Integrating Energy Management of Autonomous Smart Grids in Electricity Market Operation

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***Abstract*—This study presents a market operation model integrated with energy management programs of independent smart grids using bilevel optimization. In this framework, autonomous smart grid entities, in the lower levels, operate their own networks and send decisions to the upper level market operator that clears the day ahead market based on unit commitment and second order conic AC power flow models. A single-leader multi-follower game is thus developed, in which every smart grid derives optimal schedules of its own renewable energy resources, storage devices, and responsive demands that are interconnected through a distribution grid using mixed integer linear programming. Given the mixed integer nature of the upper and lower level decisions, we develop and customize an exact reformulation-decomposition method to compute this bilevel optimization program. Through numerical experiments performed on three test systems, we demonstrate that the proposed modeling paradigm can accurately represent the physics of the transmission and distribution grids and achieves reasonable results with significant computational efficacy.**

***Index Terms*—Smart grid, unit commitment, power market, mixed integer bilevel optimization, second-order cone programming, storage device, shiftable demand.**

I. Introduction

A. Background and Motivations

WITH the improved control and communication capabilities of today's power system infrastructures and diverse demand side technologies, it becomes feasible to replace the current power system with a more flexible one. From this and given the presence of intelligent technologies of distribution clients, more elaborate models of both market and smart grids in market operation are critical to reflect the actual network physics and attain accurate pictures of the system. Nevertheless, linear dispatch models still remain the mainstream formulation to solve the day-ahead and real-time markets because they are fast enough under such circumstances. A common alternative for the linear dispatch model is the AC optimal power flow (OPF) variant which accounts for alternating current's mathematical complexities but is nonconvex and needs much more time to solve using existing methods.

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Another challenge for accuracy improvement is the integration of load management programs and active distribution networks in the market clearing programs, with sufficient physical details, to reflect the impact and dependency of market outcomes on emerging demand-side technologies. Current practice is to use optimization routines with aggregated models of demand response programs, storage devices, and distributed generation at the transmission level. The physical distribution grid and the actual dynamics of its components (e.g., storage devices) are largely neglected. Such simplifications might be acceptable in the case of extremely large systems to reach fairly good solutions, but might produce infeasible results with other problems. Indeed, the decision-making procedure of each autonomous smart grid is a complex optimization problem with continuous and discrete decisions. It basically remains unknown how to analytically determine the operating policy of independent smart grids with demand side technologies when they are integrated in power market operation and can interact with it. Hence, an effective energy management tool, considering those critical factors and interactions, is definitely needed, to support producing optimal schedules and operating plan of an active distribution grid in modern electricity markets.

B. Literature Review

When the objectives of market participants are conflicting and the decision making process is sequential, bilevel optimization (i.e., the Stackelberg game) is naturally used to study the interactions among agents. In the context of electricity markets, bilevel optimization (and its variants) is the standard framework to capture the interaction between the ISO and market players. Here, the market operator goal is to maintain the supply-demand balance whereas the objective of smart grids or demand response aggregators, acting as profit-seeking mediators between the market and responsive consumers, is a combination of their cost of electricity and gained utility.

To address the operational challenges of active distribution networks in modern electricity markets, many advanced decision support models and sophisticated computing methods have been developed and studied. Some relevant applications and works are reviewed next. In [1], a bilevel model for a strategic aggregator (in the upper level) is proposed to determine the optimal schedules of load curtailment and flexible loads (i.e., demand response portfolio) in a wholesale electricity market. A Stackelberg model is developed in [2] to investigate the interaction between the independent system operator (ISO) and the aggregated effect of users equipped with demand-side technologies. In [3], authors present a bilevel formulation for describing the aggregator decision and management of its clients with energy market participation and show how retail prices for the customers and strategy for the wholesale market participation are determined. The work of [4] establishes a Stackelberg game between providers and end-users, where providers act as profit-seeking leaders and end-users behave as welfare-maximizing followers. Reference [5] proposed a two-stage bilevel model for energy pricing and dispatch of a smart grid retailer acting as an intermediary agent between a wholesale energy market and end consumers. Demand response of consumers, with respect to the retail price, is characterized by a Stackelberg game. The work in [6] introduces a stochastic bilevel formulation in which the ISO, in the upper level, minimizes the total operating cost, and demand response aggregators, in the lower level, maximize profit. A dynamic market mechanism is developed in [7] which introduces a state-space structure to the static market and incorporates shiftable demands, generation companies, and the ISO. A gradient play is then used to derive the dynamic evolution of the actions to reach the optimum solution of the real-time market. In [8] a single leader multiple follower Stackelberg model is formulated wherein the grid operator, as the leader, sets the tariff to ensure the balance of supply and demand, and the groups of consumers (i.e., followers) schedule their controllable loads to minimize their costs. References [9], [10], [11], [12], [13] adopt a unit commitment perspective to assess the impact of demand response on market outcomes. Specifically, [9] considers the load shifting behavior of price-sensitive consumers, and in [10] the influences of such bids on congestion and locational marginal prices in the day-ahead market are explored. Reference [11] presents a self-scheduling unit commitment model for aggregators that participate in the day-ahead energy market and seek to maximize their profits. The work in [12] incorporates demand response in co-optimized day-ahead energy and spinning reserve markets. Reference [13] designed a three-level market structure consisting of the ISO, utility, and distribution customers for market-based microgrid scheduling using mixed-integer linear programming. Authors in [14] derived optimal portfolio of demand response contracts for an aggregator which maximizes its operating surplus and satisfies the consumers' aspirations. Sadeghi-Mobarakeh *et al.* [15] developed an optimal demand biding model under uncertainty for a price-taker distribution utility using two-stage robust stochastic programming.

The aforementioned works model smart grid as a controlled or price-taker entity, operated and managed by the ISO within the wholesale market as an aggregated load or power source. Such assumptions are largely impractical recalling that distribution side technologies, like storage devices, are essentially privately-owned and clients have little incentive to let the ISO manage or operate them (due to life cycle reduction).

In some recent works, the distribution network model and the decision making independence of the smart grids are considered. The energy management issue and power exchange among a group of active distribution networks and transmission system are addressed in [16], [17], [18]. They proposed the use of a *system of systems* modeling paradigm which is a hierarchical decentralized architecture to find the optimal operating point amongst independent systems. Specifically, [16] presents such a framework and develops an iterative algorithm to coordinate a distribution company and multiple microgrids for finding the optimal operating point of these independent systems. The work in [17] extends the scope of [16] by considering a transmission system and multiple active distribution networks as a system of systems and determines unit commitment and generation dispatch decisions for the ISO and active distribution networks. The iterative solution algorithm used for solving the problem is not guaranteed to converge for the nonconvex mixed integer linear programs (MILPs) studied in these works. Reference [18] considers a similar decentralized structure composed of a leader multi-microgrid system and multiple microgrids followers, and formulates the energy management problem via bilevel optimization. Individual microgrids solve multiple-stage robust optimization to minimize the operating cost in the worst scenario while at the level of multi-microgrid system (i.e., upper level), energy management is carried out by minimizing the daily operating cost. The wholesale energy market and its interaction with the multi-microgrid system are ignored in this work.

C. Research Objectives and Paper Organization

This survey reveals that compared to the abundant literature on smart grid operation and optimization studies, there is limited research on models that consider the autonomy of smart grids and the distributed feature of demand side components in the market operation. Indeed, distribution system is a key component that interconnects various renewable resources, storage devices, and responsive loads in the area to serve the power requirements of a cluster of consumers. Hence, an analytical method that considers all those critical factors and interactions should be employed to determine the actual operating policy of smart grids. While the aforementioned approaches focus on control and operation aspects from either supply side or demand side, we develop a market based framework to study the operation and interaction of an electricity market and multiple active distribution networks that serve a group of customers. In the proposed setting, smart grids anticipate market price and can influence market quantities accordingly. That is, they play games with the market and this interaction is modeled via bilevel optimization. In our framework, the market operator (i.e., ISO) at the top level, clears the day-ahead market and a number of independent smart grids, in the lower level, act sequentially to determine their optimal schedules. Here, every smart grid is a self-organizing system that operates and manages its own network to utilize a diverse range of interconnected renewable resources, storage devices, and responsive loads in the area. This modeling approach enables us to capture the actual and discrete features of demand side technologies like storage devices, and to model the operational limits of the distribution system and responsive demands. On the market side, we assume that the ISO collects offers from the production units and clears the day ahead market using a unit commitment problem. We use a second order conic AC power flow program for the ISO problem to account for the voltage and reactive power constraints of the transmission system.

