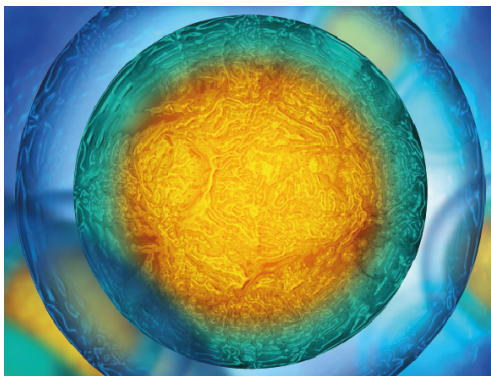


PAPER

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PAPER

Reconstructing full-field flapping wing dynamics from sparse measurements

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Supplementary material for this article is available [online](#)

Abstract

Flapping insect wings deform during flight. This deformation benefits the insect's aerodynamic force production as well as energetic efficiency. However, it is challenging to measure wing displacement field in flying insects. Many points must be tracked over the wing's surface to resolve its instantaneous shape. To reduce the number of points one is required to track, we propose a physics-based reconstruction method called system equivalent reduction expansion processes to estimate wing deformation and strain from sparse measurements. Measurement locations are determined using a weighted normalized modal displacement method. We experimentally validate the reconstruction technique by flapping a paper wing from 5–9 Hz with 45° and measuring strain at three locations. Two measurements are used for the reconstruction and the third for validation. Strain reconstructions had a maximal error of 30% in amplitude. We extend this methodology to a more realistic insect wing through numerical simulation. We show that wing displacement can be estimated from sparse displacement or strain measurements, and that additional sensors spatially average measurement noise to improve reconstruction accuracy. This research helps overcome some of the challenges of measuring full-field dynamics in flying insects and provides a framework for strain-based sensing in insect-inspired flapping robots.

1. Introduction

The dynamics of flexible insect wings has attracted considerable research interest over the past several decades. During flight, flapping insect wings bend and twist in response to aerodynamic and inertial-elastic forces [1–3]. This deformation is believed to improve the insect's aerodynamic force production [4] and to enhance its power economy [5–7]. Further, some insects rely on the neurological feedback generated by wing deformation to realize closed-loop attitude control. Hawkmoth *Manduca sexta* wings, for example, are imbued with mechanoreceptors called campaniform sensilla that are believed to trigger reflexive responses when the insect is subject to environmental perturbations [8]. Owing to the substantial role that deformation plays in biological flight, researchers have invested significant effort in measuring wing deformation and flapping kinematics in freely flying insects.

The motion of flapping wings can be decomposed into two parts: rigid body motion and elastic deformation. During rigid body motion, all points over the wing's surface maintain a fixed distance with respect to one another. This occurs during slow articulation of the wing or when the insect's body translates during flight. During elastic deformation, the relative distance between points is variable, which causes the wing to strain and store elastic potential energy. Given the high frequency flapping of insect flight, both motions occur simultaneously, and the wing's instantaneous shape and orientation can be determined by superimposing smaller elastic deformation on top of larger rigid body rotation [9, 10]. The concurrence of elastic deformation and rigid body rotation, however, makes it challenging to measure them independently.

The most common mechanism to measure flapping wing motion is through high-speed videography. By utilizing multiple cameras, one can

reconstruct the three-dimensional position of a point in space [11]. By tracking multiple points to define time-dependent planes, the kinematics of a structure can be reconstructed. Willmott and Ellington used videography to estimate flapping kinematics of the Hawkmoth *M. sexta* during hovering and forward flight in a wind tunnel [12]. Ortega-Jimenez *et al* furthered these studies by measuring *M. sexta* flapping kinematics in turbulence [13]. Greeter and Hedrick later quantified *M. sexta* wing kinematics during lateral maneuvers [14]. Similar kinematic studies have been conducted for honeybees [15], dragonflies [16], as well as other insects [17, 18] and small flapping wing robots inspired by insects [19]. While these studies have undoubtedly advanced our understanding of flapping motions, one potential issue with kinematic reconstruction stems from the motion of the specific points tracked—if these points experience out-of-plane deflection, the resolved kinematics will be influenced by both rigid body rotation and elastic deformation.

Decoupling rigid body rotation from elastic deformation generally requires sophisticated equipment or post-processing techniques in conjunction with high-speed videography. Wu *et al* utilized high-speed three-dimensional digital image correlation to measure full-field wing deformation in a flapping wing micro air vehicle (FWMAV) in air and in vacuo [20]. Hsu *et al* recorded freely flying *M. sexta* and estimated their wing deformation through a complex optimization routine interfaced to a finite element (FE) reference body [21]. Aguayo *et al* measured out-of-plane deformation in swallowtail butterfly wings using digital holographic interferometry [22]. Koehler *et al* estimated wing deformation in free flying dragonflies using videography and a template-based subdivision surface reconstruction [23]. Unlike kinematic reconstructions, which require only a handful of measurements to define planes, measuring out-of-plane elastic deformation necessitates many measurement points to resolve the wing's instantaneous shape. Full-field deformation measurements consequently demand significant data storage capabilities, and potentially large amounts of effort to post-process this data. For these reasons, full-field measurements are challenging and often impractical.

To overcome these complications, researchers often estimate full-field quantities based upon sparse measurements using physics-based reconstruction methods. Some of these reconstruction methods include Guyan condensation, improve reduced system, and system equivalent reduction expansion processes (SEREPs) [24]. Each of these methods requires a knowledge of the structure's mass and stiffness, typically determined either through FE modeling or experimental measurements, but none require explicit knowledge of the structure's forcing input. This makes them useful when a structure is subject

to uncertain loading conditions. Such physics-based reconstruction methods can be employed on flapping wings to reduce the measurement points required to estimate full-field deformation.

1.1. Research scope

The purpose of the present research is to demonstrate that sparse measurements and physics-based reconstruction techniques can be used to estimate full-field flapping wing dynamics. We show this both through a simplified physical experiment and more complex numerical simulation. To the best of our knowledge, such techniques have not previously been applied to flapping wings. Flapping wings present a unique challenge to reconstruction methods because their stiffness varies periodically throughout a wing-beat [9]. From the experimental biology perspective, this research delivers a technique that can reduce the number of points required to track when measuring wing deformation. From the robotics perspective, this research provides a simple framework to estimate instantaneous wing shape on FWMAVs in real-time if strain or displacement is measurable at select points. Instantaneous wing shape, combined with angular rates of rotation, can be used to predict aerodynamic vectors.

