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Compensation Mechanisms for Double Auctions in Peer-to-Peer Local Energy Markets

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Abstract
Purpose of Review. This paper compares different double-auction pricing mechanisms in the context of peer-to-peer trading in local electricity markets. Benchmarking against the traditional system marginal price (SMP) approach, we analyze the outcomes of the Vickrey-Clarke-Groves (VCG) and Pay-as-bid (PAB) approaches, which lead to a revenue imbalance in the market. To mitigate this imbalance, we propose a set of mechanisms that impose trading fees or subsidies with different fairness policies and discuss their impact on the behavior of market participants.
Recent Findings. Proliferation of small-scale and distributed energy resources in low-voltage distribution systems has spurred the development of platforms for peer-to-peer local electricity trading. While recent studies prove that local electricity trading can be beneficial to both the peers and system, implementation of local energy markets is still in early stages and there is no consensus on an optimal market design.
Summary. This paper provides a detailed description of market properties for a competitive LEM that is cleared using the SMP, VCG and PAB approaches. We show that the proposed compensation mechanisms effectively enforce predefined fairness policies and derive exact payments for each peer depending on market results and individual peer characteristics. The case study uses

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real-world data to illustrate that different compensation mechanisms can be used to incentivize desirable peer behavior, e.g. installation of new distributed energy resources, investment in grid supportive generation or flexible energy usage.
Keywords Local energy markets · Peer-to-peer energy trading · Double auctions · Pricing mechanisms

1 Introduction

With the proliferation of distributed energy resources (DERs) and digital, communication-based control systems (e.g. smart meters, smart appliances), there has been a growth of independent, small-scale power producers and flexible loads in low-voltage distribution systems, including behind-the-meter resources. Yet, existing wholesale electricity markets, which traditionally deal with large-scale generation resources, struggle to enable an efficient market participation of individual DERs, thus hindering their potential. Obstacles for the integration of small-scale resources include lacking means for transmission-distribution coordination and compliance with regulation and organizational requirements for small-scale resources (e.g. minimum capacity to enter the market), [32]. Alternatively, local energy markets (LEMs) are sought after to efficiently operate DERs and incentivize DER investments, [4, 14, 18, 22–24, 34, 35]. The typically small scale of trades and spatial proximity of market participants in LEMs also enables peer-to-peer based market structures, i.e. direct trading among peers (e.g. DER owners and loads) without intermediate players such as aggregators or wholesalers, [31].

Establishing a practical LEM design requires efficient mechanisms for matching producing and consuming peers and pricing each electricity transaction. While some real-world pilot projects, e.g. the *Brooklyn Microgrid* or the *SonnenCommunity*, have been successfully implemented, [35], there is no consensus on which market design is the most efficient one. This paper explores and compares the properties of various auction designs for LEM-applicable pricing and matching mechanisms, and devises compensations mechanisms to achieve desirable market properties. A well designed electricity market place must ensure resource efficiency and promotes adequate investments, [6], while also respecting operational constraints of electricity generation and transmission assets, [21]. Besides these basic requirements, the chosen strategy to match and price trades in the market can achieve additional market properties such as *revenue adequacy*, *cost recovery* and *incentive compatibility*, [7,8]. Wholesale electricity markets usually match and price trades based on an auction format with multiple buyers and sellers (so called *double auction*), where each buyer and seller pays and receives, respectively, either a uniform system marginal price (SMP) or locational marginal price (LMP). The SMP approach is, for example, applied in most European countries, [16], while the LMP approach is used in North America, [15]. (Note that this paper focuses on auction pricing mechanisms in LEMs and, therefore, only considers SMPs. The effect of

Table 1 Double Auction Market Properties Under Different Pricing Mechanisms

	Market efficiency	Revenue adequacy	Cost recovery	Incentive Compatibility
Competitive SMP	✓	✓	✓	✓
Strategic SMP	x	✓	✓	x
VCG	✓	x	✓	✓
PAB	x	x	✓	x

✓= Market property achieved x = Market property not achieved

network constraints on prices, e.g. grid-fees or distribution LMPs, [11], is beyond our scope.) SMP-based auctions are revenue adequate, i.e. the sum of payments from buyers covers the sum of payments to sellers, and achieve cost recovery, i.e. producers can at least recover their production cost. Additionally, if the market is perfectly competitive such that no participant can influence market prices, SMP-based auctions incentivize truthful bidding and are, thus, incentive compatible and, ultimately, efficient. On the other hand, if strategic bidding is possible, incentive compatibility can be hampered, leading to a reduction in the social welfare and market efficiency. Alternatively, other mechanisms such as Vickrey-Clarke-Groves (VCG) or pay-as-bid (PAB) auctions, which both lead to different prices for different trades, have been explored for electricity markets to achieve desired market properties, [2, 9, 28, 30]. However, as proven by [25], without perfect competition, no auction design can achieve all four properties simultaneously, see Table 1. Furthermore, established concepts for wholesale electricity trading require careful analyses and customization to accommodate numerous small-scale prosumers with demand preferences varying on short time scales and small-scale generation with a near-zero operating cost. Traditional electricity market designs, on the other hand, are designed for pre-dominantly fossil-fired resources with monotonic production costs and relatively long contract terms. Also, unlike in wholesale electricity markets where price manipulations are restricted by market regulations and audits, enforcing similar oversight in LEMs with numerous small-scale resources will incur prohibiting overhead costs, [32]. Finally, LEMs enable low market entry barriers to incentivize participation without investments in professional trading tools.

Enabling decentralized peer-to-peer trading through LEMs has been shown to be beneficial for both consumers and DER operators, [34]. Additionally, auction-based market designs can provably lead to efficient market outcomes, [26], and practical peer-to-peer auctions have already been adopted in other sectors, e.g. in the financial sector, enabling a micro credit system among peers, [5]. Thus, building on wholesale electricity market designs, various auction-based peer-to-peer market designs have been proposed in the recent literature, [11, 13, 17, 19, 23]. Notably, there is no consensus on what auction design and bidding mechanic will provide the right economic incentives. For example, [19] compares a PAB auction design to a random matching and pricing process.

