A Consistent Correlation between V_s, SPT, and CPT Metrics for Use in Liquefaction Evaluation Procedures

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ABSTRACT

The objective of this study is to develop a consistent relationship between small strain shear wave velocity (V_s), corrected cone penetration test (CPT) tip resistance (q_{c1Ncs}), and corrected standard penetration test (SPT) blow count ($N_{1,60cs}$) for liquefiable soils. In the absence of actual measurements of V_s in the field, it is common to use data from SPT or CPT testing to estimate V_s. However, empirical correlations between pairs of these in situ metrics can yield significantly different values of V_s. Using recent correlations between cyclic resistance ratio normalized to M7.5 ($CRR_{M7.5}$) and V_s normalized to one atmosphere of overburden (V_{s1}), q_{c1Ncs}, or $N_{1,60cs}$, a consistent relationship is developed such that reasonably similar values of V_s can be obtained using either q_{c1Ncs} or $N_{1,60cs}$. In comparison to two published V_s correlations, the correlations given in this study provide an average V_s value when using $N_{1,60cs}$ as the dependent variable but a slightly lower prediction of V_s when using q_{c1Ncs} as the dependent variable.

INTRODUCTION

Small strain shear wave velocity (V_s) can be measured in the field using several different methods, such as surface-wave, down-hole, or cross-hole measurements. However, these tests are not always performed, and thus, it is useful to be able to estimate V_s from the results of more common tests such as the standard penetration test (SPT) or the cone penetration test (CPT). Many published empirical correlations for liquefiable soils exist relating V_s and SPT or CPT data, but it is uncertain how consistent these correlations are. Accordingly, the objective of this study is to develop a consistent relationship between V_s, corrected CPT tip resistance (q_{c1Ncs}), and corrected SPT blow count ($N_{1,60cs}$) for liquefiable soils (e.g., Green and Ziotopoulou 2015).

First, a comparison of existing correlations between SPT or CPT data and V_s is given. Then a methodology for regressing a set of correlations will be proposed, and the resulting correlations will be compared with existing correlations using two liquefaction case history databases and a set of published case histories where SPT, CPT, and V_s measurements were made.

COMPARISON OF EXISTING CORRELATIONS

To assess the effectiveness and consistency of published correlations in estimating V_s for liquefiable soils, a small sampling of existing correlations was applied to two separate databases of liquefaction case histories: one in which CPT tests were performed and one in which SPT tests were performed. Both of these databases draw heavily from previously published databases (Boulanger and Idriss 2014; Idriss and Boulanger 2010), with the CPT database used having slightly different q_{c1Ncs} values due to the use of a new correlation between soil behavior index, I_c and fines content, FC:

$$FC = 80.82 I_c - 139.38 \quad (1)$$

This equation is modified for global application from a regional correlation for Christchurch, New Zealand (e.g., Maurer et al. 2019). Other pertinent modifications to the databases are outlined subsequently. The existing correlations selected for comparison are summarized in Table 1. Note that the Andrus et al. (2004) correlations compute equivalent clean sand V_s normalized to 1 atm effective overburden, $(V_{s1})_{cs}$. There is no difference between $(V_{s1})_{cs}$ and V_s normalized to 1 atm effective overburden pressure when FC is less than 5%. Thus for FC less than 5%, $(V_{s1})_{cs}$ was converted to V_s using the following relationship (e.g., Kayen et al. 2013):

$$V_s = \left(V_{s1}\right)_{cs} \left(\frac{\sigma'_v}{P_a}\right)^{0.25} \quad (2)$$

where σ'$_v$ is vertical effective stress and P_a is 1 atm pressure in the same units as σ'$_v$. Computed values of V_s from $N_{1,60cs}$ or q_{c1Ncs} using the correlations from Table 1 for the two case history databases are shown in Figure 1. Note the wide range of V_s values computed using $N_{1,60cs}$. Andrus et al. (2004) and Wair et al. (2012) propose relationships for both CPT and SPT data, so it is possible to see if these correlations provide consistent V_s estimations. There is no CPT-based counterpart to the SPT-based relationship by Tsai and Kishida (2015), and thus it cannot be used for both SPT and CPT databases. Because the SPT and CPT case history databases do not draw from identical case histories, it is not expected that the median values of V_s will be exactly the same, but it is expected that they should be similar because the case histories draw from several of the same general geographic regions associated with the same earthquakes. The relationships from Wair et al. (2012) yield noticeably different medians and distributions of V_s, whereas the Andrus et al. (2004) relationships yield nearly the same medians and reasonably similar distributions of V_s. Computed values of V_s using the Andrus et al. (2004) correlations have a median value of 144 m/s for clean sand case histories in the SPT database and 150 m/s in the CPT database, compared to median values of 130 and 166 m/s for the SPT and CPT databases, respectively, using the Wair et al. (2012) correlations.

