A Novel Framework for Capacity Analysis of Diffusion-Based Molecular Communication Incorporating Chemical Reactions

Ching-Lun Tai, *Graduate Student Member, IEEE*, and Ian F. Akyildiz, *Fellow, IEEE*

***Abstract*—In recent years, molecular communication (MC) is considered as a transformative paradigm in the communication theory and a promising solution to future nanoscale communication networks. In this article, a novel framework is introduced for diffusion-based MC and is shown how its capacity is impacted by the effects of chemical reactions which were neglected in the existing literature. Particularly, the chemical reactions corresponding to complex balanced chemical reaction networks are studied in this work. With an information-theoretic approach, the capacity is introduced where the effects of chemical reactions are taken into account. Then, the individual entropy derivations are addressed where the chemical reactions at the transmitter are considered with the chemical reaction network theory. Finally, the mutual information is derived based on these entropy derivations and the analytical capacity expressions are introduced accordingly. Numerical results exhibit the interactions between different parameters and show that the capacity actually decreases when the effects of chemical reactions are considered, implying that the capacity derived without chemical reactions was overestimated in previous studies. Consequently, the proposed framework analyzes the fundamental limits of diffusion-based MC and provides a more realistic capacity derivation comprising limitations imposed by chemical reactions, hence applicable to various more realistic MC scenarios.**

***Index Terms*—Diffusion-based molecular communication (MC), chemical reactions, capacity analysis, chemical reaction network theory.**

I. INTRODUCTION

MOLECULAR communication (MC) is an emerging communication paradigm where information is encoded into properties of substances at the molecular level and exchanged through chemical reactions and molecular transport. This paradigm is originated from the observation of nature, where the basic units of life, i.e., biological cells, generate, store, and communicate information through molecular processes in order to coordinate their collective behavior. The aim of MC theory is to quantify the flow of information in biochemical domain by modeling and engineering MC systems and devices which are envisioned to enable a plethora applications in environments challenging for classical communications, such as inside the human body [1].

Due to its tight integration within the biological environment and its feasibility at the cellular scale (nm - μm), MC is proposed as a possible tool for establishing future nanonetworks that can interact with living organisms and their biological processes. Therefore, it paves the way for a wide range of applications, mostly biomedical, but also industrial and environmental. Intelligent intrabody systems for health monitoring, biosensor networks, smart drug delivery systems, industrial monitoring and control of microbial formations for water and air filtering, organic waste disposal, and biological and chemical nanosensor networks for surveillance against biological and chemical attacks are among the potential practical applications of MC-enabled nanonetworks.

In the past decade of MC theory research, initial studies mostly driven by communication and networking engineers, are focused on identifying elements of biochemical communication processes and abstracting them into theoretical models to construct the building blocks of MC systems [2]. Numerous channel models have been proposed based on a wide range of molecule propagation processes ranging from passive diffusion of molecules [3], [4], to the advection of molecules by fluid flow [5]–[7], and to active processes where molecules are transported by other carriers such as molecular motors [8]. Further studies include design of MC devices and their parts such as transmitter/receiver [9] and studies addressing fundamental capacity limits of the aforementioned channels.

To understand the fundamental limits and the performance of an MC system, it is crucial to investigate it from an information-theoretic perspective, model and analyze the theoretical communication capacity of this system. Previous contributions in MC literature have tackled the study of capacity of some of the aforementioned channels for specific modulation and coding schemes such as time-slotted on-off keying (OOK) [10], [11] and continuous transmission [12], [13] for diffusion channels encoding information on concentration of molecules; time slotted [14] and continuous transmission [15] for channels encoding information on the timing of molecules. Other contributions addressed the problem for specific MC systems such as cardiovascular systems [5], microfluidic systems [6], molecular circuits [16], ligand-receptor binding [17], and enzymatic reactions [17], [18]. Although these

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The authors are with the School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, GA 30332 USA (e-mail: ctai32@gatech.edu; ian@ece.gatech.edu).

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studies contribute to the understanding of channel capacity for some MC scenarios, they are far from constituting a general mathematical framework that can describe the unique features of MC. Recently, [2] has addressed this research gap by providing an information-theoretic framework stemming from statistical mechanics to determine or estimate (in intractable cases) the capacity of MC channels. Starting from the Langevin stochastic differential equation (SDE) [19] describing the motion of particles, a methodology to determine or estimate the capacity of any MC system by matching the transport processes to the terms of the Langevin SDE is proposed. However, this framework only describes the propagation of molecules in the medium but falls short in depicting the peculiarities of the transmission and reception processes often occurring as chemical reactions.

Our work aims to extend the framework of [2] by adding the transmission and reception processes into the picture and provide a more complete perspective for MC capacity formulation to reveal more accurate expressions/estimations of fundamental limits of the capacity in MC.

In this article, we consider the effects of chemical reactions which correspond to complex balanced chemical reaction networks [20], [21] on the capacity of diffusion-based MC. Examples of biochemical reactions that can be modelled as complex balanced chemical reaction networks include T-cell receptor signal transduction [22] and specific classes of enzyme-driven reactions [23], [24].

The contributions of this article can be summarized as:

- We consider a diffusion-based MC model, where we propose a framework to provide stricter capacity expressions with the effects of chemical reactions at the transmitter taken into account.
- Based on the proposed framework, we adopt the chemical reaction network theory to obtain the individual entropy derivations, mutual information, and resulting capacity considering the chemical reactions at the transmitter with an information-theoretic approach.
- We perform numerical evaluations to assess the effects of chemical reactions on the resulting capacity and demonstrate the interactions between different parameters of capacity expressions. It is shown that the capacity is significantly decreased when the effects of chemical reactions are considered.

The rest of this article is organized as follows. In Section II, we review the system model for diffusion-based MC and the chemical reaction network theory. In Section III, we present a new framework to analyze the capacity of diffusion-based MC systems incorporating the effects of chemical reactions at the transmitter. In Section IV, we numerically evaluate the capacity for two different cases, namely, the capacity calculated considering only the propagation of molecules and the capacity calculated considering the chemical reactions for various parameters. Finally, in Section V, we conclude by discussing the applications of the proposed framework to different MC techniques, as well as the future research opportunities to extend this framework for a more realistic capacity bound.

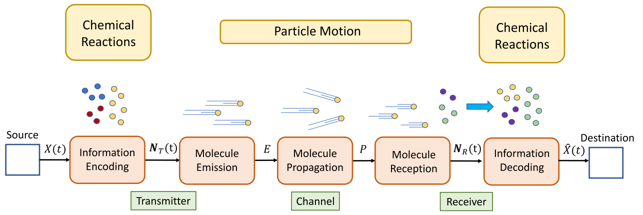

Fig. 1. Diffusion-Based MC system.

II. Preliminaries

In this section, we review several concepts that are related to this study, including the diffusion-based MC and chemical reaction network theory.

A. Diffusion-Based MC

A typical diffusion-based MC system is illustrated as in Fig. 1, consisting of the classical cascade of the source, transmitter, channel, receiver, and destination [25]. The propagation of molecules in the channel is governed by diffusion where they do not undergo chemical reactions. In addition, there are chemical reactions occurring at both the transmitter and receiver.

