Optimal Scheduling Strategy for Networked Estimation With Energy Harvesting

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***Abstract*—Joint optimization of scheduling and estimation policies is considered for a system with two sensors and two noncollocated estimators. Each sensor produces an independent and identically distributed sequence of random variables, and each estimator forms estimates of the corresponding sequence with respect to the mean-squared error sense. The data generated by the sensors are transmitted to the corresponding estimators over a bandwidth-constrained wireless network that can support a single packet per time slot. The access to the limited communication resources is determined by a scheduler that decides which sensor measurement to transmit based on both observations. The scheduler has an energy-harvesting battery of limited capacity, which couples the decision-making problem in time. Despite the overall lack of convexity of this problem, it is shown that this system admits a globally optimal scheduling and estimation strategy pair under the assumption that the distributions of the random variables at the sensors are symmetric and unimodal. Additionally, the optimal scheduling policy has a structure characterized by a threshold function that depends on the time index and energy level. A recursive algorithm for threshold computation is provided.**

***Index Terms*—Decision theory, estimation, multi-agent systems, networked control systems, optimization, quantization.**

I. Introduction

RELIABLE real-time wireless networking is an essential requirement of modern cyber-physical and networked control systems [1], [2]. Due to their large scale, these systems are typically formed by multiple physically distributed subsystems that communicate over a wireless network of limited capacity. One way to model this communication constraint is to assume that, at any time instant, only one packet can be reliably transmitted over the network to its destination. This constraint forces the system designer to use strategies that allocate the shared communication resources among multiple communicating nodes. In addition to degrading the performance of the overall system, the fact that communication among the different agents in cyber-physical systems is imperfect often leads to team-decision problems with nonclassical information structures. Such problems are usually nonconvex, and are, in general, difficult to solve.

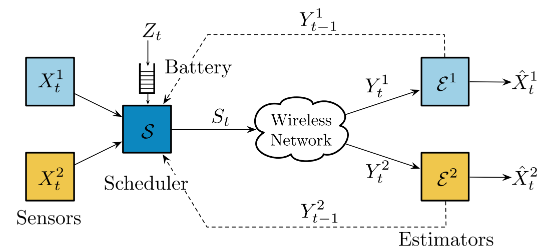

Fig. 1. Schematic diagram for the remote sensing system two sensor–estimator pairs with an energy-harvesting scheduler.

We consider a sequential remote estimation problem over a finite time horizon with noncollocated sensors and estimators. The system, shown in Fig. 1, is composed of multiple sensors, each of which has a stochastic process associated with it. Each sensor is paired with an estimator, which is interested in forming real-time estimates of its corresponding source process. The sensors communicate with their estimators via a shared communication network. Due to the limited capacity, at most, one of the sensor's observations can be transmitted at each time. To avoid collisions [3], [4], the communication is mediated by a scheduler, which observes the realization of each source. The scheduler decides at each time which of the observations (if any) gets transmitted over the communication network. In addition to the communication constraint, the framework also assumes that the scheduler operates under an energy constraint through a finite battery, which is capable of harvesting additional energy from the environment.

The designer's goal is to find scheduling and estimation strategies that jointly minimize an objective function consisting of a mean-squared estimation error criterion and a communication cost. This joint design problem is a team-decision problem with a nonclassical information structure for which obtaining globally optimal solutions is a challenging task in general [5]. However, under certain assumptions on the underlying probabilistic model, despite the difficulties imposed by lack of convexity, this problem admits an explicit globally optimal solution, whose derivation is the centerpiece of this article.

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This problem is also motivated by applications such as the Internet of Things (IoT), where there exists a necessity to coordinate access to limited communication resources by multiple heterogeneous devices in real time. In addition to that, in IoT applications, the network is expected to be able to support a massive number of users for which the traditional scheduling techniques based on random access, collision resolution, and retransmission are not feasibly implementable. Therefore, new scheduling schemes where decisions are driven by data such as the one proposed herein are becoming increasingly more relevant. This framework is also applicable to wireless body area networks, which are systems where multiple biometric sensors deployed on humans communicate with remote sensing stations over a wireless network [6]–[8]. A mobile phone is used as a hub to coordinate the access of the network among multiple sensors. The phone acts as a scheduler by collecting data from different biometric sensors and chooses in real time which one of the measurements is transmitted over the network.

