

# Complex Permittivity Extraction using Substrate Integrated Waveguide Cavity Resonator without Cross-Sectioning

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**Abstract**—In this paper, a dielectric properties extraction method for millimeter-wave applications is presented. Substrate integrated waveguide (SIW) cavity resonators with the same structure and varied thicknesses are employed to separate the dissipation factor (DF) of the substrate material for the cavity resonators. The dielectric constant and loss tangent of the dielectric substrate for the SIW is extracted at the resonance frequencies based on the unloaded Q-factors of transmission loss measurement. The DF from the unloaded Q-factors, which is highly dependent on the thickness of the substrate, is extracted using an iterative fitting process for the substrate thickness estimation without cross sectioning. To validate the extraction method, the SIW cavity resonators are fabricated using RO4003C substrate material and the dielectric properties are extracted in the X-band (8.2 to 12.4 GHz). The extracted thicknesses of the SIW resonators are validated by cross-sectioning. Additionally, the extracted dielectric properties are also verified by comparing the dielectric characteristics of the SIW resonators with the different thicknesses. With the presented method, the time expense for the conventional dielectric characterization method with cross-sectioning is reduced.

**Keywords**—Substrate Integrated Waveguide (SIW) cavity resonator, complex permittivity, DK and DF, substrate material

## I. INTRODUCTION

Printed circuit boards (PCBs) are widely used in radio frequency (RF) applications, especially in the millimeter-wave frequency bands, such as wireless communication, radar, and mobile device. To design the PCB structures in the millimeter-wave frequency bands, precise dielectric material properties are of great importance for fast and reliable design in the early stage of the product development cycle.

To extract dielectric properties in the millimeter-wave frequency bands, a number of measurement-based extraction methods have been introduced. A resonant method is used to extract the dielectric properties of a substrate materials at a single frequency while the transmission and reflection method usually provides less accurate results over a wide frequency range. Recently, substrate integrated waveguide (SIW) methods have been introduced [1]–[3]. The SIW cavity resonator method

has many advantages such as its, low cost, high accuracy, and ease of design and manufacture. However, for the SIW cavity resonator method the extracted results are highly dictated by substrate thickness. Due to this, the SIW cavity resonator method requires cross-sectioning to ensure the substrate thickness is correct. Although the thickness of the substrate is provided by the PCB manufacturer, the thickness is often different from the datasheet value after the manufacturing process. Cross-sectioning each time is resource and time consuming making the SIW cavity resonator method unattractive for practical applications.

In this work, to overcome the inefficiencies of the conventional measurement methods, a dielectric characterization method using the SIW cavity resonators with an iterative fitting process for the thickness estimation is presented. SIW cavity resonators with different thicknesses are fabricated and the dielectric constant and loss tangent are extracted at the resonance frequencies of each SIW by measuring  $S_{21}$ . To estimate the exact substrate thickness, an iterative fitting process using the effective conductivity is proposed. The effective conductivity with values lower than bulk conductivity due to surface roughness can be calculated by one-time cross-sectioning. Even though the cross-sectioning is required to calculate the effective conductivity, it could be applied to the SIW resonators with different thicknesses repeatedly. With the calculated conductivity, the exact heights of the SIW resonators can be estimated by fitting the effective conductivity and validated by additional cross-sectioning. Based on the proposed dielectric properties extraction method, it is expected that the time and cost expense of the conventional method with cross-sectioning will be reduced.