The source information $X(t)$ generated at time t goes through the processes of information encoding and molecule emission at the transmitter, where we consider there are S types of chemical substances involved in M different chemical reactions. During the information encoding, $X(t)$ is modulated through the M chemical reactions into the transmitter molecule number vector

$$\mathbf{N}_T(t) = [N_{T1}(t)\ N_{T2}(t)\ \cdots\ N_{TS}(t)]^T \in \mathbb{Z}^S_{\geq 0}, \quad (1)$$

where $N_{Ti}(t), i = 1, 2, \ldots, S$ is the number of molecules for the ith type of chemical substances at the transmitter at time t and $\mathbb{Z}^S_{\geq 0}$ is the set containing all S-dim vectors with non-negative integer entries. During the molecule emission, consider the case where a specific type of chemical substance among the S types is selected as the information carrier and subsequently emitted. Note that the characterization of intersymbol interference is beyond the scope of this study and is to be studied in future works. Associated with $\mathbf{N}_T(t)$, the transmitted signal (emitted molecules) can be expressed as [2]

$$E = \{t_n, \mathbf{l}_n(t_n)\}_n, \quad (2)$$

where n is the index of an individual emitted molecule, t_n is the time instance when the molecule indexed as n is emitted, and $\mathbf{l}_n(t_n)$ is the location of the molecule indexed as n when emitted at time t_n.

Next, the molecules emitted pass through the channel, where the particle motion can be analytically expressed with the Langevin SDE, which can be written as [19]

$$m\frac{d^2(\mathbf{l}_n(t) - \mathbf{v}_n(t)t)}{dt^2} = \mathbf{F}_n(t) - 6\pi\mu r\frac{d(\mathbf{l}_n(t) - \mathbf{v}_n(t)t)}{dt} + \mathbf{f}(t), \quad (3)$$

where m and r are the mass and radius of a single emitted molecule (which is assumed to be spherical), $\mathbf{v}_n(t)$ and

$\mathbf{F}_n(t)$ are the drift velocity and external force observed by the molecule indexed as n, respectively, μ is the viscosity of the fluid in the channel (where the fluid is assumed to be homogeneous and at infinite dilution within the propagation space), $f(t)$ is the Brownian motion force whose correlation function can be expressed as

$$\left\langle f_i(t) f_j\left(t'\right)\right\rangle = 12\pi\mu r k_B T \delta_{ij}\delta\left(t-t'\right), \tag{4}$$

where $f_i(t)$ is the component of $\mathbf{f}(t)$ in the ith dimension, $\langle\cdot\rangle$ is the average operator, k_B is the Boltzmann's constant, T is the absolute temperature, $\delta(\cdot)$ is the Dirac delta function, and

$$\delta_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}. \tag{5}$$

After passing through the channel, the emitted molecules arrive at the receiver. The received signal (received molecules) can be expressed as [2]

$$P = \{t_{n'}, \mathbf{l}_{n'}(t_{n'})\}_{n'}, \tag{6}$$

where n' is the index of an individual received molecule, $t_{n'}$ is the time instance when the molecule indexed as n' is received, and $\mathbf{l}_{n'}(t_{n'})$ is the location of the molecule indexed as n' when received at time $t_{n'}$. Then, the received signal P goes through the processes of molecule reception and information decoding at the receiver, where we suppose there are S' types of chemical substances (including the received molecules) involved in M' different chemical reactions. During the molecule reception, the received molecules in P interact with other molecules existing at the receiver through chemical reactions, which lead to the corresponding receiver molecule number vector

$$\mathbf{N}_R(t) = [N_{R1}(t)\ N_{R2}(t)\ \cdots\ N_{RS'}(t)]^T \in \mathbb{Z}_{\geq 0}^{S'}, \tag{7}$$

where $N_{Ri}(t), i = 1, 2, \ldots, S'$ is the number of molecules for the ith type of chemical substances at the receiver at time t. During the information decoding, $\mathbf{N}_R(t)$ is demodulated into the reconstructed information $\hat{X}(t)$, which is then passed to the destination.

Therefore, a diffusion-based MC system and its corresponding capacity depend on the particle motion in the channel and the chemical reactions at both the transmitter and receiver. The particle motion, which involves the behavior of information-carrying molecules in the channel, can be analyzed with the Langevin SDE as expressed in (3), and its effects on the capacity has been well studied in the literature. The chemical reactions, which consist of the biochemical interactions involved in the emission and reception of information-carrying molecules at both the transmitter and receiver, constitute an essential part of molecule transfer, but their effects on the capacity are still not clear.

B. Chemical Reaction Network Theory

In order to analyze the effects of chemical reactions on diffusion-based MC, we introduce the chemical reaction network theory [26], which investigates the evolution of the number of molecules for each chemical substance involved in various chemical reactions through time. Note that the number of chemical substances is associated with the information encoding and decoding at the transmitter and receiver, respectively.

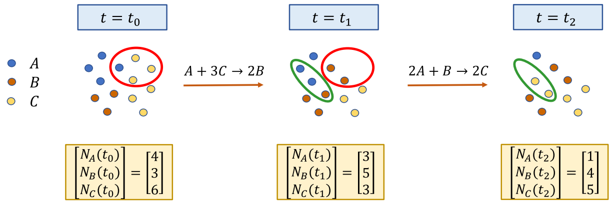

Fig. 2. An illustration of how the molecule number vector evolves with time. The red and green circles exhibit the changes in the number of molecules before and after the first state transition which occurs during (t_0, t_1) and the second state transition which occurs during (t_1, t_2), respectively.

Suppose there are $\bar{S}$ chemical substances $s_1, s_2, \ldots, s_{\bar{S}}$ involved in $\bar{M}$ different chemical reactions, where the forward and reverse reactions of a reversible chemical reaction are considered as two distinct chemical reactions. For the jth chemical reaction, we denote $\mathbf{v}_j = [v_{j1}\ v_{j2}\ \cdots\ v_{j\bar{S}}]^T$ and $\mathbf{v}'_j = [v'_{j1}\ v'_{j2}\ \cdots\ v'_{j\bar{S}}]^T$ as the coefficient vectors (also called *complexes*) of reactants and products, respectively, where v_{ji} and v'_{ji} are the coefficients of the ith type of chemical substance s_i as a reactant and a product, respectively. Assume that $\mathbf{v}_j \neq \mathbf{v}'_j, j = 1, 2, \ldots, \bar{M}$. Then, the jth chemical reaction can be expressed as

$$\sum_{i=1}^{\bar{S}} v_{ji} s_i \to \sum_{i=1}^{\bar{S}} v'_{ji} s_i, j = 1, 2, \ldots, \bar{M}. \tag{8}$$

Subsequently, the corresponding chemical reaction network of the above chemical reactions can be defined by the following definition.

Definition 1 (Chemical Reaction Network [20], [21]): Suppose there are $\bar{S}$ chemical substances $s_1, s_2, \ldots, s_{\bar{S}}$ involved in $\bar{M}$ different chemical reactions, with $\mathbf{v}_j$ and $\mathbf{v}'_j$ representing the complexes of reactants and products, respectively, in the jth chemical reaction. Denote $\mathcal{S} = \{s_i\}_{i=1}^{\bar{S}}$, $\mathcal{C} = \{\mathbf{v}_j\}_{j=1}^{\bar{M}}$, and $\mathcal{R} = \{\mathbf{v}_j \to \mathbf{v}'_j\}_{j=1}^{\bar{M}}$ as the sets of chemical substances, reactant complexes, and chemical reactions, respectively. Then, the triple $\{\mathcal{S}, \mathcal{C}, \mathcal{R}\}$ is the corresponding chemical reaction network.

