme 1). Only a few separated out the functionality and the aesthetics of the object (Theme 2). These patterns are consistent for answers defending both reversibility and non-reversibility. The number of "Not Sure" answers are too few to determine if there is a trend.

Table 2 provides more detailed results, showing the number of answers in each specific category. Note that the number of categorizations do not add up to 47 because 12 of the answers were categorized into two different codes. The detailed breakdown of counts shows that within Theme 3, the majority of answers (19 out of 32) were coded A, and a large minority of answers (10 out of 32) were coded S. Categorization into A means that students analyzed just the reversibility of an action, while categorization into S means that students took the analysis a step further - looking at any modifications to the object the action may have made, as

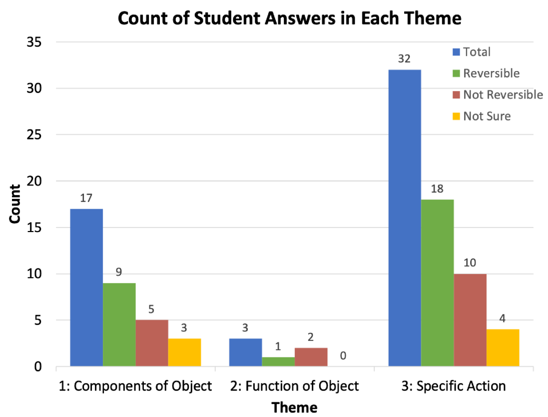

Figure 9: Number of student answers in each theme

Table 2: Categorization Results

Code	RV	NR	NS	Total
R	8	2	3	13
O	1	3	0	4
F	1	2	0	3
X	0	0	0	0
A	17	1	1	19
S	0	7	3	10
C	1	2	0	3

RV = Reversible, NR = Not reversible, NS = Not sure.

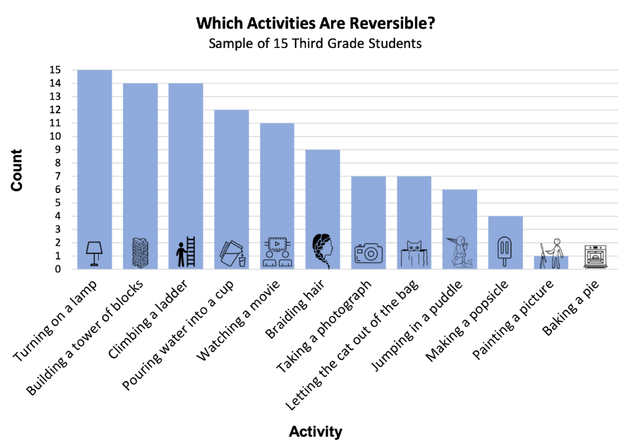

Figure 10: Number of students who categorized each activity as being reversible

well. Most student answers categorized into code A (17 out of 19) argued for reversibility of an action, and most student answers categorized into code S (7 out of 10) argued for non reversibility of an action. This suggests that students who considered how an action could modify related objects were more likely to find more nuanced reasons that an action was not reversible.

Table 3: Breakdown of Articulated General Reasoning

	Components		Function		Action		
NA	R	O	F	X	A	S	C
8	0	1	0	0	3	2	1

7.2 Assessment

For the "Is it reversible?" assessment, we first looked at how student answers varied across actions included in the assessment. Figure 10 shows the number of students (out of 15 total) who thought each of the actions shown was reversible. All students thought “turning on a lamp” was a reversible action, while no students thought “baking a pie” was a reversible action. These edge cases show that after participating in the reversibility activities, many students generally understood the concept of an action being reversible. Baking a pie is clearly a task that is impossible to reverse, and all students agreed that it was a non-reversible action. However, we also see that students did not agree on whether most of the actions were reversible or not. This indicates that students’ differences in argumentation approaches and interpretations resulted in different conclusions about reversibility.

This also shows how student impressions changed throughout the activity. At the start of the activity, when discussing the reversibility cards, many students thought that coloring a picture was a reversible action because they could use white-out on the parts they had colored - even though it would be clear that those parts had been modified. In the assessment, only 1 student thought that painting a picture was a reversible action.

We also analyzed student responses to the question, "How did you decide if an action was reversible or not?" using the categorization scheme discussed above. The results are in Table 3. Of the 15 responses, 8 were uncategorizable because they were too vague to understand (labeled NA). Examples of these types of responses include, "I pictured it in my head" and "You can reverse a lot of things." Of the remaining 7 responses, 6 fell into analyzing actions and 1 analyzed the components of the system (code O). Examples of responses categorized as action include, "You can turn on a lamp and turn off a lamp" and "If you can undo the action and make it the exact same then I think it’s reversible." The high number of uncategorizable responses suggests that it is difficult for students this age to articulate their decision mechanism in a general way.