## II. COMPLEX PERMITTIVITY EXTRACTION METHOD AND PROPOSED HEIGHT ESTIMATION PROCESS

### A. Conventional Extraction Method using SIW Cavity Resonators

The dielectric constant and loss tangent extraction using SIW cavity resonators is well established in [1]–[3]. For the completeness of the paper, the SIW cavity resonator design and complex permittivity extraction method are briefly reiterated in

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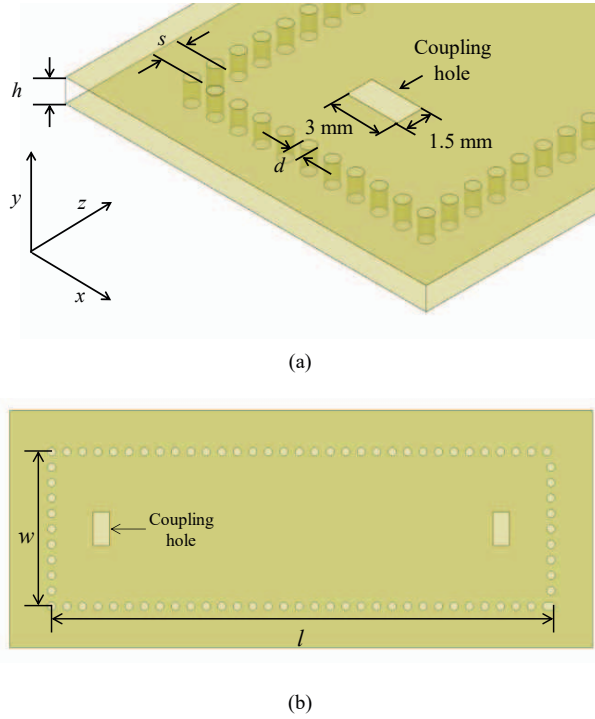


Fig. 1. Configurations of SIW cavity resonator. (a) side view (b) top view

this section. The structure of SIW is shown in Fig. 1. Due to the structure of the SIW cavity, the electric field is only distributed perpendicular to the propagation direction, which makes the TE mode dominant. When the TE mode is excited in the SIW cavity, resonance occurs according to the mode index  $m, n, k$  which is related to the physical dimensions, width, height, and length. Due to the coupling hole is positioned in the middle of the SIW cavity along the x-direction, the mode index  $m$  should be the odd number. The number of resonant modes in the SIW cavity is determined by the cavity length. For a given frequency range, the frequency interval of each mode index  $k$  is depending on the cavity length. To have a sufficient number of mode indexes,  $k$ , the length of the SIW cavity must be far longer than the width. Because the substrate thickness is much smaller than the width and length of the SIW, the mode index  $n$  should be zero. In this work, the width and length is selected as 18.6 and 96 mm, respectively. Thus, total five  $TE_{1,0,k}$  modes can be measured from the SIW in the X-band (8.2 to 12.4 GHz).

Fig. 2 depicts an  $S_{21}$  measurement setup using a vector network analyzer (VNA), conventional rectangular shape waveguides, and an SIW. With the external waveguide located at the top surface of the SIW, the unloaded Q-factors can be calculated from the  $S_{21}$  measurement results. The unloaded Q-factor is separated into three major parts; dielectric loss, conduction loss, radiation loss. When the space between adjacent VIAs satisfy  $d/s > 0.5$  [5] where  $d$  is the diameter of VIA, radiation loss is negligible, and the unloaded Q-factor,  $Q_U$  can be given as (1).

