Application of Detectability Analysis for Power System Dynamic State Estimation

Ning Zhou, *Senior Member, IEEE*, Shaobu Wang, *Senior Member, IEEE*, Junbo Zhao, *Senior Member, IEEE*, and Zhenyu Huang, *Fellow, IEEE*

Abstract— **In this paper, detectability analysis is studied and extended to guide selection of measurements for the dynamic state estimation (DSE) of a synchronous machine. To make sure that the DSE converges, past studies suggested that enough measurements must be available to make the system observable. In this paper, the convergence condition is relaxed via detectability analysis to reduce the number of required measurements. Because the objective of the DSE is to estimate the current states as time progresses, it is shown that a DSE observer can converge not only for an observable system but also for an unobservable system if the eigenvalues corresponding to the unobservable states are stable. Simulation results using the IEEE 10-machine, 39-bus system show that the unscented-Kalman-filter-based DSE can converge when measurements are chosen such that the unobservable states have stable eigenvalues. In comparison with observability analysis, the proposed application of detectability analysis can reduce the number of measurements required for the existence of a DSE observer.**

***Index Terms*—Detectability, dynamic state estimation, observability, power systems.**

I. INTRODUCTION

CONVERGENCE and accuracy of the dynamic state estimation (DSE) in the modern power grid [1] [2] depend on the proper selection of measurements. To guide the measurement selection and placement for the DSE, observability analysis has been introduced to power systems [3] [4] [5]. In [5], both the Lie-derivative-based and the linearization-based methods are proposed to quantify system observability. In [3] [4], the smallest eigenvalue, condition number, and determinant of the observability Gramian matrix are used to quantify observability. These studies suggested that to ensure the convergence and accuracy of DSE, measurements should be selected to make the dynamic system observable.

Following the guideline from the observability analysis, the authors carried out extensive case studies on the DSE in power systems. It was observed that when the selected measurements make the system observable, all the unscented Kalman filter (UKF) based observers converge. It was also found in several case studies that the UKF observers converge even when the system is not observable. The observations suggested that the "observable" requirement from observability analysis is a sufficient condition for the existence of a converged observer, but not a necessary condition. The requirements of measurement selection from the observability analysis can be relaxed to reduce the number of required measurements.

To relax the requirement of measurement selection for the DSE in a power system, application of detectability analysis [6] is advocated. Unlike observability analysis, detectability analysis suggests that a DSE observer should exist not only for an observable system but also for an unobservable system, as long as the eigenvalues corresponding to the unobservable states are stable. This means that applying detectability analysis can reduce the number of measurements required for the existence of a DSE observer in comparison with those from observability analysis because almost all the subsystems in the power grid are designed to be stable.

The rest of this paper is organized as follows. Section II introduces detectability analysis and its connection to observability analysis. Case studies are described in Section III. Finally, conclusions and future work are shown in Section IV.

II. DETECTABILITY AND OBSERVABILITY ANALYSES

The dynamic models of a synchronous machine and its associated controllers described in Appendix A of [7] are used in this paper for DSE. Due to space limitation, they are not shown in this paper. To derive an analytical relationship between observability and detectability as well as their connection to DSE, the synchronous machine model is linearized into (1). Here, $\bar{x}(t) \in R^n$ is the dynamic state vector at time *t*; $u(t) \in R^p$ is the input vector; and $y(t) \in R^q$ is the output vector. Also, $\bar{A} \in R^{n\times n}, \bar{B} \in R^{n\times p}, \bar{C} \in R^{q\times n}, and\ \bar{D} \in R^{q\times p}$ are the Jacobian matrices.

$$\frac{d\bar{x}(t)}{dt} = \bar{A}\bar{x}(t) + \bar{B}u(t) \quad (1.a)$$

$$y(t) = \bar{C}\bar{x}(t) + \bar{D}u(t) \quad (1.b)$$

One major goal of DSE is to construct an observer to estimate the dynamic states of power systems in real time. In [6] [8], the concepts of *observability* and *detectability* were used to describe whether an observer exists. When the system's initial states, i.e. *x(*0*)*, can be uniquely determined from the

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N. Zhou is with Binghamton University, State University of New York, Binghamton, NY, 13902, USA (email: ningzhou@binghamton.edu).