7.3 Discussion

This study highlights two pieces of knowledge that are important for students to learn in order to properly categorize actions as reversible or not reversible *from a quantum perspective*. The connections are challenging since QIS reversibility is normally described in very precise mathematical terms, not in terms on actions at the macro level, so we must make analogies to the everyday physical world.

First, students need to think about not only the action itself but its effect on all components in the system. For example, when considering whether “tie my shoes" or “brading hair" was reversible, some students only considered the action (“you can untie your shoes”), while others considered the result upon the shoelace (“it will be wrinkled later"). In a quantum system, to be reversible, all outputs need to be restored to their original state. Because the shoe lace and hair are inputs to the operation, they would need to be restored to their original state.

Second, when thinking about the system components, students need to recognize the need to preserve the original items, not add new things into the system through replacement or recycling. When there is a before and after picture of an action, the restoration needs to be limited to the items in that picture. In quantum computing, there is a closed system. Even something needed temporarily, like an ancilla bit, is inputted into the operation and included in the analysis of whether or not it is reversible. Therefore, any replacement items would be inputs to the original operations. Therefore, it cannot "replace" the other item. If there were two tissues initially, soiling one and replacing it with the other is not sufficient - the system went from two clean tissues to one clean and one dirty tissue.

We also see that the activities led students to think more carefully about actions being completely reversible.

The wide range of opinions on activities such as "braiding hair" show that students were divided on actions that modified the system. For example, one student who said that the action was not reversible stated that "If you can undo the action and make it the exact same then I think it's reversible" - while braiding hair is an action that can be undone, the waves left in the hair fails to make it "the exact same" as before. Meanwhile, another student who looked just at the action, saying "you sometimes untie the braid", marked the action as reversible.

Finally, we found that students seem better able to reason about the reversibility of specific actions than to generalize about the concept. This is reflected in the large number of uncategorizable responses to the assessment question. Additionally, for the few responses that were categorizable, several discussed specific actions that the students had already reasoned about rather than the general conditions that make an action reversible.

8 IMPLICATIONS

This study has two sets of implications, one on the learning trajectories themselves and another on instruction for students this age on quantum reversibility.

8.1 Learning Trajectories

These results indicate that the learning trajectory for quantum reversibility should be augmented to include a more precise definition of what makes an action reversible.

Figure 11 illustrates the new learning trajectory. There are several differences informed by this study.

First, we make a distinction between actions that "can be undone" and actions that are "reversible." We consider the former to be the real-world notion of reversibility, which could be quite flexible in how it is defined. Students have this knowledge prior to instruction, making it a lower anchor point. We then include the two core requirements for making something reversible: that the action of reversing restores all parts of the system to their original state and that additional elements from outside the system are not added to the system to assist in the reversing. Just as in our activity, these two concepts can arise in any order based on student conversations, so, informed by the Pieces of Knowledge framework, we place those in either order. Those lead to the understanding of reversible actions, as more closely defined by quantum computing.

After this, students would be ready to either go further with the real-world action analogy or tackle reversible mathematical calculations. They could learn how to make irreversible actions reversible by storing information. For example, students could have a discussion about whether the action of walking from your house to a playground is reversible. What needs to be done to reverse it? Walk the same path? Walk in the same footsteps? If you need the same path, what information do you have to store in order to get back? They would discuss adding a notepad to the system, recording your path, and then erasing it when you get back in order to reverse all elements in the system. Alternatively, students could then explore the same process with mathematical operations.

Although we did not research an activity related to reversible calculations, we have applied what we learned to that part of the trajectory. We more precisely define reversible actions as ones that can be reversed with only knowledge of the output(s) and the operation. Further research should be performed to find out whether there are specific differences students encounter in reasoning about reversible calculations.

8.2 Instruction

This activity was not merely a tool to elicit student ideas - it was an instructional tool to teach them the nuances of reversibility in an accessible way. We are able to evaluate the success of the activity in creating discussions that developed student understandings about reversibility.

All three phases of the activity were successful at getting students to discuss and evolve their understanding of reversibility. For example, when initially discussing the action of steeping tea, many students focused on returning the object to its original state and said that the action is reversible because the tea could be replaced with new water and a dry teabag. During these discussions, we pushed them to think about undoing the specific action on the specific objects, rather than simply replacing changed components of the object. The results in Figure 9 show that students began to understand analyzing the action rather than the object. Furthermore, our analysis shows that a majority of answers categorized into Theme 1 (10 out of 17) also considered the action (Theme 3). Then, leading student discussions to look at the consequences of an action and the related reversal of this action could help them deepen their analysis of reversibility.

Analysis of the assessment and worksheet data show that the activity successfully introduced ideas that help students analyze the reversibility of an action. Throughout the activity, students were able to discuss the actions themselves, the consequences of those actions, and the state of the objects related to the actions in order to start developing solid arguments for their reasoning. However, they had more difficulty reasoning generally than reasoning about specific actions.