Over the chemical reaction network as stated above, we denote the molecule number vector at time t as

$$\mathbf{N}(t) = \left[N_1(t)\ N_2(t)\ \cdots\ N_{\bar{S}}(t)\right]^T \in \mathbb{Z}_{\geq 0}^{\bar{S}}, \tag{9}$$

where $N_i(t), i = 1, 2, \ldots, \bar{S}$ is the number of molecules for the ith type of chemical substances at time t. Note that each distinct molecule number vector at a specific time instance represents a state and that all states compose a state space.

An illustration of how the molecule number vector evolves with time is shown in Fig. 2. Suppose there is a chemical reaction network $\{\mathcal{S}, \mathcal{C}, \mathcal{R}\}$, where $\mathcal{S} = \{A, B, C\}$, $\mathcal{C} = \{[1,0,3]^T, [0,2,0]^T, [2,1,0]^T, [0,0,2]^T\}$, and $\mathcal{R} = \{[1,0,3]^T \to [0,2,0]^T, [0,2,0]^T \to [1,0,3]^T, [2,1,0]^T \to [0,0,2]^T, [0,0,2]^T \to [2,1,0]^T\}$, which implies the chemical reactions of $A + 3C \rightleftharpoons 2B$ and $2A + B \rightleftharpoons 2C$. Assume that only the chemical reaction $A + 3C \to 2B$ happens between the two time instances t_0 and t_1, resulting in a state transition where one *A* molecule and three *C* molecules are converted into two *B* molecules. Consequently, the state at t_0 is transitioned to that at t_1. Now, assume that only the chemical reaction $2A + B \rightarrow 2C$ happens between the two time instances t_1 and t_2. Then, another state transition occurs in the same manner. Note that a chain of chemical reactions may occur in multiple ways with the same combination of molecules (e.g., when uni-directed chemical reactions or chemical reactions which can be triggered only in certain environments are involved).

In the chemical reaction network theory, there are two modeling approaches that are commonly used to describe the evolution of the number of molecules of chemical substances over the chemical reaction network through time, i.e., stochastic [27]–[30] and deterministic [31]–[33] approaches.

With the stochastic model, the chemical reaction network is formulated as a continuous time Markov chain where the molecule number vector at time *t* is a random process about *t* and can be expressed as [20], [21]

$$\mathbf{N}(t) = \mathbf{N}(0) + \sum_{j=1}^{\bar{M}} \left(\mathbf{v}'_j - \mathbf{v}_j\right) Y_j\left(\int_0^t \lambda_j(\mathbf{N}(\tau))\, d\tau\right), \quad (10)$$

where $Y_j(\int_0^t \lambda_j(\mathbf{N}(\tau))d\tau)$ is an independent unit-rate Poisson process [34], [35] representing the number of occurrences of the *j*th chemical reaction with the parameter being the integration from 0 to *t* of the propensity function $\lambda_j(N(\tau))$ expressed as

$$\lambda_j(\mathbf{N}(\tau)) = \kappa_j \prod_{i=1}^{\bar{S}} \frac{N_i(\tau)!}{\left(N_i(\tau) - v_{ji}\right)!} \mathbf{1}_{\{N_i(\tau) \geq v_{ji}\}}, \quad (11)$$

where κ_j is some constant associated with the *j*th chemical reaction and

$$\mathbf{1}_{\{\text{condition}\}} = \begin{cases} 1, & \text{condition} = \text{True} \\ 0, & \text{condition} = \text{False} \end{cases} \quad (12)$$

is the indicator.

With the deterministic model, the molecule number vector at time *t* is approximated as a deterministic function of *t* and can be expressed as

$$\begin{aligned}\hat{\mathbf{N}}(t) &= \hat{\mathbf{N}}(0) + \int_0^t \sum_{j=1}^{\bar{M}} r_j\left(\hat{\mathbf{N}}(\tau)\right) d\tau \\ &= \left[\hat{N}_1(t)\ \hat{N}_2(t)\ \cdots\ \hat{N}_{\bar{S}}(t)\right]^T \in \mathbb{R}_{\geq 0}^{\bar{S}}, \quad (13)\end{aligned}$$

where

$$r_j\left(\hat{\mathbf{N}}(\tau)\right) = \kappa_j \prod_{i=1}^{\bar{S}} \hat{N}_i(\tau)^{v_{ji}} \quad (14)$$

is the state-dependent rate of the *j*th chemical reaction and $\mathbb{R}_{\geq 0}^{\bar{S}}$ is the set containing all $\bar{S}$-dim vectors with non-negative entries. For notational brevity, we omit the parameter in the molecule number vector hereafter.

In the following definition, a complex balanced chemical reaction network is defined.

Definition 2 (Complex Balanced Chemical Reaction Network [20], [21]): A chemical reaction network $\{\mathcal{S}, \mathcal{C}, \mathcal{R}\}$ is called complex balanced if there exists an equilibrium value $\mathbf{c} = [c_1\ c_2\ \cdots\ c_{\bar{S}}]^T \in \mathbb{R}_{\geq 0}^{\bar{S}}$ such that for each $\eta \in \mathbb{Z}_{\geq 0}^{\bar{S}}$,

$$\sum_{j:\mathbf{v}_j = \eta} r_j(\mathbf{c}) = \sum_{j:\mathbf{v}'_j = \eta} r_j(\mathbf{c}), \quad (15)$$

where $r_j(\cdot)$ is the state-dependent rate of the deterministic model, as defined in (14).

Suppose there is a complex balanced chemical reaction network $\{\mathcal{S}, \mathcal{C}, \mathcal{R}\}$ with an equilibrium value $\mathbf{c} \in \mathbb{R}_{\geq 0}^{\bar{S}}$, which satisfies (15) relating to the state-dependent rate of the deterministic model. Then, it is derived in [20] that there exists a stationary distribution of the molecule number vector **N** in the stochastic model, as defined in (10), of this network, expressed as [20]

$$\Pr(\mathbf{N} = \mathbf{x}) = \prod_{i=1}^{\bar{S}} \frac{{c_i}^{[\mathbf{x}]_i}}{[\mathbf{x}]_i!} e^{-c_i},\ \mathbf{x} \in \mathbb{Z}_{\geq 0}^{\bar{S}}, \quad (16)$$

which is a product of $\bar{S}$ Poisson distributions, where the parameter of the *i*th Poisson distribution is c_i.

Note that there may be multiple potential equilibrium values, which correspond to different steady states (resulting from different initial conditions) from a physical perspective, in a single complex balanced chemical reaction network.

With (16), the stationary distribution of the molecule number vector can be obtained, and therefore the effects of chemical reactions on diffusion-based MC can be further analyzed.

III. FRAMEWORK FOR CAPACITY OF DIFFUSION-BASED MOLECULAR COMMUNICATION INCORPORATING CHEMICAL REACTIONS

In the existing literature, the characteristics of particle motion through the channel in diffusion-based MC have been well studied. Particularly, the mutual information between *E* and *P*, as defined in (2) and (6), respectively, and the corresponding channel capacity have been derived. However, the effects of chemical reactions are not fully considered in the capacity analysis for diffusion-based MC in previous studies, which lead to an overestimated capacity.

In this section, we derive the capacity for diffusion-based MC with the effects of chemical reactions considered, providing a more realistic capacity analysis. Note that we suppose the chemical reactions occur at both the transmitter and receiver, and no chemical reactions occur in the channel, i.e., the molecules do not undergo chemical reactions during propagation in the channel.

