Capacity of flexible loads for grid support: statistical characterization for long term planning

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***Abstract*—Flexible loads are a valuable resource for the power grid of the future to help with balancing demand and generation. A balancing authority (BA) needs to know how much flexibility a load has, meaning what type of power deviation (from the baseline demand) signals are feasible for the load. In this work we present a characterization of capacity for a flexible load in terms of the power spectral density of the power deviation. We then show how this characterization can be used for resource allocation for the grid by determining what portion of the grid's needs can be met by a collection of such loads. The key difference with prior work on flexibility characterization is that ours is posed in terms of the statistical properties grid's net load and load's demand deviation, not on specific instances of these signals. The proposed characterization can thus be used for long term planning.**

I. Introduction

The inherent variability in renewable generation sources such as solar and wind is a challenge for the power grid operators to balance demand and generation. Ramp rate constraints prevent conventional generation from handling this mismatch between demand and generation completely, while grid level storage from batteries is expensive. Thus a new resource is being investigated to help fill the mismatch: flexible loads. Flexible loads have the ability to vary power consumption over a baseline level without violating their Quality of Service (QoS). The baseline power consumption is the power consumed without grid interference. The requested amount from the grid authority, to deviate from baseline, is the *reference signal*. The tracking of a zero-mean reference signal guises, in the eyes of the grid operator, flexible loads as batteries providing storage services. This battery-like behavior of flexible loads is often referred as Virtual Energy Storage (VES) [1]. VES from flexible loads can be less expensive than energy storage from batteries [2]. Some examples of flexible loads include residential air conditioners [3], water heaters [4], refrigerators, commercial HVAC systems [5], pumps for irrigation [6] or pool cleaning [7].

For flexible loads to provide VES, the loads must track accurately the reference signal for demand deviation. From the viewpoint of the grid operator, flexible loads that do not track the reference makes them unreliable. If the grid operator expects this signal to be accurately followed, then tracking it must not cause the flexible loads to violate their QoS. From the viewpoint of loads, reference signals that continually require QoS violation are simply beyond their ability and are unacceptable. Thus, reference signals must be designed to respect the *capacity* of the collection of flexible loads.

Informally, the capacity represents limitations in aggregate behavior (e.g., the ability to track a power reference signal) due to QoS requirements for the individual load. Consequently, a key step in determining the capacity is relating the QoS requirements of each load to the grid-level power reference signal. Unfortunately, this is not a straight forward task and many varying approaches are present in the current literature [8]–[13]. The most popular approach is to develop ensemble level necessary conditions [8], [10]; reference signals that satisfy these conditions ensure the ability of all loads in the collection to satisfy QoS while tracking the reference. Other approaches include geometry based characterizations [14] and load-centric characterizations [15].

One major limitation of the above mentioned works is that the capacity characterizations are based on constraints of the reference signal rather than statistical properties of the reference signal. Thus a grid authority requires a specific reference signal to perform resource allocation; only post-facto checking if the BA's needs are within the capacity of the resource is possible. That is, these capacity characterizations cannot be used for planning. The BA's needs are exogenous, so a useful capacity characterization should allow the BA to determine the portion of its needs that can be provided by a specific class and number of loads ahead of time.

Contrarily, if one develops constraints on the statistics of the reference signal, then a notion of capacity that is useful for longer term planning can be developed. For instance, consider Figure 1 where the Power Spectral Density (PSD) of the grid level net demand is allocated to resources. This frequency based allocation does not require knowledge of a specific reference signal, but only its statistics. To avoid possible confusion with power in kW, we use Spectral Density (SD) instead of Power Spectral Density (PSD) in the rest of the paper.