Though alignment of the medians of the computed V_s values of the two databases is desirable, it is also desirable that the set of correlations relating V_s and $N_{1,60cs}$ or q_{c1Ncs} lead to similar values of predicted cyclic resistance ratio (CRR) given the same V_s normalized to 1 atm overburden, V_{s1}. As shown in Figure 2, there is poor agreement among three published CRR curves normalized to M7.5 ($CRR_{M7.5}$) (Andrus et al. 2003; Boulanger and Idriss 2012; Green et al. 2019) when using the Andrus et al. (2004) correlations to convert $N_{1,60cs}$ and q_{c1Ncs} to V_{s1}. The goal of this study is to regress a set of correlations between V_s and $N_{1,60cs}$ and between V_s and q_{c1Ncs} to align CRR curves for three types of in situ metrics: $N_{1,60cs}$, q_{c1Ncs}, and V_{s1}. The following section outlines this process.

REGRESSION OF A NEW SET OF CORRELATIONS

A third database of liquefaction case histories with V_s measurements was obtained to compare with the SPT- and CPT-based case history databases (Andrus et al. 2003). Case histories where V_s was measured using indirect surface-wave methods were removed so that the large uncertainties in the associated V_s values would not influence the correlation developed in this study. Values of cyclic stress ratio (CSR) corrected for 1 atm of overburden pressure and a

M7.5 earthquake (CSR*) were computed using the following equation:

$$CSR^{*} = 0.65 \frac{a_{max}}{g} \frac{\sigma_v}{\sigma'_{vo}} r_d \frac{1}{MSF \cdot K_\sigma} \tag{3}$$

where a_{max} is the maximum horizontal acceleration at the ground surface in g, σ_v and σ'_{vo} are the total and initial effective vertical stresses, respectively, r_d is the stress reduction factor, MSF is the magnitude scaling factor, and K_σ is the overburden correction factor. Values of a_{max} in the three databases were updated to reflect the most recent USGS ShakeMaps available (USGS "ShakeMap"), r_d and MSF were computed as per Green et al. (2019), and K_σ was computed using a new relationship by Green et al. (in preparation). Figure 3 shows CSR* vs. $N_{1,60cs}$, q_{c1Ncs}, or V_{s1} for the case histories in the three databases.

Table 1. Examples of Published V_s Correlations. Note: V_s and $(V_{s,1})_{cs}$ in m/s.

Reference	Equation	Notes
Andrus et al. (2004)	$(V_{s,1})_{cs} = 87.8\left[N_{1,60cs}\right]^{0.253}$	$(V_{s,1})_{cs}$ is equivalent clean soil V_s normalized to 1 atm overburden.
Wair et al. (2012)	$V_s = 26\left(N_{60}\right)^{0.215}\left(\sigma'_v\right)^{0.275}$	N_{60} is SPT blow count corrected to 60% energy efficiency, σ'_v is vertical effective stress in kPa. For Holocene soils.
Tsai and Kishida (2015)	$\ln(V_s) = 4.52$ $+0.22\ln\left(N_{1,60cs}\right)$ $+0.11\ln\left(\sigma'_v\right)$ $-0.03\ln(FC) + 0.02\ln(PI)$	FC is fines content in percent, PI is plasticity index, σ'_v is vertical effective stress in kPa. When FC = 0, remove FC term. When PI = 0, remove PI term.
Andrus et al. (2004)	$(V_{s,1})_{cs} = 62.6[q_{c1Ncs}]^{0.231}$	$(V_{s,1})_{cs}$ is equivalent clean soil V_s normalized to 1 atm overburden.
Wair et al. (2012)	$V_s = 118.8\log(f_s) + 18.5$ $V_s = 2.27(q_t)^{0.412}(I_c)^{0.989}(z)^{0.033}$ $V_s = \sqrt{\frac{10^{0.55I_c+1.68}(q_t - \sigma_v)}{P_a}}$	Wair et al. (2012) suggest taking the average of these three correlations: Mayne (2007), Andrus et al. (2007), and Robertson (2009) where f_s is the side friction in kPa, q_t is cone tip resistance in kPa, I_c is soil behavior index, z is depth in m, and P_a is 1 atm in the same units as σ_v and q_t.