$$\frac{1}{Q_U} = \frac{1}{Q_D} + \frac{1}{Q_C} \quad (1)$$

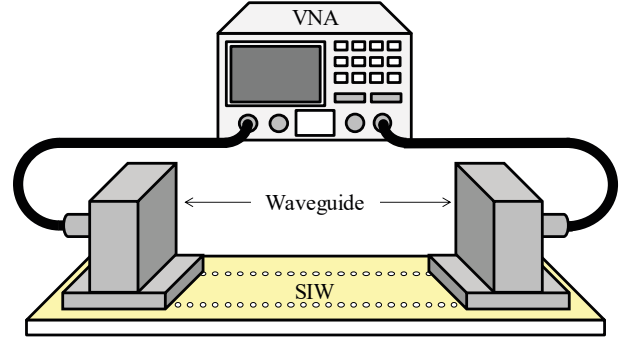


Fig. 2. S-parameter measurement of the SIW cavity resonator

$$\frac{1}{Q_U} = \tan \delta + R_s \frac{2}{\omega \mu h} \quad (2)$$

$1/Q_D$  is the dielectric loss of the substrate material caused by the dielectric loss tangent. The conduction loss, which is a function of the surface resistance  $R_s$  for the top and bottom surfaces of the SIW cavity, is derived in (1) as  $1/Q_C$  [6]. Further by knowing the unloaded Q-factor of the SIW cavity resonator, shown in (2), the dielectric loss and conduction loss can be separated using two SIWs with different thicknesses [7]. From this knowledge the dielectric loss tangent of the substrate material can be extracted from the measured  $Q_U$  as shown below

$$\tan \delta = \frac{1}{h_2 - h_1} \left( \frac{h_2}{Q_{U2}} - \frac{h_1}{Q_{U1}} \right) \quad (3)$$

$$Q_U = \frac{Q_L}{1 - |S_{21}|} \quad (4)$$

where  $h_1, h_2$  and  $Q_{U1}, Q_{U2}$  are the thicknesses and the unloaded Q-factors of different SIW cavity resonators, respectively. The unloaded Q-factors can be calculated from the measured loaded Q-factors obtained via the  $S_{21}$  measurement with (4), where  $Q_L$  is the ratio of discrete resonance frequencies to 3-dB bandwidth [6].

The inverse calculation for dielectric constant (DK) is derived in (5) where  $c_0$  is the speed of light in vacuum, and  $f$  corresponds to the measured resonant frequency of the SIW cavity [7].

$$\epsilon_r = \left( \frac{c_0}{2f} \right)^2 \left[ \left( \frac{m}{w_{eff}} \right)^2 + \left( \frac{k}{l_{eff}} \right)^2 \right] \quad (5)$$

By following (3)-(5), The dielectric constant and loss tangent of the substrate material in the SIW cavity resonators can be extracted. However, when accounting for the dielectric loss, the exact thickness of the dielectric substrate is required to achieve high extraction accuracy.

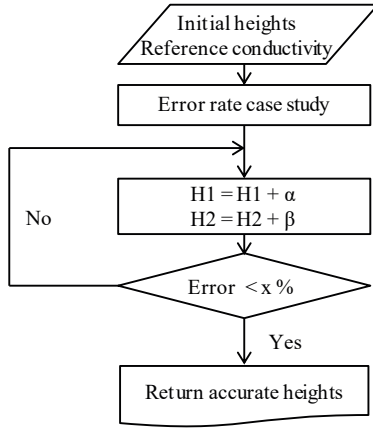


Fig. 3. Iterative fitting process for thickness estimation.

### B. Proposed Height Estimation Method

To estimate the dielectric thickness from the measured S-parameters instead of cross-section, an iterative fitting process is proposed in this section. The thickness of the SIW cavity can be estimated by calculating the effective conductivity with the initial height and comparing it to the reference conductivity measured in advanced. If the effective conductivity has a variation with the reference, the effective conductivity should be calculated again using the modified heights until it reaches a specific error rate. The effective conductivity which is the function of the height is derived in (6).

$$\sigma_{eff} = \left( \frac{1/h_1 - 1/h_2}{1/Q_{U1} - 1/Q_{U2}} \right)^2 \frac{2}{\omega\mu} \quad (6)$$