While primarily an illustrative example, it is possible to quantitatively develop the regions shown in Figure 1. There are many works that advocate for the specification of loads' abilities and/or resource allocation in the frequency domain [5], [16]–[18]. The results of real world VES ex-

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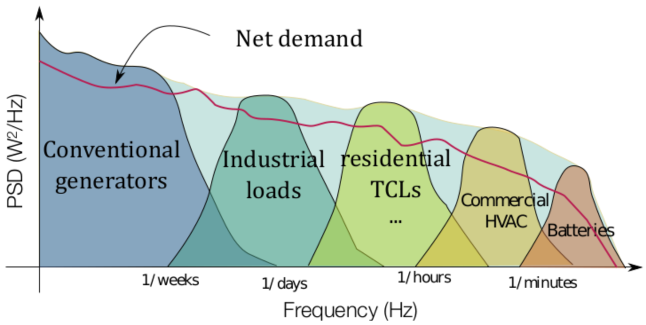

Fig. 1: An example spectral allocation of resources to meet the grids needs.

periments also suggest that specifying the spectral content of a reference signal is a feasible way to encapsulate the limitations in flexibility of a load [19]. A simplification of this concept is also widely used in today's power grid: ancillary services are classified by their response times and ramp rates [20]. Motivated by the limitations of past work and the advantages of working in frequency domain we extend the work in [16] and characterize the capacity of a flexible load in terms of the SD of reference signal for this load. The QoS constraints considered here are quite general and encapsulate operating constraints for: (i) Commercial HVAC systems, (ii) batteries, and (iii) Thermostatically Controlled Loads (TCLs). The contribution over past work is threefold: (i) we characterize capacity through constraints on the statistics of the reference signal, rather than the reference signal itself, (ii) our characterization of capacity allows for a BA to easily perform resource allocation and (iii) the time invariant SD allows for the grid authority to use our capacity characterization for long term planning.

We corroborate the advantages of our capacity characterization through numerical experiments. A convex optimization problem is setup that 'projects' the needs of the grid authority onto the set of feasible SD's. The needs of the grid is quantified as the SD of net-demand, as seen in Figure 1. The capacity characterization presented is for continuously varying loads, that is loads that can vary power consumption freely within an interval. The capacity characterization can be extended to handle discrete loads (e.g., TCLs), by applying it to the work [10].

The paper proceeds as follows, Section II describes the load model and operating constraints, Section III describes mathematical prerequisites and a characterization of individual load capacity, Section IV describes a method for determining the grid's need spectrally and how much flexible loads can contribute to grid's need using our capacity characterization. Numerical results for the method in Section IV are given in Section V.

II. Load model and Operating Constraints

Let the power consumption of a flexible load at time t be given as $P(t)$. Associated with the load is the notion of a *"storage" variable* θ that is related to the power consumption as follows,

$$\dot{\theta}(t) = \gamma\left(\theta_0(t) - \theta(t)\right) - \beta P(t), \quad (1)$$

where $\gamma > 0$ and $\beta \neq 0$ are parameters whose interpretation depends on the load and $\theta_0(t)$ is an exogenous signal. In case of an HVAC system, $\theta_0(t)$ is the ambient temperature and the storage variable is the internal temperature. The model (1), while simplistic, has been shown to agree quite well with many realistic models for certain flexible loads [21]. The nominal operation of the load is then to consume $P^*(t)$ of power to maintain the storage of the load at a fixed value θ^*. Since we are concerned with the flexibility of the load, we linearize (1) about θ^*, yielding:

$$\dot{\tilde{\theta}} = -\gamma\tilde{\theta} - \beta\tilde{P}, \quad (2)$$

where $\tilde{P}$ and $\tilde{\theta}$ are the power and storage deviation, respectively, and are defined as:

$$\tilde{P}(t) \triangleq P(t) - P^*(t), \text{ and } \tilde{\theta}(t) \triangleq \theta(t) - \theta^*. \quad (3)$$