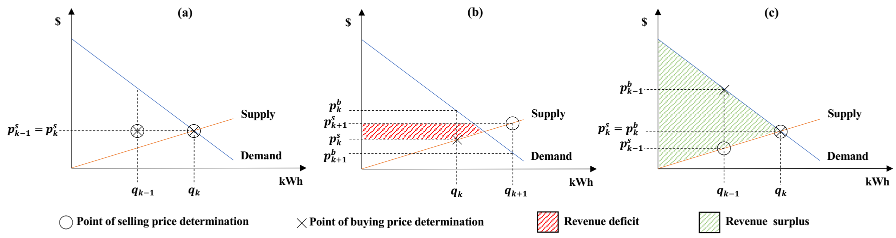

Fig. 1 Comparison of pricing mechanisms with q_k denoting the total amount traded, ◯ marking the point for selling price determination and $\times$ the point for buying price determination leading to a single clearing price in (a) SMP a uniform price for all buyers and a uniform price for all sellers in (b) VCG mechanism and individual prices for each buyer and seller in (c) PAB mechanism. Note that price indices t have been dropped for more concise notation.

In [23] and [29], marginal-price-based consensus mechanics are proposed to identify optimal bilateral contracts in a competitive environment. This concept is extended to accommodate additional grid usage fees in [11]. Further, [13] derives an optimal peer bidding strategy, assuming a VCG-type market. On the other hand, approaches that aim to achieve specific technical properties, e.g. optimal voltage control [1], battery control [14] or electric vehicle integration [10], often do not elaborate on the specifics of the underlying market design.

This paper models a double auction in the context of peer-to-peer LEMs and compares the applicability of the SMP, VCG and PAB pricing and matching mechanisms in terms of their market design properties. To investigate the desirable market design properties summarized in Tab. 1, we propose a set of compensation mechanisms, which enable the utilization of VCG and PAB auctions without creating a revenue imbalance in the market. Using real-life data provided by *Pecan Street Inc.*, [27], we then show how combinations of pricing and compensation mechanisms affect the payments and profits of the peers, and discuss how certain market outcomes such as market integration of economically weaker peers can be achieved.

2 Auction Pricing and Revenue Balancing

We investigate the effects of different pricing mechanisms and proposed compensation mechanisms using the following LEM conventions:

1) The LEM is cleared under the assumption of a double auction (i.e. both buyers and sellers submit bids into the market) and perfect competition (i.e. all peers are price takers and bid truthfully) as further described in Sec. 2.1.
2) Next, the quantities cleared in the market are priced using either the SMP, VCG or PAB pricing mechanism as detailed in Sec. 2.2.
3) Finally, buyer and seller payments are adjusted to compensate for any revenue imbalance remaining in the market. Sec. 2.3 describes two adjust-

ment approaches to calculate the resulting imbalance and four allocation approaches based on different fairness considerations.

As a result of these assumptions, the SMP approach is competitive and fulfills all desired market properties without any compensation, see Tab. 1, and therefore benchmarks the other approaches.

2.1 Market Clearing Model

For given sets $\mathcal{B}$, $\mathcal{S}$, and $\mathcal{T}$ of consumers (buyers), producers (sellers), and time steps, respectively, consider the following market clearing problem:

$$\max_{\substack{g_{i,t},DC_t,\\ d_{i,t},SR_t}} \sum_{\substack{i\in\mathcal{B},\\ t\in\mathcal{T}}}(u_{i,t}(d_{i,t})-SR_t)-\sum_{\substack{i\in\mathcal{S},\\ t\in\mathcal{T}}}(c_{i,t}(g_{i,t})+DC_t) \tag{1a}$$

$$\text{s.t. } DC_t = l_t GT, \qquad \forall t\in\mathcal{T} \tag{1b}$$

$$SR_t = s_t FT, \qquad \forall t\in\mathcal{T} \tag{1c}$$

$$\sum_{i\in\mathcal{B}} d_{i,t} + s_t = \sum_{i\in\mathcal{S}} g_{i,t} + l_t, \qquad \forall t\in\mathcal{T} \tag{1d}$$

$$\underline{g}_{i,t} \le g_{i,t} \le \overline{g}_{i,t}, \qquad \forall i\in\mathcal{S}, t\in\mathcal{T} \tag{1e}$$

$$\underline{d}_{i,t} \le d_{i,t} \le \overline{d}_{i,t}, \qquad \forall i\in\mathcal{B}, t\in\mathcal{T} \tag{1f}$$

$$s_t, l_t \ge 0, \qquad \forall t\in\mathcal{T}. \tag{1g}$$

Here $g_{i,t}$ denotes the (active) power generated by seller i at time t and $d_{i,t}$ denotes the (active) power demand of buyer i at time t. (To simplify notations we assume that each buyer and each seller is a different market participant, hence a market participant that wants to both buy and sell energy in the market is modeled as two separate entities.) Objective function (1a) maximizes social welfare, which includes the following three terms: (i) net energy consumption value (i.e., the utility of peers given by $u_{i,t}(d_{i,t})$ minus the revenue from selling energy to the grid SR_t), (ii) cost of energy procurement (i.e., production by peers at cost $c_{i,t}(g_{i,t})$), and (iii) cost of purchasing energy from the grid (i.e., deficit cost DC_t). Deficit cost DC_t is defined in (1b) as the product of the deficit power given by l_t and the grid tariff given by GT. Surplus revenue SR_t is defined in (1c) as the product of the generation surplus given by s_t and the feed-in tariff given by FT. Tariff FT defines remuneration for selling energy to the grid, and tariff GT defines the price for buying energy from the grid and, typically, $FT < GT$. The power balance between demand $d_{i,t}$, generation $g_{i,t}$, and power exchanges with the grid given by s_t and l_t is ensured in (1d). Eq. (1e) enforces the minimum and maximum production limits $\underline{g}_{i,t}$ and $\overline{g}_{i,t}$ for each seller i for all t. Eq. (1f) enforces the minimum and maximum consumption $\underline{d}_{i,t}$ and $\overline{d}_{i,t}$ for each buyer i for all t. Note that limits $\underline{g}_{i,t}$, $\overline{g}_{i,t}$, $\underline{d}_{i,t}$ and $\overline{d}_{i,t}$ are time variant to capture available generation and consumption preferences. Finally, (1g) ensures non-negativity on s_t and l_t without upper bound, i.e. the grid can always take or supply the surplus or deficit. Because s_t and l_t are only non-zero if generation and demand mismatch, generation is valued higher than FT, or demand is valued lower than GT, the maximization in (1a) and balance (1d) imply mutual exclusivity of s_t and l_t, i.e. $s_t l_t = 0$.