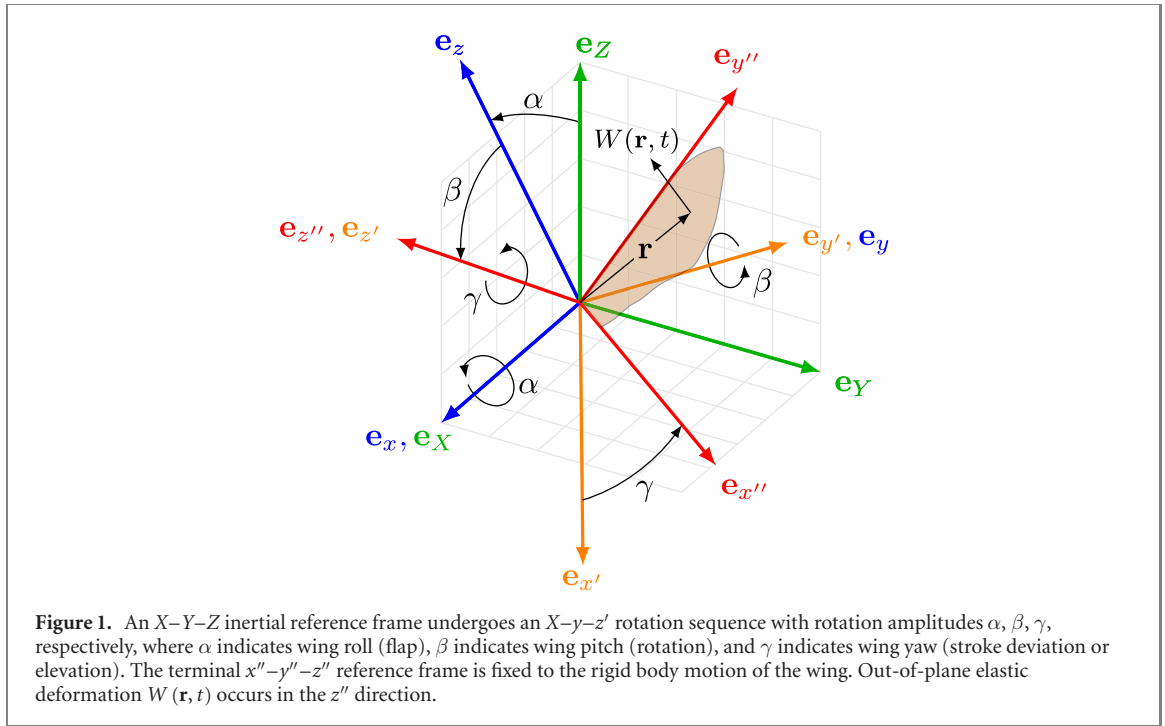
The remainder of this paper is organized as follows. First, we summarize SEREPs and normalized modal displacement method (NMD), the frameworks used to estimate wing dynamics and to optimize sensor placement, respectively. We then introduce an experiment where we flap a paper wing and predict strain at one location using measurements from two others via SEREP. Though we consider only single-degree-of-freedom (SDOF) flapping kinematics and measure strain rather than displacement, our methods are general and scalable to more complex cases. To demonstrate this, we numerically simulate the response of a more realistic insect wing FE model subject to multiple-degree-of-freedom (MDOF) flapping kinematics and reconstruct elastic deformation based on sparse measurements. We conclude by discussing the applicability of this work and the areas of research it can benefit.

2. Theory

Here, we introduce the theoretical frameworks utilized throughout this research. Referenced from a coordinate system that rotates with the wing's rigid body motion (figure 1), the spatiotemporal wing deformation $W(\mathbf{r}, t)$ is

$$W(\mathbf{r}, t) = \sum_{k=1}^{\infty} \phi_k(\mathbf{r}) q_k(t) \quad (1)$$

where the vector $\mathbf{r}$ is the planar coordinate of an arbitrary point on the wing, ϕ_k are the wing's mass normalized mode shapes, and the time dependent modal



response is $q_k(t)$. Vibration mode shapes are the way a structure moves when excited at one of its natural frequency and are analogous to eigenvectors in the mathematical eigenvalue problem. A structure's displacement field generally can be represented as a linear combination of these vibration modes. Since they are arbitrary in magnitude, they are typically scaled to the structure's mass for practical reasons. Then, a similar expansion can be derived for full-field strain replacing the mode shapes ϕ_k with the modal strains. This expansion, which allows the displacement field to be represented in terms of shapes rather than individual points, provides the foundation for the reconstruction methods that follow. Mode shapes $\phi_k(\mathbf{r})$ and modal strains can be obtained experimentally or from a FE discretization of the wing.

2.1. System equivalent reduction expansion process

Model reduction techniques are used to reduce the complexity of systems while retaining important characteristics and for reconstructing systems from sparse measurements. For this work, SEREP is used to reconstruct the full system response from a limited number of measurements [25]. This technique has proven effective for the larger fixed wings on airplanes [26]. We begin by briefly describing the theory underlying SEREP in the current context.

Though continuous in nature, insect wings can practically be treated as MDOF systems if represented by discrete measurement points. We consider reconstructions of $\mathbf{W}$, where $\mathbf{W}$ is the time-dependent vector of out-of-plane elastic deformations referenced from discrete measurement points. Although we show this methodology for displacement reconstructions,

strain reconstructions are equally valid. As an MDOF system, wing deformation $\mathbf{W}$ is

$$\mathbf{W} = \Phi \mathbf{q} \quad (2)$$

where Φ contains the mode shape ϕ_k in the k th column and $\mathbf{q}$ is the modal response vector. We choose a subset of the coordinates to retain, denoted with the subscript r , and obtain the reduced system

$$\mathbf{W}_r = \Phi_r \mathbf{q}_r \quad (3)$$

These retained coordinates correspond to the locations of sensors. Information from these sensors is used to reconstruct the full system. For a well defined problem we will need to retain at least as many coordinates as we retain modes in 1. We estimate the full displacement using the generalized inverse of Φ_r , denoted by

$$\Phi_r^g = (\Phi_r^T \Phi_r)^{-1} \Phi_r^T \quad (4)$$

to compute the least squares approximation of $\mathbf{W}$ given by

$$\mathbf{W} = \Phi \Phi_r^g \mathbf{W}_r \quad (5)$$

In the special case where we have the same number of retained modes as sensors the generalized inverse is the inverse of Φ_r . Error in the measurements $\mathbf{W}_r$ may make it beneficial to have more sensors than retained modes.

As modal contributions are shared between strains, it is also possible to mix measurements in different directions to recover the full field strain. For example, ϵ_x can be reconstructed by

$$\epsilon_x = \Phi_x \begin{bmatrix} \Phi_{x,rx} \\ \Phi_{y,rx} \end{bmatrix}^g \begin{bmatrix} \epsilon_{x,rx} \\ \epsilon_{y,rx} \end{bmatrix} \quad (6)$$

where rx and ry are the measured coordinates in x and y , respectively, and Φ_x and Φ_y contain the modal strains in x and y , respectively. Once the sensor locations are chosen, the mode shapes are used to construct a transformation matrix involving the generalized inverse matrix. This transformation matrix allows us to estimate the full field displacement or strain using equation (5) and the sensor data. Note that the transformation matrix depends only on the physical parameters of the wing and needs to be computed only once.

2.2. Weighted NMD method

Sensor placement can have a large impact on the accuracy of reconstruction. We use the weighted normalized displacement method (NMD) to determine sensor placement [27]. We first compute the driving point residues (DRP) for each node at location $\mathbf{r}$ over all retained modes k by

$$\text{DPR}_k(\mathbf{r}) = \Phi_k(\mathbf{r})^2 / \omega_k \quad (7)$$

where ω_k is the natural frequency of the k th retained mode with mode shape Φ_k . We desire sensors with high average DPR (equivalently high modal participation factors) and where each mode contributes significantly. To that end we compute a weighted NMD as

$$\text{avgNMD}(\mathbf{r}) = \frac{1}{N} \sum_{k=1}^N (|\text{DPR}_k(\mathbf{r})|) \quad (8)$$

$$\text{minNMD}(\mathbf{r}) = \min_{1 \leq k \leq N} (|\text{DPR}_k(\mathbf{r})|) \quad (9)$$

$$\text{wghNMD}(\mathbf{r}) = \text{minNMD}(\mathbf{r}) * \text{avgNMD}(\mathbf{r}) \quad (10)$$

Above, N is the total number of measurement points from which $\mathbf{W}$ is referenced. We select the measurement points with the highest wghNMD as our sensor locations. This formulation can be used to choose strain sensors locations by replacing the mode shapes with the modal strains.

3. Experimental validation

In this section, we present an experiment designed to verify the accuracy of SEREP applied to a simple flapping wing. First, we describe an SDOF mechanism used to flap a wing at 45° between 5–9 Hz. Next, we detail the paper wing used in all experimental trials, where three strain gages are used to measure the wing's dynamic response. The paper wing is structurally modeled using the FE method to determine its vibration mode shapes and modal strains. We then estimate wing strain measured at one of the gage locations based on measurements from the other two locations via SEREP.