The values in (3) are obtained by comparing (2) to (1). The dynamics (2) have transfer function from $\tilde{P}$ to $\tilde{\theta}$:

$$H(s) = -\frac{\beta}{s+\gamma}. \quad (4)$$

The concern now is, what is a feasible power deviation signal $\tilde{P}(t)$? To determine this we consider four general QoS requirements that limit the power consumption of the load: (i) power magnitude bounds, (ii) power increment bounds, (iii) load storage bounds, and (iv) energy bill bounds. By defining power change $\tilde{P}_\delta(t)$ and energy deviation $\tilde{E}(t)$ as:

$$\tilde{P}_\delta(t) \triangleq \tilde{P}(t+\delta) - \tilde{P}(t), \text{ and } \tilde{E}(t) = \int_0^t \tilde{P}(\sigma)d\sigma, \quad (5)$$

the QoS constraints are, in the order listed above:

$$\text{QoS-1:} \quad \left|\tilde{P}(t)\right| \leq c_1, \quad (6)$$

$$\text{QoS-2:} \quad \left|\tilde{P}_\delta(t)\right| \leq c_2,, \quad (7)$$

$$\text{QoS-3:} \quad \left|\tilde{\theta}(t)\right| \leq c_3,, \quad (8)$$

$$\text{QoS-4:} \quad \left|\tilde{E}(T)\right| \leq c_4, \quad (9)$$

where $\{c_i\}_{i=1}^4$ are user defined QoS limits for the respective constraints and T represents a fixed time interval, such as the length of a billing period. The constraint (9) then represents keeping the energy consumed during a period of length T close to the nominal energy consumed. Otherwise the consumer may have to pay a penalty in the form of an increase in energy bill.

Comment 1. *The constraint* (9) *is equivalent to constraining the moving averaged power deviation with window T, for all time. However, in order to avoid the notation from becoming too complicated, we utilize the form in* (9).

III. Load Capacity Characterization

A. Stochastic Setting

To develop our capacity characterization, we switch from a deterministic to a stochastic setting. In this setting we model the power deviation, $\tilde{P}$, as a stochastic process. The mean and autocorrelation function of $\tilde{P}$ are:

$$\mu_{\tilde{P}}(t) \triangleq \mathbf{E}[\tilde{P}(t)], \text{ and } R_{\tilde{P}}(s,t) \triangleq \mathbf{E}[\tilde{P}(s)\tilde{P}(t)], \tag{10}$$

where $\mathbf{E}[\cdot]$ denotes mathematical expectation. We make the following assumptions about the stochastic process $\tilde{P}$:

$$\text{A0:} \quad R_{\tilde{P}}(\tau) \text{ is continous for all } \tau, \tag{11}$$

$$\text{A1:} \quad \mu_{\tilde{P}}(t) = 0, \text{for all } t, \tag{12}$$

$$\text{A2:} \quad \tilde{P} \text{ is Wide Sense Stationary (WSS)}, \tag{13}$$

where $\tau = t - s$. The assumption A0 is technical and required for analysis. The assumption A1 comes from the fact that since $\tilde{P}$ is expressed as the difference of the power consumption from a baseline value its expectation should be set to zero. Otherwise, loads are not providing storage services. The assumption A2 is key to facilitate analysis. A2 requires the variance and mean of the process $\tilde{P}$ to be time invariant and the autocorrelation function to be a function of τ, as reflected in A0. We denote the time invariant variance as $\sigma^2_{\tilde{P}}$; the time invariant mean is already specified in A1. For a WSS stochastic process $\{X(t)\}$, the *(power) spectral density* $S_X(\omega)$ is the Fourier transform of the autocorrelation function [22]:

$$R_X(\tau) = \int_{-\infty}^{\infty} S_X(\omega) e^{(j\omega\tau)} d\frac{\omega}{2\pi}, \tag{14}$$

$$S_X(\omega) = \int_{-\infty}^{\infty} R_X(\tau) e^{(-j\omega\tau)} d\tau, \tag{15}$$

The above is referred as the Wiener-Khinchin theorem. When X has zero mean, the variance is

$$\sigma^2_X = R_X(0) = \int_{-\infty}^{\infty} S_X(\omega) d\frac{\omega}{2\pi}. \tag{16}$$

