

1 The balancing act of *Nipponites mirabilis* (Nostoceratidae, Ammonoidea):
2 managing hydrostatics throughout a complex ontogeny

3

4 Short title: Hydrostatics of *Nipponites mirabilis*

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24 Abstract

25 *Nipponites* is a heteromorph ammonoid with a complex and unique morphology that
26 obscures its mode of life and ethology. The seemingly aberrant shell of this Late Cretaceous
27 nosteroceratid seems deleterious. However, hydrostatic simulations suggest that this morphology
28 confers several advantages for exploiting a quasi-planktic mode of life. Virtual, 3D models of
29 *Nipponites mirabilis* were used to compute various hydrostatic properties through 14 ontogenetic
30 stages. At each stage, *Nipponites* had the capacity for neutral buoyancy and was not restricted to
31 the seafloor. Throughout ontogeny, horizontally facing to upwardly facing soft body orientations
32 were preferred at rest. These orientations were aided by the obliquity of the shell's ribs, which
33 denote former positions of the aperture that were tilted from the growth direction of the shell.
34 Static orientations were somewhat fixed, inferred by stability values that are slightly higher than
35 extant *Nautilus*. The initial open-whorled, planispiral phase is well suited to horizontal
36 backwards movement with little rocking. *Nipponites* then deviated from this bilaterally
37 symmetric coiling pattern with a series of alternating U-shaped bends in the shell. This
38 modification allows for proficient rotation about the vertical axis, while possibly maintaining the
39 option for horizontal backwards movement by redirecting its hyponome. These particular
40 hydrostatic properties likely result in a tradeoff between hydrodynamic streamlining, suggesting
41 that *Nipponites* assumed a low energy lifestyle of slowly pirouetting in search for planktic prey.
42 Each computed hydrostatic property influences the others in some way, suggesting that
43 *Nipponites* maintained a delicate hydrostatic balancing act throughout its ontogeny in order to
44 facilitate this mode of life.

45

46 Introduction

47 Heteromorph ammonoids are ectocochleate cephalopods whose shells undergo changes
48 in coiling throughout ontogeny. The seemingly aberrant shape of some heteromorph ammonoids
49 piques curiosity about their enigmatic modes of life and life habit. Arguably, the most bizarre
50 and conspicuous of all heteromorph genera is the Late Cretaceous (Turonian – Coniacian)
51 nosteroceratid, *Nipponites* (Fig 1). Previous research has largely focused on the biostratigraphic
52 usefulness of *Nipponites* [1–5] rather than its paleobiology [6–7] and evolutionary significance
53 [8]. The latter two areas are valuable because the morphology of this heteromorph appears
54 deleterious to survival, seemingly defying the basic principles of natural selection [9–15]. It is
55 more likely, however, that its functional morphology is obscured by a complex ontogenetic
56 trajectory in shell growth. The shell of *Nipponites* is characterized by having several open
57 planispiral (crioconic) whorls in early ontogeny, followed by a series of alternating U-bends
58 around the earlier whorls (Fig 1); denoting some degree of regularity in coiling throughout a
59 seemingly-aberrant ontogeny [1,16,17]. Okamoto [18–20] demonstrated that the coiling of
60 *Nipponites mirabilis* is, in fact, well constrained and can be approximated by a few piecewise
61 equations (alternations of sinistral and dextral helicoid phases surrounding the crioconic phase).
62 Similarly, differential geometry has proven a useful tool in modeling these complex
63 heteromorphs [21–22]. The morphology of this genus varies from the more compact *N. mirabilis*
64 to the less compact *N. occidentalis* and *N. bacchus* which both have more open whorls. The
65 current study focuses on *N. mirabilis* unless otherwise stated. This iconic type species was
66 chosen based on its regularity in coiling and the availability of data required to construct
67 hydrostatic models.
68

69 **Fig 1. Hydrostatic Parameters of *Nipponites mirabilis*.** **A**, Side view of *Nipponites* in life
70 position showing hypothetical centers of buoyancy (B), mass (M), and the horizontal axis of
71 rotation (R). The angle of the aperture (θ_a) is measured as the inclination from the vertical plane.
72 The thrust angle (θ_t) can be used to assess the directional efficiency of movement. This angle is
73 measured between the horizontal plane, and a line passing through R and the location of the
74 hyponome (source of thrust; H). **B**, Front view of *Nipponites* in life position facing the aperture.
75 This view shows the total lever arm (L) and its x-component (L_x) which is proportionate to the
76 amount of rotational movement about the vertical axis produced during jet propulsion. **C**, Top
77 view of *Nipponites* in life position showing the rotational thrust angle (θ_{tr}). This angle is
78 measured between the vertical rotation axis (vert.), which passes through B and M, and the
79 direction of the thrust vector (arrow emanating from H). Rotational thrust angles of 90° result in
80 idealized transmission of thrust into pure rotation.

81

82 The complex, meandering shell of *Nipponites* has invited several different interpretations
83 regarding potential modes of life assumed by this heteromorph. The shell morphology of
84 *Nipponites* has been compared to vermetid gastropods, and by analogy, this heteromorph has
85 been suggested to assume a sessile and benthic mode of life [23–28]. Trueman [29] also
86 considered *Nipponites* as benthic, but with some degree of mobility. Other nostoceratid genera
87 have been interpreted as negatively buoyant, benthic organisms as well [28,30,31]. By similar
88 analogy with other ‘irregularly-coiled’ mollusks, a symbiotic relationship with sponges or
89 hydrozoans occupying the free space between the whorls of *Nipponites* has been speculated [32],
90 although, no fossil evidence currently supports such a relationship. Contrasting benthic
91 interpretations, Ward & Westermann [33] suggest that *Nipponites occidentalis* was capable of a

92 planktic mode of life based on approximate calculations of organismal density. This mode of life
93 is supported by Okamoto [19] for *Nipponites mirabilis* due to the oscillation of rib obliquity of
94 the shell. Changes in rib obliquity suggests that some proper orientation of the soft body was
95 preferred, which would not matter during a negatively buoyant condition. Favoring a planktic
96 mode of life, Westermann [6] inferred that *Nipponites* was an occupant of the epipelagic, oceanic
97 waters, perhaps as a vertical migrant or planktic drifter. Its morphology is certainly not
98 streamlined, suggesting that it would have experienced considerably more hydrodynamic drag
99 than its planispiral counterparts with similar whorl expansion and width. The unique shell of this
100 genus raises questions regarding how its changes in coiling may reflect the modification of syn
101 vivo hydrostatic properties. Such a tactic was suggested for other morphologies of heteromorph
102 ammonoids [17,19,20,34-39]. These physical properties are vital to understand if *Nipponites* was
103 a benthic crawler, a planktic drifter, an active swimmer, or just an atypical genus with a
104 morphology that was not detrimental for survival or reproduction. Ultimately, an investigation of
105 this unique ammonoid can provide data to better understand heteromorph ammonoid modes of
106 life, and perhaps, the selective pressures which may have acted on particular morphotypes.