3.1. Mechanical flapper

The flapping mechanism (figure 2) described here originated in [28] and is summarized to provide

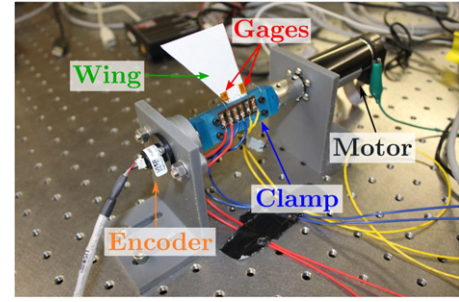
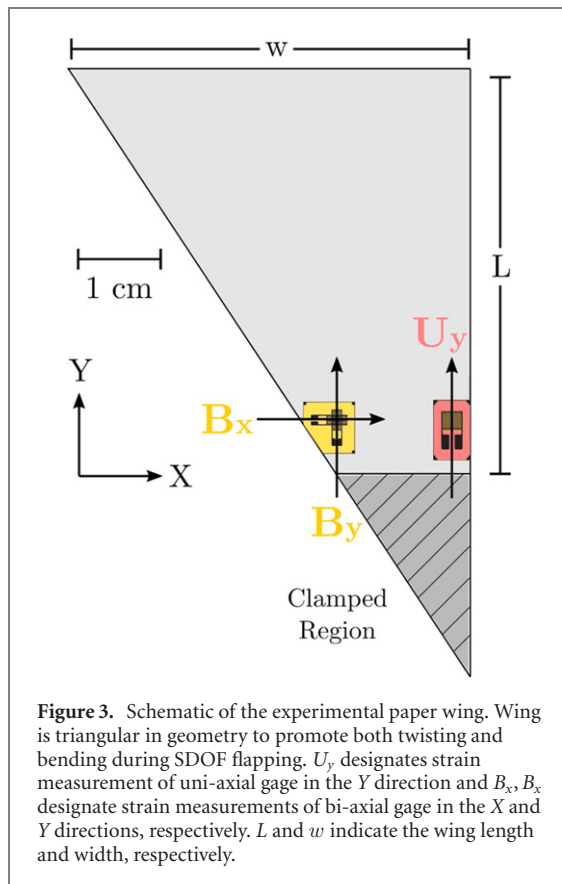


Figure 2. Diagram of flapping mechanism used for experimental trials. The mechanism flaps the paper wing from 5–9 Hz with an amplitude of 45° .

context to this work. A 60 W direct current motor (Maxon Motors, 310007) regulated by a proportional-integral-derivative controller (Maxon Motors, EPOS 24/5) drives the wing's flapping motion. The motor attaches to a fixture that clamps the base of the wing. The end of the clamping fixture is supported by a flange bearing to allow it to spin freely, and the shaft position is measured using a quantized analog encoder (US Digital, MAE3-A10-250-220-7-B). All motor brackets and fixtures are printed using a Form2 SLA 3D printer. We consider flapping frequencies from 5–9 Hz in 1 Hz intervals at an amplitude of 45° . Each flapping trial lasts 15 s, where the first 5 s are discarded from experimental results to eliminate transient responses. For high-speed video of the flapping mechanism, the reader is directed to the supplementary material (<https://stacks.iop.org/BB/16/016005/mmedia>). All experimental data is recorded using a National Instrument cDAQ-9178 acquisition system.

3.2. Experimental wing and strain measurements

We create a simple paper wing for experimental trials (figure 3). Wing properties are shown in table 1. The wing is made of card stock because it has low surface density and as a result, is influenced by both aerodynamic and inertial loads. The wing is triangular so that it will both bend and twist due to the off-set center of mass. Bending will predominately cause the wing to strain in the Y direction, whereas twisting (also torsion) causes the wing to strain in the X direction. We mount strain gages to the wing using ethyl-based cyanoacrylate, where the strain gage locations are guided by NMD. Due to the physical size of the sensors, we restrict sensor measurement grids to be placed at least 3 mm away from the edge of the wing as well as 3 mm away from one another; sensor carrier material may be placed immediately along the wing edge. Thus, sensor locations are not that exactly identified via NMD, but are in close proximity to the optimal locations given physical constraints. We mount one uni-axial gage (Omega Engineering, SGD-2/350-LY11) to measure strain in the Y (designated U_y), and one bi-axial gage (Omega Engineering, SGD-2/350-XY11) at a separate location that measures strain in



both X and Y directions (designated B_x and B_y , respectively). All strain data is recorded using a bridge input module (National Instruments, NI-9236).

The total of three measurement points implies that we can maximally retain three vibration modes for wing shape reconstruction. However, given that we will utilize only two measurements for reconstruction and the third for validation, we will retain at a maximum two modes in this experimental work. These modes are not required to begin at the first mode, nor do modes need to be adjacent—it is possible to retain the second and fourth mode, for example. Selecting the modes to retain depends on several factors, including wingbeat frequency and the spatiotemporal distribution of aerodynamic forces. For the purposes of this work, we retain either one or two vibration modes in order to demonstrate how this quantity affects the accuracy of the reconstructed signal. We always consider two measurement points. We do not consider any scenarios in which we retain more modes than measurement points, as this represents an underdetermined system of equations. While it is possible to employ SEREP even when the number of modes exceeds number of measurement points by solving a least squares optimization problem, this typically produces undesirable results.

3.3. Experimental wing structural modeling

To calculate the vibration modes for SEREP reconstruction, we create an FE model of the wing using

ABAQUS. Because the strain gages are of comparable thickness to the paper, they stiffen the overall structure and must be included in modeling efforts. Optimal sensor locations were determined before the gages were included in the FE model; it is possible that inclusion of the sensors in the FE model modestly affects optimal sensor placement. Both the paper wing and gages are modeled using quadrilateral shell elements and the material properties in table 1. The wing is discretized into 884 elements total, which is sufficient for convergence of the wing's first four natural frequencies. We then conduct a modal analysis to determine the wing's mode shapes, which includes both information regarding modal displacements and directional modal strains. The first four vibration modes are shown in figure 4. From first to fourth, the wing's natural frequencies are 21.3 Hz, 63.9 Hz, 161.3 Hz and 263.9 Hz, respectively. Natural frequencies are not required for SEREP, however they do inform which vibration modes should be retained if the input bandwidth is known. Further, natural frequencies are required to compute DRP (equation (9)). While we have utilized the FE method to determine the modal displacements and strains, these quantities may also be identified experimentally if desired.

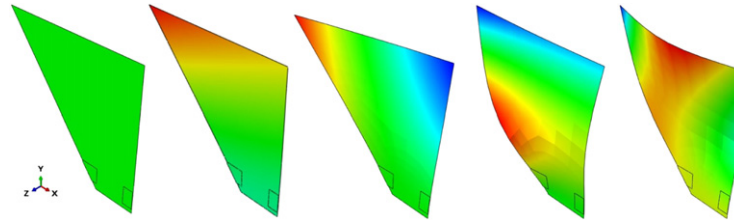
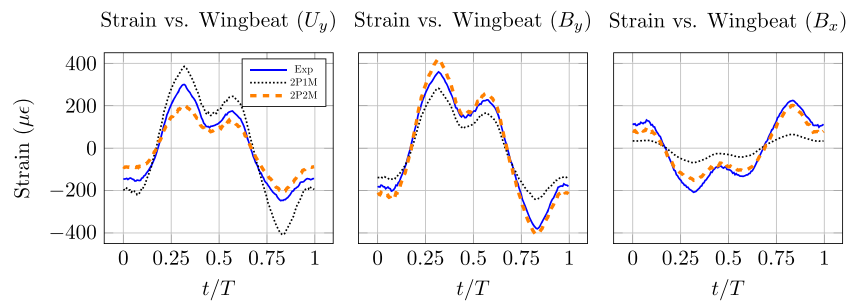
3.4. Experimental results

Here, we present the results of the SDOF flapping experiments. In general, SEREP is effective at predicting strain from sparse measurements (figures 5 and 6). Prediction accuracy appears independent of excitation frequency, and the method was effective at accounting for qualitative differences between strain waveforms. Retaining more measurement points improved the agreement between the measured and predicted strain. We show only results for 6 and 9 Hz flapping frequencies owing to their dissimilar strain responses; trends in reconstructed strain are similar for intermediate flapping frequencies.