The conductivity of the copper plate can be extracted from the conduction loss in (2). The conduction loss itself in (2) is highly dependent on the thickness of the SIW cavity. However, the effective conductivity of the top and bottom surface is regarded as the same due to the same dielectric substrate and manufacturing process. Regarding the reference conductivity of a copper, it might be much smaller than the bulk conductivity of  $5.08 \times 10^7$  S/m due to surface roughness [9], [10]. At the high-frequency band, the skin depth of copper and root mean square (RMS) heights of rough surfaces of the SIW resonator are in the micrometer range. As the skin depth falls below the RMS heights of rough surfaces, the surface roughness effects on the surface resistance of the top and bottom plates in the SIW resonator resulting in the increased resistance. Due to the increased resistance, the copper plates of the SIW resonator has the properties of smooth metal with low conductivity. This reduced conductivity is included in the conductor loss of the measured unloaded Q-factors of (6). Consequently, the reference conductivity of the top and bottom surfaces of the SIW resonator can be calculated with (6) and one-time cross-sectioning for heights measurement. The calculated reference conductivity could be applied repeatedly to iterative fitting process with other SIW resonators which have the same copper foil and manufacturing processes.

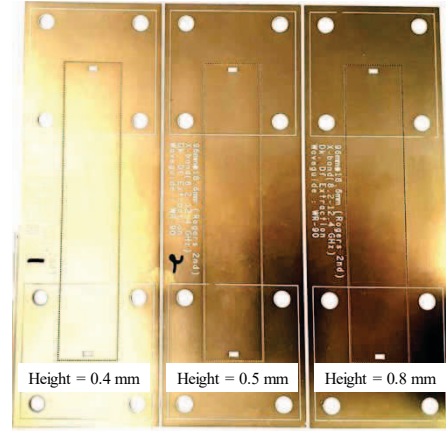


Fig. 4. Samples of SIW cavity resonators with different thicknesses.

The iterative fitting process is shown in Fig. 3. The fitting process for thickness is as follows; The heights provided by the PCB manufacturer can be used as initial values with the known conductivity used as a reference to fit the heights. The reference conductivity can be either given by the PCB manufacturer or measured independently as mentioned in the previous paragraph. With the initial heights and reference conductivity, the effective conductivity is calculated using (6) with the initial heights. If the variation between reference and effective conductivity is insufficient to satisfy the error rate, the effective conductivity should be calculated again with the modified heights until it satisfies the error rate. After the process of fitting is performed, the accurate heights of the SIW cavity are returned. Based on the iterative fitting process, the thickness of the SIW cavity for the extraction accuracy can be estimated without cross-sectioning.

## III. EXPERIMENTAL VERIFICATION

The proposed complex permittivity extraction based on height estimation is verified through measurement. Samples of SIW cavity resonators using RO4003C with three designed heights, 0.4, 0.5, and 0.8 mm are shown in Fig. 4. The DK and dissipation factor (DF) of the DUT is specified in the datasheet as 3.55 and 0.0027, measured at 2.5 GHz. The dielectric properties of the substrate are extracted in the X-band. Each SIW cavity is fed by a rectangular waveguide and the extraction is processed by using a VNA and Matlab.

### A. Complex-Permittivity Extraction Results

The effective dimensions of the SIW cavity are  $l_{eff} = 95.7$  mm and  $w_{eff} = 18.3$  mm [8]. Thus, the TE mode range,  $k$ , of the SIW cavity is from 9 to 13. The measured unloaded Q-factors are shown in Table I. The DF can be calculated by using (3) with two SIWs having a different height. The DK can be calculated by using (5) with resonant frequencies for each mode measured by the VNA as shown in Fig. 5. The mode index of  $m$  and  $k$  is specified in Table I. It is expected that the complex-permittivity of substrate material should be the same due to the same substrate and manufacturing process.

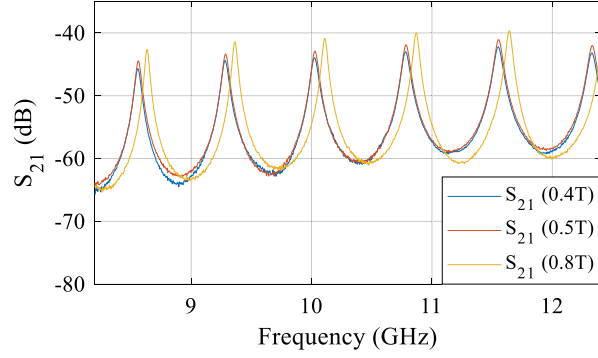


Fig. 5. Transmission loss of the SIW cavity resonators.