107 **Hydrostatic properties of heteromorph ammonoids**

108 The ability of ectocochleate cephalopods to attain neutral buoyancy is fundamental to
109 reconstruct their modes of life. The variable interpretations for nostoceratid modes of life
110 illustrate the importance of new techniques to determine the physical properties that would have
111 acted on these living cephalopods. A neutrally buoyant condition is achieved when the total
112 organismal mass is equal to the mass of the water displaced by the living animal. This depends
113 upon the body chamber to phragmocone ratio. If the phragmocone (the chambered portion of the
114 shell) is too small, the living cephalopod would not be able to compensate for its organismal

115 weight and it would become negatively buoyant [34,36,40]. This condition also depends upon
116 shell thickness and the densities assigned to each component of the living animal, which have
117 been somewhat variable in previous research [39,41].

118 Previous studies have demonstrated that heteromorph ammonoids may have been able to
119 achieve much different life orientations than their planispiral counterparts, which are not
120 restricted to horizontal or diagonally-upwards soft body positions [20,29,34–39,42–45]. These
121 living cephalopods would have assumed some static orientation when their centers of buoyancy
122 and mass were vertically aligned [41,46,47] (Fig 1). The difficulty to which these living
123 cephalopods could deviate from their static orientation depends on hydrostatic stability, which is
124 proportionate to the separation between the centers of buoyancy and mass [20]. High stability
125 would have reduced the influence of external forms of energy on orientation, but would have
126 simultaneously made it more difficult for the living cephalopod to self-modify its orientation
127 [36].

128 The directional efficiency of movement (thrust angle) depends upon the relative position
129 of the source of thrust (the hyponome) and the center of rotation (the midpoint between the
130 centers of buoyancy and mass; Fig 1A, B). Thrust energy produced by jet propulsion is more
131 efficiently transmitted into movement in the direction where the hyponome and center of rotation
132 are aligned [20,38,39,48,49]. If these two points were horizontally aligned (thrust angle of zero),
133 more energy would be transmitted to horizontal movement with minimal rocking. The rocking
134 behavior of extant nautilids is related to their sub-horizontal thrust angles and the retraction of
135 the soft body during emptying of the mantle cavity [50].

136 A rotational component of energy is increased by turning the direction of thrust out of
137 alignment with the centers of buoyancy and mass (the axis where idealized rotation would occur;

138 Fig 1B, C). An increased distance of the hyponome from these two centers would therefore
139 produce a lever arm that would impart a torque to rotate the living cephalopod about its vertical
140 axis. This type of movement is likely to have taken place for turrilitid heteromorphs [34], as well
141 as other morphotypes with their apertures positioned in a similar relative manner [37]. Idealized
142 rotation about the vertical axis would occur with a long, horizontally oriented lever arm and a
143 thrust vector adjoining its distal end with a right angle.

144 Each of these physical properties would have significantly constrained the hydrostatic
145 and hydrodynamic capabilities of living *Nipponites* throughout its ontogeny. Therefore, they
146 provide fundamental information regarding the possible modes of life and life habit for this
147 unique ammonoid, as well as possible adaptations for locomotion and feeding.

148

149 **Methods**

150 Virtual models were constructed to determine the *syn vivo* hydrostatic properties of
151 *Nipponites mirabilis*. Construction of the shell and other model components largely follow the
152 methods of Peterman et al. [37–39], although a CT scanned specimen was used as the base
153 model instead of using photogrammetry (similar to the methods of Morón-Alfonso [51]). This
154 modification from the previous methods was preferred for this species due to the complex
155 changes in shell ornamentation (rib obliquity). These ribs are parallel to the successive positions
156 of the aperture throughout ontogeny, therefore retaining vital information about life orientation
157 [20]. This method for virtual reconstruction is favorable for *Nipponites* because specimens of this
158 genus are rarely found complete; discouraging destructive sampling techniques like serial
159 grinding tomography [49,56,57]. Computed tomography (CT) scans of such specimens also lack
160 contrasts of X-ray attenuation factors to distinguish the shell from its surrounding materials [52].

161 However, each of these tomographic techniques can provide very accurate measurements of
162 hydrostatic properties and volumes when the specimens are adequate for imaging [52–59].

163 No permits were required for the described study, which complied with all relevant
164 regulations.

165 **Virtual modeling of the shell**

166 The shell of *Nipponites mirabilis* was constructed from an initial CT scan [60] of the
167 specimen INM-4-346 (Museum Park Ibaraki Prefectural Museum of Nature; Ibaraki, Japan),
168 which had a remarkable degree of preservation. Most of the ontogeny is preserved in this
169 specimen with minimal matrix on the inside (Fig 2A). However, two portions had to be virtually
170 reconstructed; 1) the crushed ~5 cm section of the adoral-most part of the body chamber, and 2)
171 the earliest crioconic whorls that are partially embedded in a remnant of the original concretion.

172 These two portions of the shell were reconstructed (Fig 2B) with array algorithms [37–40],
173 which replicate a whorl section and simultaneously translate, rotate, and scale it to build the shell
174 from the adoral direction to adapical direction (Table 1). Such arrays are similar to the
175 morphospace parameters of Raup [61]. The CT-scanned model [60] (which consists of a stack of
176 .tiff images) was converted to the tessellated .stl format required for model reconstruction and
177 volumetry using the program Molcer 1.51 [62]. The external mesh of the tessellated file was
178 isolated in order to get rid of internal features like fissures and X-ray attenuation artifacts.

179 External defects were smoothed in Meshmixer 3.3 [63] while maintaining the curvature of
180 neighboring, complete features. This external 3D-mesh served as a stencil for the reconstruction
181 of the missing and damaged portions of the shell. After the missing portions of the shell were
182 combined to the model derived from the CT-scan, the ornamentation was reconstructed by
183 matching the width and amplitude of ribs with a torus shape in Blender [64], then properly

184 oriented using the ribs present on the inner whorls. The ornamentation, reconstructed portions of
 185 the shell (Fig 2B), and the total external mesh were repaired and unified in Netfabb [65] to
 186 produce a single manifold mesh of the exterior shell. The program Blender was used to assign
 187 shell thickness to the external shell model based on measurements from specimen NMNS
 188 (National Museum of Nature and Science; Tokyo, Japan) PM35490 (Fig 3), producing a mesh
 189 denoting the entire shell without septa.

190

191 **Fig 2. Virtual Reconstruction of the Shell of *Nipponites mirabilis*.** **A**, Tessellated (.stl) 3D
 192 model generated from a CT-scan [60] of specimen INM-4-346. **B**, Reconstructed adoral portion
 193 of the body chamber and inner criocone phase with arrays algorithms (Table 1). **C**, Extruded
 194 septa generated from the suture pattern. **D**, Extruded shell and septa models unified together to
 195 produce a single, manifold 3D mesh of the entire shell.