Interestingly, retention of the first and third vibration modes (figure 4) produced the best reconstructed strain signals in all cases. This is likely due to how the aerodynamic force is distributed over the flapping wing. The second vibration mode is a torsional mode, whereas the third vibration mode is a bending mode. The aerodynamic force acting on an SDOF flapping wing will approximately scale quadratically along the Y-axis from the axis of rotation (figure 3), as well as with the square of angular velocity [28]. Because the wing's torsional mode exhibits displacement in both the positive and negative Z direction, the projection of the aerodynamic force onto the torsional mode will be close to zero. On the other hand, projecting this aerodynamic force onto the first and third vibration modes will generate a non-zero modal excitation. Thus, we believe the wing's displacement is best described via the first and third vibration modes, despite intentionally offsetting the center of

Table 1. Properties of experimental wing.

Description	Value	Unit
Wing unclamped length	5	cm
Wing width	5	cm
Wing thickness	0.17	mm
Wing Young's modulus	9.5	GPa
Wing Poisson ratio	0.33	—
Gage thickness	0.13	mm
Gage Young's modulus	2.5	GPa
Gage Poisson ratio	0.33	—
Total Mass	0.33	grams

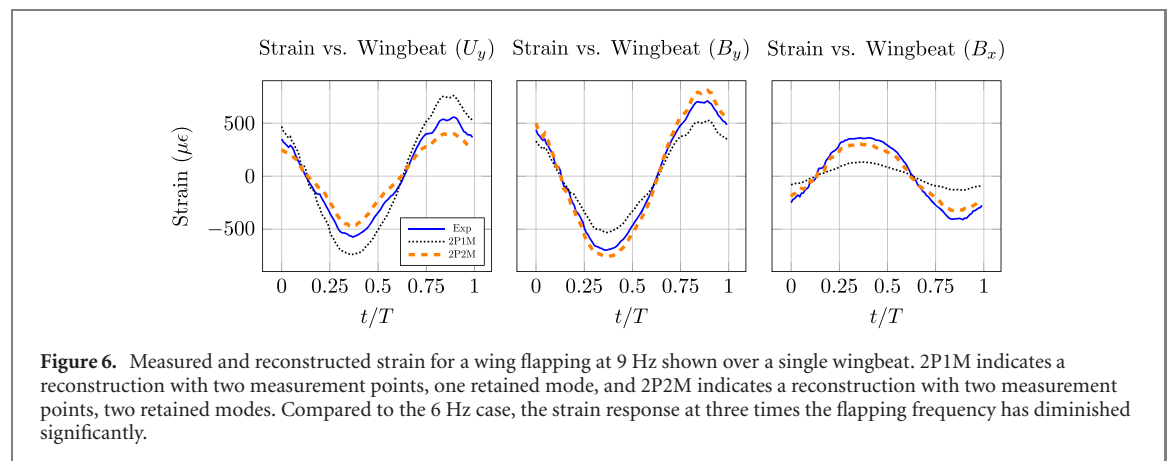
**Figure 4.** From left to right, FE model of the experimental wing in its undeformed state followed by the first four vibration modes in ascending order. Color maps indicate relative modal displacement for each vibration mode. Note that modal displacement is a dimensionless quantity, and should only be compared within a single mode rather than across distinct modes.**Figure 5.** Measured and reconstructed strain for a wing flapping at 6 Hz shown over a single wingbeat. 2P1M indicates a reconstruction with two measurement points, one retained mode, and 2P2M indicates a reconstruction with two measurement points, two retained modes. Note the significant strain response at three times the flapping frequency, which occurs due to proximity to a superharmonic resonance.

mass to promote a torsional response. For the following results where only one mode is retained, we retain the first vibration mode.

Having identified which modes are most appropriate to retain for SEREP, we turn to the reconstructed strain signals. First, consider the strain profiles depicted in figure 5 for a wing flapping at 6 Hz. For all strain sensors, the primary response occurs at the flapping frequency, where B_y and U_y are in-phase and B_x is 180° out-of-phase. Because the wing is flapping at roughly $\frac{1}{3}$ its natural frequency, it experiences a near super-harmonic resonance which contributes to a large strain response at three times the flapping frequency [28]. SEREP captures this effect despite having no direct measurement of the input. The reconstructed strain signals are generally good both in terms of magnitude and phase. Intuitively, the reconstruction is more accurate when more vibration modes are retained. When considering two retained

modes, the largest discrepancy between measured and reconstructed strain occurs in U_y , which maximally has an error of about 30% in magnitude.

Next, we turn our attention to 9 Hz flapping (figure 6). Similar to the 6 Hz case, B_y and U_y are in-phase and B_x is 180° out-of-phase. The strain response at three times the flapping frequency has diminished significantly, because the third harmonic of the flapping frequency is no longer in proximity to the wing's first resonant frequency. Strain reconstructions are of similar accuracy to the 6 Hz case. The largest error remains in U_y , but is reduced by about 5%. The reduction in error likely comes from a change in modal participation factors; it is possible that unretrained modes more notably influenced the wing while it was flapping at 6 Hz, but that those modes are not as influential when the flapping frequency is increased to 9 Hz.



Consequently, within the experimental context, we contend that unretained vibration modes are the most appreciable source of error. Though the first and third mode dominate the wing's deformation, the second torsional mode does have an influence under certain conditions. For the second vibration mode, modal strains are higher at gages B_x and B_y relative to U_y . Because the strain at U_y is predicted from measurements at B_x and B_y , the greater torsional influence at these points adversely impacts the U_y reconstruction. This error can be reduced by including more strain measurements, which will (1) enable us to retain more vibration modes, including the second torsional mode, and (2) facilitate in averaging measurement noise. Furthermore, including additional measurement points helps reconcile uncertainty in the FE model. Suppose that the strain gages in the experiment are placed a fraction of a millimeter away from where they are located in the FE model. This will affect the predicted modal strains, and thus, the reconstruction accuracy. If there are numerous sensors in the physical experiment and each are located a small, random distance from their modeled FE location, the bias of each sensor will average each other out to improve the reconstruction accuracy. Therefore, increasing sensor count will increase reconstruction accuracy by reducing noise and enabling additional modes to be retained.

4. Numerical simulation

In the previous section, we demonstrated that SEREP effectively reconstructs strain for a simplified flapping wing. However, insect wings represent a more significant challenge. They are made up of a thin membrane reinforced by thicker veins [29], have a cambered surface to promote geometric stiffening [30], and experience MDOF rotational flapping [12]. Further, because the natural frequencies in some insect wings are closely spaced [31], it is likely that several vibration modes contribute to the wing's overall deformation.

To further investigate SEREP's ability to estimate full-field flapping wing dynamics, we numerically simulate the response of a more realistic insect wing with conventional flapping kinematics. First, we derive an FE model of an *M. sexta* forewing that considers both camber and venation. Then, we utilize an inertial-elastic model to simulate the full-field response of the flapping wing. Lastly, we examine how sensor type, number and placement influence the accuracy of reconstruction methods.

4.1. Simulation wing and kinematics

We develop the *M. sexta* forewing model using ABAQUS (figure 7). The planar wing geometry is based off a micro computed tomography image from [32]. The membrane is modeled using shell elements and the veins are modeled using beam elements, where vein cross sections are assumed solid and circular and have effective diameters ranging from 100–400 μm . Note that this assumed vein cross-section is a simplification. Insect veins often have more intricate cross-sectional areas and may be hollow or contain hemolymph [33], and these factors must be considered when developing high fidelity FE models. Then, we apply a curvature to both the membrane and veins using reported results from [34]. The membrane and veins are restricted to move together by tie constraints. All material properties for the wing are approximated from [30] and are detailed in table 2. We assume the materials to be linear and isotropic. All degrees of freedom are fixed at the wing root. The model consists of 774 elements in total, which is sufficient for convergence of the wing's first four natural frequencies.