A. Information-Theoretic Formulation

To begin with, note that the effects of chemical reactions reflect on the molecule number vectors $\mathbf{N}_T$ and $\mathbf{N}_R$, as defined in (1) and (7), at both the transmitter and receiver, respectively.

In order to take the effects of chemical reactions at both the transmitter and receiver into consideration, the capacity of diffusion-based MC in this study is defined as [36]

$$C = \max_{f_{\mathbf{N}_T}(\mathbf{n}_T)} I(\mathbf{N}_T; \mathbf{N}_R), \quad (17)$$

where $I(\mathbf{N}_T;\mathbf{N}_R)$ is the mutual information between $\mathbf{N}_T$ and $\mathbf{N}_R$. Note that (17) is a simplified formulation neglecting the channel memory, which is to be integrated in future works.

To better distinguish the effects of chemical reactions at the transmitter and receiver individually, we focus on the chemical reactions at the transmitter in this study, while the chemical reactions at the receiver are to be studied in future works. Therefore, we simplify $\mathbf{N}_R$ as a deterministic function of the received signal *P*.

First, note that the conditional entropy of a deterministic function of a process given that process is zero. Therefore,

$$H(\mathbf{N}_R|P) = 0. \tag{18}$$

In order to compute the capacity *C* defined in (17), we need to address the mutual information term $I(\mathbf{N}_T;\mathbf{N}_R)$, which can be expressed as

$$\begin{aligned} I(\mathbf{N}_T;\mathbf{N}_R) &= H(\mathbf{N}_T) - H(\mathbf{N}_T|\mathbf{N}_R) \\ &= H(\mathbf{N}_T) - H(\mathbf{N}_T|\mathbf{N}_R) + H(\mathbf{N}_R) - H(\mathbf{N}_R) \\ &\quad - H(\mathbf{N}_R|P), \end{aligned} \tag{19}$$

where the second equality holds due to (18).

According to the definition of mutual information, the third and fifth terms in (19) can be rewritten as

$$H(\mathbf{N}_R) - H(\mathbf{N}_R|P) = I(\mathbf{N}_R;P) = H(P) - H(P|\mathbf{N}_R). \tag{20}$$

Note that the sum of the two conditional entropy derivations, $H(\mathbf{N}_T|\mathbf{N}_R)$ and $H(P|\mathbf{N}_R)$, is equal to the joint conditional entropy $H(\mathbf{N}_T,P|\mathbf{N}_R)$, since $\mathbf{N}_T$ and *P* are conditionally independent given $\mathbf{N}_R$. Hence,

$$H(\mathbf{N}_T|\mathbf{N}_R) + H(P|\mathbf{N}_R) = H(\mathbf{N}_T,P|\mathbf{N}_R). \tag{21}$$

With (20) and (21), the mutual information term $I(\mathbf{N}_T;\mathbf{N}_R)$ can be rewritten as

$$\begin{aligned} I(\mathbf{N}_T;\mathbf{N}_R) &= H(\mathbf{N}_T) + H(P) - H(\mathbf{N}_T,P|\mathbf{N}_R) - H(\mathbf{N}_R) \\ &= H(\mathbf{N}_T) + H(P) - H(\mathbf{N}_T,P,\mathbf{N}_R) \\ &= H(\mathbf{N}_T) + H(P) - H(\mathbf{N}_T,P) \\ &= I(\mathbf{N}_T;P) \\ &= H(\mathbf{N}_T) - H(\mathbf{N}_T|P), \end{aligned} \tag{22}$$

where the third equality holds given that $\mathbf{N}_R$ is a deterministic function of *P*.

Note that the mutual information term $I(\mathbf{N}_T;\mathbf{N}_R)$ is therefore equal to the difference between the two entropy derivations, $H(\mathbf{N}_T)$ and $H(\mathbf{N}_T|P)$. Next, we will address the two terms, $H(\mathbf{N}_T)$ and $H(\mathbf{N}_T|P)$, respectively.

B. Entropy of Molecule Number Vector at Transmitter

As expressed in (22), the first term to be addressed is $H(\mathbf{N}_T)$, which is the entropy of the molecule number vector at the transmitter $\mathbf{N}_T$.

Suppose there are *S* types of chemical substances involved in *M* different chemical reactions, as described in Section II-A, at the transmitter, where the *M* chemical reactions correspond to a complex balanced chemical reaction network, as defined in Definition 2, with an equilibrium value $\mathbf{c} \in \mathbb{R}^S_{\geq 0}$, which is derived from the state-dependent rate of the deterministic model. It is assumed that the supply of molecules (after molecule emission) is sufficiently fast so that the equilibrium value can be considered as unchanged in a large time scale.

Now, we would like to obtain the stationary distribution of $\mathbf{N}_T$ in the stochastic model of this complex balanced chemical reaction network. According to (16), this can be expressed as

$$\Pr(\mathbf{N}_T = \mathbf{x}) = \prod_{i=1}^{S} \frac{{c_i}^{[\mathbf{x}]_i}}{[\mathbf{x}]_i!} e^{-c_i}, \mathbf{x} \in \mathbb{Z}^S_{\geq 0}, \tag{23}$$

which is a product of *S* Poisson distributions, where the *i*th distribution can be expressed as

$$Z_i \sim \text{Pois}(c_i), i = 1, 2, \ldots, S. \tag{24}$$

With (23) and (24), the entropy of $\mathbf{N}_T$ can be approximated as

$$H(\mathbf{N}_T) \approx \sum_{i=1}^{S} H(Z_i), \tag{25}$$

which is the sum of the *S* entropy derivations corresponding to the *S* Poisson distributions.

C. Conditional Entropy of Molecule Number Vector at Transmitter Given Received Signal

Subsequently, the second term of (22) to be addressed is $H(\mathbf{N}_T|P)$, which is the conditional entropy of $\mathbf{N}_T$ given *P*.

Assume that the *q*th type of chemical substances, among the *S* types at the transmitter, are used for diffusion-based MC, where the particles of the *q*th type are released during molecule emission, pass through the channel, and are received during molecule reception.

Suppose the *S* Poisson distributions defined in (24) are conditionally independent given *P* (due to the assumption that no chemical reaction occurs in the channel). Then,

$$\begin{aligned} H(\mathbf{N}_T|P) &= \sum_{i=1}^{S} H(Z_i|P) \\ &= \sum_{i=1}^{q-1} H(Z_i) + H\big(Z_q|P\big) + \sum_{i=q+1}^{S} H(Z_i), \end{aligned} \tag{26}$$

where the second equality holds since we only use the *q*th type of chemical substances so that *P* is only dependent on the *q*th Poisson distribution Z_q and is independent of the other $S-1$ Poisson distributions. According to (26), the conditional entropy $H(\mathbf{N}_T|P)$ is the sum of the conditional entropy $H(Z_q|P)$ and the $S-1$ entropy derivations corresponding to the $S-1$ Poisson distributions except Z_q.

Now, we first focus on the *q*th term of (26), i.e., the conditional entropy $H(Z_q|P)$. Suppose the diffusion coefficient in the channel is *D*, and the receiver is a sphere with radius R_r.