Proposition 1. *[22] Let X be a WSS and m.s. continuous stochastic process, then for all $t \geq 0$,*

$$\textit{(i)} \quad \mathbf{E}\left[\int_0^t X(\sigma)d\sigma\right] = \int_0^t \mu_X(\sigma)d\sigma = t\mu_X.$$

$$\textit{(ii)} \quad \mathbf{E}\left[\left(\int_0^t X(\sigma)d\sigma\right)^2\right] = 2\int_0^t R_X(\sigma)\left(T-\sigma\right)d\sigma.$$

Proposition 2. *[22] Let X be a WSS stochastic process and input to the linear time invariant BIBO stable system $G(s)$ with output Y, then Y is WSS, X and Y are jointly WSS, and*

$$\textit{(i)} \quad \mathbf{E}[Y] = G\left(j\omega\right)\Big|_{\omega=0}\mathbf{E}[X],$$

$$\textit{(ii)} \quad S_Y(\omega) = |G(j\omega)|^2 S_X(\omega),$$

where S_X is the SD of X, S_Y is the SD of Y, and $G(j\omega)$ is the frequency response of $G(s)$.

Furthermore, the Chebyshev inequality for a random variable X will be useful:

$$\mathcal{P}\big(|X-\mu_X| \geq k\big) \leq \frac{\sigma^2_X}{k^2}, \quad \forall\ k > 0, \tag{17}$$

where $\mathcal{P}(\cdot)$ denotes probability.

B. Inequality Constraints: Spectral Characterization

The QoS constraints (6)-(9) are characterized probabilistically in the following way. The inequalities in (6)-(9) turn into probabilistic inequalities; the probability of the QoS constraint *not* being met is required to be small:

$$\mathcal{P}\left(\left|\tilde{P}(t)\right| \geq c_1\right) \leq \epsilon_1, \tag{18}$$

$$\mathcal{P}\left(\left|\tilde{P}_\delta(t)\right| \geq c_2\right) \leq \epsilon_2, \tag{19}$$

$$\mathcal{P}\left(\left|\tilde{\theta}(t)\right| \geq c_3\right) \leq \epsilon_3, \tag{20}$$

$$\mathcal{P}\left(\left|\tilde{E}(T)\right| \geq c_4\right) \leq \epsilon_4. \tag{21}$$

The quantities $\{\epsilon_i\}_{i=1}^4$ set the tolerance level for satisfying the respective constraint and are chosen to be small.

In order to pose the inequality constraints (18)-(21) in terms of $S_{\tilde{P}}$, two steps are taken. The first step is to utilize the Chebyshev inequality (17) to bound the probabilities in (18)-(21) as a function of the variance of the given random variable. The second step is then to use the Wiener-Khinchin theorem (14) to express the variance as the integral of $S_{\tilde{P}}$.

Lemma 1. *Let $\tilde{P}$ satisfy A0-A2, then for all t*

$$\mathbf{E}[\tilde{E}(T)] = 0,\ \mathbf{E}[\tilde{P}_\delta(t)] = 0,\ \mathbf{E}[\tilde{\theta}(t)] = 0.$$

Proof. Apply the result of Proposition 1 and 2 for $\mathbf{E}[\tilde{E}(T)]$ and $\mathbf{E}[\tilde{\theta}(t)]$, respectively. The linearity of expectation suffices for $\mathbf{E}[\tilde{P}_\delta(t)]$. □

With the result in Lemma 1 and Chebyshev's inequality (17) we formulate sufficient conditions for the inequality constraints (18)-(21) as follows,

$$\sigma^2_{\tilde{P}} \leq c_1^2\epsilon_1, \quad \sigma^2_{\tilde{\theta}} \leq c_2^2\epsilon_2, \tag{22}$$

$$\sigma^2_{\tilde{E}_T} \leq c_3^2\epsilon_3, \quad \sigma^2_{\tilde{P}_\delta} \leq c_4^2\epsilon_4, \tag{23}$$