TABLE I. MEASURED UNLOADED Q-FACTORS

Resonant modes	$h = 0.4 \text{ mm}$	$h = 0.5 \text{ mm}$	$h = 0.8 \text{ mm}$
$TE_{1,0,9}$	152.81	169.43	220.27
$TE_{1,0,10}$	155.45	169.91	222.58
$TE_{1,0,11}$	154.94	170.53	222.79
$TE_{1,0,12}$	157.61	171.38	225.32
$TE_{1,0,13}$	159.26	172.35	225.85

The extracted dielectric properties of the SIW cavity are shown in Fig. 6. The DK of substrate materials shows a good correlation, within a 2% variation, however, the DF shows a large variation depending on the thicknesses of the SIW cavity which due to variations the substrate thickness.

### B. Improved Extraction Results

To achieve high accuracy on DF extraction with the iterative fitting process, knowing the effective conductivity is required. The effective conductivity calculated by using (6) is  $1.08 \times 10^7 \text{ S/m}$  in the X-band. The effective conductivity of copper plates of the SIW resonators has been reduced in the measured frequency range due to the surface roughness. Based on the known conductivity, the thicknesses of the SIW cavities have been calculated using (6) and unloaded Q-factors shown in Table I. The calculated thicknesses of the SIW cavities with 0.2% error rate are 0.408, 0.478, 0.818 mm. To validate the calculated thicknesses of the SIW cavities, additional cross-

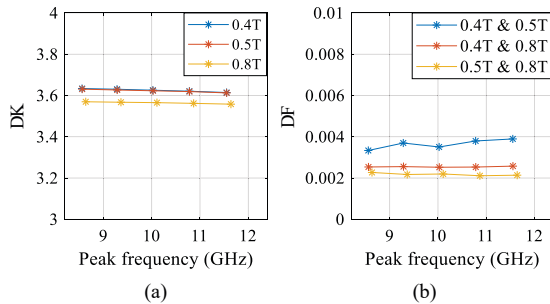


Fig. 6. Extracted dielectric properties of SIW cavity resonator (a) dielectric constant. (b) dielectric loss.

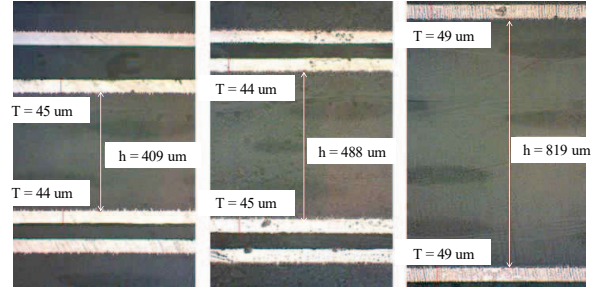


Fig. 7. Cross-sectioning results for each SIW cavity.

sectioning was carried out. The cross-section results are shown in Fig. 7. The thicknesses of SIWs measured by an optical microscope are 0.409, 0.488, 0.819 mm, respectively. Based on the fitting and cross-section results, it is clear that the accurate estimation of heights can be carried out using the proposed fitting method.

TABLE II. EXTRACTED DF

Extraction group	0.408/0.478	0.408/0.818	0.478/0.818
$TE_{1,0,9}$	0.0021	0.0025	0.0026
$TE_{1,0,10}$	0.0026	0.0026	0.0025
$TE_{1,0,11}$	0.0024	0.0025	0.0026
$TE_{1,0,12}$	0.0028	0.0025	0.0025
$TE_{1,0,13}$	0.0030	0.0026	0.0025