Assume that the realizations of $Z_q|P$ are independent at different probabilistic trials [37], each of which is statistically uncorrelated to each other, and bandlimited within an effective bandwidth *W* [38], which corresponds to the symbol transmission rate (number of molecule emissions per second). In other words, the bandwidth W is the reciprocal of the symbol time (time duration of each molecule emission) and can be altered by changing the environment where chemical reactions occur (e.g., by adding catalysts). Then, according to the Shannon-Hartley theorem [36], [39], which expresses the capacity as a product of the maximum sampling rate and number of bits in a symbol, it is implied that the conditional entropy $H(Z_q|P)$ can be expressed as

$$H\big(Z_q|P\big) = 2WH\big(Z_q|\mathcal{P}\big), \tag{27}$$

where $2W$ is the maximum sampling rate, and $\mathcal{P}$ is the received signal per time sample and can be expressed as

$$\mathcal{P} = \sum_{k=1}^{\frac{1}{2W\tau_p}} p_k, \tag{28}$$

where τ_p is the time interval between the measurements for their statistical independence and can be expressed as [40]

$$\tau_p = \frac{{R_r}^2}{D}, \tag{29}$$

$\frac{1}{2W\tau_p}$ is the number of independent measurements within a time sample, and p_k is the kth independent measurement of the number of received particles within a single time sample. Suppose each independent measurement of the number of received particles within a single time sample follows a binomial distribution [41], i.e.,

$$p_k \sim \mathrm{Binomial}\big(\bar{Z}_q, F(d)\big),\ k = 1, 2, \ldots, \frac{1}{2W\tau_p}, \tag{30}$$

where $\bar{Z}_q$ is the average number of emitted particles within a time sample and can be expressed as

$$\bar{Z}_q = \frac{E\big[Z_q\big]}{2W} = \frac{c_q}{2W}, \tag{31}$$

and $F(d)$ is the successful probability of each particle being received in terms of the distance d. In general, the successful probability $F(d)$ depends on the characteristics of the channel and is adaptable toward different application scenarios. Note that each independent measurement in (30) can be approximated as a Poisson distribution [37], expressed as

$$p_k \sim \mathrm{Pois}\big(\bar{Z}_q F(d)\big),\ k = 1, 2, \ldots, \frac{1}{2W\tau_p}, \tag{32}$$

when $\bar{Z}_q$ is sufficiently large and $F(d)$ is sufficiently small. The expected value of each independent measurement defined in either (30) or (32) can be obtained as

$$E[p_k] = \bar{Z}_q F(d),\ k = 1, 2, \ldots, \frac{1}{2W\tau_p}. \tag{33}$$

In the following, we use the binomial model in (30) for derivations.

From the Bayes' theorem, the connection between the prior and posterior probability distributions is established. If the prior and posterior probability distributions fall into the same probability distribution family, then the prior (posterior) probability distribution is the conjugate of the posterior (prior) probability distribution [42]. Note that the binomial distribution is the conjugate of the beta distribution [43]. Therefore, according to the Bayesian inference theory [43], the independent measurements following a binomial distribution in (30) correspond to $Z_q|\mathcal{P} = p$ following a beta distribution, which can be expressed as

$$Z_q|\mathcal{P} = p \sim \mathrm{Beta}\bigg(p, \frac{1}{2W\tau_p}\bar{Z}_q - p\bigg), \tag{34}$$

whose corresponding probability density function can be expressed as [43]

$$f_{Z_q|\mathcal{P}}\big(z_q|p\big) = \frac{z_q^{p-1}\big(1 - z_q\big)^{\frac{1}{2W\tau_p}\bar{Z}_q - p - 1}}{B\Big(p, \frac{1}{2W\tau_p}\bar{Z}_q - p\Big)}, \tag{35}$$

where $B(\cdot,\cdot)$ is the beta function defined as [44]

$$B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha + \beta)}, \tag{36}$$

where $\Gamma(\cdot)$ is the gamma function defined as [45]

$$\Gamma(\alpha) = (\alpha - 1)!,\ \alpha \in \mathbb{N}. \tag{37}$$

Note that when the two parameters of the beta function are both positive integers, the beta function can be rewritten as [44]

$$B(\alpha, \beta) = \frac{(\alpha - 1)!(\beta - 1)!}{(\alpha + \beta - 1)!},\ \alpha, \beta \in \mathbb{N}, \tag{38}$$

according to the definition of gamma function defined in (37). Substituting (31) into (34), (34) can be rewritten as

$$Z_q|\mathcal{P} = p \sim \mathrm{Beta}\bigg(p, \frac{c_q}{4W^2\tau_p} - p\bigg). \tag{39}$$

Subsequently, the conditional entropy $H(Z_q|\mathcal{P} = p)$ can be computed through its definition expressed as

$$H\big(Z_q|\mathcal{P} = p\big) = -\int f_{Z_q|\mathcal{P}}\big(z_q|p\big)\log_2 f_{Z_q|\mathcal{P}}\big(z_q|p\big)\,dz_q, \tag{40}$$

which is an explicit function of the probability density function in (35).

Therefore, with (35) the conditional entropy $H(Z_q|\mathcal{P} = p)$ can be obtained as [46]

$$\begin{aligned} H\big(Z_q|\mathcal{P} = p\big) &= \ln B\bigg(p, \frac{c_q}{4W^2\tau_p} - p\bigg) - (p-1)\psi(p) \\ &\quad - \bigg(\frac{c_q}{4W^2\tau_p} - p - 1\bigg)\psi\bigg(\frac{c_q}{4W^2\tau_p} - p\bigg) \\ &\quad + \bigg(\frac{c_q}{4W^2\tau_p} - 2\bigg)\psi\bigg(\frac{c_q}{4W^2\tau_p}\bigg), \end{aligned} \tag{41}$$

where $\psi(\cdot)$ is the digamma function defined as [44]

$$\psi(\alpha) = \frac{\Gamma'(\alpha)}{\Gamma(\alpha)}, \tag{42}$$

where $\Gamma'(\cdot)$ is the derivative of the gamma function.

In addition, the statistics of the received signal per time sample $\mathcal{P}$ is also of interest. Note that the expected value of $\mathcal{P}$ can be computed as

$$E[\mathcal{P}] = \sum_{k=1}^{\frac{1}{2W\tau_p}} E[p_k] = \frac{1}{2W\tau_p}\bar{Z}_q F(d) = \frac{c_q}{4W^2\tau_p}F(d), \tag{43}$$

where the second equality holds due to (33) and the third equality holds due to (31).

For the conditional entropy $H(Z_q|\mathcal{P})$, we approximate it by fixing $\mathcal{P}$ at its expected value. Therefore, substituting $E[\mathcal{P}]$ for p in (41), the conditional entropy $H(Z_q|\mathcal{P})$ can be approximated as

$$\begin{aligned} H\big(Z_q|\mathcal{P}\big) &\approx H\big(Z_q|\mathcal{P}=E[\mathcal{P}]\big) \\ &= \ln B\bigg(E[\mathcal{P}], \frac{c_q}{4W^2\tau_p}-E[\mathcal{P}]\bigg) \\ &\quad - (E[\mathcal{P}]-1)\psi(E[\mathcal{P}]) \\ &\quad - \bigg(\frac{c_q}{4W^2\tau_p}-E[\mathcal{P}]-1\bigg)\psi\bigg(\frac{c_q}{4W^2\tau_p}-E[\mathcal{P}]\bigg) \\ &\quad + \bigg(\frac{c_q}{4W^2\tau_p}-2\bigg)\psi\bigg(\frac{c_q}{4W^2\tau_p}\bigg). \end{aligned} \tag{44}$$