so that the probability of exceeding the inequality constraints (18)-(21) will be less than the respective specified amount, $\{\epsilon_i\}_{i=1}^4$. Before transforming the LHS of (22)-(23) in terms of $S_{\tilde{P}}$ it is necessary to compute the variance $\sigma^2_{\tilde{P}_\delta}$ and $\sigma^2_{\tilde{E}}$, for which we partly rely on Proposition 1, the linearity of expectation, and the Wiener-Khinchin theorem:

$$\sigma^2_{\tilde{P}_\delta} = \mathbf{E}\left[\left(\tilde{P}_\delta(t)\right)^2\right] = 2\left(R_{\tilde{P}}(0) - R_{\tilde{P}}(\delta)\right), \tag{24}$$

$$\sigma^2_{\tilde{E}_T}(T) = \mathbf{E}\left[\left(\tilde{E}_T(T)\right)^2\right] = 2\int_0^T R_{\tilde{P}}(\tau)\left(T-\tau\right)d\tau$$

$$= 2\int_0^T (T-\tau)\left(\int_{-\infty}^{\infty} S_{\tilde{P}}(\omega)e^{(j\omega\tau)}d\frac{\omega}{2\pi}\right)d\tau, \tag{25}$$

$$= \int_{-\infty}^{\infty} S_{\tilde{P}}(\omega)\left(\int_0^T (T-\tau)\cos\left(\omega\tau\right)d\tau\right)d\frac{\omega}{\pi}. \tag{26}$$

The integrands in (25) are positive, so that the integrability and absolute integrability are equivalent and the integrand satisfies both conditions for a fixed T. Thus the change of the order of integration is valid according to Fubini's theorem [23]. Furthermore, the inner integral in (26) can be evaluated by parts. Now applying the Wiener-Khinchin theorem (14) to the LHS of (22)-(23), the constraints (18)-(21) are converted into constraints on the SD of $\tilde{P}$:

$$\int_0^\infty S_{\tilde{P}}(\omega)d\frac{\omega}{\pi} \le c_1^2\epsilon_1, \tag{27}$$

$$2\int_0^\infty S_{\tilde{P}}(\omega)\left(1-\cos\left(\omega\delta\right)\right)d\frac{\omega}{\pi} \le \epsilon_2 c_2^2, \tag{28}$$

$$\int_0^\infty S_{\tilde{\theta}}(\omega)d\frac{\omega}{\pi} = \int_0^\infty |H(j\omega)|^2 S_{\tilde{P}}(\omega)d\frac{\omega}{\pi} \le c_3^2\epsilon_3, \tag{29}$$

$$2\int_0^\infty S_{\tilde{P}}(\omega)\frac{1-\cos(T\omega)}{\omega^2}d\frac{\omega}{\pi} \le c_4^2\epsilon_4, \tag{30}$$

where (27) is achieved from (16), and (29) is achieved through Proposition 2. The above integrals have been transformed from $(-\infty,\infty)$ to $[0,\infty)$ due to the symmetry of the above integrands about the point $\omega = 0$.

The *capacity* of a load is then characterized by the set

$$\mathcal{S} \triangleq \left\{S_{\tilde{P}}\middle|S_{\tilde{P}} \text{ satisfies constraints } (27)-(30)\right\}. \tag{31}$$

The load is capable of tracking any $\tilde{P}$ whose SD is in $\mathcal{S}$.

IV. Resource Allocation for Flexible Loads

We illustrate here how the capacity of a flexible load characterized by (27)-(30) can be used by a Balancing Authority (BA) for resource allocation. We consider a collection of *homogeneous* loads to be used as a resource to provide grid support, which is equivalent to a larger flexible load. Conceptually, our proposed method projects the need of a BA onto the constraint set of this one large flexible load. First, we describe how a BA can determine its needs spectrally and how to define the constraint set for this larger flexible load. Second, we combine the constraint set for this flexible load and the BA's spectral needs into a resource allocation optimization problem.