The most difficulty that naturally arises from modeling physical characteristics of smart grid components into this type of bilevel optimization is the discrete feature of the decisions at the lower level problems. Note that this is in addition to the generators' binary commitment decisions in the upper level which gives rise to bilevel optimization with mixed integer structures in both levels. This presents a significant computational challenge for which the most of available solution techniques primarily rely on heuristics, approximations, or modeling simplifications that lack computational efficiency and might lead to sub-optimality. Notice that the conventional reformulations that use strong duality or KKT conditions to replace the lower level problem are not applicable to this type of bilevel optimization. Different from the existing methods, we extend and customize an exact solution technique, i.e., the *reformulation-and-decomposition* scheme [19], to solve this challenging bilevel optimization. This method implements the *column-and-constraint generation algorithm* [20], and has demonstrated a strong solution capacity on a few bilevel mixed integer linear programs [21], [22], [23]. Overall, the proposed platform allows us to analyze the integration of smart grids in modern electricity market and to gain a deeper understanding of the impacts of distribution side technologies (e.g., storage devices, shiftable demands, etc.) on market operation, which could not be obtained by the conventional aggregated smart grid models.

In view of the above discussion and to demonstrate notable advantages over the existent methods in the literature, this paper makes the following contributions:

(a) From a formulation perspective, we present a bilevel mixed integer conic model for the day ahead market clearing integrated with the energy management problem of autonomous smart grids. This is a multi-period optimization model accounting for renewable generation, storage devices, and responsive demands interconnected through the distribution gird and can exchange power with the wholesale market.

(b) From an algorithmic viewpoint and to the best of our knowledge, we develop and customize the first exact decomposition method for computing a bilevel optimization program with mixed integer conic upper level and multiple mixed integer linear lower level problems. The presented method will provide a novel tool to analytically study the operation management and optimization of autonomous smart grids that are integrated in the electricity market and are connected to the main transmission grid.

The rest of the paper is organized as follows: it begins with problem statement in Section II, where we describe the notation, the market structure, and modeling assumptions. The overall bilevel optimization is also presented in this section. In Section III, the solution algorithm is outlined. Experimental results are presented and discussed in Section IV and the concluding remarks are provided in Section IV.

II. Problem Statement

In this section we present the bilevel optimization program for clearing the day ahead electricity market considering optimal operation of independent smart grids using mixed integer conic and linear programming. The notation and assumptions are introduced first.

A. Notational Conventions

The following notation is used in the modeling. For a typical transmission network, Δ is the set of network nodes (indexed by i, j) and $\Delta(i)$ is the set of nodes directly connected to node i. The set of transmission branches is Ω indexed by ij. Series conductance, series susceptance, and shunt susceptance in the $\pi-$model of branch ij are denoted by $g_{ij}, b_{ij}, b_{ij}^{sh}$, respectively. Subscript $t \in T$ denotes time period. The nodal active/reactive demands are d_i^p/d_i^q and the power imported to the smart grid are p_i^d/q_i^d. The complex voltage at transmission node i is $V_i = e_i + jf_i$. The minimum and maximum node voltages are V_i^{min} and V_i^{max}, and the rated branch current is I_{ij}^{max}. The offer cost of generation is c_i^g, and the active and reactive power output variables are indicated with p_i and q_i. The generator's no-load cost, start-up cost, and shut-down cost are indicated with C_i^{nl}, C_i^u, and C_i^d. The real power curtailment variable and its unit cost are r_i and c_i^c, respectively. Parameter σ_i is a constant related to the power factor of the real power curtailed at node i. The active and reactive power flows of branch ij are p_{ij} and q_{ij}, and their corresponding upper limits are p_{ij}^{max} and q_{ij}^{max}. The squared branch current is I_{ij}^{sq}. The lower and upper bounds of active and reactive power generation are given by p_i^{min}, p_i^{max}, q_i^{min}, and q_i^{max}, respectively. Binary variables v_i, u_i, s_i are used for modeling commitment, start-up, and shut-down of generators.

For every distribution grid connected to transmission node i via its main substation transformer, we denote by Φ_i the set of grid nodes and by $\Phi_i(n)$ the set of nodes connected to node n. The main substation node of the distribution gird is denoted by 0 and hence, $\Phi_i(0)$ represents the set of nodes connected to the substation. The set of network branches is $\mathcal{N}_i$. Distribution nodes are indexed by n, m, l. The fixed part of the real and reactive demand at node n are represented with p_n^0 and q_n^0. The shiftable part of demand is denoted by d_n^f and its upper bound by $\bar{d}_n^f$. The power factor of the load is γ_n^d. Resistance and reactance of branch nm are given by r_{nm} and x_{nm}, respectively. The squared voltage magnitude of node n is shown by U_n and its upper and lower limits by U_n^{max} and U_n^{min}, respectively. The feeder's active and reactive power flows are p_{nm} and q_{nm} and the corresponding limits are p_{nm}^{max} and q_{nm}^{max}. The real power output of a renewable (distributed) energy resource is g_n^r and its power factor is γ_n^r. The energy level of the storage device (battery) is e_n^b and its upper and lower bounds are e_n^{bmax} and e_n^{bmin}. The charging and discharging power levels of the storage are b_n^c and b_n^d, and the upper bounds are b_n^{cmax}

and b_n^{dmax}. We assume that the storage device efficiency in the charging or discharging mode is η_n^b. The binary variable z_n indicates the charging/discharging operating mode of the storage. The allowable number of changes in the operation state of the storage at node n is N_n^b. The total energy consumed by the flexible demand over the scheduling period is E_n^f with α_n and β_n being the start and the end operating time intervals. For notational simplicity, we assume that generation and consumption exist at every distribution node and one smart grid is connected to each transmission node i.

B. Structure and Modeling Assumptions

Let us suppose a day ahead energy market run by the ISO for energy trade. Following the current practice in most power markets in the U.S., which perform unit commitment, we assume that production units submit complex offers to the ISO which embody their actual operational cost and constraints [2], [6], [9], [13]. In this work, the operational cost data includes variable (running) cost, start-up cost, shut-down cost, and no-load cost. The operational constraint include the up and down ramping rates, the minimum up-time and downtime, and the minimum and maximum generation. The ISO then decides on the starting-up and shutting-down of the units once offers are collected.

In the proposed setting, smart grids are part of and connected to the main transmission grid and their real and reactive power exchanges are treated as dependent variables whose precise values are computed by the lower-level (smart grids') problems in an interactive fashion with the ISO problem. Observe that smart grids will accordingly influence market outcomes by adjusting their operating plan and dispatch decisions. Indeed, they play games with the market and their interactions are modeled by means of bilevel optimization. This is the standard reasoning behind a Stackelberg leader-follower game formulation.

We model each smart grid as a self-organizing independent entity run by an operator (or aggregator). It collects information from renewable generation resources, storage devices, and consumers connected to the distribution system nodes. It then performs an optimal power flow problem and computes the self-consumption and dispatch decisions of its network components [16], [18]. Renewable generation resources are considered non-dispatchable with their output power profiles being known and given. The consumers at the distribution nodes have either fixed (non-shiftable) or shiftable demands. In this context, shiftable demand refers to a distribution user that can shift consumption to cheaper time slots but its cumulative consumption across the time horizon, denoted as E_n^f, is fixed and given. This parameter specifies the daily target power consumption of the consumer. The beginning and the end of the operating time intervals are also specified for shiftable demands. Therefore, every shiftable demand d_{nt}^f is modeled as a nonnegative time-dependent variable in the smart grid problem [15], [24]. In this paper, we do not consider bids from consumers connected to the transmission system.