Now that we know the particular differences between student initial understandings and the quantum-inspired definition of reversibility, however, we can provide more targeted instruction to help move students along the learning trajectory. From a high level,

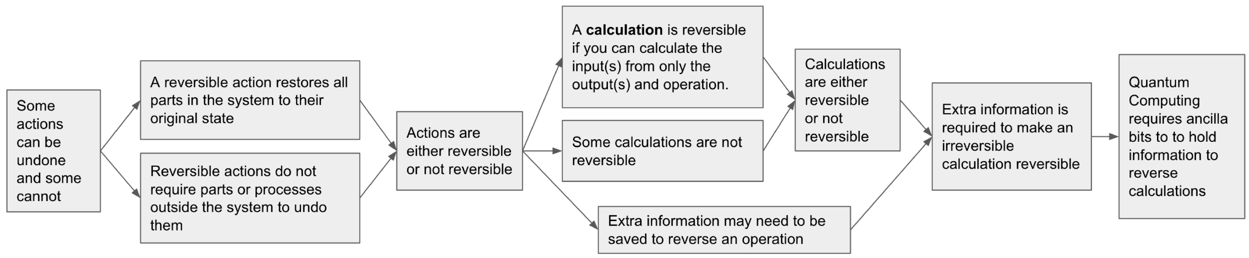

Figure 11: Revised Quantum Reversibility Learning Trajectory

one of the most major challenges for students was thinking of not only the action but of all of the items in the system. There are a few ways that this could be accomplished.

One way to do this would be to have students be given a concrete action to reverse, such as cutting a piece of paper. They could then discuss whether different approaches (e.g. taping or gluing it back together) would result in all items being just like they started. Including this concrete example and discussion would allow the facilitator to clarify the that it is important to identify all the parts of the system that the action is working on in order to determine if the action is reversible.

In addition, existing parts of the activity could be augmented to draw student attention to the items in the system. For the middle exercise, in which students identify an action that is or is not reversible, students would be asked to identify all of the items in the system, the action that took place, and then draw before and after pictures that include all of the items in the system.

Finally, when students categorize items in the final assessment, instead of just having students identify something as reversible or not reversible, we could have them categorize non-reversible actions based on the reversibility rules they violate. This would bring those elements to the forefront of their decision process and help them generalize their reasoning.

9 LIMITATIONS

This study involves the consented subset of a single classroom of students from a single school, resulting in only 15 students participating. While we have documented interesting ideas and thoughts from those students, we are unable to make any statements about whether this sample is representative of the general population of 8-9 year old students.

10 CONCLUSIONS

This report study the development and evolution of a learning trajectory and activity for teaching quantum reversibility to a non-technical audience. We found that even third-grade students are able to reason about actions in a way that has synergy with quantum reversibility. However, we identified two common patterns of reasoning that do not match quantum reversibility - focusing on the action but not requiring all items to be restored to their original states and allowing substantial extra resources to be added to the system to provide an equivalent, but not the same, item after reversal. This was used to revise the initial learning trajectory. This revised learning trajectory can become a starting point for research on how to move student understanding even closer to what is necessary for making quantum computing contributions.

11 ACKNOWLEDGEMENTS

We thank the teacher and all of her students for being eager, engaged participants in our quantum information science activities. This material is based upon work supported by a National Science Foundation Expedition in Computing, under Grant Nos. 1730449 and 1730088.