Each buyer's utility is modeled as a function parametrized by its maximum desired consumption $\overline{d}_{i,t}$ and flexibility preference $\alpha_{i,t}$ such that:

$$u_{i,t}(d_{i,t}) = \alpha_{i,t}\overline{d}_{i,t} - \alpha_{i,t}(d_{i,t} - \overline{d}_{i,t})^2 \quad \forall i \in \mathcal{B}, t \in \mathcal{T}, \tag{2}$$

where lower values of parameter $\alpha_{i,t}$ correspond to a greater flexibility and $u_{i,t}(d_{i,t})$ is maximized when $d_{i,t} = \overline{d}_{i,t}$. In other words, the loss in utility from not consuming at the maximum desired level can be interpreted as the cost of not consuming and $\alpha_{i,t}$ is the marginal cost per unit of an incremental demand reduction. Note that parameter $\alpha_{i,t}$ can vary for all buyers and every time slot, i.e. individual flexibility can vary over time depending on time-dependent preferences.

The production cost of each seller is modeled using its *levelized costs of electricity* $LCOE_i$ such that:

$$c_{i,t}(g_{i,t}) = g_{i,t} LCOE_i, \quad \forall i \in \mathcal{S}, t \in \mathcal{T}. \tag{3}$$

Using LCOEs instead of a short-term marginal cost, as in traditional energy-only markets, ensures that sellers are able to eventually recover their initial investments. Otherwise, in renewable-dominant systems, marginal-cost and scarcity-based pricing lead to reduced payments to DERs and thus discourage investments, [3].

2.2 Pricing Mechanisms

Fig. 1 offers a schematic overview of the prices resulting from the (a) SMP, (b) VCG, and (c) PAB mechanisms. The depicted demand functions show the stacked marginal utility functions, representing the bids made by buyers, whereas the supply function depicts the stacked marginal cost functions, i.e. the offers made by sellers. The marginal trade, with the buyer and seller indexed by k, is located either on the intersection of the demand and supply functions or toward the left from the intersection. All bids and all offers with prices lower and greater than the marginal price remain unsettled, i.e. no payments and electricity are exchanged. Index $k-1$ marks the settled peers with the second highest utility and the second lowest cost, while index $k+1$ marks the unsettled peers with the highest utility and the lowest cost, respectively. Note that the welfare maximization in (1a) ensures that the offers of sellers are sorted in a ascending order, and that the offers of buyers are sorted in a descending order.

Under the SMP mechanism every buyer and every seller pays and receives the same price per unit traded, respectively. This price is determined by the intersection of marginal utility and marginal cost, as indicated in Fig. 1(a). All

Table 2 Pricing mechanisms by revenue imbalance

	SMP	VCG	PAB
Market Revenue Imbalance $\Delta R_{t,p} = \sum_{i \in \mathcal{B}} P^b_{i,t} - \sum_{i \in \mathcal{S}} P^s_{i,t}$	0	≤ 0	≥ 0

buyers with a greater marginal utility and all sellers with a lower marginal cost relative to the market-clearing price settle their transactions at the market-clearing price. The resulting payment (cost) of buyer i defined as $P^b_{i,t}$ and payment (remuneration) of seller i defined as $P^s_{i,t}$ at time t are computed as:

$$P^b_{i,t} = p^b_{k,t} d_{i,t} \qquad \forall i \in \mathcal{B}, t \in \mathcal{T} \tag{4}$$

$$P^s_{i,t} = p^s_{k,t} g_{i,t} \qquad \forall i \in \mathcal{S}, t \in \mathcal{T}, \tag{5}$$

where $p^b_{k,t} = p^s_{k,t}$ and $p^b_{k,t}$ denotes the bidding price of buyer k and $p^s_{k,t}$ denotes the asking price of seller k at time t. Note that here the price arises naturally as the dual multiplier of market clearing condition (1d). Furthermore, the SMP mechanism does not cause a revenue imbalance in the market, [33], see Tab. 2 and Fig. 1(a).

The VCG mechanism, instead of using the system-wide price as in the SMP mechanism, settles trades via two different prices for buying and selling. While this approach ensures incentive compatibility also in non-competitive markets, there remains a revenue deficit (see Tab. 1). Prices and payments are determined by comparing the marginal trade and nearest unsettled trade. The price per unit for all buyers is defined as $\max(p^b_{k+1,t}, p^s_{k,t})$ and the selling price for all sellers is defined by $\min(p^s_{k+1,t}, p^b_{k,t})$, with $p^b_{k+1,t}$ and $p^s_{k+1,t}$ denoting the bid and ask price of buyer $k+1$ and seller $k+1$. In the example given in Fig. 1(b), all buyers pay $p^b_{k+1,t}$ per unit, whereas all sellers receive $p^b_{k,t}$ per unit and payments are computed as:

$$P^b_{i,t} = \max(p^b_{k+1,t}, p^s_{k,t}) d_{i,t} \qquad \forall i \in \mathcal{B}, t \in \mathcal{T} \tag{6}$$

$$P^s_{i,t} = \min(p^s_{k+1,t}, p^b_{k,t}) g_{i,t} \qquad \forall i \in \mathcal{S}, t \in \mathcal{T}. \tag{7}$$

Since the price per unit received by sellers is greater than the price per unit paid by buyers, the resulting revenue deficit is $p^b_{k,t}$ - $p^b_{k+1,t}$ per unit traded. The total revenue deficit, indicated by the red area in Fig. 1(b), can be calculated as shown in Tab. 2.

Although the VCG mechanism achieves incentive compatibility, its revenue inadequacy is problematic because market liquidity to settle all trades is not guaranteed *a priori*. In the existing literature there is no consensus on how to efficiently address the revenue imbalance in VCG double auctions. One recent approach is presented in [8] where the imbalance is allocated proportionally to individual peer contributions towards revenue adequacy.