196 **Fig 3. Thickness Measurements used for Virtual Model Extrusion.** Thicknesses of the shell
 197 (black) and septa (grey) as a function of whorl height. Measurements were recorded from
 198 specimen NMNS PM35490 and used to define thickness in the virtual model.

199

200 **Table 1. Reconstruction of the Shell**

Terminal Body Chamber		Translation (mm)			Rotation (degrees)			Scale		
Array #	# Replications	X	Y	Z	X	Y	Z	X	Y	Z
1*	67	-0.035	0.018	-0.265	-0.70	0.59	0.40	1.000	1.000	1.000
2*	29	-6.722	-6.548	-14.237	-0.70	1.00	-0.60	1.000	1.001	1.002
3*	40	-11.103	-12.910	-16.980	0.20	0.19	-1.70	1.002	0.999	1.001
Criocone Phase		Translation (mm)			Rotation (degrees)			Scale		
Array #	# Replications	X	Y	Z	X	Y	Z	X	Y	Z
1*	55	-30.050	-11.085	2.920	0.53	1.10	0.70	0.996	0.997	0.996
2	100	-30.057	-11.171	2.960	0.37	1.30	0.88	0.996	0.996	0.996

3*	122	-14.760	-14.063	5.757	0.39	0.88	0.69	0.998	0.998	0.998
4*	83	-25.009	-11.595	5.269	-0.15	1.80	0.70	0.997	0.997	0.997
5*	59	-20.306	-15.710	10.206	0.38	1.00	0.50	0.998	0.998	0.998
6	154	-20.286	-15.722	10.188	0.45	1.50	1.00	0.996	0.996	0.996

201 Array instructions used to reconstruct the juvenile criocone phase and the adoral portion of the
 202 terminal body chamber. These arrays were used in a piecewise manner to replicate the whorl
 203 section from the adoral direction to adapical direction by translation, rotation, and scaling in the
 204 x, y, and z directions. Asterisks denote arrays that had their origins reset to their current locations
 205 before replication. If origins were not reset, the origins of their previous arrays were used.

206

207 Septa were constructed by recording a suture pattern from specimen NMNS PM35490
 208 (Fig 4). The external shell of this specimen (Fig 4B, C) was removed with air abrasives and
 209 pneumatic tools under a stereoscopic microscope and the suture (Fig 4D) was recorded with a
 210 digital camera lucida. This suture was imported in the Blender workspace and the curve modifier
 211 was used to wrap it around the whorl section of the shell. This suture was then replicated and
 212 placed along the majority of the phragmocone so that adjacent lobules and folioles were almost
 213 tangential. Ontogenetic changes in the suture pattern were not considered because they probably
 214 represent only small differences in mass and its distribution. That is, each suture had the same
 215 degree of complexity and its expanded portion was placed adjacent to the venter throughout
 216 ontogeny. The crioconic, bilaterally symmetric juvenile phase was reconstructed with array
 217 algorithms, which allowed septa to be duplicated with the same equations. The septa within the
 218 majority of the phragmocone were constructed by extruding the suture patterns inwards to a
 219 single point, then refining and smoothing the interior in order to approximate minimum curvature
 220 surfaces. A body chamber ratio of approximately 42% the total curvilinear length was measured
 221 from a remarkably complete specimen of *Nipponites mirabilis* from a private collection. This

222 specimen was 3D scanned with an Artec Space Spider (to allow comparisons with the CT
223 scanned specimen) and is housed in the morphosource database [66]. A nearly complete
224 specimen of *Nipponites mirabilis* (MCM-A0435; Mikasa City Museum, Mikasa, Japan; Fig 4E)
225 was also compared in this manner and stored in the database [66], which yielded an approximate
226 body chamber ratio of 36%. This ratio would be slightly higher if the aperture was not partially
227 crushed. The proper number of septa to maintain the body chamber ratio of around 42% were
228 placed in the phragmocone and extruded based on measured thicknesses from specimen NMNS
229 PM35490 (Fig 3). These final septa (Fig 2C) were merged with the extruded, external shell to
230 produce a single, manifold 3D mesh of the entire, septate shell (Fig 2D).

231

232 **Fig 4. Specimens Used for Shell Reconstruction.** **A**, Original concretion containing NMNS
233 PM35490. Umbilical view (**B**) and ventral view (**C**) of the shell section used to record the suture
234 pattern (**D**). **E**, 3D scan of the Mikasa City Museum Specimen (MCM-A0435) used to
235 approximate the body chamber ratio. Scale bar = 2 cm.

236

237 **Virtual modeling of the soft body and camerae**

238 The shell constrains the size and shape of other model components that influence
239 hydrostatics. A model of the soft body was constructed by isolating the interface between the
240 shell and the body chamber, and similarly, the camerae were isolated from the phragmocone of
241 the shell. The faces of the isolated body chamber and camerae meshes were inverted so that the
242 normals (vectors denoting the outside) were pointed outwards. The ammonoid soft body is
243 largely unknown; however, due to phylogenetic bracketing [67], the presence of ten arms can be
244 inferred [68–70] with a possibly reduced (compact) soft body. A soft body resembling the

245 consensuses of Klug & Lehmann [70] and Landman et al. [71] was constructed for *Nipponites*
246 and unified to the repaired, isolated internal body chamber mesh. The camerae were later
247 partitioned into fractions of cameral liquid and cameral gas for hydrostatic calculations. For the
248 sake of model simplicity, both cameral liquid and cameral gas were assumed be evenly
249 distributed in the phragmocone. This assumption may be reasonable based on the retention of
250 cameral liquid via the pellicle and surface tension along septal margins [72]. However, the
251 distribution of cameral liquid likely changed throughout ontogeny, having been located behind
252 the soft body as new septa formed. The hydrostatic influence of this assumption is small as long
253 as the percentage of cameral liquid in the phragmocone is low. This process yielded mass
254 distributions of the fractions of cameral liquid and cameral gas that have the same centers as the
255 center of volume for all camerae.

256 **Modeling changes in shell morphology throughout ontogeny**

257 The final hydrostatic model of the adult *Nipponites mirabilis* was used to derive a total of
258 14 models representing different ontogenetic stages. This was accomplished by deleting the septa
259 in the phragmocone and deleting the adoral portion of the body chamber so that the proper body
260 chamber ratio was maintained throughout ontogeny. The apertures of these stages were chosen to
261 terminate near the apices and inflection points between the U bends of the shell in order to
262 sample a variety of former aperture positions. The total curvilinear distance along the venter
263 from the apex to the aperture was normalized by this same distance for the terminal stage,
264 yielding a proxy for the age of each model. This distance was measured virtually in Blender, by
265 isolating a curved line along the venter in 3D space and measuring the sum of the tens of
266 thousands of straight line segments that closely approximate this curve. Such a metric for age in
267 only an estimate because both absolute and relative time cannot be quantified. Therefore, the

268 normalized value reported is relative, and represents the percentage of the way through an
 269 individual's lifespan if the shell's linear growth rate is held constant. It is unknown how this
 270 growth rate changes throughout ontogeny, but it likely would have changed in some way. It is
 271 also possible that the body chamber ratio of *Nipponites* subtly changed throughout ontogeny,
 272 which has been reported for other ammonoids [73].