In Fig. 1 the structure of the proposed bilevel model is depicted. In the following subsections, we model each market

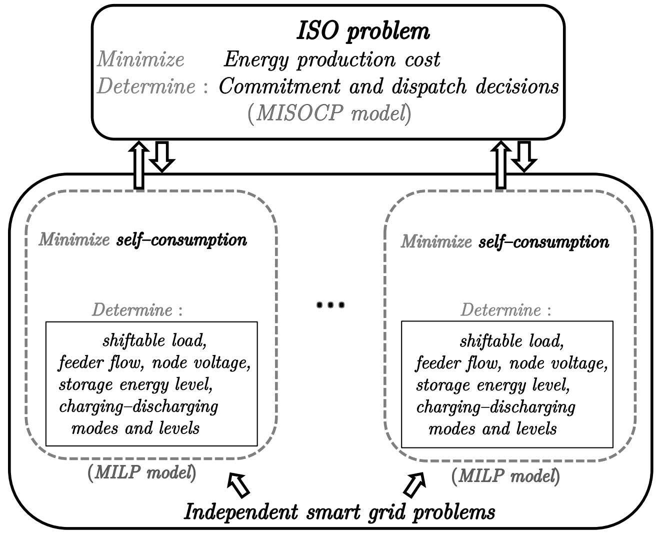

Fig. 1. Framework of proposed bilevel model.

entity involving generators, smart grid and its components, and the ISO. We consider a deterministic situation in this study, and the extension to a stochastic model to describe random demand fluctuations and renewable generation is left for future research and more advanced algorithm development. The corresponding problem constraints and the overall bilevel optimization are described next.

C. Bilevel Market Clearing Model Integrated With Optimal Smart Grid Operation

As indicated earlier, the ISO receives complex offers from production units which include operational cost data and operational constraints. It then performs a multiperiod optimization to clear the day ahead energy market. This is the upper level problem whose solution yields the starting-up, shutting-down, and dispatch decisions of the conventional production units. The demands of the smart grids connected to the transmission system are treated as dependent variables to be determined by the lower level problems in an iterative way.

Using a second order conic AC power flow model, the upper level market clearing problem is thus written as:

$$\min \sum_{t\in T}\sum_{i\in\Delta} c_i^g p_{it} + c_i^c r_{it} + v_{it}C_i^{nl} + u_{it}C_i^u + s_{it}C_i^d \tag{1a}$$

$$\text{s.t.}\ p_{it} + \sum_{j\in\Delta(i)} p_{jit} - \sum_{j\in\Delta(i)} p_{ijt} = d_{it}^p + p_{it}^d - r_{it} \quad \forall i\in\Delta, t\in T \tag{1b}$$

$$q_{it} + \sum_{j\in\Delta(i)} q_{jit} - \sum_{j\in\Delta(i)} q_{ijt} = d_{it}^q + q_{it}^d - \sigma_i r_{it} \quad \forall i\in\Delta, t\in T \tag{1c}$$

$$v_{it}p_i^{min} \le p_{it} \le v_{it}p_i^{max},\ v_{it}\in\{0,1\}\ \forall i\in\Delta, t\in T \tag{1d}$$

$$v_{it}q_i^{min} \le q_{it} \le v_{it}q_i^{max} \quad \forall i\in\Delta, t\in T \tag{1e}$$

$$p_{ijt} = g_{ij}\left(C_{iit} - C_{ijt}\right) + b_{ij}S_{ijt} \quad \forall ij\in\Omega, t\in T \tag{1f}$$

$$q_{ijt} = \left(-b_{ij} - b_{ij}^{sh}/2\right)C_{iit} + b_{ij}C_{ijt} + g_{ij}S_{ijt} \quad \forall ij\in\Omega, t\in T \tag{1g}$$

$$I_{ijt}^{sq} = F_{ij}^{l_1}C_{iit} + F_{ij}^{l_2}C_{jjt} - F_{ij}^{l_3}S_{ijt} - F_{ij}^{l_4}C_{ijt}$$

$$\forall ij \in \Omega, t \in T \tag{1h}$$

$$I_{ijt}^{sq} \leq \left(I_{ij}^{max}\right)^2 \quad \forall ij \in \Omega, t \in T \tag{1i}$$

$$p_{it+1} - p_{it} \leq R_i^u \quad \forall i \in \Delta, t \in T \tag{1j}$$

$$p_{it} - p_{it+1} \leq R_i^d \quad \forall i \in \Delta, t \in T \tag{1k}$$

$$\sum_{k=t-UT_i+1}^{t} u_{ik} \leq v_{it} \ \forall i \in \Delta, t \in \{UT_i, \ldots, T\} \tag{1l}$$

$$\sum_{k=t-DT_i+1}^{t} s_{ik} \leq 1 - v_{it} \ \forall i \in \Delta, t \in \{DT_i, \ldots, T\} \tag{1m}$$

$$v_{it} - v_{it-1} = u_{it} - s_{it}, \quad u_{it}, s_{it} \in \{0, 1\}$$

$$\forall i \in \Delta, t \in T \tag{1n}$$

$$0 \leq r_{it} \leq r_i^{max} \quad \forall i \in \Delta, t \in T \tag{1o}$$

$$\left(V_i^{min}\right)^2 \leq C_{iit} \leq \left(V_i^{max}\right)^2 \forall i \in \Delta, t \in T \tag{1p}$$

$$C_{iit} = 1 \quad i = ref. \ \forall t \in T \tag{1q}$$

$$C_{ijt} = C_{jit}, \ S_{ij} = -S_{ji}, \ C_{ijt} \geq 0 \ \forall ij \in \Omega, t \in T \tag{1r}$$

$$C_{ijt}^2 + S_{ijt}^2 \leq C_{iit} C_{jjt} \quad \forall ij \in \Omega, t \in T \tag{1s}$$

$$\left(p_{it}^d, q_{it}^d\right) \in \text{argmin}\{\text{MILP-OPF } [(3a) - (3o)]\} \ \forall i, t \tag{1t}$$

where variables $C_{ij} = e_i e_j + f_i f_j$ and $S_{ij} = e_i f_j - e_j f_i$ are used to formulate the conic AC power flow model [25]. The fixed parameters $F_{ij}^{l_1}, F_{ij}^{l_2}, F_{ij}^{l_3}$, and $F_{ij}^{l_4}$ in (1h) are related to the branch admittance defined below:

$$F_{ij}^{l_1} = g_{ij}^2 + \left(b_{ij} + b_{ij}^{sh}/2\right)^2, \quad F_{ij}^{l_2} = g_{ij}^2 + b_{ij}^2 \tag{2a}$$

$$F_{ij}^{l_3} = 2g_{ij}^2 + 2b_{ij}^2 + b_{ij} b_{ij}^{sh}, \quad F_{ij}^{l_4} = g_{ij} b_{ij}^{sh}. \tag{2b}$$

The upper level problem (1a)-(1j) is a unit commitment (UC) model based on second order conic AC power flow equations. Specifically, (1b)-(1c) are nodal power balance equations. Constraints (1d)-(1e) enforce the allowable range of power production. Equations (1f)-(1h) define active and reactive branch flows and branch current, respectively. The branch ampacity is stated by (1i). The ramp-up and ramp-down rate limits are enforced by (1j)-(1k). The minimum up and down times of the generators are stated by (1l)-(1m). Equation (1n) states the logical relationships between binary commitment variables. The maximum load curtailment at each transmission node is given by (1o). Constraints (1p)-(1q) indicate the voltage range of each transmission node as well as the reference bus magnitude. Constraints (1r)-(1s) define the governing equations for the AC power flow variables in second order cone format. Finally, constraint (1t) states that p_{it}^d and q_{it}^d in power balance equations (1b)-(1c) are calculated by the lower level problem associated with the smart grid connected to transmission node i.