Finally, combining (26), (27), (43), and (44), we obtain the conditional entropy $H(\mathbf{N}_T|P)$ as

$$\begin{aligned} H(\mathbf{N}_T|P) &= \sum_{i=1}^{q-1} H(Z_i) + \sum_{i=q+1}^{S} H(Z_i) + 2W\ln B \\ &\quad \times \bigg(\frac{c_q}{4W^2\tau_p}F(d), \frac{c_q}{4W^2\tau_p}(1-F(d))\bigg) \\ &\quad - 2W\bigg(\frac{c_q}{4W^2\tau_p}F(d)-1\bigg)\psi\bigg(\frac{c_q}{4W^2\tau_p}F(d)\bigg) \\ &\quad - 2W\bigg(\frac{c_q}{4W^2\tau_p}(1-F(d))-1\bigg) \\ &\quad \times \psi\bigg(\frac{c_q}{4W^2\tau_p}(1-F(d))\bigg) \\ &\quad + 2W\bigg(\frac{c_q}{4W^2\tau_p}-2\bigg)\psi\bigg(\frac{c_q}{4W^2\tau_p}\bigg). \end{aligned} \tag{45}$$

D. Resulting Capacity

From the above derivations, we explicitly obtain the two terms, $H(\mathbf{N}_T)$ and $H(\mathbf{N}_T|P)$, as expressed in (25) and (45), respectively. Note that $H(\mathbf{N}_T)$ and $H(\mathbf{N}_T|P)$ share the common terms $\sum_{i=1}^{q-1} H(Z_i) + \sum_{i=q+1}^{S} H(Z_i)$.

Substituting (25) and (45) into (22), the mutual information term $I(\mathbf{N}_T;P)$ can be rewritten as

$$\begin{aligned} I(\mathbf{N}_T;P) &= H\big(Z_q\big) - 2W\ln B \\ &\quad \times \bigg(\frac{c_q}{4W^2\tau_p}F(d), \frac{c_q}{4W^2\tau_p}(1-F(d))\bigg) \\ &\quad + 2W\bigg(\frac{c_q}{4W^2\tau_p}F(d)-1\bigg)\psi\bigg(\frac{c_q}{4W^2\tau_p}F(d)\bigg) \\ &\quad + 2W\bigg(\frac{c_q}{4W^2\tau_p}(1-F(d))-1\bigg) \\ &\quad \times \psi\bigg(\frac{c_q}{4W^2\tau_p}(1-F(d))\bigg) \\ &\quad - 2W\bigg(\frac{c_q}{4W^2\tau_p}-2\bigg)\psi\bigg(\frac{c_q}{4W^2\tau_p}\bigg). \end{aligned} \tag{46}$$

Note that the common terms $\sum_{i=1}^{q-1} H(Z_i) + \sum_{i=q+1}^{S} H(Z_i)$ are eliminated during the subtraction in (22).

Next, we focus on the first term of (46), i.e., the entropy $H(Z_q)$. Recall that Z_q follows a Poisson distribution with parameter c_q and can be expressed as

$$Z_q \sim \operatorname{Pois}\big(c_q\big), \tag{47}$$

whose corresponding probability mass function is

$$\Pr\big(Z_q = l\big) = e^{-c_q}\frac{{c_q}^l}{l!}. \tag{48}$$

Then, the entropy $H(Z_q)$ can be computed through its definition expressed as

$$H\big(Z_q\big) = -\sum \Pr\big(Z_q = l\big)\log_2 \Pr\big(Z_q = l\big), \tag{49}$$

which is an explicit function of the probability mass function in (48).

Hence, with (48) the entropy $H(Z_q)$ can be obtained as [47]

$$H\big(Z_q\big) = c_q\big(1-\log_2 c_q\big) + e^{-c_q}\sum_{l=0}^{\infty}\frac{{c_q}^l\log_2 l!}{l!}. \tag{50}$$

Suppose c_q is sufficiently large so that the entropy $H(Z_q)$ can be approximated as [47]

$$H\big(Z_q\big) \approx \frac{1}{2}\log_2\big(2\pi e c_q\big). \tag{51}$$

Substituting (51) into (46), the mutual information $I(\mathbf{N}_T;P)$ can be finally obtained as

$$\begin{aligned} I(\mathbf{N}_T;P) &= \frac{1}{2}\log_2\big(2\pi e c_q\big) - 2W\ln B \\ &\quad \times \bigg(\frac{c_q}{4W^2\tau_p}F(d), \frac{c_q}{4W^2\tau_p}(1-F(d))\bigg) \\ &\quad + 2W\bigg(\frac{c_q}{4W^2\tau_p}F(d)-1\bigg)\psi\bigg(\frac{c_q}{4W^2\tau_p}F(d)\bigg) \\ &\quad + 2W\bigg(\frac{c_q}{4W^2\tau_p}(1-F(d))-1\bigg) \\ &\quad \times \psi\bigg(\frac{c_q}{4W^2\tau_p}(1-F(d))\bigg) \\ &\quad - 2W\bigg(\frac{c_q}{4W^2\tau_p}-2\bigg)\psi\bigg(\frac{c_q}{4W^2\tau_p}\bigg). \end{aligned} \tag{52}$$

Now that the mutual information term $I(\mathbf{N}_T;P)$ is derived in (52), we subsequently address the capacity C which is defined in (17).

According to (52), the only term in $I(\mathbf{N}_T;P)$ that depends on the probability density function $f_{\mathbf{N}_T}(\mathbf{n}_T)$ is c_q. Note that there may be multiple equilibrium values for $\mathbf{c}$, depending on $f_{\mathbf{N}_T}(\mathbf{n}_T)$, in the complex balanced chemical reaction network at the transmitter, where each equilibrium value corresponds to a distinct c_q. Therefore, the capacity C can be achieved with the probability density function $f_{\mathbf{N}_T}(\mathbf{n}_T)$ leading to the equilibrium value $\mathbf{c}^*$ which corresponds to the maximum of all possible c_q, denoted by c_q^*.

Finally, we can derive the capacity C, which is formally stated as follows: Suppose c_q^* is sufficiently large and the molecule supply is sufficiently fast. Given that $\mathbf{N}_R$ is a deterministic function of P, that the S Poisson distributions defined in (24) are conditionally independent given P, that realizations of $Z_q|P$ are independent at different probabilistic trials and bandlimited within W, and that independent measurements p_k follow the binomial distribution, the capacity C can be obtained by substituting c_q^* for c_q in (52), expressed as

$$\begin{aligned} C = {} & \frac{1}{2}\log_2\left(2\pi e c_q^*\right) \\ & - 2W\ln B\left(\frac{c_q^*}{4W^2\tau_p}F(d), \frac{c_q^*}{4W^2\tau_p}(1-F(d))\right) \\ & + 2W\left(\frac{c_q^*}{4W^2\tau_p}F(d)-1\right)\psi\left(\frac{c_q^*}{4W^2\tau_p}F(d)\right) \\ & + 2W\left(\frac{c_q^*}{4W^2\tau_p}(1-F(d))-1\right) \\ & \times \psi\left(\frac{c_q^*}{4W^2\tau_p}(1-F(d))\right) \\ & - 2W\left(\frac{c_q^*}{4W^2\tau_p}-2\right)\psi\left(\frac{c_q^*}{4W^2\tau_p}\right). \end{aligned} \tag{53}$$

IV. NUMERICAL EVALUATIONS

In this section, we present the numerical results to assess the effects of chemical reactions on the capacity of diffusion-based MC systems. Specifically, the capacity analysis in this work takes into account the effects of chemical reactions at the transmitter, which are not considered in the existing literature calculating the capacity based on solely the propagation of molecules in the channel (e.g., [12]). Therefore, we evaluate the effects of the chemical reactions in the transmission process, as described in Section II-A, by comparing the capacity expression (53) obtained in Section III with that derived in [12], where the former addresses the chemical reactions and the latter does not. We also demonstrate the interactions between different parameters and their impacts on the resulting capacity.