273 **Hydrostatic calculations**

274 Neutral buoyancy occurs when the sum of organismal mass is equal to the mass of water
 275 displaced. The proportion of camerae to be emptied of cameral liquid relative to the total
 276 available cameral volume (Φ) that satisfies a neutrally buoyant condition was computed with the
 277 following equation (after Peterman et al., 2019a):

$$278 \Phi = \frac{\left(\frac{V_{wd}\rho_{wd} - V_{sb}\rho_{sb} - V_{sh}\rho_{sh}}{V_{ct}} \right) - (\rho_{cl})}{(\rho_{cg} - \rho_{cl})} \quad (1)$$

279 Where V_{wd} and ρ_{wd} are the volume and density of the water displaced, V_{sb} and ρ_{sb} are the volume
 280 and density of the soft body, V_{sh} and ρ_{sh} are the volume and density of the shell, ρ_{cl} is the density
 281 of cameral liquid, ρ_{cg} is the density of cameral gas, and V_{ct} is the total cameral volume of the
 282 phragmocone. A soft body density of 1.049 g/cm³ is preferred based on the measurement of
 283 *Nautilus* soft body by Hoffmann & Zachow [74] that was later averaged by a seawater-filled
 284 mantle cavity and thin mouthparts by Peterman et al. [38]. A shell density of 2.54 g/cm³ was
 285 adopted from Hoffman & Zachow [74]. The cameral liquid density of 1.025 g/cm³ [75] and
 286 cameral gas density of 0.001 g/cm³ are used in the current study.

287 The total center of mass is weighted according to each material of unique density (i.e., the
 288 soft body, shell, cameral liquid, and cameral gas in the current study). Each individual center of

289 mass for the soft body, shell, cameral liquid, and cameral gas were computed in MeshLab [76]
 290 and the total center of mass was computed with the equation:

291
$$M = \frac{\sum(L*m_o)}{\sum m_o} \quad (2)$$

292 Where M is the total center of mass in a principal direction, L is the center of mass of a single
 293 object measured with respect to an arbitrary datum in each principal direction, and m_o is the
 294 mass of any particular object that has a unique density. Equation 2 was used in the x, y, and z
 295 directions to compute the coordinate position of the center of mass.

296 The center of buoyancy (B) is equal to the center of volume of the medium displaced by
 297 the external model. A model denoting the exterior interface of *Nipponites* was constructed from
 298 the external shell and soft body protruding from the aperture and its center was computed in
 299 MeshLab.

300 The static orientation of the total model occurs when B and M are vertically aligned. The
 301 hydrostatic stability index is computed from these centers.

302
$$S_t = \frac{|B-M|}{\sqrt[3]{V}} \quad (3)$$

303 The separation between the centers of buoyancy (B) and mass (M) is normalized by the cube root
 304 of the organismal volume (V ; equal to the volume of seawater displaced) in order to be applied
 305 to ectocochleates with irregular coiling [20].

306 Apertural angles (θ_a) were measured with respect to the vertical (Fig 1A). That is, angles
 307 of zero correspond to a horizontally facing soft body, angles of $+90^\circ$ correspond to an upward
 308 facing soft body, and angles of -90° correspond to a downward facing soft body.

309 Thrust angles (θ_t) were measured with respect to the horizontal (Fig 1B) between the
 310 point source of thrust and the rotational axis. Therefore, as the thrust angle approaches zero,
 311 more energy is transmitted into horizontal movement with a lower rotational component.

312 Rotational thrust angles (θ_{tr}) were measured between the thrust vector (perpendicular to
313 the aperture) and the rotational axis (Fig 1C). A rotational thrust angle of 90° would allow pure
314 rotation to take place, while angles of 0° and 180° would result in translational movement.

315

316 **Results**

317 The unknown soft body can produce errors in buoyancy calculations depending upon its
318 total volume. By comparing the soft body used herein with a soft body that terminates at the
319 aperture, there is only a 0.5% difference in the proportion of the phragmocone to be emptied for
320 neutral buoyancy (Φ). Similarly, the mass distribution is not significantly different between
321 either model (a 0.7 % difference in hydrostatic stability), partially because the soft body density
322 is similar to that of seawater.

323 Because the body chamber ratio was somewhat variable (~6% different) on measured
324 specimens, this ratio was manipulated by removing one septum and adding one septum to the
325 terminal stage model with a body chamber ratio of 42%. Removing one septum increases the
326 total body chamber ratio to 46%. This change yields a 16% increase in Φ (to 84.6%) and a 7%
327 increase in S_t (to 0.0786). Adding one septum decreases the body chamber ratio to 37%, yielding
328 a 10% decrease in Φ (to 65.7%) and an 8% decrease in S_t (to 0.0676). These changes suggest that
329 a small error (~10%) in body chamber ratio would not significantly alter calculations of
330 buoyancy or the characteristics of the mass distribution. Small deviations from the ideal body
331 chamber ratio took place (Table 2) during model construction. However, the body chamber ratio
332 test suggests that their hydrostatic influences are minimal. Similarly, changes in the true body
333 chamber ratio throughout ontogeny would result in minimal error if such differences were
334 sufficiently low (approximately less than 10%).

335 The densities assumed in the hydrostatic calculations could potentially adjust the total
 336 mass and its distribution, therefore, directly influence the conditions for neutral buoyancy (Φ)
 337 and hydrostatic stability (S_t). Using historically higher density values for the soft body with an
 338 estimate of aptychi (1.068 g/cm^3 [38]) and the shell (2.62 g/cm^3 [36]), Φ is increased by 9.9%
 339 and S_t is increased by 11.9%. While values of 1.049 g/cm^3 and 2.54 g/cm^3 are preferred for the
 340 soft body and shell, respectively [38], this test demonstrates that hydrostatic calculations are
 341 sensitive to density values, which have varied historically [41].