Let us suppose a number of independent smart grids that are connected to the transmission system through substation transformers and exchange power with the main network. The operator of each smart grid anticipates market price (at the connection node) and computes optimal operating decisions by minimizing its real and reactive power consumption using a mixed integer linear program. The operational and technical constraints of the distribution network and various demand-side technologies (storage, responsive demand, etc) are modeled in the smart grid problem. So, every smart grid at transmission node i solves the following optimal power flow program, written for all $i \in \Delta$:

$$\min \sum_{t \in T} \left(p_{it}^d + q_{it}^d\right) \tag{3a}$$

$$\sum_{m \in \Phi_i(n)} p_{nmt} - \sum_{l \in \Phi_i(n)} p_{lnt} = g_{nt}^r + b_{nt}^d - b_{nt}^c - d_{nt}^f - p_{nt}^0$$

$$\forall n \in \Phi_i, t \in T \tag{3b}$$

$$\sum_{m \in \Phi_i(n)} q_{nmt} - \sum_{l \in \Phi_i(n)} q_{lnt} = \gamma_n^r g_{nt}^r - \gamma_n^d d_{nt}^f - q_{nt}^0$$

$$\forall n \in \Phi_i, t \in T \tag{3c}$$

$$U_{mt} = U_{nt} - 2(r_{nmt} p_{nmt} + x_{nmt} q_{nmt})$$

$$\forall n \in \Phi_i, \quad m \in \Phi_i(n), nm \in \mathcal{N}_i, t \in T \tag{3d}$$

$$U_n^{min} \leq U_{nt} \leq U_n^{max} \quad \forall n \in \Phi_i, t \in T \tag{3e}$$

$$| \, p_{nmt} \, | \leq p_{nm}^{max}, \quad | \, q_{nmt} \, | \leq q_{nm}^{max} \quad \forall nm \in \mathcal{N}_i, t \in T \tag{3f}$$

$$p_{it}^d = \sum_{m \in \Phi_i(0)} p_{0mt}, \quad q_{it}^d = \sum_{m \in \Phi_i(0)} q_{0mt} \quad \forall t \in T \tag{3g}$$

$$e_{nt}^b = e_{nt-1}^b + b_{nt}^c \times \eta_n^b - b_{nt}^d \times 1/\eta_n^b \quad \forall n \in \Phi_i, t \in T \tag{3h}$$

$$e_n^{bmin} \leq e_{nt}^b \leq e_n^{bmax} \quad \forall n \in \Phi_i, t \in T \tag{3i}$$

$$0 \leq b_{nt}^c \leq z_{nt} b_n^{cmax} \quad \forall n \in \Phi_i, t \in T \tag{3j}$$

$$0 \leq b_{nt}^d \leq (1 - z_{nt}) b_n^{dmax} \quad \forall n \in \Phi_i, t \in T \tag{3k}$$

$$\sum_{t \in T} \left| z_{nt} - z_{nt-1} \right| \leq N_n^b \quad \forall n \in \Phi_i, t \in T \tag{3l}$$

$$0 \leq d_{nt}^f \leq \bar{d}_n^f \quad \forall n \in \Phi_i, t \in T \tag{3m}$$

$$\sum_{t=\alpha_n}^{\beta_n} d_{nt}^f = E_n^f \quad \forall n \in \Phi_i, t \in T \tag{3n}$$

$$z_{nt} \in \{0, 1\} \quad \forall n \in \Phi_i, t \in T \tag{3o}$$

The lower level problem (3) minimizes the net consumption of real and reactive power in objective function (3a). Nodal power balance equations are given in (3b)-(3c). Constraints (3d)-(3e) define the governing equations for voltage and branch flow of the distribution network as well as the allowable voltage range, respectively. The maximum branch flow is enforced by (3f). The power balance at the substation of the distribution network is given by (3g). The energy balance of the storage device is stated by (3h) considering its efficiency, η_n^b. Constraints (3i)-(3k) set lower and upper bounds on the energy level and charging and discharging power of the storage. Note that binary variable z_{nt} is introduced in (3j)-(3k) to avoid charging and discharging simultaneously. The number of changes in the operation state of the storage is limited through constraint (3l). The upper and lower bounds of shiftable demand is enforced in (3m). Constraint (3n) states that the total energy consumed by the shiftable demand, over the time horizon, is known and fixed. Finally, (3o) introduces the binary variable z_{nt}. As previously indicated, we consider fixed and shiftable nodal demands. For shiftable demands, d_{nt}^f indicates a non-negative time-dependent variable whose accumulate value over the time horizon has a designated constant value, as stated by (3n). Note that this is a temporally-coupled constraint which couples the power consumption across the time horizon and makes the problem hard to solve because it cannot be independently solved for each time slot.

In the above formulation, the duration of each time slot is set to 1 hour and it is further assumed that the storage devices operate in unity power factor mode with negligible reactive power generation. The set of decision variables of each lower level smart grid's problem is defined as $\mathcal{Y}_L^i = \{z_n, p_i^d, q_i^d, d_n^f, e_n^b, b_n^c, b_n^d, U_n, p_{nm}, q_{nm}\}$.

Remark 1: The nonlinear constraint (3l) can be readily linearized by introducing binary variable $\overline{z}_{nt}$. This way, (3l) can be equivalently replaced by $z_{nt} - z_{nt-1} \leq \overline{z}_{nt}$, $z_{nt-1} - z_{nt} \leq \overline{z}_{nt}$, and $\sum_{t\in T} \overline{z}_{nt} \leq N_n^b$.

Remark 2: With substation's active and reactive flows being equal to $p_{it}^d = \frac{C_{iit}-U_{0t}}{2r_s} - \frac{x_s}{r_s} q_{it}^d$ and $q_{it}^d = \frac{C_{iit}-U_{0t}}{2x_s} - \frac{r_s}{x_s} p_{it}^d$, the objective function (3a) can be expressed as: $A_i^d(C_{iit} - U_{0t})$, where U_{0t} is the substation voltage, C_{iit} denotes the transmission node voltage, and A_i^d is a constant parameter defined as $A_i^d = \frac{(x_s+r_s)-2(x_s^2+r_s^2)}{x_s r_s}$. In these equations, $r_s + jx_s$ is the series impedance of the substation transformer. The objective function (3a) and hence, each smart grid's lower level problem depend on the upper level variable C_{iit}.

It is seen that the lower level problem (3) is essentially a mixed integer linear program. The operator of each smart grid solves this model to compute the optimal self-consumption quantities and the dispatch decisions of its resources and demands. Given this and as indicated earlier, the conventional reformulation methods that employ KKT conditions or strong duality to replace the lower level problem with the equivalent counterpart are not applicable here. In the following, we develop and customize the exact reformulation and decomposition method to compute this problem iteratively.

III. Solution Methodology

The optimization model (1)-(3) is composed of a single upper level program in mixed integer second order cone programming (MISOCP) format and multiple MILP lower level problems. To compute this bilevel program, we use and extend the *reformulation-decomposition algorithm* developed in [19], which partitions the original model into a master problem and multiple subproblems.

To make the exposition in this section more accessible, we first provide a compact matrix-based representation of the bilevel program (1)-(3), as written below:

$$\min \quad ax + by \tag{4a}$$
$$s.t. \quad By + Cr_i = d \tag{4b}$$
$$Ax + Dy \geq l, \quad x \in \{0,1\} \tag{4c}$$
$$\|Hy\| \leq hy \tag{4d}$$
$$\text{where} \quad r_i \in \text{argmin}\{\, e_i(r_i + y_i): \tag{4e}$$
$$s.t. \quad G_i w_i + E_i r_i + O_i y_i = f_i \tag{4f}$$
$$F_i w_i + K_i r_i \leq k_i \tag{4g}$$
$$r_i \geq 0, \quad w_i \in \{0,1\}\} \;\; \forall i \in \Delta \tag{4h}$$

where x and y denote the upper level binary and continuous variables, and r_i and w_i indicate the lower level continuous and binary variables associated with the smart grid at transmission node i. The coefficient matrices and/or vectors $(A, a, B, b, E_i, e_i, l, D, d, F_i, f_i, G_i, H, h, K_i, k_i, O_i)$ with proper dimensions are related to these variables. In constraint (4d), we let symbol $\|.\|$ denote the l_2-norm for vectors (matrices). Equation (4e) represents the objective function and (4f)-(4h) are the primal constraints of the lower level problem associated with each smart grid at transmission node i.