With the model built, we conduct a modal analysis to determine the wing's first four natural frequencies and mode shapes (figure 7). The first and second natural frequencies agree fairly well with those reported for *M. sexta* [31], however the third and fourth are considerably off. This is likely due to some of the modeling simplifications, for example linear isotropic material behaviors, as well as the inherent complexity of modeling higher-order vibration modes accurately.

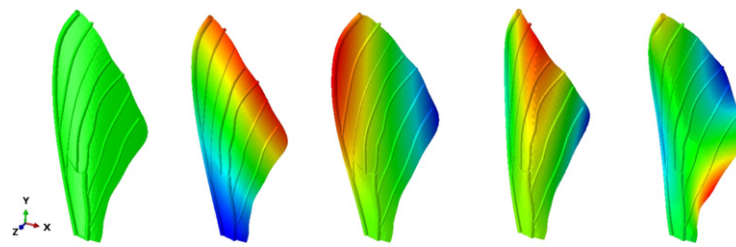


Figure 7. From left to right, FE model of the idealized insect wing in its undeformed state followed by the first four vibration modes in ascending order. Color maps indicate relative modal displacement for each vibration mode. Vein diameters are magnified by $3\times$ to facilitate visualization.

Table 2. Properties of simulation wing.

Description	Value	Unit
Membrane density	1500	kg m^{-3}
Membrane Young's modulus	4.0	GPa
Membrane Poisson ratio	0.33	—
Membrane thickness	50	μm
Vein density	1500	kg m^{-3}
Vein Young's modulus	8.0	GPa
Wing span	50.5	mm
Wing max chord width	20	mm

Table 3. Natural frequencies of the FE insect wing are calculated and are subsequently adjusted to match measurements reported in the literature [31]. All frequencies are in Hz.

	ω_1	ω_2	ω_3	ω_4
Original value	57.9	94.4	275.4	391.5
Adjusted value	60	84	107	142

We manually adjust the FE wing natural frequencies to agree with reported values prior to calculating the wing displacement during flapping. The original natural frequency values, as well as the adjusted values, are shown in table 3. Lowering the third and fourth natural frequencies to be in closer proximity to the flapping frequency will increase their contributions to the overall wing deformation.

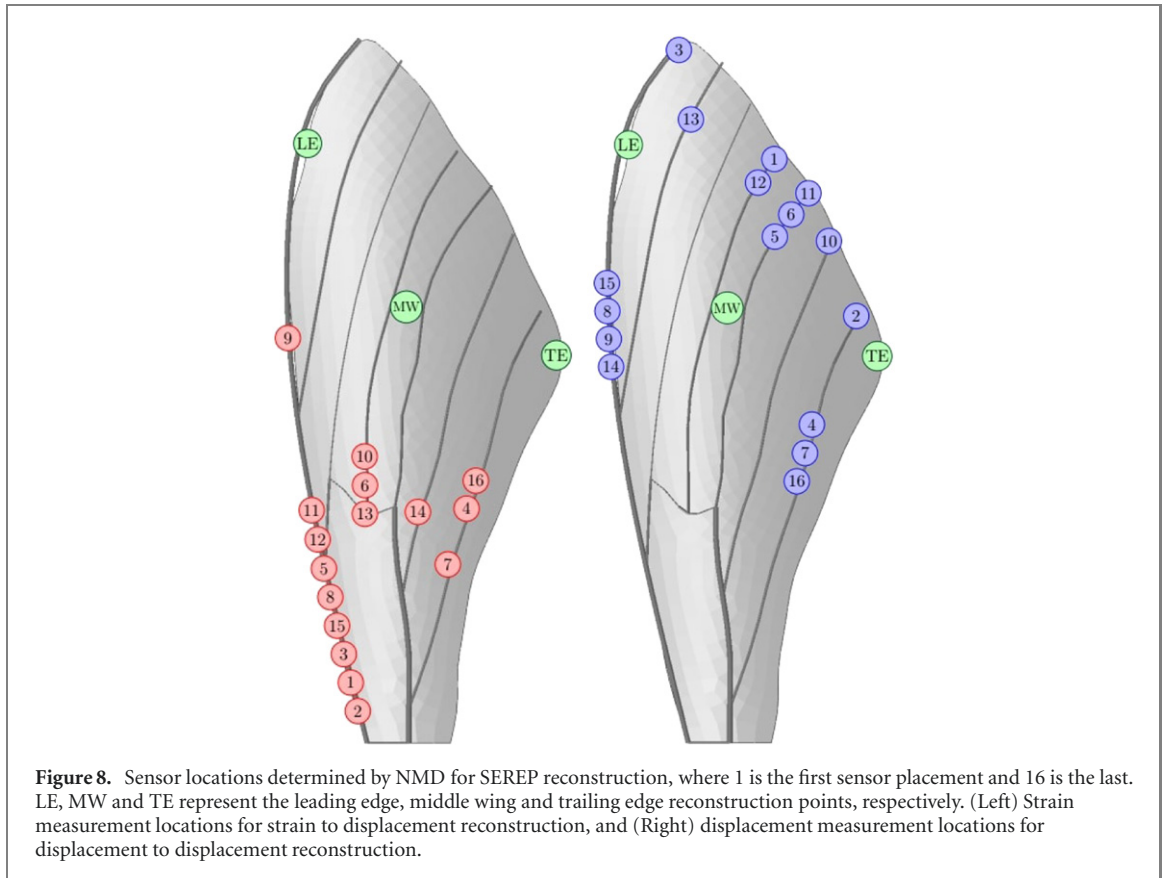
Lastly, to estimate wing deformation, we implement the model developed in [35]. This model assumes the rigid body rotations are known at the base of the wing (figure 1). We simplify these rotations from [12], and assume a roll amplitude of 60° , a pitch amplitude of 45° , and a pitch-roll phase difference of $\frac{\pi}{2}$ rad. We assume all rotations are harmonic with a flapping frequency of 25 Hz. Yaw rotation angle, also called stroke deviation or elevation, is not considered. We assume the response can be represented via a linear combination of the four modes calculated above.

To reduce computational costs, we neglect aerodynamic loading. While this will affect the wing's response, it will not in general affect the accuracy of SEREP or NMD since these methods rely only on the FE model and not dynamic inputs. However, we emphasize that the numerical model uses

only four modes to calculate wing displacement. This implies that, even if aerodynamic forces are included, they cannot excite any vibration modes except those used as the basis for the model. This contrasts the experimental scenario, where the modes that contributed to wing deformation are not known *a priori*. Within the numerical simulation, we have knowledge all modes that contribute the wing deformation, though we do not know individual contributions of each mode before applying SEREP. For the *M. sexta*, previous literature shows only the first four vibration modes can be measured reliably, which in part suggests higher order vibration modes do not contribute significantly to the wing's deformation [31]. If a higher order vibration mode does contribute to a structure's response and is not included in the dynamic reconstruction, this will adversely affect the reconstruction's accuracy.

4.2. Simulation results

We estimate full-field wing displacement using sparse strain measurements and displacement measurements. Sensor locations determined by NMD for both cases are shown in figure 8. We place up to sixteen sensors, where sensor one corresponds to the highest priority and sensor sixteen the lowest. For simplicity, we enforce that the sensor placements occur



on the vein structure. Strain sensors are assumed to measure bending strain in the vein's axial direction. We select three points on the wing's membrane to assess reconstruction accuracy, where the points are located at wing's leading edge, trailing edge, and midway between the two (figure 8). Midway point reconstructions are particularly useful in practical settings since it is more challenging to track points on the interior of the wing than points along the wing's perimeter. Because noiseless measurements result in perfect reconstruction, we add random Gaussian noise of ± 0.5 mm to displacement measurements ($\approx 10\%$ maximal displacement) and $\pm 5\mu\epsilon$ to strain measurements ($\approx 0.5\%$ maximal strain) in order to demonstrate the benefit to including additional measurement points in reconstruction efforts. While these noise levels are relatively low, all results in this section are shown unaltered by signal processing, both the measurements used for reconstructions or the reconstructions themselves.

