Non-Hermitian Dynamics Mimicked by a Fully Parametric Hybrid Nonlinear Optical System

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ABSTRACT: We demonstrate that hybridization of OPA and idler SHG produces desirable nonlinear dynamics equivalent to OPA with idler loss, despite being a fully conservative system. Both systems take the form of a damped Duffing oscillator.

Many emerging technologies are enhanced by the introduction of loss. Changing a Hermitian system to a non-Hermitian one can usefully alter its dynamics, for example, by introducing an attractive and static steady state to a system that normally exhibits only oscillatory behavior. This paradigm was recently demonstrated in the context of optical parametric amplifiers. Referred to as quasi-parametric amplification (QPA) [1], or dissipative optical parametric amplification [2], loss is introduced at the idler wavelength to inhibit the recombination of signal and idler photons to the pump, resulting in a unidirectional flow of energy from pump to signal.

An inherent disadvantage of a dissipative system, however, is that useable energy is lost. Further, the removed energy is often dissipated as heat, which sets a limitation on the scalability of non-Hermitian devices. Here we show that a fully conservative nonlinear system consisting of coupled nonlinear processes can mimic the dynamical behavior of a dissipative nonlinear system. Idler mediated second harmonic amplification (SHA), a hybridization of second harmonic generation (SHG) and optical parametric amplification (OPA), mimics the unidirectional energy flow dynamics of QPA by displacing idler photons to their second harmonic field. This novel approach is fully parametric and provides a route to efficient amplification of a signal field while retaining the energy displaced from the idler in a useable optical field. We rigorously demonstrate the dynamical equivalency of the Hermitian and non-Hermitian systems by showing that SHA and QPA are each governed by the damped Duffing oscillator equation with parameters dependent on the method of idler displacement. We further show that the Duffing oscillator model unites the descriptions of QPA and SHA with conventional OPA.

The non-Hermitian process of QPA is described by three coupled wave equations:

$$dA_s/dz = i\kappa_s A_p A_i^* e^{i\Delta k_{OPA} z} \; (1),$$

$$dA_p/dz = i\kappa_p A_s A_i e^{-i\Delta k_{OPA} z} \; (2),$$

$$dA_i/dz = i\kappa_{i,OPA} A_p A_s^* e^{i\Delta k_{OPA} z} - \alpha A_i \; (3a),$$

where κ_j, and A_j are the electric field coupling constants, and amplitudes, respectively, for signal (s), pump (p), and idler (i). These are the usual equations for OPA with an additional idler loss term. When the loss coefficient α is set to zero, the equations model conventional OPA. The OPA wave-vector mismatch is given by $\Delta k_{OPA} = k_p - k_s - k_i$.

To describe the fully conservative, hybridized nonlinear system of SHA, we replace eq. 3a for the idler with two equations that include the coupling of idler both to the signal and pump and to its second harmonic (SH):

$$dA_i/dz = i\kappa_{i,OPA} A_p A_s^* e^{i\Delta k_{OPA} z} + i\kappa_{i,SHG} A_{2i} A_i^* e^{i\Delta k_{SHG} z} \; (3b),$$

$$dA_{2i}/dz = i\kappa_{2i} A_i^2 e^{-i\Delta k_{SHG} z} \; (4),$$

where A_{2i} is the idler SH field amplitude. The SHG wave-vector mismatch is given by $\Delta k_{SHG} = k_{2i} - 2k_i$. Nondimensionalizing the equations for QPA (1, 2, 3a) and SHA (1, 2, 3b, 4) in terms of photon flux and using the Manley-Rowe relations, we can derive an equation for the pump field for each case:

$$d^2u_p/d\zeta^2 = -\left(1 + \left|\mathrm{u_{p,0}}\right|^2 - \mathrm{n_d}(\zeta)\right) u_p + 2u_p^3 - \gamma(\zeta)\, du_p/d\zeta \,, \qquad (5)$$

which describes a damped Duffing oscillator with fractional photon amplitude $u_j = \sqrt{2n_j\epsilon_0 c/\hbar\omega_j F_0}\, A_j$ and nondimensional propagation coordinate $\zeta = \Gamma_{OPA} z$, where $\Gamma_{OPA} = \left(\hbar\omega_p\omega_s\omega_i d_{eff}^2 F_0/2n_p n_s n_i \epsilon_0 c^3\right)^{1/2}$ is the nonlinear coupling strength and F_0 is the total initial photon flux. The parameters $n_d(\zeta)$ and $\gamma(\zeta)$ represent the fraction of photons displaced from the idler field and a damping coefficient, respectively, and they satisfy the relationship $dn_d(\zeta)/d\zeta = 2\gamma(\zeta)|u_i|^2$. These parameters are tabulated in Fig. 1a for OPA, QPA and SHA.

