Inverse Optimal Control with Sit-To-Stand Data

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I. Summary

Investigating the underlying optimality criteria used by humans during motion can inform theories for motor control as well as aid in designing robot controllers. Inverse optimal control generates cost functions that are minimized by a set of observed state and control trajectories, allowing for provable claims about optimality. This work applies a recent approach by [1] to experimental Sit-To-Stand (STS) data to obtain individualized cost functions for 11 subjects. When used in optimal control synthesis, the cost functions are found to re-create the observed angular trajectories in a leave-one-out analysis with a maximum normalized error of 7.6% across subjects. The normalized coefficients of these cost functions between subjects are all within a Euclidean distance of 0.25. This result suggests that the study participants exhibit similar underlying optimality criteria during STS.

II. Introduction

A common assumption is that humans and animals behave optimally during locomotion by minimizing some cost, such as the cost of transport [2]. Unfortunately, many claims about underlying optimality criteria involve comparing human data to trajectories produced by hand-picked cost functions, e.g., [3]. Inverse optimal control (IOC) aims to extract a cost function from a set of observed state and input trajectories. IOC has been implemented in various forms for a range of human motions, including manipulation tasks [4], seated posture [5], and walking [6]. These methods either make assumptions about the control strategy being used, do not ensure that the globally optimal cost function is found, or use high-dimensional models which introduce difficulties in real-world implementation.

In this work, we use a formulation developed by [1] applied to the Sit-To-Stand (STS) motion, a task necessary for maintaining independence and quality of life [7]. This semidefinite programming (SDP) method guarantees the recovery of a globally optimal solution. We solve for individualized polynomial cost functions of various degrees using kinematic data from 11 subjects. These cost functions are then used to generate optimal controllers and simulate trials in a leave-one-out context. We compare error between simulated and real data and analyze the cost function coefficients across all subjects.

III. Methods

Motion capture data were collected from 11 subjects as they performed the STS task. For the nominal trials, each subject stood from a comfortable seated position five times. Next, the subject shifted their feet forward, then backward from the nominal position in two-inch increments.

We model the STS motion using an inverted pendulum model (IPM), which represents the angle x_1 and angular velocity x_2 of each subject's center of mass (COM) relative to the pendulum base throughout the STS trajectory. A torque u is applied to the pendulum at its base.

We used a formulation developed by [1], restricting the functions to polynomials with coefficients in $[-1, 1]$. First, we performed a leave-one-out analysis by running IOC with all of a subject's nominal and footshifted trials except one. We found these individualized polynomial cost functions of state and input for degrees 2, 4, and 6. Next, we simulated the left-out trajectory using these cost functions, and calculated the root mean square error E between the simulated angle ($\hat{x}_1$) and actual angle (x_1). The error values were normalized by the total range of x_1-values. For each degree of cost function, we averaged the leave-one-out E across all subjects and trials. Then, we ran IOC for all of a subject's trials to create one cost function per subject and degree. Treating the set of coefficients as vectors, we normalized each to have a unit norm and used a Euclidean distance metric to measure the distance d between subjects' sets of coefficients.

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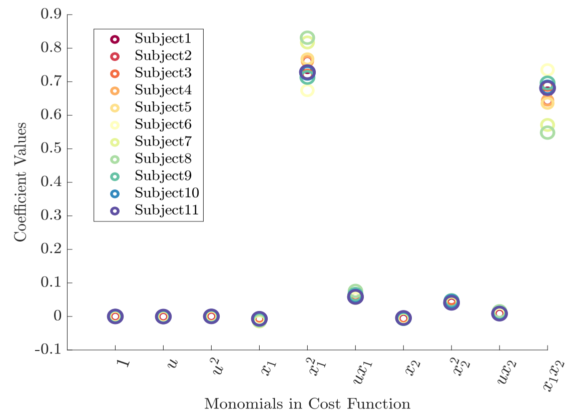

Fig. 1: Weights on all monomials of the best fitting cost function for each subject, normalized to have unit norm, shown here for degree 2.

	deg2	deg4	deg6
E ($\mu \pm \sigma$)	(7.6 ± 1.8)%	(5.6 ± 1.3)%	(5.8 ± 1.4)%
max. d	0.25	0.13	0.18

TABLE I: For cost-function degrees 2, 4, and 6: Normalized leave-one-out root mean square error, averaged across trials of all subjects, and maximum Euclidean distance between any two subjects' set of coefficients with unit norm.

IV. Results and Discussion

Fig. 1 shows the coefficients of each subject's degree-2 cost function plotted for each monomial of u, x_1, and x_2. Table I reports the average leave-one-out E-values across subjects and trials as well as the maximum d between any two subjects' sets of coefficients. For comparison, the maximum possible d between any two unit norm vectors is 2. For all three degrees of cost function, the coefficients are strikingly similar across subjects, suggesting the study participants had similar optimiality criteria during STS. Future work will aim to connect these polynomial cost functions to physical meanings as well as develop strategies for incorporating these cost functions into high-level controllers for wearable robotics.

References

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