REFERENCES

[1] F. Arute, K. Arya, and R. Babbush. 2019. Quantum supremacy using a programmable superconducting processor. *Nature* 574 (2019), 505–510.
[2] Ashok Basawapatna, Kyu Han Koh, Alexander Repenning, David C. Webb, and Krista Sekeres Marshall. 2011. Recognizing Computational Thinking Patterns. In *Proceedings of the 42nd ACM Technical Symposium on Computer Science Education (SIGCSE '11)*. Association for Computing Machinery, New York, NY, USA, 245–250. https://doi.org/10.1145/1953163.1953241
[3] M. T. Battista. 2011. Conceptualizations and issues related to learning progressions, learning trajectories, and levels of sophistication. *The Mathematics Enthusiast* 8, 3 (2011), 507–570.
[4] Frederic T. Chong, Diana Franklin, and Margaret Martonosi. 2017. Programming languages and compiler design for realistic quantum hardware. *Nature* 549, 7671 (Sept. 2017), 180–187.
[5] J. Confrey, A. P. Maloney, and A. K. Corley. 2014. Learning trajectories: A framework for connecting standards with curriculum. *ZDM - International Journal on Mathematics Education* 46, 5 (2014), 719–733.
[6] Brian A. Danielak. 2019. Deprecating Misconceptions Through Context-Dependent Accounts of Productive Knowledge. In *Proceedings of the 2019 ACM Conference on International Computing Education Research (ICER '19)*. ACM, New York, NY, USA, 91–100. https://doi.org/10.1145/3291279.3339424
[7] Andrea A DiSessa. 1988. Knowledge in pieces. (1988).
[8] National Governors Association Center for Best Practices and Council of Chief State School Officers. 2010. Common Core State Standards - Mathematics.
[9] Lov K. Grover. 1996. A Fast Quantum Mechanical Algorithm for Database Search. In *Proceedings of the Twenty-eighth Annual ACM Symposium on Theory of Computing (STOC '96)*. ACM, New York, NY, USA, 212–219. https://doi.org/10.1145/237814.237866
[10] D. Hammer and T. Sikorski. 2015. Implications of complexity for research on learning progressions. *Science Education 99* 3 (2015), 424–431.
[11] John Loeffler. 2019. Google's Quantum Processor May Achieve Quantum Supremacy in Months. *Interesting Engineering* (June 2019). https://interestingengineering.com/googles-quantum-processor-may-achieve-quantum-supremacy-in-months
[12] Erik Lucero. 2019. A New Law to Describe Quantum Computing's Rise? *Quanta Magazine* (June 2019).
[13] Ashley Montanaro. 2016. Quantum algorithms: an overview. *npj Quantum Information* 2 (Jan. 2016). https://doi.org/10.1038/npjqi.2015.23
[14] J. Moreno-Leon, G. Robles, and M. Roman-Gonzalez. 2015. Dr. Scratch: Automatic Analysis of Scratch Projects to Assess and Foster Computational Thinking. *RED: Revista de Educacion a Distancia* 46 (September 2015).
[15] Alberto Peruzzo, Jarrod McClean, Peter Shadbolt, Man-Hong Yung, Xiao-Qi Zhou, Peter J. Love, Alán Aspuru-Guzik, and Jeremy L. O'Brien. 2014. A variational eigenvalue solver on a photonic quantum processor. *Nature Communications* 5 (Jul 2014). https://doi.org/10.1038/ncomms5213

[16] Markus Reiher, Nathan Wiebe, Krysta M. Svore, Dave Wecker, and Matthias Troyer. 2017. Elucidating Reaction Mechanisms on Quantum Computers. In *Proceedings of the National Academy of Sciences*. National Academy of Sciences. https://www.microsoft.com/en-us/research/publication/elucidating-reaction-mechanisms-on-quantum-computers/

[17] Kathryn M. Rich, T. Andrew Binkowski, Carla Strickland, and Diana Franklin. 2018. Decomposition: A K-8 Computational Thinking Learning Trajectory. In *Proceedings of the 2018 ACM Conference on International Computing Education Research (ICER '18)*. ACM, New York, NY, USA, 124–132. https://doi.org/10.1145/3230977.3230979

[18] Kathryn M. Rich, Carla Strickland, T. Andrew Binkowski, and Diana Franklin. 2019. A K-8 Debugging Learning Trajectory Derived from Research Literature. In *Proceedings of the 50th ACM Technical Symposium on Computer Science Education (SIGCSE '19)*. ACM, New York, NY, USA, 745–751. https://doi.org/10.1145/3287324.3287396

[19] Kathryn M. Rich, Carla Strickland, T. Andrew Binkowski, Cheryl Moran, and Diana Franklin. 2017. K-8 Learning Trajectories Derived from Research Literature: Sequence, Repetition, Conditionals. In *Proceedings of the 2017 ACM Conference on International Computing Education Research (ICER '17)*. ACM, New York, NY, USA, 182–190. https://doi.org/10.1145/3105726.3106166

[20] J. Sarama, D. H. Clements, J. Barrett, D. W. Van Dine, and J. S. McDonel. 2011. Evaluation of a learning trajectory for length in the early years. *ZDM - Mathematics Education* 43 (2011), 667–680.

[21] Linda Seiter and Brendan Foreman. 2013. Modeling the Learning Progressions of Computational Thinking of Primary Grade Students. In *Proceedings of the Ninth Annual International ACM Conference on International Computing Education Research (ICER '13)*. ACM, New York, NY, USA, 59–66. https://doi.org/10.1145/2493394.2493403

[22] Peter W. Shor. 1997. Polynomial-Time Algorithms for Prime Factorization and Discrete Logarithms on a Quantum Computer. *SIAM J. Comput.* 26, 5 (Oct. 1997), 1484–1509. https://doi.org/10.1137/S0097539795293172

[23] Peter W. Shor. 2003. Why Haven'T More Quantum Algorithms Been Found? *J. ACM* 50, 1 (Jan. 2003), 87–90. https://doi.org/10.1145/602382.602408

[24] M. A. Simone. 1995. Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education* 26, 2 (1995), 114–145.