S. Wang and Z. Huang are with Pacific Northwest National Laboratory, Richland, WA 99354, USA (email: shaobu.wang@pnnl.gov, zhenyu.huang@pnnl.gov).

J. Zhao is with the Department of Electrical and Computer Engineering, Mississippi State University, Starkville, MS, 39759 USA (email: junbo@ece.msstate.edu).

system inputs and outputs, the system is considered to be ***observable***. In contrast, when its current states, i.e. *x(t)*, can be asymptotically estimated as time progresses, the system is considered to be ***detectable***. The connections between the two concepts and DSE are discussed below.

A. Review on Observability Analysis

Following the linearization method in [5], the observability matrix $\tilde{O}$ can be constructed as (2). The following two metrics can be used to quantify the observability of (1):

a) The rank of $\tilde{O}$, i.e., $r = rank(\tilde{O}, tolerance)$. If $r=n$, the system is fully observable. If $r<n$, the system not observable.

b) The smallest singular value (SSV) of $\tilde{O}$, i.e., $\sigma_{min} = S_{n,n}$ where $S = svd(\tilde{O})$ and $S_{n,n}$ denotes the element at the n^{th} row and n^{th} column of matrix S. The larger the σ_{min}, the higher the degree of system observability.

$$\tilde{O} = \begin{bmatrix} \bar{C} \\ \bar{C}\bar{A} \\ \vdots \\ \bar{C}\bar{A}^{n-1} \end{bmatrix} \tag{2}$$

Note that the two metrics are related to each other because $r < n$ if only if $\sigma_{min} < tolerance$.

B. Observable and Unobservable States

When the system (1) is not observable (i.e., $r < n$), one may apply a transform $T \in R^{n \times n}$ to change the coordinates of the states so that the observable and unobservable states can be separated [6]. More specifically, assuming that $v_{r+1} \ \cdots \ v_n$ are the basis of kernel$(\tilde{O})$ and $w_1 \ \cdots \ w_r$ are the complementary basis, the transformation matrix T can be constructed as $T = [v_{r+1} \ \cdots \ v_n \ w_1 \ \cdots \ w_r]$. Applying $\bar{x}(t) = T^T x(t)$, one can have the transformed system matrices $A = T\bar{A}T^T$, $B = T\bar{B}$, $C = \bar{C}T^T$, and $D = \bar{D}$. The transformed system has a canonical observability form, a.k.a., staircase form, as in (3).

$$\begin{bmatrix} \frac{dx_{no}(t)}{dt} \\ \frac{dx_o(t)}{dt} \end{bmatrix} = \begin{bmatrix} A_{no} & A_{12} \\ 0 & A_o \end{bmatrix} \begin{bmatrix} x_{no}(t) \\ x_o(t) \end{bmatrix} + \begin{bmatrix} B_{no} \\ B_o \end{bmatrix} u(t) \tag{3.a}$$

$$y(t) = [0 \quad C_o] \begin{bmatrix} x_{no}(t) \\ x_o(t) \end{bmatrix} + [D]u(t) \tag{3.b}$$

Here, $x_o(t) \in R^r$ represents the *observable* states; and $x_{no}(t) \in R^{n-r}$ represents the *unobservable* states.

C. Detectability Analysis

The observable states in x_o are also detectable because $x_o(t)$ can be derived from $x_o(0)$ using (3.a). On the other hand, the fact that the states in x_{no} are not observable does not necessarily indicate that they are not detectable. The detectability of x_{no} can be determined as follows.