The DF is calculated again using the heights estimated by the proposed method. In Fig. 8(b), the variation on DF is greatly reduced. The extraction results are summarized in Table II. The DF shown in blue has a small variation as compared to the other cases. This variation is caused by the variation of the Q-factors. According to (3), the DF is highly dependent on the accuracy of unloaded Q-factors and the thickness of the substrate. If the difference between  $h_1$  and  $h_2$  is insufficient, the error of Q-factors could be amplified during the extraction process. Thus, in order to avoid the error of DF with varying thicknesses, a sufficient thickness difference is required. Based on the iterative fitting and extraction results, it is confirmed that the dielectric constant and loss tangent of the dielectric substrate on SIW can be extracted without cross-sectioning.

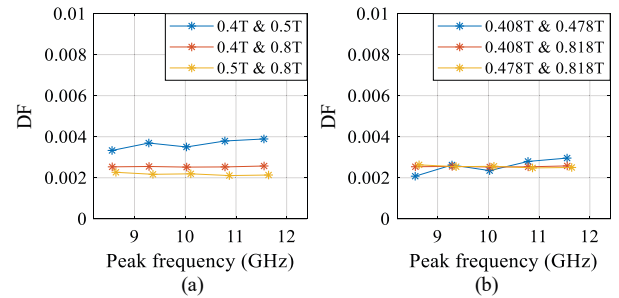


Fig. 8. Re-extracted dielectric loss with varying thicknesses. (a) with initial height of SIW cavity. (b) with fitted height of SIW cavity.



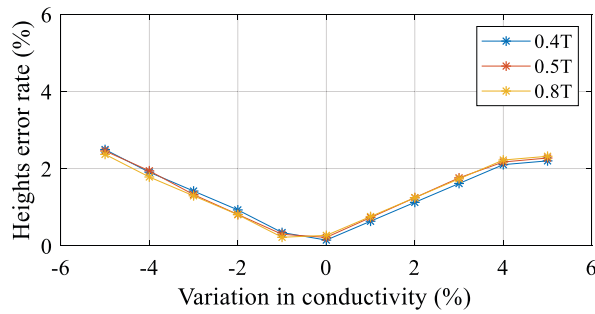


Fig. 9. Error sensitivity analysis results.

The error sensitivity analysis for thickness estimation is carried out to further validate the iterative fitting process. To analyze the sensitivity of the fitting process, a variation of 10% is intentionally applied to the reference conductivity provided by the manufacturer. Using the varied conductivity, the thicknesses are calculated through the iterative fitting process. After the iterative fitting process, the calculated thicknesses, using the varied conductivity, are compared to the cross-sectioning results. The error sensitivity analysis results are shown in Fig. 9. In comparison with the thicknesses calculated by the fitting process and those obtained through cross-sectioning, the maximum error rate is 2.5% which is much smaller than the variation in conductivity. The iterative fitting process is found to be precise and reliable through this error sensitivity analysis.

#### IV. CONCLUSION

In this paper, a dielectric properties extraction method for millimeter-wave applications is presented. The presented method employed a SIW, designed for the X-band in which the  $TE_{m,0,k}$  mode is excited. The complex-permittivity of the substrate material has been extracted by using a VNA and a rectangular waveguide. During the extraction process, the exact heights of the SIW resonators which can be obtained by cross-sectioning are required for accurate DF calculation. Therefore, to manage the quality of manufactured PCBs in a big volume, a number of cross-section measurements must be conducted which is inefficient in terms of time and cost. To overcome the inefficiencies, the iterative fitting process for height estimation using the known conductivity is proposed. Even though the

effective conductivity of the top and bottom copper of the SIW resonators can be calculated with the exact heights from the cross-sectioned results, the calculated conductivity could be applied to the iterative fitting process with other SIW resonators without additional cross-sectioning. With the iterative fitting results, the dielectric properties of the substrate material are successfully extracted. Based on these findings, it is expected that the complex-permittivity extraction in the early stage of the product development cycle can be performed in a cost-effective way.

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