The common parameters shared by all evaluations include the system temperature T, which is set as the standard state temperature 298 K, and the radius of the (assumed spherical) receive volume R_r, which is set as 30 μm. Note that the Boltzmann constant is $K_b = 1.38 \times 10^{-23}$ J/K. For diffusion-based MC systems, the successful probability is set as $F(d) = \frac{4}{3}\frac{{R_r}^3}{Dd}$ [12]. For a fair comparison, the average transmit power, denoted as $\bar{P}_{\mathcal{H}}$, used in [12] (for the case without chemical reactions) is set as the amount needed for the emission of c_q^* molecules, i.e.,

$$\bar{P}_{\mathcal{H}} = \frac{3}{2}K_b T c_q^*. \tag{54}$$

For all plots in this section, blue curves (labeled as "w/ chemical reactions") correspond to the capacity expression (53) in this article, while red curves (labeled as "w/o chemical reactions") correspond to the capacity expression (59) in [12].

A. *Capacity Versus Distance Under Different c_q^**

The relationship between the capacity and the distance under different c_q^* conditions with and without chemical reactions considered is shown in Fig. 3. The diffusion coefficient D and the bandwidth W are set as 10^{-9} m^2/s and 30 Hz, respectively. The distance d is evaluated ranging from 40 to 80 μm, and the c_q^* is evaluated across $\{5 \times 10^4, 10^5\}$.

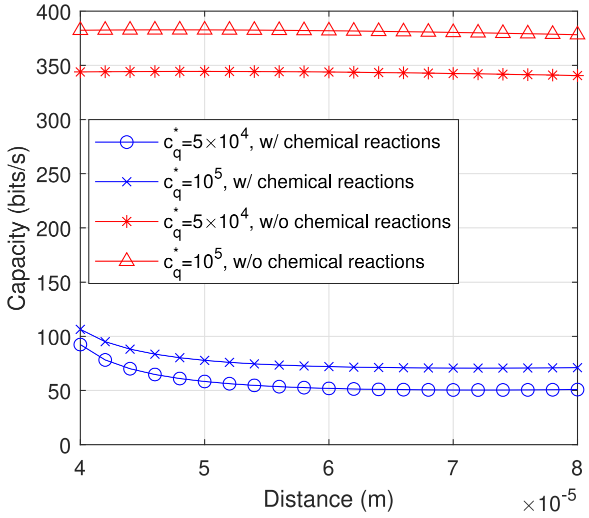

Fig. 3. Relationship between the capacity and the distance under different c_q^* conditions with and without chemical reactions considered.

From the plots in Fig. 3, it can be observed that the capacity decreases as the distance increases in all cases due to the enhanced molecular noise with the increased distance. Note that the decrease is more significant when chemical reactions are considered and is less significant when chemical reactions are not considered. In addition, an increase in c_q^* leads to an increased capacity, since more information bits can be transmitted with more molecules emitted.

B. *Capacity Versus Bandwidth Under Different c_q^**

In Fig. 4, we show the relationship between the capacity and the bandwidth under different c_q^* conditions with and without chemical reactions considered. The diffusion coefficient D and the distance d are set as 10^{-9} m^2/s and 50 μm, respectively. The bandwidth W is evaluated ranging from 15 to 30 Hz, and the c_q^* is evaluated across $\{5 \times 10^4, 10^5\}$.

According to Fig. 4, it can be found that the capacity increases as the bandwidth increases in all cases, since more information bits can be transmitted within a larger bandwidth. However, it can be observed that the impacts of the bandwidth on the capacity is less significant in the case when chemical reactions are not considered than in the case when chemical reactions are considered, which implies that the bandwidth is not dominant in the case when the effects of chemical reactions are taken into account.

C. *Capacity Versus Diffusion Coefficient Under Different c_q^**

The relationship between the capacity and the diffusion coefficient under different c_q^* conditions with and without chemical reactions considered is shown in Fig. 5. The bandwidth W and the distance d are set as 30 Hz and 50 μm, respectively. The diffusion coefficient D is evaluated ranging from 10^{-9} to 10^{-8} m^2/s, and the c_q^* is evaluated across $\{5 \times 10^4, 10^5\}$.

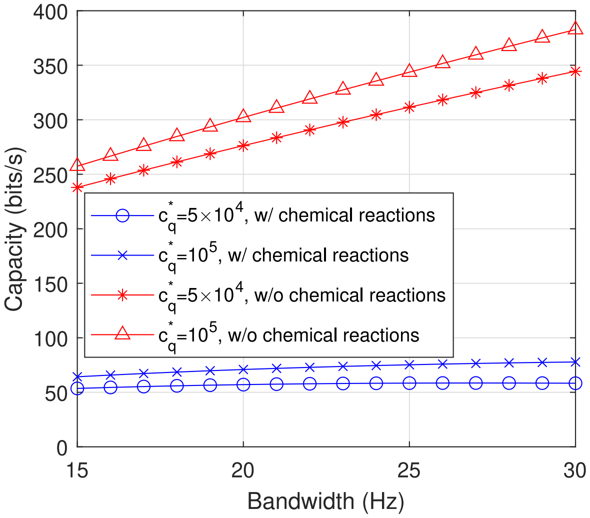

Fig. 4. Relationship between the capacity and the bandwidth under different c_q^* conditions with and without chemical reactions considered.

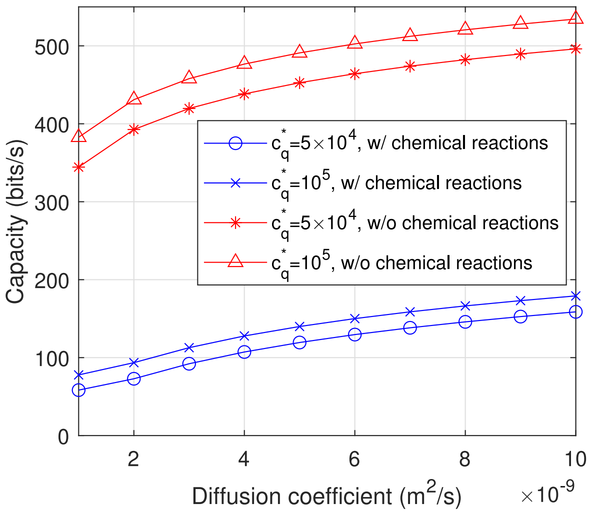

Fig. 5. Relationship between the capacity and the diffusion coefficient under different c_q^* conditions with and without chemical reactions considered.

Based on Fig. 5, it can be observed that the capacity increases with an increase in the diffusion coefficient in all cases, since molecules diffuse further and transmit more information bits in an environment with a larger diffusion coefficient.

D. Capacity Versus Distance Under Different Bandwidth

In Fig. 6, we investigate the relationship between the capacity and the distance under different bandwidth conditions with and without chemical reactions considered. The c_q^* and the diffusion coefficient D are set as 10^5 and 10^{-9} m^2/s, respectively. The distance d is evaluated ranging from 40 to 80 μm, and the bandwidth W is evaluated across $\{15, 30\}$ Hz.