Three $CRR_{M7.5}$ curves as a function of $N_{1,60cs}$, q_{c1Ncs}, or V_{s1} were drawn so that they generally followed the lower bound of the CSR* values computed for the case histories in which liquefaction was observed, as shown in Figure 3. In each case, the $CRR_{M7.5}$ curve was modified from a published curve to better fit the new CSR* values for the case histories computed in this study. These published curves were Andrus et al. (2003; Aea03) for the V_s case histories and Green et al. (2019; Gea19) for the CPT and SPT case histories. The Gea19 $CRR_{M7.5}$ curve was modified for the SPT-based case histories, where $N_{1,60cs}$ values were converted to q_{c1Ncs} values using the following relationship derived from expressions relating relative density (D_r) to q_{c1Ncs} and $N_{1,60cs}$ given in Idriss and Boulanger (2010):

$$q_{c1Ncs} = \left[0.308 \cdot \left(N_{1,60cs}\right)^{0.5} + 2.224\right]^{3.788} \quad (4)$$

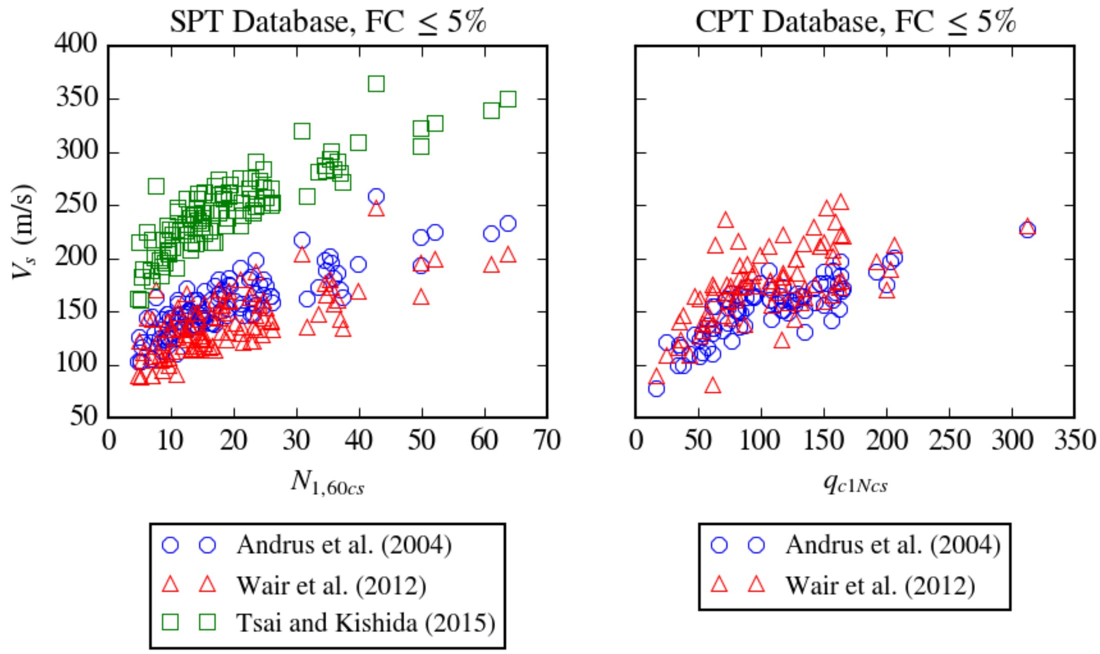

Figure 1. Computed V_s vs $N_{1,60cs}$ and q_{c1Ncs} using published correlations for two different liquefaction case history databases.