Alternatively, the PAB mechanism determines individual prices for every buyer and seller and, thus, leads to a revenue surplus in the market that ensures market liquidity. Under the PAB, every seller receives its individual ask price, while every buyer pays its individual bid price. In the example given in Fig. 1(c), buyer i pays $p^b_{i,t}$ and seller i receives $p^s_{i,t}$, respectively, whereas buyer k pays $p^b_{k,t}$ and seller k receives $p^s_{k,t}$. Hence, the resulting payments are:

$$P^b_{i,t} = p^b_{i,t} d_{i,t} \qquad \forall i \in \mathcal{B}, t \in \mathcal{T} \tag{8}$$

$$P^s_{i,t} = p^s_{i,t} g_{i,t} \qquad \forall i \in \mathcal{S}, t \in \mathcal{T}. \tag{9}$$

The PAB mechanism leads to a revenue surplus because the amount payed by buyers is greater than the amount received by sellers. The total surplus, indicated by the green area in Fig. 1(c), can be calculated as shown in Tab. 2.

2.3 Compensation Mechanisms

Any revenue surplus or deficit in the market must be balanced to avoid market inefficiency and illiquidity, [8]. Therefore, we introduce an *ex post* compensation payment (CP_i) for each peer i. For a given market revenue deficit we denote CP_i as a trading fee, which has to be paid to compensate this deficit. Alternatively, for a given market revenue surplus, CP_i is a trading subsidy, which buyers and sellers receive to fully redistribute this surplus.

Computing CP_i for each peer depends on two main factors: (i) the amount of revenue imbalance and (ii) the allocation factor, which decides how the imbalance is allocated among the peers:

$$\mathrm{CP}_i = \underbrace{\text{Revenue Imbalance}}_{\substack{\text{Determination of imbalance}\\ \text{amount to allocate:}\\ \text{EDS, CDS}}} \cdot \underbrace{\text{Peer Allocation of Imbalance}_i}_{\substack{\text{Determination of individual}\\ \text{peer shares of imbalance:}\\ \text{S-AM, A-AM, SF-AM, WC-AM}}}.$$

We introduce two schemes to compute these imbalance factors, called Equal Distribution Scheme (EDS) and Contribution Distribution Scheme (CDS). For the calculation of the allocation factors, we propose four different allocation mechanisms called Simple Allocation Mechanism (S-AM), Amount-based Allocation Mechanism (A-AM), Welfare Contribution Allocation Mechanism (WC-AM), and Social Fairness Allocation Mechanism (SF-AM). The combinatorial options are depicted by Fig. 2, showing eight different ways to compute CP_i.

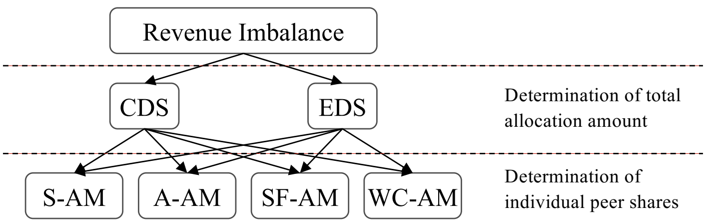

Fig. 2 Overview of Compensation Mechanisms

Table 3 Overview over different compensation mechanisms to address revenue inadequacy

Compensation Mechanism	Abbreviation	Allocation Criterion	x_i (Sellers)	x_i (Buyers)	Rational
Simple Allocation Mechanism	S-AM	Number of peers	1	1	Simple mechanism, treating all peers equally. Imbalance is allocated equally over all peers
Amount-based Allocation Mechanism	A-AM	Amount of electricity traded	$g_{i,t}$	$d_{i,t}$	Payments depend on usage of LEM platform. Imbalance is allocated individually according to amount of electricity traded
Welfare Contribution Allocation Mechanism	WC-AM	Marginal cost and marginal utility	$c_{i,t}(g_{i,t})$	$u_{i,t}(d_{i,t})$	Addressing fairness by rewarding peers depending on their contribution to social welfare
Social Fairness Allocation Mechanism	SF-AM	Marginal cost and marginal utility	$c_{i,t}(g_{i,t})$	$u_{i,t}(d_{i,t})$	Addressing fairness by supporting peers with low benefit from local trading

The EDS and CDS differ in how the cause of the revenue imbalance is accounted, i.e. how much of the total imbalance is attributed to the buyers and to the sellers. Under the EDS, as its name suggests, buyers and sellers are treated equally, so that the scheme is myopic to the contribution of each side to the resulting market revenue imbalance and this imbalance is allocated among all peers. Hence, if the imbalance is not caused equally by buyers and sellers, one side is discriminated in these compensation payments, leading to a disadvantage compared to the SMP results. At the same time, the other side has an advantage compared to the SMP results. Alternatively, the CDS addresses the issue of fairness between buyers and sellers. Under the CDS, the revenue imbalance caused by buyers is only allocated to buyers and the imbalance caused by sellers is only allocated to sellers. We denote the total revenue imbalance as $\Delta R_{t,p}$ and the contributions of buyers and sellers as $\Delta R^b_{t,p}$ and $\Delta R^s_{t,p}$, respectively. Note that $\Delta R_{t,p} = \Delta R^b_{t,p} + \Delta R^s_{t,p}$ and that $\Delta R^s_{t,p}$ ($\Delta R^b_{t,p}$) can be computed as the difference between the sum of seller (buyer) payments under the SMP and the sum of seller (buyer) payments under the VCG or PAB, see Appendix A.

The four allocation mechanisms then determine how the imbalance is allocated among the peers. Each allocation mechanism is designed to address the issue of fairness in the allocation in a different way. Tab. 3 summarizes these four allocation mechanisms and their allocation criteria The S-AM allocates the imbalance equally among the peers. The A-AM allocates the imbalance based on the amount each peer traded. The two payment-based mechanisms WC-AM and SF-AM are based on the utility and costs of peers. The WC-AM rewards peers with the greatest contribution to the social welfare, i.e. they receive a greater compensation from the market revenue surplus or have to pay a lower fee for the market revenue deficit. Hence, sellers with a lower cost and buyers with a greater utility benefit the most. The SF-AM reverses this logic by leading to greater compensations and lower fees to peers with a lower utility or a greater cost, thus essentially subsidizing low-competitive peers.