To provide a decomposable structure for algorithm development, we follow [19] to reformulate bilevel program (4) by duplicating the lower level variables and constraints in the upper level problem and adding one additional constraint (5e), as written below:

$$\min \quad ax + by \tag{5a}$$
$$s.t. \quad [(4b) - (4d)] \tag{5b}$$
$$G_i w_i' + E_i r_i' + O_i y_i = f_i, \quad F_i w_i' + K_i r_i' \leq k_i \tag{5c}$$
$$r_i' \geq 0, \quad w_i' \in \{0,1\} \tag{5d}$$
$$e_i(r_i' + y_i) \leq \min \;\{\, e_i(r_i + y_i): \tag{5e}$$
$$s.t. \; G_i w_i + E_i r_i + O_i y_i = f_i, \quad F_i w_i + K_i r_i \leq k_i \tag{5f}$$
$$r_i \geq 0, \quad w_i \in \{0,1\}\} \;\; \forall i \in \Delta \tag{5g}$$

where the duplicated lower level variables in the upper level are denoted by r_i' and w_i'. Note that (5c)-(5d) are defined for each smart grid. Because of (5e), we conclude that problem (5) is equivalent to the original bilevel optimization (1)-(3). Although more complicated than (1)-(3), the reformulated problem (5) provides a convenient representation to derive non-trivial bounds to problem (1)-(3). Let $\mathcal{W}$ be the collection of all possible realization of w_i and $\widetilde{w}_i$ be a particular realization of w_i. Next, by enumerating w_i and introducing its continuous variables $\hat{r}_i^{\widetilde{w}_i}$, we can rewrite (5e)-(5g) as:

$$e_i\big(r_i' + y_i\big) \leq \min \;\Big\{ e_i\Big(\hat{r}_i^{\widetilde{w}_i} + y_i\Big): \tag{6a}$$
$$s.t. \quad G_i \hat{r}_i^{\widetilde{w}_i} + E_i \widetilde{w}_i + O_i y_i = f_i \tag{6b}$$
$$F_i \widetilde{w}_i + K_i \hat{r}_i^{\widetilde{w}_i} \leq k_i \tag{6c}$$
$$\hat{r}_i^{\widetilde{w}_i} \geq 0 \Big\} \;\; \widetilde{w}_i \in \mathcal{W}, \; i \in \Delta \tag{6d}$$

Observe that once $\widetilde{w}_i$ is given, the right-hand side of (6a)-(6d) is a linear program. Moreover, instead of having a complete enumeration, (6a)-(6d) developed based on a subset $\widetilde{\mathcal{W}} \subseteq \mathcal{W}$ results in a relaxation of (5), or equivalently the original bilevel optimization (1)-(3). These features enable us to develop the decomposition algorithm using *column-and-constraint generation* method [20], as described next.

A. Subproblems

For a given upper level decision (x^*, y^*), we formulate and compute the following subproblem $\mathrm{SP1_i}$ for each smart grid connected to transmission node $i \in \Delta$:

$$\mathbf{SP1_i}\text{: } \Psi_i(x^*, y_i^*) = \min \;\; e_i(r_i + y_i^*) \tag{7a}$$
$$s.t. \quad G_i w_i + E_i r_i + O_i y_i^* = f_i : \lambda_i \tag{7b}$$

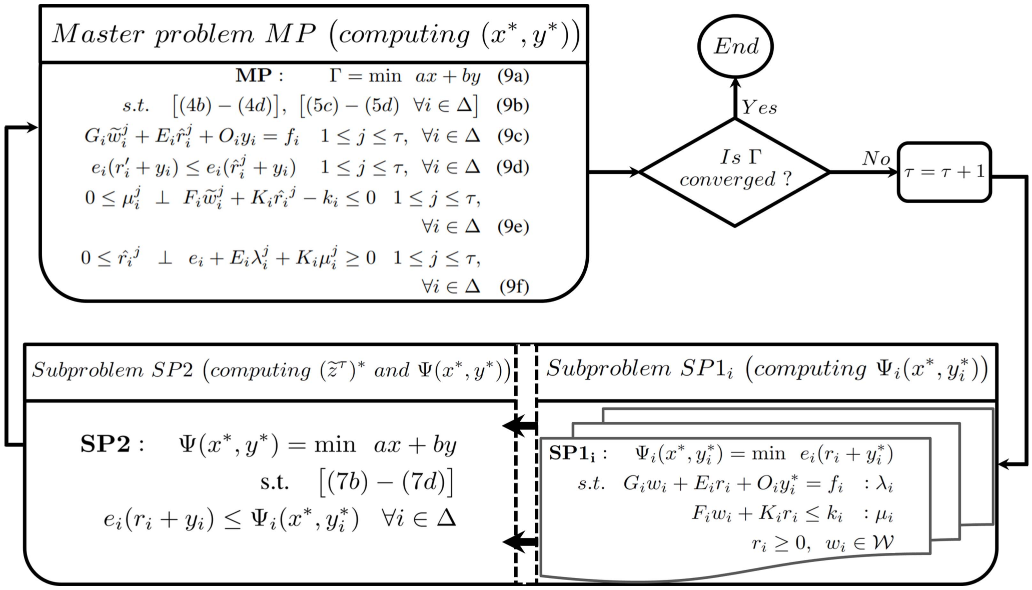

Fig. 2. Flowchart of the proposed reformulation-decomposition method.

$$F_i w_i + K_i r_i \le k_i : \mu_i \quad (7c)$$
$$r_i \ge 0,\ w_i \in \mathcal{W} \quad (7d)$$

Although problem $\mathrm{SP1_i}$ is an MILP, for a fixed w_i, the remaining problem is a linear program. The dual variables of constraints (7b) and (7c) are indicated by λ_i and μ_i. Clearly, $\mathrm{SP1_i}$ provides an optimal solution of lower level model (3) for dispatch plan (x^*, y_i^*). However, it might have multiple solutions. The second subproblem, i.e., SP2, derives one that is in favor of the upper level model, as written below:

$$\textbf{SP2}: \Psi(x^*, y^*) = \min \ ax + by \quad (8a)$$
$$\text{s.t.} \ [(7\text{b}) - (7\text{d})] \quad (8b)$$
$$e_i(r_i + y_i) \le \Psi_i(x^*, y_i^*) \ \forall i \in \Delta \quad (8c)$$

It is seen that SP2 is an easily computable MILP problem.

B. Master Problem

With formulations (5)-(6), the master problem is then constructed by: 1) duplicating the lower level variables (denoted by w_i' and r_i') and constraints in the upper level problem; and 2) replacing the lower level problem (in iteration j) of a fixed realization $\widetilde{w}_i^j \in \widetilde{\mathcal{W}} \subseteq \mathcal{W}$ by its KKT conditions (continuous primal variables are indicated by $\hat{r}_i^j$, and dual variables by λ_i^j and μ_i^j). The compact form of the master problem is provided within the algorithm description (see (9a)-(9f) below and also [19]). Next, we provide steps of our customized *column-and-constraint generation decomposition algorithm* to solve the overall bilevel optimization. Let LB and UB be the lower and upper bounds, τ be the iteration index, and ϵ be the optimality tolerance.

This algorithm dynamically provides stronger upper bounds (from subproblems) and lower bounds (from the master problem) and, in each iteration, adds new variables and constraints to the master problem until the difference between bounds is not larger than optimality tolerance ϵ. The mathematical proof of finite convergence of this algorithm to the optimal value can be found in [19], [20]. Fig. 2 illustrates the proposed iterative algorithm.