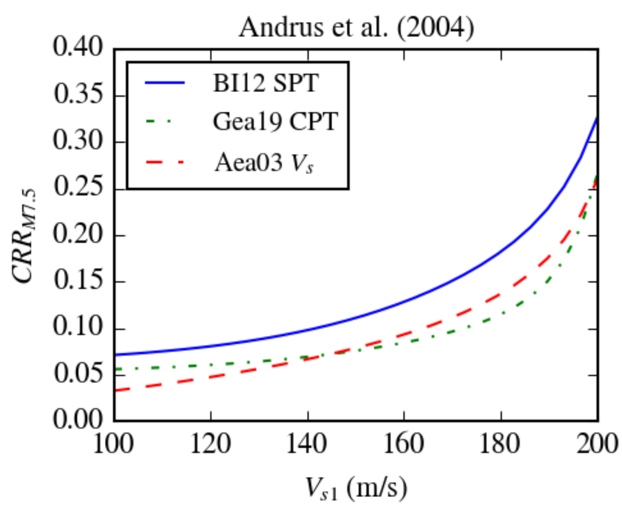

Figure 2. Comparison of $CRR_{M7.5}$ curves (Andrus et al. 2003; Boulanger and Idriss 2012; Green et al. 2018) when Andrus et al. (2004) is used to convert $N_{1,60cs}$ and q_{c1Ncs} to V_{s1}.

Two correlations were regressed of the form $V_{s,1} = A(N_{1,60cs})^B$ and $V_{s,1} = A(q_{c1Ncs})^B$ such that the Gea19 $CRR_{M7.5}$ curves were in accord with the Aea03 $CRR_{M7.5}$ curve in $CRR_{M7.5}$-$V_{s,1}$ space. The regression was targeted within the range of $V_{s,1}$ values where liquefied case histories were observed (i.e., extreme trends in the $CRR_{M7.5}$ curves were ignored). The resulting regressed correlations are:

$$V_{s,1} = 61.89\left(N_{1,60cs}\right)^{0.360} \tag{5}$$

$$V_{s,1} = 16.88\left(q_{c1Ncs}\right)^{0.489} \tag{6}$$

The desired value from these correlations is V_s, which can be computed from these equations as:

$$V_s = 61.89\left(N_{1,60cs}\right)^{0.360}\left(\frac{\sigma_v'}{P_a}\right)^{0.25} \tag{7}$$

$$V_s = 16.88\left(q_{1Ncs}\right)^{0.489}\left(\frac{\sigma_v'}{P_a}\right)^{0.25} \tag{8}$$