A generalized notation for computing CP_i under the EDS is given as follows:

$$CP_{i,t} = \Delta R_{t,p} \frac{x_i}{\sum_{i \in \mathcal{B} \cup \mathcal{S}} x_i} \quad \forall i \in \mathcal{B} \cup \mathcal{S}, t \in \mathcal{T}, p \in \mathcal{P}, \tag{10}$$

where set $\mathcal{P} = \{\text{SMP, VCG, PAB}\}$ denotes pricing mechanisms. Variable x_i captures the allocation criterion as indicated in Tab. 3. Therefore, the sum of

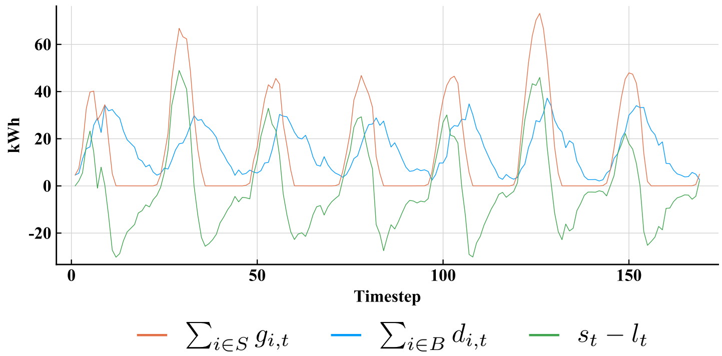

Fig. 3 Aggregated local generation $\sum_{i\in\mathcal{S}} g_{i,t}$ and consumption $\sum_{i\in\mathcal{B}} d_{i,t}$ within 58-Peer local energy market over one week.

all x_i in the denominator is used to share the the imbalance and compute the allocation factor for peer i. Similarly, CP_i for the CDS is computed as follows:

$$CP^s_{i,t} = \Delta R^s_{t,p} \frac{x_i}{\sum_{i\in S} x_i} \quad \forall i \in \mathcal{S}, t \in \mathcal{T}, p \in \mathcal{P} \tag{11}$$

$$CP^b_{i,t} = \Delta R^b_{t,p} \frac{x_i}{\sum_{i\in B} x_i} \quad \forall i \in \mathcal{B}, t \in \mathcal{T}, p \in \mathcal{P} \tag{12}$$

Using (10)–(12) and the allocation criteria x_i given in Tab. 3, the exact CP_i equations for all combinations shown in Fig. 2 can be obtained. The resulting expressions are presented in Appendix B.

3 Case Study and Impact on Peer Incentives

We examine the effects of different pricing and compensation mechanisms by conducting a numerical case study using real-world data provided by *Pecan Street Inc.*, [27]. The data set includes one week from early fall 2019 with moderate load levels and moderate available solar generation for a community of 58 peers. The load and solar generation profile have an hourly resolution. The market is cleared as modeled in (1) based on the buyers utility and sellers cost given in (2) and (3), respectively. Additionally, we model three flexibility levels – *low* ($\alpha_{i,t} = 3$), *medium* ($\alpha_{i,t} = 2$) and *high* ($\alpha_{i,t} = 1$) – which are randomly assigned to buyers at each time step t. Similarly, the values of LCOEs are randomly assigned for each peer ranging between 0.151 $/kWh and 0.242 $/kWh, which is consistent with the recent LCOE estimates for rooftop solar installations by *Lazard Ltd.*, [12]. Finally, electricity can be purchased from the utility grid at the fixed tariff of $GT = 0.30$ $/kWh and the feed-in-tariff is set to $FT = 0.12$ $/kWh.

Fig. 3 shows the resulting sold and purchased $g_{i,t}$ and $d_{i,t}$ and the net surplus computed as $s_t - l_t$. Compared to the traditional fixed tariff environment with tariffs GT and FT, the simulated perfect LEM with SMP leads to cost

Table 4 Simulation results: Total local payments and market revenue

	SMP	VCG	PAB
Buyers' payments $\sum_{i\in\mathcal{B}}\sum_{t\in\mathcal{T}} P_{i,t}$	\$268.45	\$267.16	\$967.45
Sellers' payments $\sum_{i\in\mathcal{S}}\sum_{t\in\mathcal{T}} P_{i,t}$	\$268.45	\$325.51	\$214.99
Compensation payments $\sum_{i\in\mathcal{B}}\sum_{j\in\mathcal{S}}\sum_{t\in\mathcal{T}} (P_{i,t} - P_{j,t})$	\$0	\$-58.35	\$752.46
Market revenue $\sum_{i\in\mathcal{B}}\sum_{t\in\mathcal{T}}(P_{i,t}+CP_{i,t}) - \sum_{i\in\mathcal{S}}\sum_{t\in\mathcal{T}}(P_{i,t}+CP_{i,t})$	\$0	\$0	\$0
Buyers' relative payments increase without buyer flexibility	36.4%	36.6%	15.1%
Sellers' relative payments increase without buyer flexibility	36.4%	39.6%	33.8%

savings of 15.52 % for buyers and increases the profit of sellers by 37.97 %. Tab. 4 itemizes the total payments among peers, i.e. without grid payments, and the necessary compensation to balance the market revenue. Additionally, Tab. 4 shows relative savings of the market with flexibility over the market with fixed bids. Notably, inflexible demand yields a greater total trading volume and thus greater payments for both buyers and sellers.

Given the uniform SMP approach, the buyer and seller payments are identical and, thus, automatically achieve revenue adequacy. With the VCG approach, payments charged to the buyers decrease slightly, while the revenue of sellers increases, leading to the revenue deficit of \$58.35. Finally, the PAB approach significantly increases payments charged to the buyers and decrease the revenue of sellers, which yields the total market revenue surplus of \$752.46.