Numerical integration of the coupled wave equations for $\Delta k_{OPA} = 0$, $\alpha = 0$, and $|\Delta k_{SHG}| \gg 0$ shows the dynamics of an ordinary OPA (Fig. 1b): an oscillatory exchange of photons between pump, signal, and idler fields. We note that in this case the equation for the pump photon flux, eq. 5, becomes an undamped Duffing oscillator, which generates the well known Jacobi elliptic function solutions of OPA. Fig. 1c depicts what happens in QPA, i.e., when a loss is introduced, such that $\alpha > 0$ and $|\Delta k_{SHG}| \gg 0$. As the idler field grows, it loses energy to the medium, stifling the flow of energy from the signal and idler back into the pump. Eq. 5 has become the equation of motion of a damped oscillator.

In the same fashion, in SHA (setting $\Delta k_{SHG} = 0$ and $\alpha = 0$) (Fig. 1d), idler photons are displaced to the idler SH field as the idler field strength grows, and due to the unidirectional flow of energy for perfectly phase-matched SHG, energy cannot return to the idler field, thus suppressing back-conversion to the pump. Moreover, in SHA, the solutions to eq. 5 converge to a steady state with full photon conversion from the pump to signal field under all ordinary initial conditions of an OPA (i.e., $|u_p| > |u_s|, |u_i| = |u_{2i}| = 0$) and independent of the initial value $|u_p|$. Even though the damping in SHA depends on the monotonically increasing idler SH field, the solutions exhibit dynamics typical of a damped oscillator with underdamping occurring for $\gamma(\infty) < 1$, critical damping for $\gamma(\infty) = 1$, and overdamping for $\gamma(\infty) > 1$ (Fig. 1e-g) where $\gamma(\infty) = \frac{\Gamma_{SHG}}{2\Gamma_{OPA}}|u_{p,0}|^2$.

While the irreversibility of the non-Hermitian QPA system is due to the coupling of the idler field to a thermodynamic heat bath, the Hermitian SHA system gains its irreversibility from the unidirectional flow of power from fundamental to SH fields in a perfectly phase-matched SHG process. The Hermitian nature of SHA becomes clear when either Δk_{SHG} or $\Delta k_{OPA} \neq 0$. In this case (not shown), there is an eventual reversal of the dynamics. This surprising outcome that non-Hermitian dynamics can be mimicked by a fully parametric system solves the problem of lost useable energy without sacrificing the desired unidirectional flow of photons from pump to signal. This can be used to overcome the spatiotemporally non-uniform amplification in OPA, and is a step toward beating the quantum efficiency limit. An analysis of high efficiency SHA devices and of the possibility of simultaneous phase matching of OPA and SHG (which we find to be surprisingly flexible for ultrafast applications in common OPA crystals) will be presented elsewhere.

More generally, the discovery of dissipative-system-like dynamics enabled by coupled hybridized parametric systems offers a qualitatively new behavioral space for nonlinear photonics applications. It also motivates further investigation of hybridized nonlinear optical systems in order to look for similar or other qualitatively new dynamical behaviors in conservative systems. We believe an effort to tailor nonlinear optical evolution dynamics in this fashion could be important to the advancement of optical technologies.

(a)

Nonlinear Process	$n_d(\zeta)$	$\gamma(\zeta)$
OPA	0	0
QPA	$\frac{2\alpha}{\Gamma_{OPA}}\int_0^{\zeta}|u_i|^2 d\zeta'$	$\frac{\alpha}{\Gamma_{OPA}}$
SHA	$2|u_{2i}|^2$	$\frac{\Gamma_{SHG}}{\Gamma_{OPA}}|u_{2i}|$

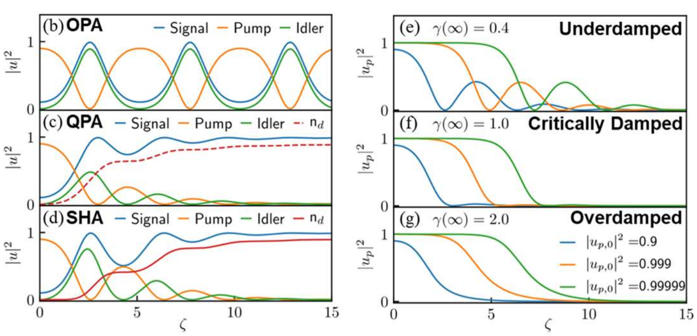

Fig. 1: (a) Table of Duffing oscillator photon displacement n_d and damping γ parameters. (b) OPA dynamics of fractional photon flux showing conversion-back-conversion cycles between the pump field and the signal and idler fields with the pump having the form of an undamped Duffing oscillator (with Jacob elliptic function solution). (c) QPA dynamics showing a unidirectional flow of pump energy to the signal field and an increasing number of photons lost from the idler field (red dashed). (d) SHA dynamics showing a similar unidirectional flow of energy from pump to signal with idler energy being displaced to its SH field. For both (c) and (d), $\gamma(\zeta) = 0.7$ as $\zeta \to \infty$. Both QPA and SHA have fractional pump photon flux evolution dynamics in the form of a damped Duffing oscillator. Depending on $\gamma(\zeta)$, (e) underdamping, (f) critical damping, and (g) overdamping dynamics can be observed, as shown for the fractional pump photon flux in SHA for the cases of 10, 30, and 50 dB gain ($|u_{p,0}|^2 =$ 0.9, 0.999, and 0.99999, respectively).

REFERENCES

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