For a system described by (3), an observer can be constructed as in (4) [9]. Here $K \in R^{n \times q}$ is a design parameter that users can determine to make the estimated state $\hat{x}$ converge to the true state x. For Kalman filter applications, K is selected to make $\hat{x}$ the minimum-variance unbiased estimator of x.

$$\frac{d\hat{x}(t)}{dt} = A\hat{x}(t) + Bu(t) + K\{y(t) - C\hat{x}(t) - Du(t)\} \tag{4}$$

To study the detectability of the states, denote $\Delta x(t) = \hat{x}(t) - x(t)$. Equation (5) can be obtained by taking the difference between (3.a) and (4) then plugging in *y(t)* from (3.b). Equation (5) suggests that if $(A - KC)$ is stable (i.e., its eigenvalues are all on the left side of the imaginary axis of the *s*-plane), $\lim_{t \to \infty} \Delta x(t) = 0$, i.e., $\lim_{t \to \infty} [\hat{x}(t) - x(t)] = 0$, and the system is detectable.

$$\frac{d\Delta x(t)}{dt} = (A - KC)\, \Delta x(t) \tag{5}$$

To study the implication of (5) to power system DSE, remarks on some special cases are summarized as follows:

a) If all the eigenvalues of matrix A are stable, the system will always be detectable no matter which outputs are available. This conclusion can be easily verified by setting K=0 to make $A - KC = A$. The above observation suggests that if terminal voltage phasors are used as the input of a subsystem in the power grid, a DSE observer will exist for the subsystem because these subsystems are designed to be stable when they are connected to an infinite bus. The infinite bus can be considered a voltage-phasor input to the subsystem. As such, using terminal voltage phasors as input can guarantee the stability of all the states and in turn the detectability of the subsystem.

b) If the system is fully observable, i.e., $r=n$, K can be selected to place the eigenvalues of $(A - KC)$ at desired locations [10] to make the system detectable. Thus, the observability requirement proposed in [3] [5] for the DSE of a power system is a sufficient condition for the existence of a DSE observer.

c) The system is detectable if all the eigenvalues corresponding to the unobservable states are stable. If the system is not fully observable, (6) can be obtained by substituting A and C of (3) into (5). In (6), the eigenvalues of $(A - KC)$ come from two matrices: $(A_o - K_o C_o)$ and A_{no}. The eigenvalues of $(A_o - K_o C_o)$, which correspond to the observable state $x_o(t)$, can be placed at a stable location by choosing the right K_o. By contrast, K cannot influence the eigenvalues of A_{no}. Thus, the convergence of the DSE depends on the stability of the eigenvalues of A_{no}. If an eigenvalue is not stable, the states in $x_{no}(t)$ are not detectable because the corresponding estimation errors $\Delta x_{no}(t)$ will continue to grow. If all the eigenvalues of A_{no} are stable, the states in $x_{no}(t)$ are detectable because the corresponding estimation errors $\Delta x_{no}(t)$ will be damped out. On the *s*-plane, the rightmost eigenvalue determines the lowest damping effect on the estimation errors. Thus, less damping of the rightmost eigenvalue should indicate lower convergence speed and larger estimation errors of the participating states.

$$\begin{aligned} A - KC &= \begin{bmatrix} A_{no} & A_{12} \\ 0 & A_o \end{bmatrix} - \begin{bmatrix} K_{no} \\ K_o \end{bmatrix} [0 \quad C_o] \\ &= \begin{bmatrix} A_{no} & A_{12} - K_{no}C_o \\ 0 & A_o - K_o C_o \end{bmatrix} \end{aligned} \tag{6}$$

For power system DSE applications, detectability can be more relevant than observability because DSE aims to estimate the current states based on historical evolution.

III. Case Studies

In this section, the observability and detectability analyses are carried out on synchronous machine number 5 in the IEEE 10-machine 39-bus system, of which the description can be found in [11]. The model and simulation setups are the same as those used in [12]. The differential equations for modeling the synchronous machine are described in Appendix A of [7].

To isolate machine number five from the rest of the system for DSE, it is assumed that a phasor measurement unit is placed at the generator bus to measure the voltage phasors (V) and current phasor (I). The real power (P_e) and reactive power (Q_e) are derived from current and voltage phasors. To implement the proposed method, the Jacobian matrices in (1) are derived using the perturbation method. MATLAB® function *obsvf.m* is used to transform the model into the canonical observability form in (3). The UKF is implemented to estimate the nine dynamic states of the synchronous machine, which consist of four states for its generator ($\delta, \omega_r, E'_d, E'_q$), three states for its exciter (E_{fd}, V_F, V_R), and two states for its turbine-governor model (T_M, P_{SV}).