A. Spectral Needs of the BA

In the following we provide an example procedure for a BA to spectrally determine its needs, as illustrated in Figure 1. The BA first estimates the SD, Φ^{ND}, of its net demand, i.e., demand minus renewable generation. It can estimate this quantity from time series data of demand and renewable generation, or through a modeling effort, or a combination thereof. The next step for the BA is to fit a parameterized model to Φ^{ND}, which is termed S^{ND}. All controllable resources, including generators, flywheels, batteries, and flexible loads, together have to supply Φ^{ND} (or its parameterized model S^{ND}). The third step is obtain the portion of S^{ND} that flexible loads have to provide (similar to what is shown in Figure 1) by "filtering" S^{ND}. Letting $F(j\omega)$ be an appropriate filter, then the SD of the signal the grid authority would like flexible loads to contribute is:

$$S^L(\omega) = |F(j\omega)|^2 S^{ND}(\omega). \tag{32}$$

In the numerical example in this paper, we empirically estimate Φ^{ND} from time series data (from BPA, a balancing authority in the Pacific Northwest), and then obtain S^{ND} by fitting an ARMA(p,q) model to Φ^{ND}. An example of S^{ND} is shown as the red line in Figure 1, with S^L being any of the shaded regions in Figure 1.

Comment 2. *We have introduced a procedure for the BA to determine its needs in the spectral density domain. This procedure is completely independent from our characterization of capacity presented. The next step is to use the results of the procedure, the BA's spectral needs, to find the closest SD of the loads to the grid's need.*

B. Capacity of a collection of homogeneous loads

We aggregate a collection of smaller homogeneous loads to develop a larger (aggregate) flexible load. Homogeneous loads, by definition, have identical models (4), QoS constraints (6)-(9) and parameters, and consequently have the same SD $S_{\tilde{P}}(\omega)$. The total power deviation of a collection of N homogeneous loads is $\bar{P}(t) \triangleq N\tilde{P}(t)$, and the SD of the larger flexible load specified in terms of the common SD:

$$\bar{S}_{\tilde{P}}(\omega) = N^2 S_{\tilde{P}}(\omega). \tag{33}$$

where $\bar{S}_{\tilde{P}}(\omega)$ is the SD of $\bar{P}(t)$. To develop the constraints (27)-(30) on the SD $\bar{S}_{\tilde{P}}(\omega)$ it suffices to multiply the constraints (27)-(30) by N^2 and substitute in $\bar{S}_{\tilde{P}}(\omega)$ using its representation. This results in a set of constraints on the SD for the larger flexible load,

$$\begin{aligned}\mathcal{S}_{agg} \triangleq \Bigg\{\bar{S}_{\tilde{P}}\Bigg|\ &\bar{S}_{\tilde{P}}(\omega) \ge 0,\ \int_0^\infty \bar{S}_{\tilde{P}}(\omega)d\frac{\omega}{\pi} \le N^2 c_1^2\epsilon_1, \\ &\int_0^\infty 2\bar{S}_{\tilde{P}}(\omega)\left(1-\cos\left(\omega\delta\right)\right)d\frac{\omega}{\pi} \le N^2\epsilon_2 c_2^2, \\ &\int_0^\infty |H(j\omega)|^2\bar{S}_{\tilde{P}}(\omega)d\frac{\omega}{\pi} \le N^2 c_3^2\epsilon_3, \\ &\int_0^\infty 2\bar{S}_{\tilde{P}}(\omega)\frac{1-\cos(T\omega)}{\omega^2}d\frac{\omega}{\pi} \le N^2 c_4^2\epsilon_4, \Bigg\}.\end{aligned} \tag{34}$$

Comment 3. *$\mathcal{S}_{agg}$ is the capacity of the aggregate of N homogeneous flexible loads.*