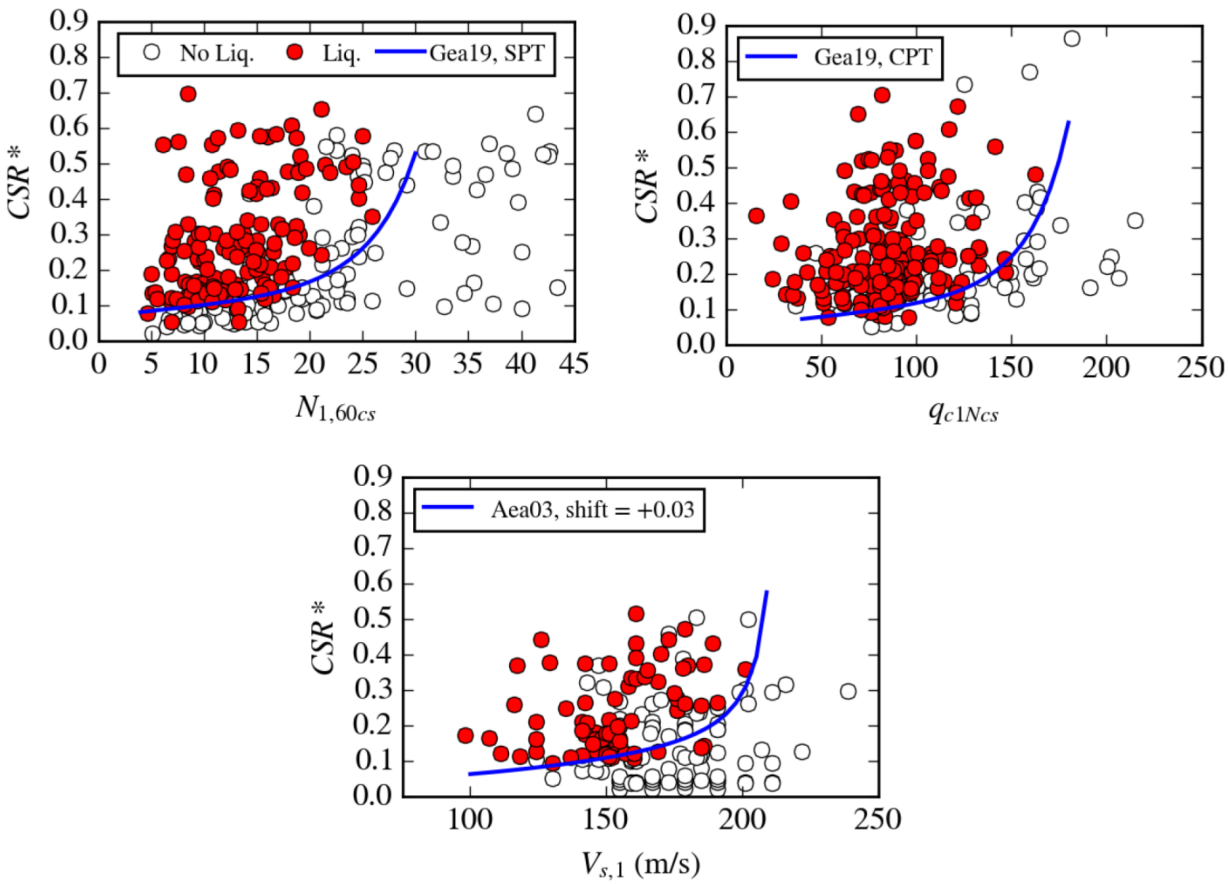

Figure 3. CSR* vs. in situ metrics for three liquefaction case history databases and selected $\mathbf{CRR_{M7.5}}$ curves. CSR* are updated values as computed in this study. Liq.: liquefaction was observed; No Liq.: no liquefaction was observed.

RESULTS

The three selected $CRR_{M7.5}$ curves align almost perfectly when using the correlations given in this study, as shown in Figure 4. However, these $CRR_{M7.5}$ curves begin to diverge outside the range of V_{s1} from 100 to 200 m/s. This is not a significant concern because this is the limit of the range of V_{s1} values for case histories where liquefaction was observed, and therefore the trends of the $CRR_{M7.5}$ curves outside of this range are less significant to the purpose of this study.

The set of correlations given in this study are also mutually consistent with the D_r–based correlations from Idriss and Boulanger (2010). This is because these D_r–based correlations were used to modify the CPT-based Gea19 $CRR_{M7.5}$ curve to compute $CRR_{M7.5}$ using $N_{1,60cs}$. As shown in Figure 5, for the ranges of q_{c1Ncs} and $N_{1,60cs}$ in which liquefaction is a concern, the correlations given in this study better align with the q_{c1Ncs} and $N_{1,60cs}$ values produced by the Boulanger and Idriss (2014) (BI14) D_r-based correlations. In addition, the median values of V_s using the correlations from this study are reasonably similar between the CPT and SPT case history databases: 125 and 132 m/s, respectively.

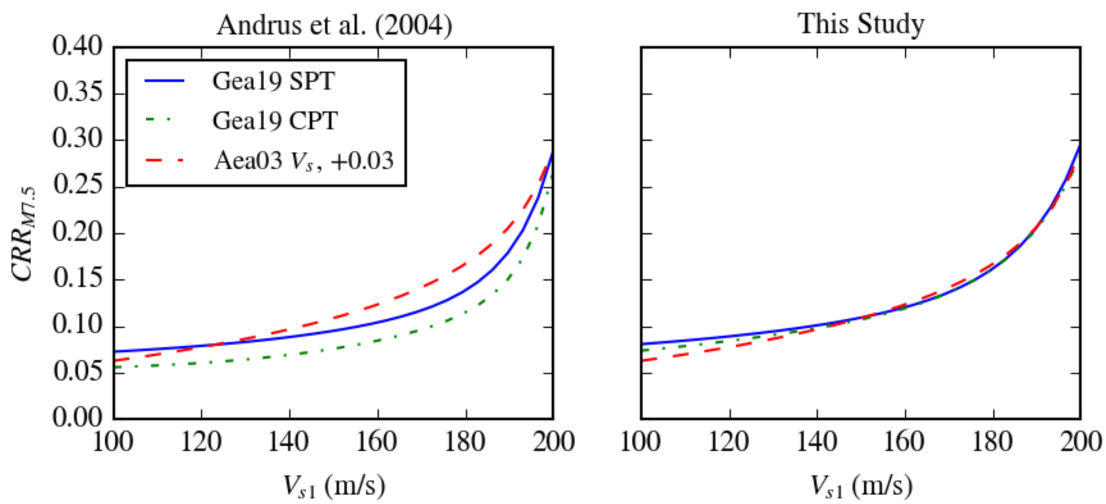

Figure 4. Comparison of $CRR_{M7.5}$ curves when correlations from Andrus et al. (2004) and this study are used to convert $N_{1,60cs}$ and q_{c1Ncs} to V_{s1}.