Algorithm 1 Algorithm for Solving MIP Bilevel Problem $(1)-(3)$

1. **Step 1**. Set $LB = 0$, $UB = \infty$, and $\tau = 0$.
2. **Step 2**. Solve the following master problem:

$$\textbf{MP}: \quad \Gamma = \min \ ax + by \quad (9a)$$
$$s.t. \quad [(4b)-(4d)],\ [(5c)-(5d)\ \ \forall i \in \Delta] \quad (9b)$$
$$G_i\widetilde{w}_i^j + E_i\hat{r}_i^j + O_i y_i = f_i \quad 1 \le j \le \tau,\ \forall i \in \Delta \quad (9c)$$
$$e_i(r_i' + y_i) \le e_i(\hat{r}_i^j + y_i) \quad 1 \le j \le \tau,\ \forall i \in \Delta \quad (9d)$$
$$0 \le \mu_i^j \ \perp \ F_i\widetilde{w}_i^j + K_i\hat{r}_i^j - k_i \le 0 \quad 1 \le j \le \tau,\ \forall i \in \Delta \quad (9e)$$
$$0 \le \hat{r}_i^j \ \perp \ e_i + E_i\lambda_i^j + K_i\mu_i^j \ge 0 \quad 1 \le j \le \tau,\ \forall i \in \Delta \quad (9f)$$

   Report optimal solution (x^*, y^*) and update $LB = \Gamma$.
3. **Step 3**. Given y_i^*, solve $\mathrm{SP1_i}$ for $i \in \Delta$ and report $\Psi_i(x^*, y_i^*)$.
4. **Step 4**. Solve SP2 for given (x^*, y_i^*) and $\Psi_i(x^*, y_i^*)$. Report optimal $(\widetilde{z}^\tau)^*$ and $\Psi(x^*, y^*)$. Update $UB = \min\{\Psi(x^*, y^*), UB\}$.
5. **Step 5**. If $UB - LB \le \epsilon$, stop with a solution associated with UB. Otherwise, $\tau = \tau + 1$ and go back to step 2.

It is worthwhile to emphasize that master problem MP is a complementary program that can be converted into a regular mixed integer second order cone program by linearizing (9e)-(9f) using big-M method and binary variables. So, existing commercial MISOCP solvers can be used to compute all subproblems and master problem. Although MP could be

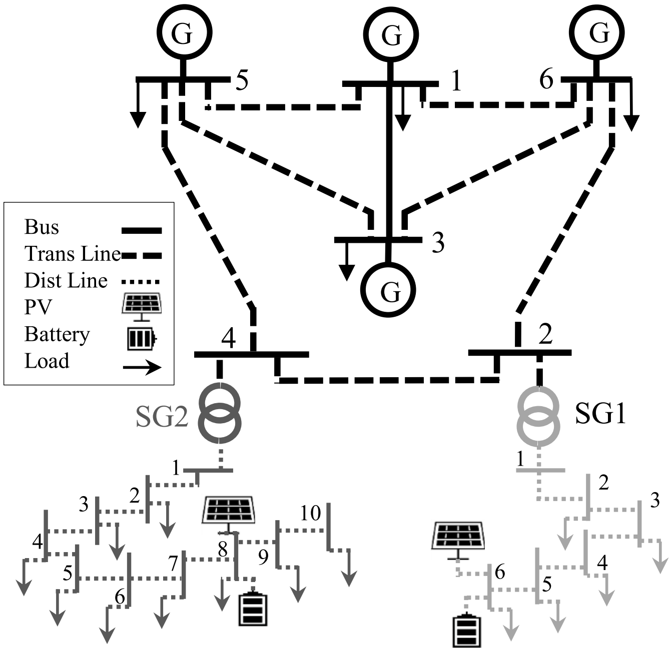

Fig. 3. Example system: a transmission network and two smart grids.

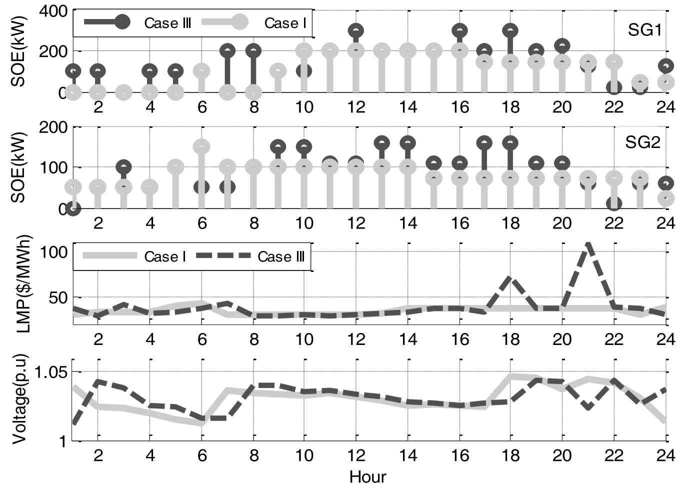

Fig. 4. Results of case I and case III: state of energy (SOE) of storage devices, mean value of nodal LMPs, and voltage profile of the transmission system.

cumbersome as its dimension increases each iteration, the critical advantage of this algorithm is that it produces optimal solutions within a very small number of iterations and algorithmic operations. In our experiments, in most cases, the algorithm provides optimal solutions after 3 iterations and needed a maximum of 4 iterations in the most demanding case study.

IV. Experimental Results

In this section, numerical results of three test systems including an example network, IEEE 30-bus system, and IEEE 57-bus system are presented and discussed. The example system together with various case studies are used to illustrate the main features of the proposed formulation and computing methodology. The time horizon consists of 24 hours and the duration of each time slot is set to one hour. They are presented next.

A. Example System

This is a six-bus transmission network with four generators and two smart grids, labeled SG1 and SG2. smart grids are connected respectively to node 2 and node 4, as shown in Fig. 3. Each smart grid has one photovoltaic (PV) generation and one storage device connected to the same node of the distribution network. The following scenarios, in a 24-hour time horizon, are considered:

(I) It is assumed that shiftable demand at each node is equal to 10% of the fixed nodal demand. That is, if the fixed nodal demand is p^0_{nt}, then we set $0 \leq d^f_{nt} \leq 0.1p^0_{nt}$. Also, the number of charging/discharging time of the storage devices is limited.

(II) The location/node of the PV generation–storage is changed and the reset conditions are as in case I.

(III) The charging/discharging time of the storage is assumed unlimited. That is, a big N^b_n is applied to constraint (3l). The rest conditions are similar to case I.

(IV) Similar to case I with the exception that the shiftable demand at each node is increased to 15%.

(V) Similar to case IV with the exception that 50% of the distribution nodes have shiftable demand and the rest nodal demands are fixed. The nodes with shiftable demand are chosen randomly.

(VI) The topology of the distribution network is changed. In smart grid 1, the switches on lines 2, 3, and 5 are opened and switches 6, 7, and 8 are closed. Also, in smart grid 2, the switches on lines 4, 5, and 8 are opened and switches 10, 11, and 12 are closed.

(VII) The distribution networks are not modeled and the load-generation conditions are similar to case III. This case basically represents an aggregated smart grid model.

The optimization results of these case studies are summarized in Table I. The impact of PV generation–storage location on the optimal operation of the smart grids and transmission network is analyzed in cases I and II. Notice that the losses of these networks and the unit commitment cost reduce in case I. The effect of limiting the number of charging/discharging time on the energy level of the storage devices in case I and case III is shown in Fig. 4. In addition, the average LMP over the 24-hour period and the mean voltage profile of the transmission nodes are plotted in this figure. It is worthwhile pointing out that N^b_n is set to 5 in case I while it is equal to a large number in case III so as to relax/remove constraint (3l). It is seen from Fig. 4 that the stored energy level is reduced in case I in both smart grids and curves become flatter indicating reduction in the number of charging/discharging times. These results clearly show that the inclusion of actual characteristics and limitation of storage device are critical for deriving the true optimal schedules of the smart grids.