C. Allocation through projection

Resource allocation is performed by projecting the SD $S^L(\omega)$ - what the grid needs - onto the set $\mathcal{S}$ - what the load can provide. The solution of the projection problem determines the SD $\bar{S}_{\tilde{P}}$ for the flexible load. The projection problem is,

$$\min_{\{\bar{S}_{\tilde{P}}\}} \quad \int_0^\infty \left(\bar{S}_{\tilde{P}}(\omega) - S^L(\omega)\right)^2 d\omega \tag{35}$$

$$\text{s.t.} \quad \bar{S}_{\tilde{P}} \in \mathcal{S}_{agg}, \tag{36}$$

$$\bar{S}_{\tilde{P}}(\omega) \le S^L(\omega), \quad \forall\ \omega \in [0,\infty). \tag{37}$$

Doing so allocates the needs of the grid to the flexible loads; the loads will cover as much of the BA's needs as they can while maintaining their QoS. In other words, the projection computes the regions shown in Figure 1 corresponding to constraints of the flexible loads. Constraint (37) is included to ensure that the capacity is not over scheduled, i.e., the load should contribute more than the BA requires.

In order to implement (35) on a computer, two steps are taken: (i) the problem is converted from continuous to discrete time and (ii) the resulting discrete time integrals in the objective and constraints are approximated with the trapezoidal integration. The first step converts the region of integration from $[0, \infty)$ to $[0, \pi]$. The second step casts the problem to a finite dimensional convex optimization problem.

D. Long term resource allocation

The problem (35) can be utilized by a BA for long term resource allocation. To do so, the BA can solve the problem (35) for varying system conditions (e.g., number of loads or type of filter F in (32)). This allows the BA to answer questions such as: (i) What is the VES capacity of 50000 HVAC systems, or (ii) How many flexible loads would be required to meet the energy storage needs of the grid?

V. Numerical Examples

Here we consider N homogeneous commercial building HVAC systems. The baseline power consumption for each building is time varying, but the nominal value 40kW. For such an example, $\tilde{\theta}$, γ, and β in (2) represent internal temperature deviation, time constant of temperature dynamics in response to change in power consumption, and a scaled coefficient of performance, respectively. Table I shows the parameters used to model HVAC systems and their QoS.

We present two numerical experiments: (i) solving the problem (35) with for a fixed $N = 3600$, and (ii) a parametric study that varies N over a range. The purpose of (i) is to illustrate the solution process of the problem (35). The purpose of (ii) is to illustrate how much of the grids requirements are met with a varying number of loads.

To aid exposition of the results, we define aggregate capacity index ζ as:

$$\zeta = \frac{\int_0^\infty \bar{S}_{\tilde{P}}(\omega) d\omega}{\int_0^\infty S^L(\omega) d\omega} \times 100\%, \tag{38}$$

so as to show the percentage of capacity required by the BA that can be covered by the loads.

In all scenarios we solve the discrete time finite dimensional version of the convex optimization problem (35) with CVX [24]. All relevant simulation parameters, if not specified otherwise, can be found in Table I.

A. BA's spectral needs

The net load data is collected from Bonneville Power Administration (BPA: www.bpa.gov). The SD of the net demand is determined empirically from time series data; see

TABLE I: Simulation parameters

Par.	Unit	Value	Par.	Unit	Value
c_1	kW	40	γ	1/hour	177.6
c_2	kW	8	β	$^\circ C$/kWh	0.0450
c_3	$^\circ C$	1.11	T	Day	1
c_4	kWh	5	ω^H	1/hour	60
δ	Sec	10	ω^L	1/hour	2
$\{\epsilon_i\}_{i=1}^4$	N/A	0.05			

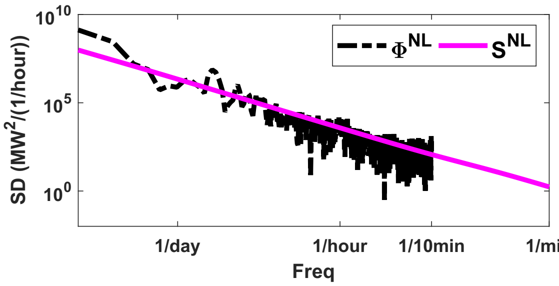

Fig. 2: Empirical net demand SD and modeled SD for BPA.