Tab. 5 itemizes the payments with and without compensation mechanisms for two buyers with high and low flexibility preferences and two sellers with high and low LCOE values. At the chosen time step, the maximum desired demand ($\sum_i \overline{d}_{i,t}$) and maximum available generation ($\sum_i \overline{g}_{i,t}$) are almost equal and, therefore, there is only a modest electricity consumption from the grid. Without any compensation, the results are qualitatively similar to the aggregated results in Tab. 4, corroborating the analysis above that the VCG approach yields lower payments for buyers and greater payments for sellers and the PAB approach yields greater payments for buyers and lower payments for sellers. Indeed, under the VCG approach, 4.2 % of the market revenue deficit can be attributed to the buyers and 95.8 % to the sellers. Thus, applying the EDS under the VCG approach leads to greater additional fees for buyers relative to the CDS because the market deficit caused by sellers is also allocated among the two buyers. Under the PAB approach, on the other hand, 80.0 % and 20.0 % of the market revenue surplus can be attributed to the buyers and sellers, respectively. Therefore, the EDS leads to lower returns for buyers compared to the CDS because sellers receive some of the market revenue surplus generated by buyers.

Furthermore, as Tab. 5 shows, greater demand flexibility ($\alpha_{i,t} = 1$) is systematically rewarded under all compensation mechanisms, sometimes even leading to negative costs for flexible peers under the PAB approach. Thus, comparing payments charged to the buyers under the PAB approach highlights

Table 5 Simulation results: Individual peer payments

	B10	**B16**	**S9**	**S18**
Trading Amount	44.96Wh	1526.94Wh	78Wh	556Wh
$\alpha_{i,t}$ / LCOE	1	3	0.229$/kWh	0.151$/kWh
SMP	$0.0104	$0.3537	$0.0181	$0.1288
VCG	$0.0102	$0.3491	$0.0234	$0.1668
PAB	$0.0110	$1.0532	$0.0178	$0.0842
VCG+S-AM (EDS)	$0.0159	$0.3547	$0.0178	$0.1612
VCG+A-AM (EDS)	$0.0119	$0.3507	$0.0206	$0.1469
VCG+WC-AM (EDS)	$0.0318	$0.3536	$0.0203	$0.1521
VCG+SF-AM (EDS)	$0.0308	$0.3534	$0.0214	$0.1458
PAB+S-AM (EDS)	$-0.0105	$1.0316	$0.0394	$0.1057
PAB+A-AM (EDS)	$0.0049	$0.8446	$0.0285	$0.9486
PAB+WC-AM (EDS)	$-0.0714	$1.0357	$0.0253	$0.1650
PAB+SF-AM (EDS)	$-0.0674	$1.0365	$0.0297	$0.1404
VCG+S-AM (CDS)	$0.0107	$0.3491	$0.1261	$0.1560
VCG+A-AM (CDS)	$0.0104	$0.3537	$0.0181	$0.1288
VCG+WC-AM (CDS)	$0.0113	$0.3509	$0.0166	$0.1343
VCG+SF-AM (CDS)	$0.0103	$0.3589	$0.0193	$0.1230
PAB+S-AM (CDS)	$-0.0235	$1.0187	$0.0264	$0.0928
PAB+A-AM (CDS)	$0.0012	$0.7194	$0.0221	$0.1145
PAB+WC-AM (CDS)	$0.0996	$0.3447	$0.2108	$0.1192
PAB+SF-AM (CDS)	$-0.0642	$0.9234	$0.0233	$0.1101

Table 6 Governance implications of allocation mechanisms

	S-AM	A-AM	WC-AM	SF-AM
Energy Efficiency Promotion	x	✓	x	x
Equipment Discrimination Prevention	✓	✓	x	x
Investment Incentivization	x	x	✓	✓
Strategic Bidding Resistance	✓	✓	x	x

✓= Achievable x = Not achievable

a strong effect of the PAB mechanism on the results for peers with different utilities and costs. While costs of inflexible buyers are systematically greater relative to flexible buyers, sellers with lower LCOEs systematically receive a lower payment. Notably, the amount based allocation (A-AM) using the CDS, exactly leads to the payments resulting from the optimal competitive SMP pricing. This is always the case for the A-AM, because the revenue imbalance resulting from the CDS only depends on the traded amount.

The case study results also imply that some compensation mechanisms can be leveraged to advance certain policies by fostering a desired behavior or attract peers to participate in the LEM. Tab. 6 summarizes some possible implications, independent from the specific results of this case study. Promotion of energy efficiency refers to rewarding a lower consumption and greater flexibility. Equipment discrimination prevention refers to a policy that does not privilege or handicap a peer based on its equipment, e.g. penalize a producer with an older PV system with a greater LCOE. A compensation mechanism achieves investment incentivization, if it enables governance on investment behavior of peers, i.e. peers are rewarded based on cost and utility. For example, this includes lower production costs, greater flexibility or market supportive behavior (e.g. PV systems with generally low efficiency and, thus, high LCOE, but increased generation in evening hours due to their orientation relative to solar irradiation). Finally, strategic bidding resistance refers to a mechanism where the compensation allocation can not be influenced by strategic bidding. For example, investment incentives can be achieved through the payment-based mechanisms WC-AM and SF-AM. While the WC-AM supports lower costs and greater utilities, the SF-AM rewards greater flexibility and subsidizes more production from DERs even with greater LCOEs (e.g. due to imperfect solar panel orientation, see above). The A-AM compensation leads to a fee or payment proportional to the amount traded in the market, which rather rewards buyers and sellers with greater energy efficiency. Also, since the compensation only depends on the traded amount, it can not be influenced by strategic bidding, whereas the SF-AM and WC-AM could raise additional incentive-compatibility issues as the compensation payments could be influenced by the bid and offer price.

4 Conclusion

This paper describes and compares functionality and performance of market design options for double-auction LEMs. Moreover, to address revenue imbalances resulting from some double-auction mechanisms, different compensation mechanisms are introduced. The case study reveals how these compensation mechanisms can be leveraged to foster certain peer behavior within a LEM, while simultaneously achieving ex-post revenue adequacy for the PAB and VCG approaches. Future work will consider strategic bidding, where buyers and sellers seek to influence prices by not acting according to their declared preferences. Further, compensation mechanisms considering physical aspects such as grid supporting behavior (e.g. voltage regulation and reactive power support) can be developed. In addition, uncertainty in generation and demand values, e.g. along the lines of [20], will be considered in future work.