A. DSE with Different Measurement Configurations

The dynamic states are estimated using the UKF through the following four combinations of inputs and outputs: **(a)** V as inputs, P_e and Q_e as outputs; **(b)** V as inputs, I is not available due to measurement loss (i.e., no output and C=0); **(c)** I as inputs, P and Q as outputs; **(d)** I as inputs, V is not available due to measurement loss (i.e., no output and C=0). The DSE results are summarized in Fig. 1. Because space is limited, only the results of relative rotor angle δ_{5-1}, rotor speed ω_{r5}, exciter field voltage E_{fd5}, and governor mechanical power T_{M5} are shown. It is observed that the UKF can estimate the states reasonably well for cases (a), (b), and (c), whereas it diverges for case (d).

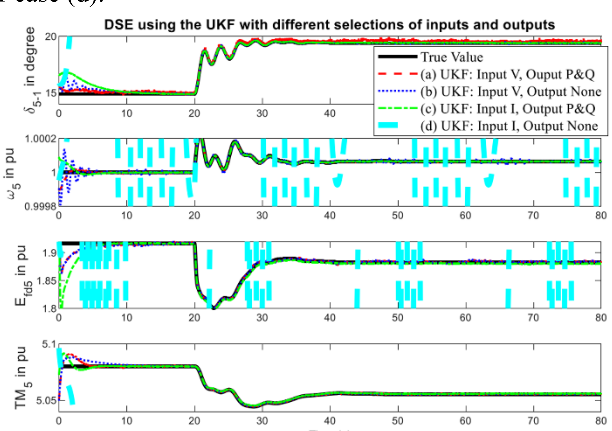

Fig. 1. DSE results from different selections of inputs and outputs.

B. Observability Analysis

To assess the observability, the system is linearized around each operational point. The median and median absolute deviation (MAD) of the SSVs of the observability matrices are summarized in Table I for different input/output selections. Table I shows that when both P and Q are used as the outputs for cases (a) and (c), the SSVs are significant and therefore the system is considered observable (i.e., all the states are observable). By contrast, when there is no output, such as cases (b) and (d), the observability matrices and all their singular values are zero. As such, the system is not observable (i.e., all the states are unobservable). Similar observability behaviors are observed using Lie's method [5] and the unobservability index method [4] for observability analysis. In addition, case studies are carried out using P_e and Q_e individually as the output, and observability analysis results are similar to cases (a) and (c). Details are not included to stay concise. Note that even though the system is not observable for (b), its states can still be estimated reasonably well, as shown in Fig. 1.

TABLE I. SSVs for Different Measurement Selections

Inputs	***Outputs***	***SSV*** $(\boldsymbol{Median}) \pm (\boldsymbol{MAD})$	***Convergence of the UKF***
(a) V	$\boldsymbol{P_e}$ **&** $\boldsymbol{Q_e}$	$(0.0942) \pm (0.0006)$	Yes
(b) V	**None**	0 ± 0	Yes
(c) I	$\boldsymbol{P_e}$ **&** $\boldsymbol{Q_e}$	$(0.0357) \pm (0.0002)$	Yes
(d) I	**None**	0 ± 0	No

C. Detectability Analysis

To explain why the dynamic states can still be estimated when all the states of the synchronous machine are unobservable in case (b), the detectability analysis is carried out. Following the method described in subsection II.C, the rightmost eigenvalues of A_{no} are estimated for case (b) and their real parts are in the range of -0.0605 ± 0.0002. In other words, the unobservable states have stable eigenvalues and the system is detectable. Even though the system is not observable and its initial states cannot be uniquely determined, the current states will converge to their true states as time progresses. Because measured voltage phasors are used as the model inputs in case (b), the stability of the synchronous machine model is expected because it is designed to be stable when connected to an infinite bus. By contrast, for case (d), the rightmost eigenvalues of A_{no} are in the range of 0.772 ± 0.003, which is unstable. Thus, the system is neither observable nor detectable. As a result, the UKF diverges in case (d).