Section IV-A. We choose an ARMA(2,1) model to fit the empirically estimated SD. Note that because the sampling rate of the data is 5 min, the highest frequency of the empirical SD is $1/10$ minutes. Since we might expect HVAC systems to provide ancillary service at frequencies higher than $1/10$min [19], we extrapolate the net load SD to higher frequencies. The empirical SD (denoted Φ^{ND}) and the modeled SD (denoted S^{ND}) are shown in Figure 2.

B. Meeting the BA's needs with 3600 buildings

The reference SD, denoted S^L, is obtained by "filtering" S^{ND} obtained from the previous subsection. We choose the lower band of this filter as $\omega^L = 1/30$min, so to reflect the large penetration of hydroelectric generation in the pacific northwest; hydro has minimal ramping constraints so that VES from flexible loads is required at higher frequencies.

The SD S^L is then projected onto $\mathcal{S}$ by solving (35) and the resulting SD for the flexible load considered is $\bar{S}_{\tilde{P}}$. All three SDs, S^L, S^{ND}, and $\bar{S}_{\tilde{P}}$ are shown in Figure 3. At $N = 3600$, the buildings are able to meet the requirements of the grid at that frequency band.

C. Parametric Study

To examine the effect of number of loads on the VES capability, we vary N and solve problem (35) for each. Effectively, this is varying the size of the equivalent larger flexible load. We vary N from $N = 1000$ to $N = 4500$, while the rest of the parameters are held fixed. We evaluate the results by computing the aggregate capacity index (38) for each N. The results are shown in Figure 4. As expected, as N increases ζ increases too: if more flexible loads are used, more of the BA's requirements is fulfilled. In particular, a collection of $N = 3600$ commercial building HVAC systems is enough to fulfill the needs of the grid in the range of time scales selected by the filter F.

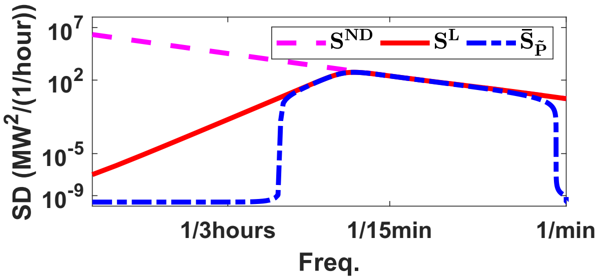

Fig. 3: Net load SD, reference SD, and the load capacity for problem (35) for $N = 3600$.

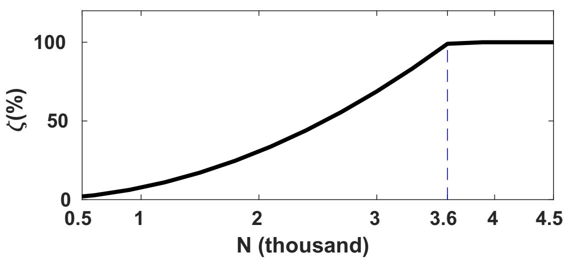

Fig. 4: Capacity index as the number of commercial building HVAC systems is varied.

VI. Conclusion

We present a characterization of capacity for flexible loads that allows a balancing authority to quantify the amount of its energy storage needs that a flexible load can meet. The capacity is characterized as a set of constraints that the load's power spectral density of its demand deviation must satisfy in order maintain the load's quality of service. We then use this capacity characterization to determine the "optimal portion" of a grid's energy storage needs that a collection of homogeneous loads can provide.

In contrast to past works that largely focus on determining if a power demand signal is feasible for a collection of loads, our characterization is in terms of the *statistics* of the power deviation signal. Our method is thus useful for long term planning. In particular, the method allows one to answer question such as "how many flexible HVAC systems does a grid need to meet its requirements", or "what fraction of the grid's needs can be met by 1000 water heaters"?

Extending the method to heterogeneous flexible loads is the next task. Another avenue is to consider additional QoS metrics, especially cycling or lockout constraints for on/off loads.

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