342 The number of septa in the phragmocone and their positions were based on a single
 343 septum constructed from a suture pattern in the adapertural region of the adult phragmocone.
 344 Variability in septal morphology throughout ontogeny could have resulted in an inappropriate
 345 number of septa, which would have altered shell mass. To better understand the influence of this
 346 assumption, five septa from the middle of the phragmocone (whorl height of $\sim 1 \text{ cm}$) were added
 347 and subtracted from the final adult model. This modification resulted in minor differences of
 348 about $\pm 1.7\%$ in Φ and $\pm 1.5\%$ in S_t (similar to the experiment of Hoffmann et al. [53])

349

350 **Table 2. Hydrostatic properties of *Nipponites mirabilis*.**

Stage	Age (%)	BC Ratio	Φ	S_t	θ_a	θ_{ao}	L (mm)	L_x (mm)	L norm	L_x norm	θ_t	θ_{tr}
(Crio) 1	0.18	42.7	97.3	0.099	69.5	69.0	11.65	11.43	1.236	1.212	11.1	7.7
2	0.23	38.6	82.6	0.101	74.3	49.5	10.59	10.59	0.902	0.902	1.0	117.8
3	0.29	40.1	83.5	0.094	13.8	14.5	10.04	9.43	0.726	0.682	-20.1	117.1
4	0.33	39.7	76.4	0.093	99.5	99.5	9.58	8.61	0.616	0.554	26.0	98.2
5	0.38	42.8	77.4	0.069	2.0	-30.3	13.28	11.19	0.773	0.651	-32.6	135.5
6	0.41	42.7	75.5	0.082	22.9	24.1	14.61	12.23	0.787	0.659	-33.1	139.1
7	0.49	42.6	81.7	0.070	24.1	5.0	16.64	14.32	0.765	0.658	-30.6	124.5
8	0.55	41.6	79.8	0.083	31.5	22.4	19.51	16.20	0.821	0.682	-33.9	143.0
9	0.60	42.2	79.8	0.083	31.6	18.8	18.88	16.49	0.753	0.658	-29.1	113.4
10	0.71	41.8	81.3	0.079	34.5	15.1	21.92	21.72	0.758	0.752	-7.6	123.2

11	0.77	41.2	76.3	0.075	17.3	-12.7	22.18	17.38	0.720	0.564	-38.4	122.0
12	0.83	43.5	75.4	0.076	50.1	39.8	27.54	27.38	0.835	0.830	-6.2	154.2
13	0.88	39.1	72.2	0.072	-8.2	-11.1	27.66	24.86	0.807	0.726	-26.0	105.9
(Term) 14	1.00	42.1	73.1	0.073	19.9	30.0	28.10	23.44	0.781	0.651	-33.5	131.1

351 Hydrostatic properties computed for the 14 ontogenetic stages examined. Crio = criocone phase;
 352 Term = terminal phase; Age% = curvilinear length for that stage normalized by the curvilinear
 353 length of the terminal specimen; BC Ratio = curvilinear length of body chamber normalized by
 354 the total curvilinear length at a particular stage; Φ = the proportion of the phragmocone to be
 355 emptied of liquid for a neutrally buoyant condition; S_t = hydrostatic stability index; θ_a = apertural
 356 angle; θ_{ao} = apertural orientation if rib obliquity was ignored (normal to shell growth direction);
 357 L = total lever arm; L_x = x-component of the lever arm, norm = normalized by the cube root of
 358 water displaced for each particular stage; θ_t = thrust angle; θ_{tr} = rotational thrust angle.

359

360 Ontogenetic changes in hydrostatics

361 Hydrostatic properties were computed for 14 life stages (Figs 5 and 6; Table 2) in order
 362 to assess changes throughout the ontogeny of *Nipponites mirabilis* and other species sharing
 363 similar morphologies. *Nipponites mirabilis* has the capacity for neutral buoyancy at all life
 364 stages, retaining liquid between approximately 3% and 28% of the total cameral volumes. After
 365 the juvenile criocone phase, Φ decreases and stabilizes at its lower values (Fig 7). Hydrostatic
 366 stability (S_t) follows a similar decreasing trend and does not significantly oscillate (Fig 7). These
 367 hydrostatic stability index values ranging between approximately 0.10 and 0.07 are sufficiently
 368 large enough to orient the living cephalopod to maintain some static orientation during all of the
 369 examined ontogenetic stages. As already suggested by Okamoto [20,21], the orientation of the
 370 aperture (θ_a) oscillates in a complicated fashion throughout ontogeny, ranging between

371 approximately -11 and 99 degrees in the current study (Fig 7). Apertural orientations
372 significantly turned downwards are not observed. The juvenile criocone phase has apertural
373 angles of about 70°, followed by complex oscillations as the alternating U-shaped bends develop.
374 Afterwards, there is some degree of regularity in orientation, mostly exhibiting horizontal and
375 diagonally upwards directions (Fig 7). These properties likely vary, based on the possibility of a
376 slightly changing body chamber ratio throughout ontogeny.

377

378 **Fig 5. Final hydrostatic models of the first eight ontogenetic stages (A-H) of *Nipponites***
379 ***mirabilis*.** All models are oriented so that the ventral margin of the aperture faces towards the
380 right. The tip of the upper cone corresponds to the center of buoyancy while the tip of the lower
381 cone is the center of mass. At rest, these two centers are vertically aligned, denoting the proper
382 static orientation assumed by living *Nipponites mirabilis*.

383 **Fig 6. Final hydrostatic models of the last six ontogenetic stages (A-F) of *Nipponites***
384 ***mirabilis*.** All models are oriented so that the ventral margin of the aperture faces towards the
385 right. The tip of the upper cone corresponds to the center of buoyancy while the tip of the lower
386 cone is the center of mass. At rest, these two centers are vertically aligned, denoting the proper
387 static orientation assumed by living *Nipponites mirabilis*.

388 **Fig 7. Hydrostatic Properties Computed Throughout Ontogeny.** The proportion of the
389 phragmocone to be emptied of cameral liquid for neutral buoyancy (Φ ; circles), hydrostatic
390 stability index (S_t ; squares), and apertural angles (θ_a ; triangles) as a function of age (proxied by
391 the curvilinear length for that stage normalized by the curvilinear length of the terminal
392 specimen). Dashed lines denote interpolations between the 14 measured stages.

393

394 **Rib obliquity and static orientation**

395 While apertural orientations during the ontogeny of *Nipponites mirabilis* vary, horizontal
 396 to upward orientations are preferred. This is further supported by comparing the apertural angles
 397 (as denoted by the orientation of the ribs on the shell) with the same angle if ribs were not
 398 oblique (i.e., if the aperture was perfectly perpendicular to the direction of shell growth). The
 399 obliquity of the ribs generally enhances the apertural orientation by about 10° in the upwards
 400 direction (Fig 8). Unless otherwise stated, the apertural angles reported herein are measured
 401 parallel to the ribs and represent the aperture positions that would have occurred during life at
 402 each ontogenetic stage.

403

404 **Fig 8. The Influence of Rib Obliquity on Orientation.** Apertural angles with observed rib
 405 obliquity (θ_a ; triangles) and the angles normal to the direction of shell growth (zero obliquity;
 406 circles) as a function of age (proxied by the curvilinear length for that stage normalized by the
 407 curvilinear length of the terminal specimen). Light grey shading and dark grey shading denote
 408 rib obliquity that boosts θ_a in the upwards direction and downward directions, respectively.