First, consider the strain sensor placements (figure 8, left). The highest priority gages congregate near the base of the leading edge vein where the strain is greatest during flapping. The fourth, sixth and seventh gages are placed near the trailing edge and wing's torsional lines, which are locations that experience large strains when the wing twists. The remaining gages are placed in similar high strain regions. Interestingly, many of the optimized strain sensor locations coincide with the campaniform sensilla locations on the *M. sexta* forewing [8], though

there is little evidence that insects can sense their instantaneous wing shape. Instead, the similarities in optimized and actual sensor locations is more likely because these are the regions that experience greatest strain during flight.

Next, consider the displacement sensor placements (figure 8, right). Most sensors are located in the distal region of the wing, where it experiences the largest deformations. Sensors one through three are located between the wingtip and trailing edge, near the trailing edge and at the wingtip, respectively. Each of these points experience significant motion as the wing's response tends to be dominated by the first and second vibration modes. Beyond, sensors tend to cluster in the distal region of the wing, with another grouping emerging on the leading edge vein. While it is possible to define a minimal distance that sensors must be placed with respect to one another to prevent clustering, we did not consider this here.

With measurement points selected, we turn our attention to reconstructed displacements. The leading and trailing edge point reconstructions, superimposed on exact displacements, are shown in figures 9 and 10 for four and sixteen sensors. To quantify the difference between the reconstructed and exact displacements, we define an error proxy as

$$\text{Error} = 20 \log_{10} \left(\int_0^t |W - W_r| dt \right) \quad (11)$$

where W is evaluated at the trailing edge, leading edge, or middle wing reconstruction point and the error

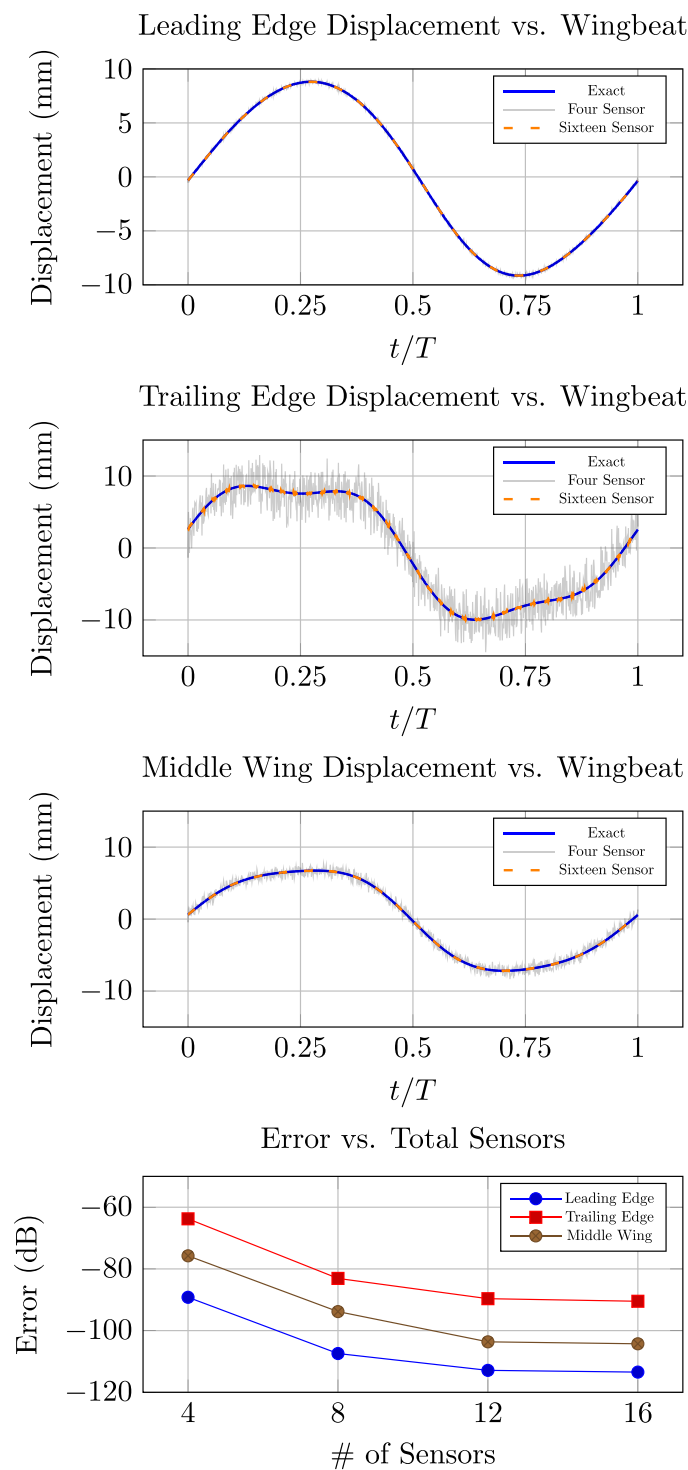


Figure 9. Full-field displacement reconstructions from strain measurements for the simulated insect wing. Top and middle show leading edge and trailing edge displacements and displacement reconstructions for four and sixteen sensors. Bottom shows the error in reconstruction as a function of the number of sensors.

units are in decibels. The error is shown as a function of sensor count in figures 9 and 10. For strain reconstructions, SEREP predicts leading edge displacement well for either four and sixteen sensors. This likely has to do with the high concentration of sensors along the leading edge vein, and the proximity of the tracked point to this vein. In contrast, the trailing edge is well-

predicted by sixteen sensors but experiences significant noise for four sensors. In this case, many sensors are required to average the measurement noise artificially to produce a good reconstruction, at least without applying a low-pass or moving mean filter to the reconstructed signal. Middle wing displacement is predicted with accuracy greater than the trailing

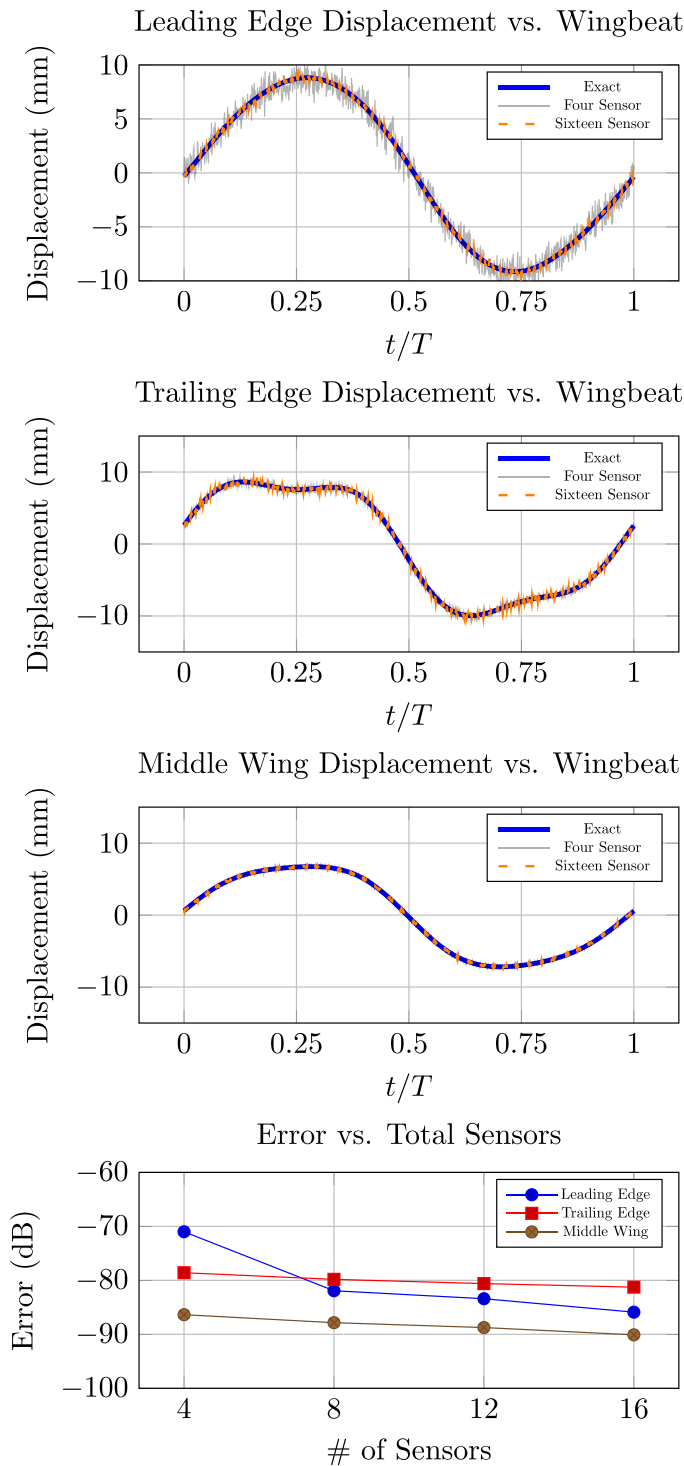


Figure 10. Full-field displacement reconstructions from displacement measurements for the simulated insect wing. Top and middle show leading edge and trailing edge displacements and displacement reconstructions for four and sixteen sensors. Bottom shows the error in reconstruction as a function of the number of sensors.