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Appendix

A. Allocation Amounts

Eqs. (13a), (13b) and (13c) show the exact computation of the revenue imbalance under EDS $\Delta R_{t,p}$ and CDS $\Delta R^s_{t,p}$, $\Delta R^b_{t,p}$:

$$\Delta R_{t,p} = |\sum_{i\in S} P^{b,p}_{i,t} - \sum_{i\in B} P^{s,p}_{i,t}| \qquad \forall t \in \mathcal{T}, p \in \mathcal{P} \tag{13a}$$

$$\Delta R^s_{t,p} = |\sum_{i\in S} (P^{s,SMP}_{i,t} - P^{s,p}_{i,t})| \qquad \forall i \in \mathcal{S}, t \in \mathcal{T}, p \in \mathcal{P} \tag{13b}$$

$$\Delta R^b_{t,p} = |\sum_{i\in S} (P^{b,SMP}_{i,t} - P^{b,p}_{i,t})| \qquad \forall i \in \mathcal{B}, t \in \mathcal{T}, p \in \mathcal{P}. \tag{13c}$$

B.Compensation Payments

This section shows the computation of compensation payments CP_i for individual peers for each of the eight possible compensation methods resulting from the two distributions schemes and four allocation mechanisms, see Fig. 2.

Payments CP_i for S-AM under EDS are computed as:

$$CP^s_{i,t} = \frac{1}{N} \Delta R_{t,p} \qquad \forall i \in \mathcal{S}, t \in \mathcal{T}, p \in \mathcal{P} \tag{14a}$$

$$CP^b_{i,t} = \frac{1}{N} \Delta R_{t,p} \qquad \forall i \in \mathcal{B}, t \in \mathcal{T}, p \in \mathcal{P}, \tag{14b}$$

and under CDS as:

$$CP^s_{i,t} = \frac{1}{N^s} \Delta R^s_{t,p} \qquad \forall i \in \mathcal{S}, t \in \mathcal{T}, p \in \mathcal{P} \tag{15a}$$

$$CP_{i,t}^{b} = \frac{1}{N^b} \Delta R_{t,p}^{b} \qquad \forall i \in \mathcal{B}, t \in \mathcal{T}, p \in \mathcal{P}, \tag{15b}$$

where superscripts s and b indicate the payments for sellers and buyers, respectively, and N, N^b and N^s denote the total number of peers, the number of buyers and the number of sellers, respectively.

Payments CP_i for A-AM under EDS are computed as:

$$CP_{i,t}^{s} = \frac{g_{i,t}}{\sum_{i \in S} g_{i,t} + \sum_{i \in B} d_{i,t}} \Delta R_{t,p} \qquad \forall i \in \mathcal{S}, t \in \mathcal{T}, p \in \mathcal{P} \tag{16a}$$

$$CP_{i,t}^{b} = \frac{d_{i,t}}{\sum_{i \in S} g_{i,t} + \sum_{i \in B} d_{i,t}} \Delta R_{t,p} \qquad \forall i \in \mathcal{B}, t \in \mathcal{T}, p \in \mathcal{P}, \tag{16b}$$

and under CDS as:

$$CP_{i,t}^{s} = \frac{g_{i,t}}{\sum_{i \in S} g_{i,t}} \Delta R_{t,p}^{s} \qquad \forall i \in \mathcal{S}, t \in \mathcal{T}, p \in \mathcal{P} \tag{17a}$$

$$CP_{i,t}^{b} = \frac{d_{i,t}}{\sum_{i \in B} d_{i,t}} \Delta R_{t,p}^{b} \qquad \forall i \in \mathcal{B}, t \in \mathcal{T}, p \in \mathcal{P}. \tag{17b}$$

For the payment based compensation mechanisms additional sets need to be defined. Set $\mathcal{B}^{\uparrow}$ is the set of buyers sorted by marginal utility (μ_i) in ascending order and $B_n^{\uparrow}$ is the n-th element of $\mathcal{B}^{\uparrow}$ containing all elements of $\mathcal{B}$ such that $\mu_1 \leq \mu_2 \leq ... \leq \mu_n$. Set $\mathcal{B}^{\downarrow}$ is the set of buyers sorted by μ_i in descending order and $B_n^{\downarrow}$ is the n-th element of $\mathcal{B}^{\downarrow}$ containing all elements of $\mathcal{B}$ such that $\mu_1 \geq \mu_2 \geq ... \geq \mu_n$. In analogy, $\mathcal{S}^{\uparrow}$ and $S^{\downarrow}$ are the sets of sellers sorted by their marginal cost (mc_i) in ascending and descending order, respectively.

For VCG pricing, payments CP_i for WC-AM under EDS are computed as:

$$CP_{i,t}^{s} = \frac{c_{i,t}(g_{i,t})}{\sum_{r \in S} c_{r,t}(g_{r,t}) + \sum_{m \in B^{\uparrow}, n \in B^{\downarrow}} \frac{u_{n,t}(d_{n,t})}{d_{n,t}} d_{m,t}} \Delta R_{t,VCG} \qquad \forall i \in \mathcal{S}, t \in \mathcal{T} \tag{18a}$$

$$CP_{i,t}^{b} = \frac{\frac{u_{j,t}(d_{j,t})}{d_{j,t}} d_{i,t}}{\sum_{r \in S} c_{r,t}(g_{r,t}) + \sum_{m \in B^{\uparrow}, n \in B^{\downarrow}} \frac{u_{n,t}(d_{n,t})}{d_{n,t}} d_{m,t}} \Delta R_{t,VCG} \qquad \forall i \in \mathcal{B}^{\uparrow}, j \in \mathcal{B}^{\downarrow}, t \in \mathcal{T}, \tag{18b}$$

and under CDS as:

$$CP_{i,t}^{s} = \frac{c_{i,t}(g_{i,t})}{\sum_{r \in S} c_{r,t}(g_{r,t})} \Delta R_{t,VCG}^{s} \qquad \forall i \in \mathcal{S}, t \in \mathcal{T} \tag{19a}$$

$$CP_{i,t}^{b} = \frac{\frac{u_{j,t}(d_{j,t})}{d_{j,t}} d_{i,t}}{\sum_{m \in B^{\uparrow}, n \in B^{\downarrow}} \frac{u_{n,t}(d_{n,t})}{d_{n,t}} d_{m,t}} \Delta R_{t,VCG}^{b} \qquad \forall i \in \mathcal{B}^{\uparrow}, j \in \mathcal{B}^{\downarrow}, t \in \mathcal{T}. \tag{19b}$$