To explore the effect of shiftable demand on the distribution and transmission systems, the losses of these networks and the unit commitment cost are depicted in Fig. 5 for case I and

TABLE I
OPTIMAL TRANSMISSION AND SMART GRID DECISIONS IN A 24-HOUR PERIOD: EXAMPLE SYSTEM

case	I	II	III	IV	V	VI	VII
Smart grid 1							
PV-Storage node	**6**	**3**	**6**	**6**	**6**	**6**	**any node**
Peak demand (kW, kVAr)	4494, 2549	4494, 2549	4657, 2542	4244, 2749	4342, 2797	4115, 2512	4570, 2500
Losses (kW, kVAr)	856, 443	975, 512	844, 443	874, 452	826, 427	108, 97	0.0, 0.0
Objective (kW, kVAr)	67347, 36121	67600, 36268	66731, 36200	65607, 34671	68994, 36434	66670, 35757	65783, 36373
Smart grid 2							
PV-Storage node	**8**	**4**	**8**	**8**	**8**	**8**	**any node**
Peak demand (kW, kVAr)	2849, 1978	2849, 1978	3090, 1899	2787, 1979	2849, 2045	3014, 1825	3075, 1892
Losses (kW, kVAr)	239, 134	314, 174	203, 114	195, 109	231, 129	37, 23	0.0, 0.0
Objective (kW, kVAr)	57097, 33456	57264, 32790	56624, 31015	54309, 30409	57773, 33708	56797, 33104	56302, 32852
Transmission network							
Losses (MW)	16.5	16.7	14.9	16.0	17.1	16.2	15.9
Mean LMP ($/MWh)	34.7	35.4	39.1	36.5	36.5	37.2	34.8
Objective ($)	5412	5423	5376	5168	5504	5460	5323

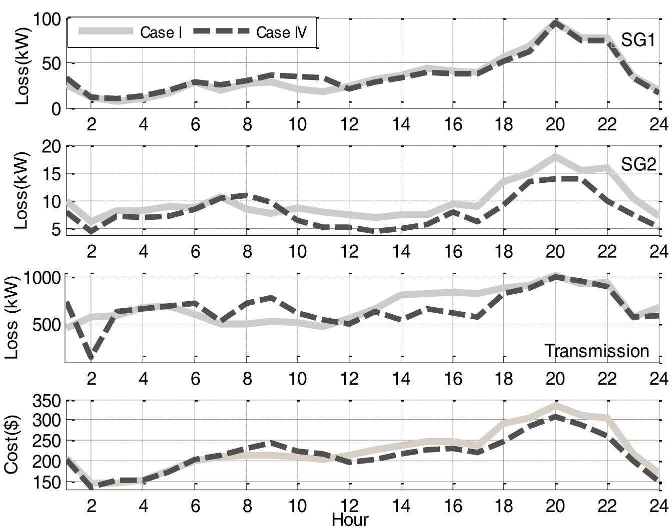

Fig. 5. Results of case I and case IV: losses and production cost of units.

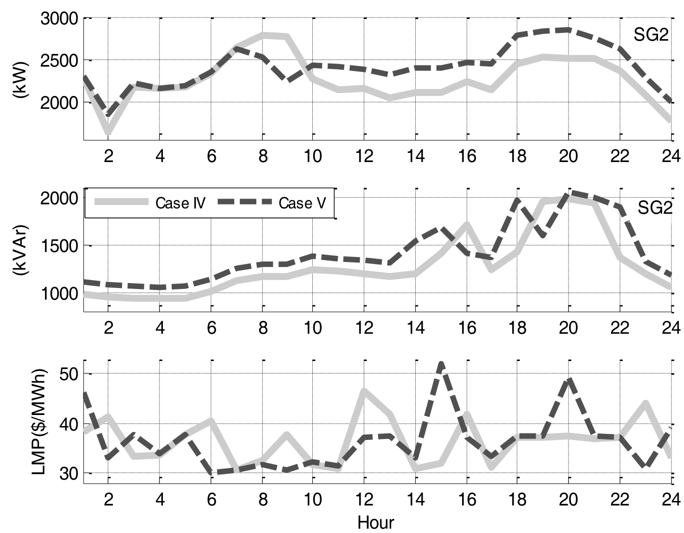

Fig. 6. Results of case IV and case V: real and reactive power imports of SG2 and mean nodal LMP of the transmission system.

case IV. Note that network losses and commitment cost reduce in most hours by increasing the demand responsiveness (being shiftable) by 5% in case IV. Fig. 6 exhibits the real and reactive power imported to smart grid 2 and the mean nodal LMP of the transmission network for case IV and V. As previously indicated, in case V only 50% of the distribution nodes have shiftable demands, and as a result, the power import to the smart grid increases in most hours which raises the LMP of the transmission network, on average. This experiment clearly illustrates how the location of the responsive demand in an active distribution network can have noticeable effects on the market outcomes. Indeed, such impacts can be captured provided that distribution network and its operational constraints are modeled in the problem. In cases VI, we assumed that the operator of each smart grid can change the grid configuration, as is common practice in the distribution network for mitigating violations or reducing losses. Here, the impacts of such acts on the market outcomes are observed. For example, the mean LMP in case VI, comparing with case I, increases by 7%. In case VII, the distribution network is not modeled and an aggregated load-generation model is thus considered. From Table I, it is seen that in this case study, transmission losses increase and the peak demand of the smart grids is reduced. Consequently, the commitment cost of generation unrealistically decreases, compared to case III and V in which distribution networks are modeled. For better illustration, the mean nodal LMP and losses of the transmission system in cases III and VII are depicted in Fig. 7. Observe the sharp increase/change in the mean LMP and losses in some hours in case III, which are largely eliminated in case VII due to oversimplification.

The results presented in this case study clearly indicate that for market outcomes to be reasonable, the formulation approach is critical, specially for larger networks, as will be discussed later in this section. Finally, Fig. 8 shows the hourly commitment status of the generators (labeled G_i for $i = 1, \ldots, 4$) and charging/discharging mode of the storage

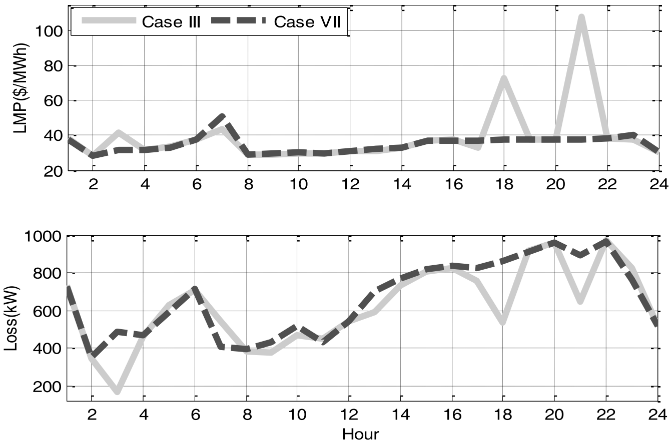

Fig. 7. Mean LMP and losses of transmission system in case III and case VII.

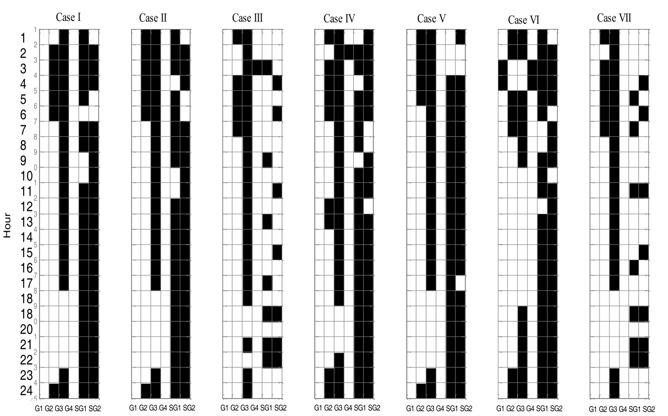

Fig. 8. Hourly on/off status of the production units and charging/discharging operating mode of the storage devices. White squares indicate on or charging status and black ones denote off or discharging status.

devices (labeled SG_i for $i = 1, 2$) in the cases studied in this section. It simply exhibits how changes in load-generation conditions or modeling details of the lower level problems (i.e., smart grids) lead to differing commitment schedules in the upper level problem.