edge but less than the leading edge, which again relates to the proximity of this point relative to the high concentration of sensors near the leading edge.

For the displacement reconstructions, the leading edge point requires more sensors to reconcile noise. Again, this is likely because the four initial sensors are relatively distant from the reconstruction point. Trailing edge displacement is well represented by four sensors, though inclusion of additional sensors does not

significantly reduce the noise that does exist. The middle wing point has the best reconstruction accuracy for all sensor amounts because its central location puts it in close proximity to all measurement points. In all cases, increasing sensors decreases error monotonically. It appears that the error at both measurement points will eventually converge asymptotically, though at a greater sensor amount than is considered here.

5. Discussion

Deformation is integral to the function of flapping wings, however measuring deformation over an entire wing surface is challenging or impractical in many contexts. Through this research, we demonstrate how SEREP can estimate full-field quantities using sparse measurements, where the optimal locations of the measurement points are informed by NMD. SEREP is general in the sense that it can estimate displacement from strain or visa versa, as well as strain from strain. Through experiment, we show that we are able to estimate a strain waveform at one sensor location based upon two other measurements with good accuracy. Further numerical simulation on a more complex insect wing subject to realistic flapping kinematics showed that wing deformation can effectively be reconstructed from sparse strain or displacement measurements, and that additional sensors reduce reconstruction noise via spatial averaging.

5.1. Applications in biology

The methods developed in this research can help overcome some of the challenges of measuring flapping wing deformation. First, SEREP can help reconcile issues associated with landmark occlusion. Throughout a flight sequence, it is possible that a tracked point moves out of the camera field of view because it is occluded by the insect's body or other wing. Occlusion can be eliminated if a sufficient number of cameras are used for point tracking, but there tends to be a practical limit due to monetary or spatial constraints. Because SEREP requires sampling at only a handful of points to determine full-field wing deformation, it provides a good mechanism to estimate the wing shape in video frames where measurement points cannot be seen. Second, if an accurate FE model is available, SEREP can facilitate reconstructions inclusive of mesoscale wing features such as wing corrugation and venation. These features are difficult to track without a high density of measurement points [18]. Optical methods have also been proposed as a means to incorporate mesoscale features into wing reconstructions [23]. Lastly, reconstructions based upon sparse measurements reduce the computational time associated with estimating full-field dynamics. SEREP is algebraic, and the reconstruction matrix in equation (5) is pre-computed from the wing's stationary vibration modes. As a result, reconstructions can be conducted efficiently, including in real time if desired.

Nonetheless, there are limitations to the methods presented in this research as well. First, even if the optimal wing measurement points are determined via NMD, they are not necessarily convenient to track. Researchers often use clearly distinguishable landmarks when measuring wing deformation [29],

and there are no guarantees that the points identified by NMD coincide with these landmarks. Artificial markers may be applied to wings but at the risk of affecting the wing's mass and dynamical properties. Furthermore, both SEREP and NMD require the wing's vibration modes to be known. Modes are typically approximated through FE modeling or measured directly via experimental modal analysis. FE models have been developed for some of the most commonly studied flying insects such as the *M. sexta* [30, 31], dragonflies [36, 37], locust [38] and hoverflies [39]. Mode shape and natural frequency data has been measured directly for the *M. sexta* [31] and honeybee [40]. Analogous FE models or experimental modal data are not readily accessible for less common insects. FE models are challenging to develop for some insects due to a lack of information regarding geometric and material properties. Measuring vibration modes experimentally is straightforward, however these modes are not scaled with respect to the wing's mass. Mass scaling is not required by SEREP, though many sensor placement algorithms necessitate vibration modes to be normalized with respect to the wing's mass. While there are experimental techniques to estimate mass normalized modes, they often require systematic addition and removal of weight to the structure [41] which may be prohibitive for lightweight wings. Fortunately, many of these challenges are circumvented when considering the wings of bio-inspired robotics rather than insects; since robotic wings are designed through a forward process, there is substantially less uncertainty and FE models can be developed with a high level of precision and accuracy.

5.2. Applications in robotics

The instantaneous wing shape of small FWMVs influences the vehicle's aerodynamics. If strain is measurable, SEREP can be used to estimate this shape. Strain gages themselves are infeasible for centimeter-scale aircraft—in addition to adding weight and affecting passive wing deformation, the wiring network would quickly become cumbersome if multiple sensors were employed. On the other hand, advances in printable piezoelectrics may soon enable both the strain sensor and wiring network to be printed directly to the wing [42]. This would facilitate inclusion of dense sensor networks, similar to those observed in real insect wings [8]. In addition to wing shape, these networks of strain sensors may also facilitate identification of aerodynamic environment the vehicle is navigating [43].

Of course, instantaneous shape alone is not sufficient to predict aerodynamic forces. This would require additional knowledge of the flapping kinematics, such as the components of angular velocity. While we have not attempted to predict rotation

rates from strain in this work, Eberle *et al* showed that this mapping is possible [44]. However, even if wing shape and rotation rates are known, flapping wing fluid–structure interaction (FSI) models are still too high-order to make predictions in real time; most models capable of predicting aerodynamics efficiently neglect wing deformation altogether [45–47]. Though flapping wing FSI models continue to improve, most typically they employ limiting assumptions such as a rigid leading edge [48] or uni-lateral coupling between fluid and structure [35]. Nonetheless, the research presented here provides an approach that may ultimately improve aerodynamic sensing in FWMVs moving forward.