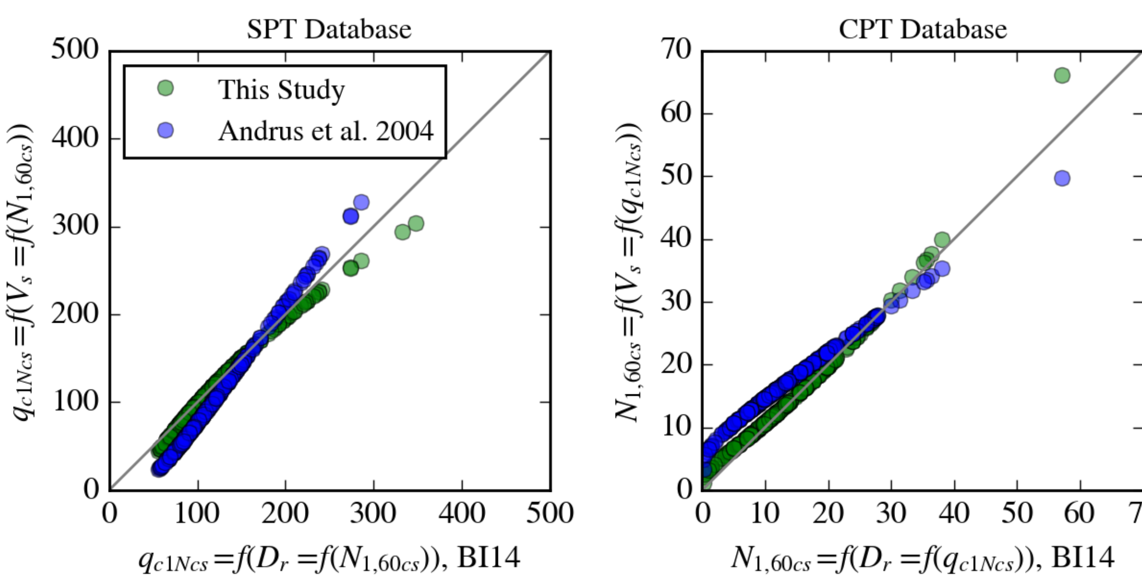

Figure 5. Direct comparisons of q_{c1Ncs} values (or $N_{1,60cs}$ values) converted from $N_{1,60cs}$ values (or q_{c1Ncs} values) using V_s-based correlations and those converted using D_r-based correlations.