For PAB pricing, payments CP_i for WC-AM under EDS are computed as:

$$CP^s_{i,t}=\frac{\frac{c_{j,t}(g_{j,t})}{g_{j,t}}g_{i,t}}{\sum_{m\in S^{\uparrow},n\in S^{\downarrow}}\frac{c_{n,t}(g_{n,t})}{g_{n,t}}g_{m,t}+\sum_{r\in B}u_{r,t}(d_{r,t})}\Delta R_{t,PAB} \quad \forall i\in\mathcal{S}^{\uparrow}, j\in\mathcal{S}^{\downarrow}, t\in\mathcal{T} \tag{20a}$$

$$CP^b_{i,t}=\frac{u_{i,t}(d_{i,t})}{\sum_{m\in S^{\uparrow},n\in S^{\downarrow}}\frac{c_{n,t}(g_{n,t})}{g_{n,t}}g_{m,t}+\sum_{r\in B}u_{r,t}(d_{r,t})}\Delta R_{t,PAB} \quad \forall i\in\mathcal{B}, t\in\mathcal{T}, \tag{20b}$$

and under CDS as:

$$CP^s_{i,t}=\frac{\frac{c_{j,t}(g_{j,t})}{g_{j,t}}g_{i,t}}{\sum_{m\in S^{\uparrow},n\in S^{\downarrow}}\frac{c_{n,t}(g_{n,t})}{g_{n,t}}g_{m,t}}\Delta R^s_{t,PAB} \quad \forall i\in\mathcal{S}^{\uparrow}, j\in\mathcal{S}^{\downarrow}, t\in\mathcal{T} \tag{21a}$$

$$CP^b_{i,t}=\frac{u_{i,t}(d_{i,t})}{\sum_{r\in B}u_{r,t}(d_{r,t})}\Delta R^b_{t,PAB} \quad \forall i\in\mathcal{B}, t\in\mathcal{T}. \tag{21b}$$

For VCG pricing, payments CP_i for SF-AM under EDS are computed as:

$$CP^s_{i,t}=\frac{\frac{c_{j,t}(g_{j,t})}{g_{j,t}}g_{i,t}}{\sum_{m\in S^{\uparrow},n\in S^{\downarrow}}\frac{c_{n,t}(g_{n,t})}{g_{n,t}}g_{m,t}+\sum_{r\in B}u_{r,t}(d_{r,t})}\Delta R_{t,VCG} \quad \forall i\in\mathcal{S}^{\uparrow}, j\in\mathcal{S}^{\downarrow}, t\in\mathcal{T} \tag{22a}$$

$$CP^b_{i,t}=\frac{u_{i,t}(d_{i,t})}{\sum_{m\in S^{\uparrow},n\in S^{\downarrow}}\frac{c_{n,t}(g_{n,t})}{g_{n,t}}g_{m,t}+\sum_{r\in B}u_{r,t}(d_{r,t})}\Delta R_{t,VCG} \quad \forall i\in\mathcal{B}, t\in\mathcal{T}, \tag{22b}$$

and under CDS as:

$$CP^s_{i,t}=\frac{\frac{c_{j,t}(g_{j,t})}{g_{j,t}}g_{i,t}}{\sum_{m\in S^{\uparrow},n\in S^{\downarrow}}\frac{c_{n,t}(g_{n,t})}{g_{n,t}}g_{m,t}}\Delta R^s_{t,VCG} \quad \forall i\in\mathcal{S}^{\uparrow}, j\in\mathcal{S}^{\downarrow}, t\in\mathcal{T} \tag{23a}$$

$$CP^b_{i,t}=\frac{u_{i,t}(d_{i,t})}{\sum_{r\in B}u_{r,t}(d_{r,t})}\Delta R^b_{t,VCG} \quad \forall i\in\mathcal{B}, t\in\mathcal{T}. \tag{23b}$$

For PAB pricing, payments CP_i for SF-AM under EDS are computed as:

$$CP^s_{i,t}=\frac{c_{i,t}(g_{i,t})}{\sum_{r\in S}c_{r,t}(g_{r,t})+\sum_{m\in B^{\uparrow},n\in B^{\downarrow}}\frac{u_{n,t}(d_{n,t})}{d_{n,t}}d_{m,t}}\Delta R_{t,PAB} \quad \forall i\in\mathcal{S}, t\in\mathcal{T} \tag{24a}$$

$$CP^b_{i,t}=\frac{\frac{u_{j,t}(d_{j,t})}{d_{j,t}}d_{i,t}}{\sum_{r\in S}c_{r,t}(g_{r,t})+\sum_{m\in B^{\uparrow},n\in B^{\downarrow}}\frac{u_{n,t}(d_{n,t})}{d_{n,t}}d_{m,t}}\Delta R_{t,PAB}$$
$$\forall i\in\mathcal{B}^{\uparrow}, j\in\mathcal{B}^{\downarrow}, t\in\mathcal{T}, \qquad (24b)$$

and under CDS as:

$$CP^s_{i,t}=\frac{c_{i,t}(g_{i,t})}{\sum_{r\in S}c_{r,t}(g_{r,t})}\Delta R^s_{t,PAB} \qquad \forall i\in\mathcal{S}, t\in\mathcal{T} \qquad (25a)$$

$$CP^b_{i,t}=\frac{\frac{u_{j,t}(d_{j,t})}{d_{j,t}}d_{i,t}}{\sum_{m\in B^{\uparrow},n\in B^{\downarrow}}\frac{u_{n,t}(d_{n,t})}{d_{n,t}}d_{m,t}}\Delta R^b_{t,PAB} \qquad \forall i\in\mathcal{B}^{\uparrow}, j\in\mathcal{B}^{\downarrow}, t\in\mathcal{T}. \qquad (25b)$$