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References

- [1] Roccia B A, Preidikman S and Balachandran B 2017 Computational dynamics of flapping wings in hover flight: a co-simulation strategy *AIAA J.* **55** 1806–22
- [2] Fitzgerald T, Valdez M, Vanella M, Balaras E and Balachandran B 2011 Flexible flapping systems: computational investigations into fluid-structure interactions *Aeronaut. J.* **115** 593–604
- [3] Takizawa K, Tezduyar T E and Kostov N 2014 Sequentially-coupled space-time FSI analysis of bio-inspired flapping-wing aerodynamics of an MAV *Comput. Mech.* **54** 213–33
- [4] Mountcastle A M and Combes S A 2013 Wing flexibility enhances load-lifting capacity in bumblebees *Proc. R. Soc. B* **280** 20130531
- [5] Yin B and Luo H 2010 Effect of wing inertia on hovering performance of flexible flapping wings *Phys. Fluids* **22** 111902
- [6] Reid H E, Schwab R K, Maxcer M, Peterson R K D, Johnson E L and Jankauski M 2019 Wing flexibility reduces the energetic requirements of insect flight *Bioinspir. Biomim.* **14** 056007
- [7] Jankauski M, Guo Z and Shen I Y 2018 The effect of structural deformation on flapping wing energetics *J. Sound Vib.* **429** 176–92
- [8] Dickerson B H, Aldworth Z N and Daniel T L 2014 Control of moth flight posture is mediated by wing mechanosensory feedback *J. Exp. Biol.* **217** 2301–8
- [9] Jankauski M and Shen I Y 2014 Dynamic modeling of an insect wing subject to three-dimensional rotation *Int. J. Micro Air Vehicles* **6** 231–51
- [10] Jankauski M and Shen I 2016 Experimental studies of an inertial-elastic rotating wing in air and vacuum *Int. J. Micro Air Vehicles* **8** 53–63
- [11] Hedrick T L 2008 Software techniques for two- and three-dimensional kinematic measurements of biological and biomimetic systems *Bioinspir. Biomim.* **3** 034001
- [12] Willmott A P and Ellington C P 1997 The mechanics of flight in the hawkmoth *Manduca sexta*. i. kinematics of hovering and forward flight *J. Exp. Biol.* **200** 2705–22
- [13] Ortega-Jimenez V M, Greeter J S M, Mittal R and Hedrick T L 2013 Hawkmoth flight stability in turbulent vortex streets *J. Exp. Biol.* **216** 4567–79
- [14] Greeter J S M and Hedrick T L 2016 Direct lateral maneuvers in hawkmoths *Biol. Open* **5** 72–82
- [15] Altshuler D L, Dickson W B, Vance J T, Roberts S P and Dickinson M H 2005 Short-amplitude high-frequency wing strokes determine the aerodynamics of honeybee flight *Proc. Natl Acad. Sci.* **102** 18213–8
- [16] Wang H, Zeng L, Liu H and Yin C 2003 Measuring wing kinematics, flight trajectory and body attitude during forward flight and turning maneuvers in dragonflies *J. Exp. Biol.* **206** 745–57
- [17] Hsu S-J, Thakur N and Cheng B 2019 Speed control and force-vectoring of bluebottle flies in a magnetically levitated flight mill *J. Exp. Biol.* **222** jeb187211
- [18] Walker S M, Thomas A L R and Taylor G K K 2009 Photogrammetric reconstruction of high-resolution surface topographies and deformable wing kinematics of tethered locusts and free-flying hoverflies *J. R. Soc. Interface* **6** 351–66
- [19] Whitney J P and Wood R J 2010 Aeromechanics of passive rotation in flapping flight *J. Fluid Mech.* **660** 197–220
- [20] Wu P, Stanford B, Bowman W, Schwartz A and Ifju P 2009 Digital image correlation techniques for full-field displacement measurements of micro air vehicle flapping wings *Exp. Tech.* **33** 53–8
- [21] Hsu T-S S, Fitzgerald T, Nguyen V P, Patel T and Balachandran B 2017 Motion visualization and estimation for flapping wing systems *Acta Mech. Sin.* **33** 327–40
- [22] Aguayo D D, Mendoza Santoyo F, De la Torre-I M H, Salas-Araiza M D, Caloca-Mendez C and Gutierrez Hernandez D A 2010 Insect wing deformation measurements using high speed digital holographic interferometry *Opt. Express* **18** 5661–7
- [23] Koehler C, Liang Z, Gaston Z, Wan H and Dong H 2012 3d reconstruction and analysis of wing deformation in free-flying dragonflies *J. Exp. Biol.* **215** 3018–27
- [24] Avitabile P 2005 Model reduction and model expansion and their applications—part 1 theory *Proc. of the Twenty-Third Int. Modal Analysis Conf.* (Orlando, FL, USA)
- [25] O'Callahan J, Avitabile P and Riemer R 1988 System equivalent reduction expansion process *Proc. 7th Inter. Modal Analysis Conf.*
- [26] Chan-gi P 2016 Wing shape sensing from measured strain *AIAA J.* **54** 1068–77
- [27] Kammer D C 1991 Sensor placement for on-orbit modal identification and correlation of large space structures *J. Guid. Control Dyn.* **14** 251–9
- [28] Schwab R K, Reid H E and Jankauski M 2020 Reduced-order modeling and experimental studies of bilaterally coupled fluid–structure interaction in single-degree-of-freedom flapping wings *J. Vib. Acoust.* **142** 021012
- [29] Meresman Y, Husak J F, Ben-Shlomo R and Ribak G 2020 Morphological diversification has led to inter-specific variation in elastic wing deformation during flight in scarab beetles *R. Soc. Open Sci.* **7** 200277
- [30] Combes S A and Daniel T L 2003 Flexural stiffness in insect wings i. Scaling and the influence of wing venation *J. Exp. Biol.* **206** 2979–87
- [31] Norris A G, Palazotto A N and Cobb R G 2013 Experimental structural dynamic characterization of the hawkmoth (*Manduca sexta*) forewing *Int. J. Micro Air Vehicles* **5** 39–54

- [32] Norris A G 2013 Experimental characterization of the structural dynamics and aero-structural sensitivity of a Hawkmoth wing toward the development of Design Rules for Flapping wing micro air vehicles *Theses and Dissertation* Air Force Institute of Technology 839
- [33] Song Z, Tong J, Yan Y, Wu W and Sun J 2020 Effects of microfluid in the veins of the deployable hindwings of the Asian ladybeetle on flight performance *Comput. Biol. Med.* **121** 103817
- [34] O'Hara R P and Palazotto A N 2012 The morphological characterization of the forewing of the *Manduca sexta* species for the application of biomimetic flapping wing micro air vehicles *Bioinspir. Biomim* **7** 046011
- [35] Schwab R, Johnson E and Jankauski M 2019 A novel fluid–structure interaction framework for flapping, flexible wings *J. Vib. Acoust.* **141** 061002
- [36] Darvizeh M, Darvizeh A, Rajabi H and Rezaei A 2009 Free vibration analysis of dragonfly wings using finite element method *Int. J. Multiphys.* **3** 101–10
- [37] Jongerius S R and Lentink D 2010 Structural analysis of a dragonfly wing *Exp. Mech.* **50** 1323–34
- [38] Herbert R C, Young P G, Smith C W, Wootton R J and Evans K E 2000 The hind wing of the desert locust (*Schistocerca gregaria* Forskal). iii. a finite element analysis of a deployable structure *J. Exp. Biol.* **203** 2945–55
- [39] Kesel A B, Philippi U and Nachtigall W 1998 Biomechanical aspects of the insect wing: an analysis using the finite element method *Comput. Biol. Med.* **28** 423–37
- [40] Clark C J, Mountcastle A M, Mistick E and Elias D O 2017 Resonance frequencies of honeybee (*Apis mellifera*) wings *J. Exp. Biol.* **220** 2697–700
- [41] Fernández P, Reynolds P and López-Aenlle M 2011 Scaling mode shapes in output-only systems by a consecutive mass change method *Exp. Mech.* **51** 995–1005
- [42] Xu W, Huang H-L, Liu Y, Luo C, Cao G Z and Shen I Y 2016 Fabrication and characterization of PZT-silane nano-composite thin-film sensors *Sensors Actuators A* **246** 102–13
- [43] Manohar K, Brunton S L and Kutz J N 2017 Environment identification in flight using sparse approximation of wing strain *Journal of Fluids and Structures* **70** 162–80
- [44] Eberle A L, Dickerson B H, Reinhall P G and Daniel T L 2015 A new twist on gyroscopic sensing: body rotations lead to torsion in flapping, flexing insect wings *J. R. Soc. Interface* **12** 20141088
- [45] Jankauski M, Daniel T L and Shen I Y 2017 Asymmetries in wing inertial and aerodynamic torques contribute to steering in flying insects *Bioinspir. Biomim* **12** 046001
- [46] Sane S P and Dickinson M H 2002 The aerodynamic effects of wing rotation and a revised quasi-steady model of flapping flight *J. Exp. Biol.* **205** 1087–96
- [47] Nakata T, Liu H and Bomphrey R J 2015 A CFD-informed quasi-steady model of flapping-wing aerodynamics *J. Fluid Mech.* **783** 323–43
- [48] Wang Q, Goosen J F L and van Keulen F 2017 An efficient fluid–structure interaction model for optimizing twistable flapping wings *J. Fluids. Struct.* **73** 82–99