409

410 **Directional efficiency of movement**

411 During its juvenile crioconic phase, *Nipponites mirabilis* was well suited for horizontal
 412 backwards movement (denoted by the near zero thrust angles; θ_b). This trend somewhat persists
 413 into later ontogenetic stages, while slightly decreasing and remaining above -40°. However, after
 414 the crioconic phase, the rotational thrust angle (θ_{tr}) dramatically increases as the U-shaped bends
 415 in the shell develop, suggesting that there is a strong rotational component of movement when
 416 thrust is produced normal to the aperture (Fig 9). While the normalized lever arm lengths seem to

417 decrease during ontogeny, sufficient torques for rotation can only be produced when the
418 rotational thrust angle is high. Furthermore, the x-component of the normalized lever arm is not
419 significantly lower than the total normalized lever arm during ontogeny, suggesting that the
420 subhorizontal declination of the total lever arms would still provide significant rotational
421 movement in ontogenetic stages after the crioconic phase (Fig 9).

422

423 **Fig 9. The Directional Efficiency of Movement.** Thrust angles in the vertical direction (θ_t ;
424 black dashed line), rotational thrust angles (θ_{tr} ; grey dashed line), and lever arms as a function of
425 age (proxied by the curvilinear length for that stage normalized by the curvilinear length of the
426 terminal specimen). The total lever arm (grey solid line) and x-component of that lever arm (X
427 Lever Arm; solid black line) are both normalized by the cube root of the volume of water
428 displaced (V_{wd}) for each stage. Idealized rotation would take place with high, relative x-
429 components of the lever arm and θ_{tr} of 90°. Idealized horizontal movement would occur with θ_t
430 of 0° and θ_{tr} of 0° or 180°.

431

432 Discussion

433 The mode of life of *Nipponites*

434 Hydrostatic simulations reveal that *Nipponites mirabilis* had the capacity for neutral
435 buoyancy throughout its ontogeny (Φ values < 100%), retaining some amount of cameral liquid
436 in the shell to compensate for residual buoyancy (Fig 7). These results support the buoyancy
437 calculations of Ward and Westermann [33], who report a similar scenario for *N. occidentalis*.
438 The inferences drawn from rib obliquity most likely functioning in a neutrally buoyant setting
439 [19] are also supported by the hydrostatic results. While the coiling of *Nipponites* is complex and

440 somewhat resembles gastropods such as vermetids and *Opisthostoma* [23–27], considerable
441 negative buoyancy and resultant benthic modes of life are unlikely.

442 Hydrostatic stability is large enough for living *Nipponites* to assume static, syn vivo
443 orientations throughout its entire ontogeny (excluding some short time after hatching when the
444 relative effects of viscous forces are higher). While the hydrostatic stability index slightly
445 decreases throughout ontogeny, the computed values are all larger than the stability index of
446 extant *Nautilus* (~0.05 [40]), suggesting that living *Nipponites* probably was not able to
447 significantly modify its own apertural orientation (in terms of its vertical orientation). The
448 highest stability in ectocochleates occur in orthocones, especially those without cameral deposits
449 [36,40]. Lower stability values should occur for morphotypes with larger body chambers that
450 wrap around the phragmocone (e.g., serpenticones [6,47,49]). At first glance, *Nipponites* seems
451 to fall into this latter category at later ontogenetic stages because of the series of alternating U-
452 bends surrounding the earlier crioconic phase and somewhat large body chamber. However, the
453 sigmoidal soft body (which heavily influences the total mass distribution) actually appears to be
454 somewhat confined in the vertical directions (Figs 5 and 6). That is, most of the soft body is still
455 distributed below the phragmocone, lowering the center of mass relative to the center of
456 buoyancy and increasing hydrostatic stability. In most cases, uncoiling of the shell tends to
457 generally increase hydrostatic stability compared to planispiral ectocochleates
458 [20,36,37,39,45,77].

459 Due to sufficient hydrostatic stability throughout ontogeny, fixed static orientations are
460 assumed by living *Nipponites mirabilis*. That is, upward to horizontally facing orientations are
461 preferred, while downward facing orientations were not observed in any of the examined
462 ontogenetic stages (Fig 7). These observed orientations may have accommodated a lifestyle of

463 feeding upon small prey in the water column, which has been proposed for other nostoceratid
464 heteromorphs [37,45,78]. There is some period of time between about 20% and 40% of the
465 lifespan of *Nipponites mirabilis* (after the crioconic phase and prior to the establishment of
466 regularly alternating U-bends) where orientation oscillates between upward facing and
467 horizontally facing. These somewhat rapid changes may have been an awkward time for these
468 living heteromorphs. On the other hand, this irregularity infers that *Nipponites* was able to
469 assume a functioning lifestyle regardless of these particular differences in orientation. This
470 indifference further suggests that this heteromorph assumed a low energy lifestyle that does not
471 demand athletic predation or predator evasion.

472 If the ribs of *Nipponites mirabilis* were not oblique, the static orientation of this species
473 would be about 10° less (downward) for many of the examined ontogenetic stages (Fig 8). The
474 obliquity of the ribs (which oscillates in magnitude throughout ontogeny [19]), therefore, assists
475 in maintaining a generally horizontal to diagonally upward facing orientation of the soft body.
476 Rib obliquity also suggests that the evasion of downward orientations was required to effectively
477 function for feeding and perhaps locomotion for most stages. Another example of rib obliquity
478 adjusting apertural orientation and growth direction can be observed in certain terrestrial
479 heteromorph gastropods (e.g., *Opisthostoma vermiculum* [79]). However, this likely constrains
480 orientation more for neutrally buoyant ectocochleates than for terrestrial/benthic gastropods.

481 **Locomotion of *Nipponites mirabilis***

482 The juvenile crioconic phase of *Nipponites mirabilis* would have been well suited to
483 horizontal backwards movement with minimal rocking due to its low thrust angles and
484 positioning of the hyponome (and thrust vector) relative to the vertical rotational axis (Fig 1A, B;
485 Fig 9). Similar hydrostatic properties are likely for criocone morphotypes with similar