B. IEEE 30-Bus System

The transmission system is the modified IEEE 30-bus system. Four smart grids are considered including the 4-node system, 18-node system, the 22-node system, and the 33-node system. They are connected to transmission nodes 20, 18, 30, and 15, respectively. The data of the transmission and distribution networks were adopted from matpower database [26] with some modifications. These smart grids have two, three, four, and five PV-storage systems. Assuming a 24-hour period, the following cases were studied:

(I) Assuming 10% of nodal demands are shiftable. A limited number of charging/discharging time (i.e., a small N_n^b) is applied to all storage devices in the smart grids.

(II) The original topology is changed and the PV-storage location is preserved as in case I. Compared with the original network topology, in smart grid 1, switches on lines 2 and 3 are opened. In smart grid 2, switches on lines 8, 12, 13, and 17 are opened. In smart grid 3, switches on lines 3, 8, 11, and 15 are opened. In smart grid 4, switches on lines 7, 9, 14, 32, and 37 are opened. The original topology of these networks are given in the matpower's database.

(III) The locations of the PV-storage are changed and the reset conditions are identical with case I.

TABLE II
OPTIMAL OPERATING DECISIONS OF TRANSMISSION SYSTEM AND SMART GRIDS IN A 24-HOUR PERIOD: IEEE 30-BUS SYSTEM

Case	I	II	III
Smart grid 1 (4-node network)			
PV-Storage node	2,4	2,4	2,3
Peak demand (kW, kVAr)	2796, 2705	2739,2702	2784, 27061
Losses (kW, kVAr)	528, 285	360, 240	540, 293
Objective (MW, Mvar)	44.7, 37.4	44.7, 37.3	44.8, 37.4
Smart grid 2 (18-node network)			
PV-Storage node	7,12,13	7,12,13	5,6,7
Peak demand (kW, kVAr)	9847, 7838	9619, 6536	9776, 7838
Losses (kW, kVAr)	2900, 15543	2566, 7118	3093, 15474
Objective (MW, Mvar)	190, 142	189, 135	191, 142
Smart grid 3 (22-node network)			
PV-Storage node	14,16,18,22	14,16,18,22	10,11,12,13
Peak demand (kW, kVAr)	550, 636	531, 631	545, 636
Losses (kW, kVAr)	153, 78	12.0, 9.0	168, 86
Objective (MW, Mvar)	11.0, 10.6	10.5, 10.5	10.8, 10.6
Smart grid 4 (33-node network)			
PV-Storage node	6,14,24,30,32	6,14,24,30,32	7,8,9,10,11
Peak demand (kW, kVAr)	3697, 2398	3878, 2399	3513, 2413
Losses (kW, kVAr)	1147, 775	1035, 770	1446, 994
Objective (MW, Mvar)	39.6, 29.0	39.3, 28.9	40.3, 29.4
Transmission network			
Losses (MW)	50.7	50.2	50.3
Mean LMP ($/MWh)	29.0	29.0	28.8
Objective ($)	64036	63826	64383

The optimization results are presented in Table II. Observe the variations in market outcomes and smart grid schedules due to changes in the PV-storage location or in the configuration of the distribution system. For instance, the objectives of smart grids are higher in case I and case III with respect to case II. Also, topology changes in case II reduce the distribution system losses and the objective function of the upper level problem (i.e., the social cost in the ISO problem). Thus, the simplified aggregated models of smart grids at the transmission level will underestimate the market operation costs and lead to inaccurate outcomes, as seen from the indices of the transmission system in Table II.

C. IEEE 57-Bus System

The proposed methodology was applied to the IEEE 57-bus system. This system composed of seven generators and eighty lines. We assume that four smart grids are connected to this system. The distribution systems include the 13-bus system at node 3, 18-bus system at node 9, 22-bus system at node 52, and 33-bus system at node 42. The required data of the transmission and distribution systems were adopted from matpower [26] with some modifications.

TABLE III
OPTIMAL OPERATING DECISIONS OF TRANSMISSION SYSTEM AND SMART GRIDS IN A 24-HOUR PERIOD: IEEE 57-BUS SYSTEM

Case	I	II	III
Smart grid 1 (13-node network)			
PV-Storage node	8,13	5,6	8,13
Peak demand (kW, kVAr)	10991, 6150	10713, 6566	10863, 5910
Losses (kW, kVAr)	7625, 5982	7409, 5818	3606, 2852
Objective (MW, Mvar)	171, 89	170, 88	167, 85
Smart grid 2 (18-node network)			
PV-Storage node	10,15,18	4,8,12	10,15,18
Peak demand (kW, kVAr)	11057, 6928	10999, 6856	11013, 7014
Losses (kW, kVAr)	2829, 16810	3427, 17561	2824, 9393
Objective (MW, Mvar)	214, 127	213, 126	214, 119
Smart grid 3 (22-node network)			
PV-Storage node	4,6,8,9	19,20,21,22	4,6,8,9
Peak demand (kW, kVAr)	474, 478	475, 478	488, 485
Losses (kW, kVAr)	288, 146	255, 129	20.8, 16
Objective (MW, Mvar)	9.5, 9.4	9.4, 9.4	9.4, 9.4
Smart grid 4 (33-node network)			
PV-Storage node	14,15,16, 17,18	10,14,20, 24,28	14,15,16, 17,18
Peak demand (kW, kVAr)	3557, 2256	3632.9, 2319	3565, 2266
Losses (kW, kVAr)	1695, 1210	1291, 864	1703, 1231
Objective (MW, Mvar)	54, 34	53, 33	54, 34
Transmission network			
Losses (MW)	666.4	665.6	666.9
Mean LMP ($/MWh)	21.3	21.3	21.3
Objective (M$)	418520	417930	418450

Optimization results for three cases are presented in Table III. These cases are similar to those in the previous 30-bus system. In case I it is assumed that 10% of nodal demands are shiftable with the number of charging/discharging times of the storage devices being limited. In case II, the PV-storage locations are changed and the rest conditions are the same as in case I. In case III, the topology of the distribution networks with respect to cases I are modified and the remaining conditions are similar to those in case I.

From Table III, it is seen that variations in the optimal decisions and schedules of the smart grids are reflected in the market outcomes. For instance, transmission losses and social cost reach to their lowest values when the PV-storage locations in the distribution networks are as in case II. This highlights the fact that actual market outcomes depend on the reaction and operating decisions of the smart distribution networks and the level of physical details considered in the modeling of these systems, as demonstrated in the cases studied in this sections.

V. CONCLUSION

In this paper we develop a modeling framework for integrating optimal energy management problem of autonomous smart grids in the day-ahead market operation. We formulate a bilevel optimization program to capture the interaction of the ISO and multiple independent smart grids. The upper level problem formulates the day-ahead market clearing procedure using a second order conic AC power flow model. The lower level problems represent energy consumption management of autonomous smart grids in which the mixed integer nature of the decisions pertaining to the storage devices and shiftable demands are considered.

Given such mixed integer structure in both upper and lower level problems, for which no analytical solution methodology exists, we develop and customize an exact reformulation-decomposition method to compute the problem iteratively using a master problem and multiple subproblems.

On the illustration of the proposed scheme on three test systems, we observed that the aggregated modeling approach of smart grids which neglects or uses a simplified model of distribution system and actual characteristics of storage devices, will result in unrealistic or underestimation of market outcomes. The proposed method improves solution accuracy by capturing physical transmission and distribution systems, the discrete nature of decision making procedure at the lower level problems, and the interaction between the power market and smart grids.

Possible directions for future research include incorporation of more features of demand side technologies such as response time or ramp time of responsive demands, electric vehicles, etc., and evaluation of their impacts on market operation and outcomes.

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