D. Effects of the Eigenvalues of Unobservable States

To study the effects of the eigenvalues of the unobservable states on the DSE, the damping of the rightmost eigenvalue in case (c) is modified and its effect on DSE is examined. To identify the most sensitive parameters that can change the rightmost eigenvalue, the states are sorted according to the magnitudes of their participation factors (PFs) [13]. The dominant PFs of $(0.9102) \pm (0.0003)$ and $(0.07584) \pm (0.0003)$ are associated with E'_q and E'_d respectively, which indicate that states E'_q and E'_d actively interact with the eigenvalue. Other states have fairly small PFs and therefore do not noticeably participate the oscillation of the eigenvalue. Because the PF is equal to the sensitivity index of the eigenvalue with respect to the diagonal element of the differential equation [13], the sensitivity of eigenvalue damping with respect to the open-circuit time constant T'_{d0} is positive. Note that according to differential equation (28) in [7], the diagonal element of E'_q is determined by $-\frac{1}{T'_{d0}}$.

Setting $T'_{d0} = 3.3, 33, and\ 330\ s$, the corresponding positions of the rightmost eigenvalues are $\lambda_{3.3} = -0.213$,

$\lambda_{33} = -0.061, and\ \lambda_{333} = -0.007$. In other words, increasing the open circuit time constant T'_{d0} can move the eigenvalue to the right, thus decreasing the damping of the eigenvalue. This observation is consistent with results from the above sensitivity analysis of the eigenvalue. The state estimation errors of E'_q and E'_d are plotted as time series in Fig. 2 for different values of time constant T'_{d0} . Fig. 2 shows that, at the very beginning, estimation errors are similar. Also, the estimation errors decrease faster for smaller T'_{d0} (i.e., better damping of the eigenvalue) and the mean squared errors (MSE) are smaller for smaller T'_{d0}. The other states do not noticeably participate the oscillation of the eigenvalue and the effects of the eigenvalue damping on the estimation errors of other states are not noticeable. These observations suggest that the increasing damping of the rightmost eigenvalue of the unobservable states can speed up the convergence of DSE, and that the effects are determined by the participating states.

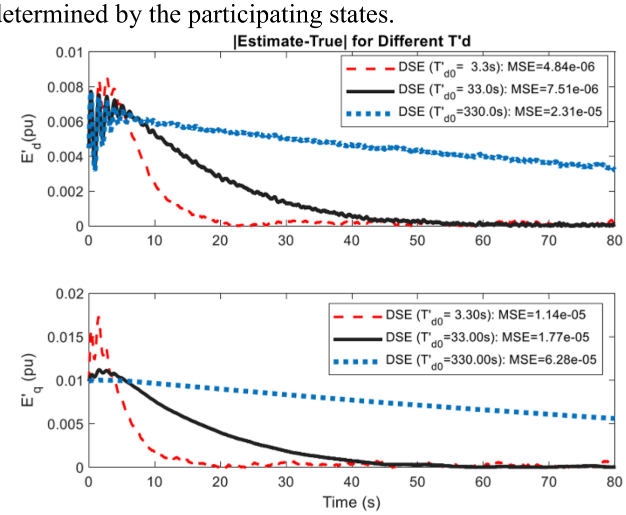

Fig. 2. Estimation errors of the DSE from different T'_{d0} values.

IV. Conclusions

In this paper, detectability analysis is applied to guide selection of measurements for the DSE of a synchronous machine. To ensure the existence of a DSE observer, early studies adopted observability analysis, which suggests that measurements must be selected to make all the states observable. This paper shows that the requirement of observability is a sufficient condition and can be relaxed by applying detectability analysis, which suggests that DSE can converge not only for an observable system but also for an unobservable system if the eigenvalues corresponding to the unobservable states are stable. The simulation results show that better-damped eigenvalues of the unobservable states will result in faster convergence of the DSE. In addition, it is suggested that the terminal voltage phasor is a good candidate for the input of a subsystem in the power grid because that selection can guarantee the stability of the subsystem, and in turn its detectability and the convergence of a DSE observer. Relaxing the requirement via detectability analysis can reduce the number of required measurements relative to those from observability analysis. Future work will be to extend the detectability concept to deal with a strongly nonlinear system.

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