486 proportions. Thrust angles decrease throughout ontogeny with some degree of oscillation but
487 remain above -40°. These thrust angles at later stages suggest that significant amounts of thrust
488 energy would still be transmitted into horizontal backwards movement, though with some degree
489 of rocking. The subzero thrust angles post-crioconic phase result in the point of thrust located
490 below the horizontal rotational axis suggesting that movement would be rather complicated, with
491 oscillations in apertural angles about some horizontal axis and vertical axis, simultaneously. By
492 examining the lever arms (normalized for each ontogenetic stage), the horizontal components of
493 the lever arms are not much lower than the total lever arms, suggesting that rotational torque
494 about the vertical axis during jet propulsion would be significant. After the crioconic phase, the
495 alternating U-bends in the shell allow the thrust vector to be rotated out of alignment with the
496 vertical rotational axis that passes through the centers of buoyancy and mass. This misalignment
497 after the crioconic phase allows rotation about the vertical axis to take place if thrust is produced
498 normal to the orientation of the aperture. However, this rotational thrust angle is not as ideal as in
499 torticonic (helical) heteromorphs like the turrilitids [34] and the intermediate ontogenetic stages
500 of *Didymoceras* [37], which are closer to 90°. Instead, these rotational thrust angles of the post-
501 crioconic stages, fall between pure rotation (90°) and pure translation (180°) at around 135°
502 (with some amount of variation throughout ontogeny). If the hyponome was able to bend 45°
503 right or left [80], then living *Nipponites* may have been able to select between pure rotational
504 movement and pure translational movement (influenced by some superimposition of chaotic
505 rocking and hydrodynamic drag). This scenario depends upon the largely unknown ammonoid
506 soft body [69,70] and propulsive mechanisms [68]. If the hyponome was not able to sufficiently
507 bend, then jet thrust for post-criocone phase individuals would be transmitted into a combination
508 of translation and rotation about the vertical axis.

509 The thrust angles and directional efficiency of movement provide useful information
510 about the locomotion and feeding of living *Nipponites*. The lateral movement (and perhaps
511 dispersal potential) of crioconic juveniles would have been on par with planispiral ammonoids
512 (albeit with higher hydrodynamic drag), but afterwards, movement is complicated and some
513 amount of rocking and rotation would occur. This rotational movement (pirouetting), however,
514 could have been useful in feeding, perhaps improving the amount of space through which the
515 living ammonoid could have searched for and captured small planktic prey. These hydrostatic
516 properties further support a quasi-planktic, low energy mode of life for *Nipponites*.

517 **Complex heteromorphy in an evolutionary context**

518 Okamoto [81] suggests that *Nipponites* originated from the nostoceratid
519 *Eubostrychoceras* based on comparisons of shell sculpture, early shell morphology, and
520 stratigraphic occurrence. In a theoretical framework, the juvenile crioconic coiling of both
521 *Eubostrychoceras japonicum* and its probable descendent *Nipponites mirabilis* are very similar.
522 After this phase, the former species retains helical coiling throughout most of its ontogeny except
523 for the adult whorl while the latter species alternates sinistral and dextral helical coiling [17–
524 21,81]. While the details of the rather-sudden appearance of *Nipponites* remain unclear, the
525 simulations of the current study infer significant differences in hydrostatic properties between
526 these two nostoceratid genera. *Eubostrychoceras japonicum* undergoes similar coiling patterns to
527 the nostoceratid, *Didymoceras*, but has a longer, stretched out helical phase. Earlier hydrostatic
528 simulations [37] reveal that *Didymoceras* was poorly suited for lateral movement, yet adept at
529 rotating about its vertical axis. These properties are likely analogous to *Eubostrychoceras*. While
530 *Nipponites* has a similar ability to rotate about its vertical axis after the criocone phase,
531 horizontal to diagonally upwards orientations are assumed instead of the likely downward

532 diagonal orientations of *Eubostrychoceras*. For *Eubostrychoceras* to attain *Nipponites*-like
533 orientations, its shell would have to coil upwards, compromising its helical coiling. Furthermore,
534 the alternating U-bends in *Nipponites* retain some degree of lateral movement potential.
535 Therefore, the seemingly-aberrant coiling of *Nipponites* might represent adaptations to
536 maintaining preferred orientations and effective directions of locomotion. Perhaps, this
537 morphology was useful to exploit favorable trophic opportunities in the water column [82].

538 The hydrostatic simulations of *Nipponites mirabilis* also provide a frame of reference for
539 other nostoceratid heteromorphs. *N. occidentalis*, for example, exhibits a larger degree of
540 uncoiling [33], and therefore, may have had higher stability and a larger lever arm for rotation.
541 Similarly, throughout the late Turonian and Coniacian, a larger degree of uncoiling takes place
542 for specimens found in successively younger strata [81]. These specimens cluster into three
543 distinguished morphotypes [81] that may have become more stable and adept at rotation as they
544 progressed through this time interval. Another morphological constraint is the degree of
545 compactness, which served as a selective factor in ammonoids [83]. *Nipponites mirabilis* is quite
546 compact, while this feature varies in other species and morphotypes.

547 **The stigma of heteromorphy**

548 Heteromorph ammonoids have been commonly regarded as bizarre evolutionary
549 experiments or degenerates [8–15], and their unique coiling schemes are enigmatic in terms of
550 their functional morphology and potential modes of life. While the inevitable phylogenetic
551 extinction of heteromorphs (i.e., typolysis) is now rebutted [8], the stigma of this concept has
552 persisted and is further propagated by their seemingly aberrant coiling schemes. Heteromorph
553 ammonoids, however, were very diverse, disparate, and successful throughout the Cretaceous
554 [45,82,84–86]. Furthermore, the coiling schemes of several morphotypes of heteromorph

555 ammonoids suggest that they exploited unique solutions to manage the physical properties that
556 constrained their modes of life by modifying their shells to serve primarily as specialized
557 hydrostatic devices [36–39]. Additionally, some heteromorphs may have been released from
558 certain constraints, allowing freedom to experiment. The hydrostatic simulations of the current
559 study reveal that the coiling of *Nipponites*, which seems biologically absurd, does in fact confer
560 an advantage for specific *syn vivo* orientations and with rotational capabilities. As suggested for
561 several heteromorphs, the general mode of life assumed by *Nipponites* may parallel extant
562 cranchid squids, feeding upon small prey as slow, but not immobile, quasi-planktic cephalopods
563 [6,33,77,87].

564

565 **Conclusions**

566 Hydrostatic analyses support a quasi-planktic mode of life for *Nipponites mirabilis* with
567 unique forms of movement that could have enabled a planktotrophic feeding strategy. This
568 species and other heteromorphs with similar proportions had the capacity for neutral buoyancy
569 and were not restricted to the benthos. Throughout the ontogeny of *Nipponites*, horizontally
570 facing to upwardly facing soft body orientations were occupied. These orientations were likely
571 preferred for feeding on small plankton in the water column. This behavior is supported by the
572 tendency for rib obliquity to oscillate [19], which was primarily found to upwardly tilt apertural
573 orientations from the direction of shell growth. Somewhat larger hydrostatic stability values,
574 relative to *Nautilus*, suggest that the vertical component of the apertural orientation would not
575 have significantly changed during locomotion or interaction with external forms of energy. A
576 change in hydrostatics takes place between the juvenile criocone stage and the later stages
577 consisting of alternating U-bends in the shell, specifically regarding the directional propensity

578 for movement. Although the criocone phase of *Nipponites* likely experienced more
579 hydrodynamic drag than planispiral ammonoids of similar size, this morphology was stable, and
580 proficient at backwards horizontal movement. As the alternating U-bends develop, *Nipponites* is
581 better suited for rotational movement about its vertical axis, while possibly maintaining the
582 option to move horizontally backwards by changing the direction of its hyponome. These syn
583 vivo physical properties do not support a benthic or immobile mode of life for *Nipponites*. Its
584 forms of movement were likely slow, however, suggesting that *Nipponites* assumed a low energy
585 lifestyle while pirouetting to scan for small prey in the water column. The hydrostatic properties
586 throughout the ontogeny of *Nipponites* contrast with those of its probable ancestor,
587 *Eubostrychoceras* [81]. These differences in morphology along with the hydrostatic analyses in
588 the current study infer that the seemingly convoluted coiling scheme of *Nipponites* represents
589 unique adaptive solutions to several hydrostatic constraints, rather than random morphological
590 aberration.