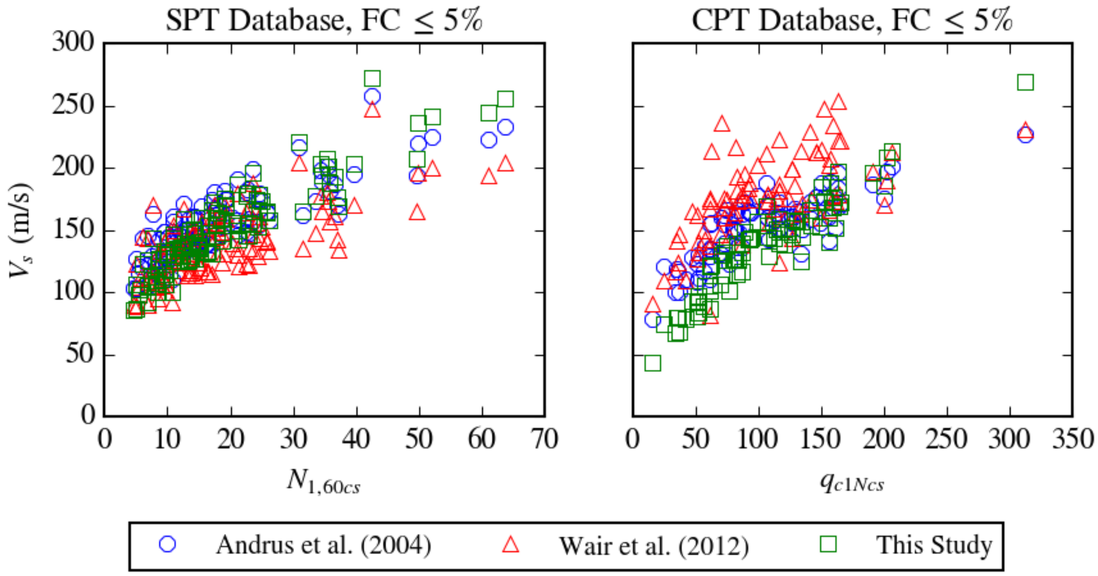

Figure 6. Computed V_s vs $N_{1,60cs}$ and q_{c1Ncs} using published correlations and correlations given in this study for two different liquefaction case history databases.

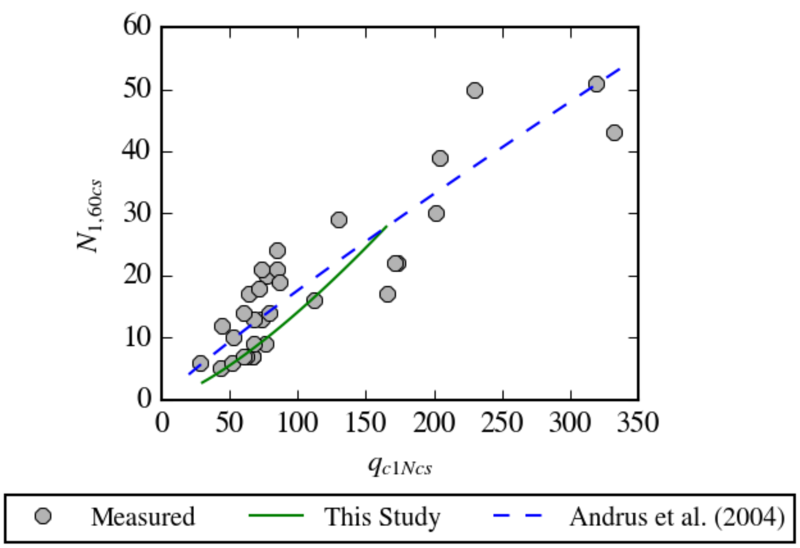

Figure 7. Pairs of $N_{1,60cs}$ and q_{c1Ncs} from the same sites given in Andrus et al. (2004) compared to the correlations developed in this study and those developed by Andrus et al.

DISCUSSION

Figure 6 compares the correlations provided in this study to two other published V_s correlations for the SPT and CPT databases. In comparison to the other two V_s correlations, the correlations given in this study provide an average V_s value when using $N_{1,60cs}$ as the dependent variable but a slightly lower prediction of V_s when using q_{c1Ncs} as the dependent variable. Figure 7 shows pairs of q_{c1Ncs} and $N_{1,60cs}$ from tests performed at the same location (Andrus et al. 2004). Using the set of V_s correlations given in this study to convert values of q_{c1Ncs} to $N_{1,60cs}$ matches the paired data fairly well, but not as well as the Andrus et al. (2004) set of correlations. Note that the correlations provided in this study only apply to a limited range of $N_{1,60cs}$, q_{c1Ncs}, and $V_{s,1}$ values corresponding to the portion of the $CRR_{M7.5}$ curves that were aligned to produce these correlations. Thus these correlations should be used with caution when outside the following ranges: $N_{1,60cs}$ = 6 to 27 blws/30 cm, q_{c1Ncs} = 50 to 155 atm, $V_{s,1}$ = 110 to 205 m/s, and σ'_v = 19 to 120 kPa.

CONCLUSIONS

A set of correlations to estimate V_s from $N_{1,60cs}$ or q_{c1Ncs} were developed for liquefiable soils. The correlations given in this study better align the $CRR_{M7.5}$ curves for three types of in situ metrics ($N_{1,60cs}$, q_{c1Ncs}, and $V_{s,1}$), align with D_r-based correlations with $N_{1,60cs}$ and q_{c1Ncs}, and yield reasonably similar V_s distributions between CPT and SPT databases. Thus, the correlations given in this study meet the desired criteria and are appropriate for use with liquefiable soils. However, these correlations should be used with caution when the vertical effective stresses and in situ test metrics are outside the ranges represented in the case history databases used in this study.

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