According to Fig. 6, it can be found that the capacity decreases as the distance increases and the bandwidth decreases. In addition, the bandwidth is more dominant in the resulting capacity when chemical reactions are not taken into consideration. These observations are in accordance with those obtained in Sections IV-A and IV-B.

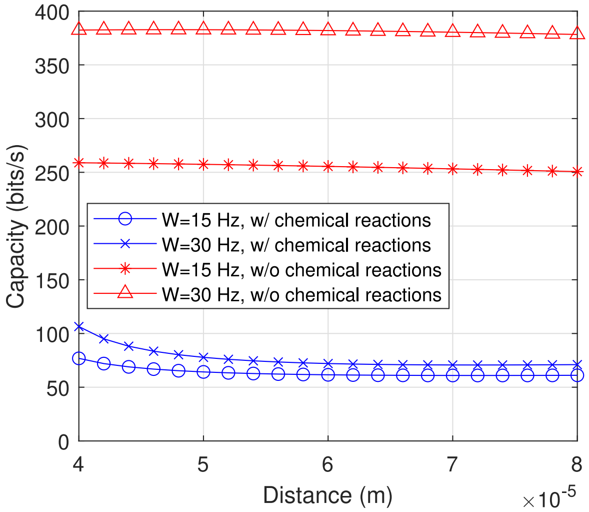

Fig. 6. Relationship between the capacity and the distance under different bandwidth conditions with and without chemical reactions considered.

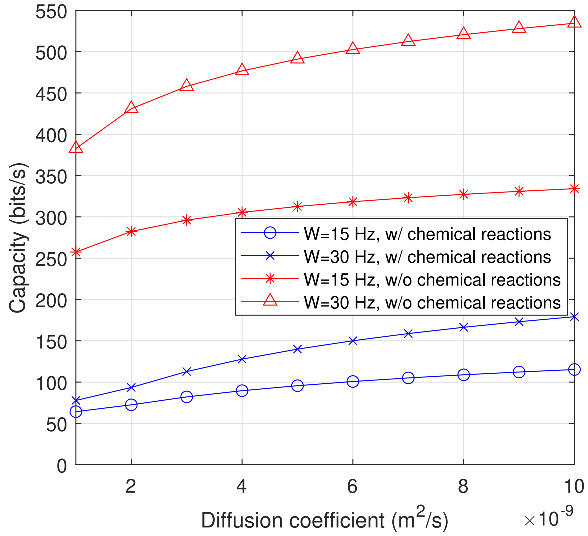

Fig. 7. Relationship between the capacity and the diffusion coefficient under different bandwidth conditions with and without chemical reactions considered.

E. Capacity Versus Diffusion Coefficient Under Different Bandwidth

The relationship between the capacity and the diffusion coefficient under different bandwidth conditions with and without chemical reactions considered is shown in Fig. 7. The c_q^* and the distance d are set as 10^5 and 50 μm, respectively. The diffusion coefficient D is evaluated ranging from 10^{-9} to 10^{-8} m^2/s, and the bandwidth W is evaluated across $\{15, 30\}$ Hz.

From the Fig. 7, it can be observed that the capacity increases as the diffusion coefficient increases and the bandwidth increases. Similarly, the bandwidth has a larger effect on the resulting capacity when the chemical reactions are not considered. These observations are consistent with those obtained in Sections IV-B, IV-C, and IV-D.

F. Impact of Chemical Reactions on the Resulting Capacity

From the results shown in the figures, it can be observed that there is an approximate four-fold decrease in the resulting capacity with chemical reactions considered compared with that without chemical reactions considered, which implies that the capacity would be overestimated if the effects of chemical reactions are not taken into account. Moreover, this observation can be intuitively interpreted with the famous data processing inequality [25], which explains the decrease in the resulting capacity when the process of chemical reactions are additionally integrated into the capacity derivation.

V. CONCLUSION

In this article, we propose a framework to investigate the effects of chemical reactions on the capacity of diffusion-based molecular communication in an information-theoretic manner. Particularly, we study the chemical reactions which correspond to complex balanced chemical reaction networks. Within the capacity analysis, we adopt the chemical reaction network theory and obtain the individual entropy derivations, mutual information, and final capacity expressions with the chemical reactions at the transmitter taken into account. Numerical results demonstrate the relationships between different parameters and show that there is a significant decrease in the resulting capacity when chemical reactions are considered compared with the case when they are not considered, which implies that the capacity is overestimated when the effects of chemical reactions are not included.

The proposed framework provides a methodology using the chemical reaction network theory incorporated with the information theory, leading to a stricter capacity expression. The effects of transmission processes are represented by chemical reactions and incorporated in the capacity derivations through the state of the complex balanced chemical reaction network which can easily be adapted to various application scenarios. Future directions around this topic include the integration of effects of chemical reactions at the receiver, the interaction between the particle motion and chemical reactions during molecule propagation in the channel (in systems such as reaction-diffusion systems), and multi-type molecules used as information carriers.

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Ching-Lun Tai (Graduate Student Member, IEEE) received the B.S. degree in electrical engineering from National Taiwan University, Taipei, Taiwan, in 2018. He is currently pursuing the Ph.D. degree in electrical and computer engineering with the Georgia Institute of Technology, Atlanta, GA, USA. From 2018 to 2019, he was a Research Assistant with the Institute of Information Science, Academia Sinica, Taipei. His current research interests include communication theory and signal processing.

Ian F. Akyildiz (Fellow, IEEE) received the B.S., M.S., and Ph.D. degrees in electrical and computer engineering from the University of Erlangen–Nürnberg, Germany, in 1978, 1981, and 1984, respectively.

He is the Ken Byers Chair Professor of Telecommunications with the Georgia Institute of Technology, and the Director of the Broadband Wireless Networking Laboratory. Since June 2020, he serves on the advisory board of the Technology Innovation Institute, Abu Dhabi, UAE, a newly established research center. He has been the President of the Truva, Inc., since March 1989, and the CTO for the newly established company Airanaculus since April 2020. Since 2017, he has also been serving as a Consulting Professor with the Department of Computer Engineering, University of Cyprus. He has been a Megagrant Research Leader with the Institute for Information Transmission Problems, Russian Academy of Sciences, Moscow, Russia, since May 2018. He has been a Visiting Distinguished Professor with the SSN College of Engineering, Chennai, India, since October 2019, and an Adjunct Professor with the Department of Electrical Engineering, University of Iceland, since September 2020. According to Google Scholar, as of October 2020, his H-index is 125 and the total number of citations to his papers is more than 120,000. His current research interests are in Internet of bionanothings, molecular communication, reconfigurable intelligent surfaces, terahertz communications, nanonetworks, 5G/6G wireless systems, Internet of space things/CUBESATs, and wireless sensor networks in challenged environments, such as underground and underwater. He was the Editor-in-Chief Emeritus of *Computer Networks* (Elsevier) from 1999 to 2019, and the Founding Editor-in-Chief Emeritus of the *Ad Hoc Networks* (Elsevier) from 2003 to 2019, *Physical Communication* (Elsevier) from 2008 to 2017, and *Nano Communication Networks* (Elsevier) from 2010 to 2017. He is an ACM Fellow and has received numerous awards from the IEEE, ACM and other professional organizations.