591

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602

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823 **Figure and Table Captions**

824 **Fig 1. Hydrostatic Parameters of *Nipponites mirabilis*.** **A**, Side view of *Nipponites* in life
825 position showing hypothetical centers of buoyancy (B), mass (M), and the horizontal axis of
826 rotation (R). The angle of the aperture (θ_a) is measured as the inclination from the vertical plane.
827 The thrust angle (θ_t) can be used to assess the directional efficiency of movement. This angle is
828 measured between the horizontal plane, and a line passing through R and the location of the
829 hyponome (source of thrust; H). **B**, Front view of *Nipponites* in life position facing the aperture.
830 This view shows the total lever arm (L) and its x-component (L_x) which is proportionate to the
831 amount of rotational movement about the vertical axis produced during jet propulsion. **C**, Top
832 view of *Nipponites* in life position showing the rotational thrust angle (θ_{tr}). This angle is
833 measured between the vertical rotation axis (vert.), which passes through B and M, and the
834 direction of the thrust vector (arrow emanating from H). Rotational thrust angles of 90° result in
835 idealized transmission of thrust into pure rotation.

836 **Fig 2. Virtual Reconstruction of the Shell of *Nipponites mirabilis*.** **A**, Tessellated (.stl) 3D
837 model generated from a CT-scan [60] of specimen INM-4-346. **B**, Reconstructed adoral portion
838 of the body chamber and inner criocone phase with arrays algorithms (Table 1). **C**, Extruded
839 septa generated from the suture pattern. **D**, Extruded shell and septa models unified together to
840 produce a single, manifold 3D mesh of the entire shell.

841 **Fig 3. Thickness Measurements used for Virtual Model Extrusion.** Thicknesses of the shell
842 (black) and septa (grey) as a function of whorl height. Measurements were recorded from
843 specimen NMNS PM35490 and used to define thickness in the virtual model.

844 **Fig 4. Specimens Used for Shell Reconstruction.** **A**, Original concretion containing NMNS
845 PM35490. Umbilical view (**B**) and ventral view (**C**) of the shell section used to record the suture
846 pattern (**D**). **E**, 3D scan of the Mikasa City Museum Specimen (MCM-A0435) used to
847 approximate the body chamber ratio. Scale bar = 2 cm.

848 **Fig 5. Final hydrostatic models of the first eight ontogenetic stages (A-H) of *Nipponites***
849 ***mirabilis*.** All models are oriented so that the ventral margin of the aperture faces towards the
850 right. The tip of the upper cone corresponds to the center of buoyancy while the tip of the lower
851 cone is the center of mass. At rest, these two centers are vertically aligned, denoting the proper
852 static orientation assumed by living *Nipponites mirabilis*.

853 **Fig 6. Final hydrostatic models of the last six ontogenetic stages (A-F) of *Nipponites***
854 ***mirabilis*.** All models are oriented so that the ventral margin of the aperture faces towards the
855 right. The tip of the upper cone corresponds to the center of buoyancy while the tip of the lower
856 cone is the center of mass. At rest, these two centers are vertically aligned, denoting the proper
857 static orientation assumed by living *Nipponites mirabilis*.

858 **Fig 7. Hydrostatic Properties Computed Throughout Ontogeny.** The proportion of the
859 phragmocone to be emptied of cameral liquid for neutral buoyancy (Φ ; circles), hydrostatic
860 stability index (S_t ; squares), and apertural angles (θ_a ; triangles) as a function of age (proxied by
861 the curvilinear length for that stage normalized by the curvilinear length of the terminal
862 specimen). Dashed lines denote interpolations between the 14 measured stages.

863 **Fig 8. The Influence of Rib Obliquity on Orientation.** Apertural angles with observed rib
864 obliquity (θ_a ; triangles) and the angles normal to the direction of shell growth (zero obliquity;
865 circles) as a function of age (proxied by the curvilinear length for that stage normalized by the

866 curvilinear length of the terminal specimen). Light grey shading and dark grey shading denote
 867 rib obliquity that boosts θ_a in the upwards direction and downward directions, respectively.

868 **Fig 9. The Directional Efficiency of Movement.** Thrust angles in the vertical direction (θ_t ;
 869 black dashed line), rotational thrust angles (θ_{tr} ; grey dashed line), and lever arms as a function of
 870 age (proxied by the curvilinear length for that stage normalized by the curvilinear length of the
 871 terminal specimen). The total lever arm (grey solid line) and x-component of that lever arm (X
 872 Lever Arm; solid black line) are both normalized by the cube root of the volume of water
 873 displaced (V_{wd}) for each stage. Idealized rotation would take place with high, relative x-
 874 components of the lever arm and θ_{tr} of 90° . Idealized horizontal movement would occur with θ_t
 875 of 0° and θ_{tr} of 0° or 180° .

876 **Table 1.** Array instructions used to reconstruct the juvenile criocone phase and the adoral portion
 877 of the terminal body chamber. These arrays were used in a piecewise manner to replicate the
 878 whorl section from the adoral direction to adapical direction by translation, rotation, and scaling
 879 in the x, y, and z directions. Asterisks denote arrays that had their origins reset to their current
 880 locations before replication. If origins were not reset, the origins of their previous arrays were
 881 used.

882 **Table 2.** Hydrostatic properties computed for the 14 ontogenetic stages examined. Crio =
 883 criocone phase; Term = terminal phase; Age% = curvilinear length for that stage normalized by
 884 the curvilinear length of the terminal specimen; BC Ratio = curvilinear length of body chamber
 885 normalized by the total curvilinear length at a particular stage; Φ = the proportion of the
 886 phragmocone to be emptied of liquid for a neutrally buoyant condition; S_t = hydrostatic stability
 887 index; θ_a = apertural angle; θ_{ao} = apertural orientation if rib obliquity was ignored (normal to
 888 shell growth direction); L = total lever arm; L_x = x-component of the lever arm, norm =

889 normalized by the cube root of water displaced for each particular stage; θ_t = thrust angle; θ_{tr} =
890 rotational thrust angle.

891