A. Related Literature, Connections with Prior Work, and Contributions

Over the last few years, the problem of scheduling transmissions over limited capacity networks shared by multiple estimators/control loops has received a lot of attention [9]–[11] and references therein. To the best of our knowledge, the works of Shi and Zhang [12] and Xia *et al.* [13] were among the pioneers in characterizing the tradeoffs between communication frequency and the estimation error covariance for event-triggered scheduling schemes. Molin *et al.* [14] proposed a dynamic priority scheme for scheduling real-time data over a shared network for state estimation using the notion of Value of Information. Recently, the works of Knorn and Quevedo [15] and Knorn *et al.* [16] incorporated the features of energy-harvesting, energy-sharing, and energy-leaking sensor batteries in the computation of optimal transmission scheduling schemes. Guo *et al.* [17] addressed the critical issue of security and corresponding robustness concerning cyber-attacks in the remote estimation of multisystems scheduled over a shared collision channel.

There is a vast literature on scheduling in point-to-point communication between a single sensor and estimator. The work of Imer and Basar [18] and, subsequently, Lipsa and Martins [19], Nayyar *et al.* [20], and Wu *et al.* [21] were among the first to address the issues related to the joint design of scheduling and estimation strategies. Since then, critical new features have been incorporated into the base model. Leong *et al.* [22] characterized structural results of the optimal transmission scheduling function, displaying a threshold in the estimation error covariance and the battery's energy level. Wu *et al.* [23] and Leong *et al.* [24] studied the issue of learning the optimal scheduling strategy when the probability of packet drop by the channel is unknown. The works of Leong *et al.* [25] and Lu *et al.* [26] studied the optimal design of a threshold strategy for remote estimation in the presence of an eavesdropper under a secrecy constraint, also showing that the optimal scheduling strategy has a threshold structure.

Our work relates and contributes to the existing literature in the following aspects. The problem formulation considered herein can be seen as a generalization of the system studied in [18] to the case of multiple sensors with the addition of an energy-harvesting scheduler. Unlike other results that make structural assumptions on the estimator (linearity or piecewise linearity), our approach is to perform joint optimization without making any structural assumptions, which often leads to intractable optimization problems (see II-D). Our results, however, make assumptions on the probabilistic model of the sources similar to the ones in [19], [20], and [27]. Nonetheless, despite the simplicity of the system model, our results do not follow from trivial or any existing arguments.

Our approach is to first relax the problem by expanding the information sets at the estimators. We proceed by solving the relaxed problem using the common information approach [28]. We investigate the value functions of the dynamic program and completely characterize the jointly optimal scheduling and estimation strategies for the relaxed problem. We show that the globally optimal solution for the relaxed problem is independent of the additional information introduced in the expansion, and, therefore, it is also optimal for the original problem.

The main contributions of this work are as follows:

1) We establish the joint optimality of a pair of scheduling estimation strategies for a sequential problem formulation with independent and identically distributed (i.i.d.) sources and an energy-harvesting scheduler under symmetry and unimodality assumptions of the observations' probability density functions (pdfs).
2) We provide a proof strategy that uses a combination of the expansion of information structures and the common information approach.
3) We illustrate our theoretical results with numerical examples.

B. Notation

We adopt the following notation: Random variables and random vectors are represented using upper case letters, such as X. Realizations of random variables and random vectors are represented by the corresponding lower case letter, such as x. We use $X_{a:b}$ to denote the collection of random variables $(X_a, X_{a+1}, \dots, X_b)$. The pdf of a continuous random variable X, provided that it is well defined, is denoted by π. Functions and functionals are denoted using calligraphic letters such as $\mathcal{F}$. We use $\mathcal{N}(m, \sigma^2)$ to represent the Gaussian probability distribution of mean m and variance σ^2, respectively. The real line is denoted by $\mathbb{R}$. The set of natural numbers is denoted by $\mathbb{N}$. The set of non-negative integers is denoted by $\mathbb{Z}_{\geq 0}$. The probability of an event $\mathfrak{E}$ is denoted by $\mathbb{P}(\mathfrak{E})$; the expectation of a random variable Z is denoted by $\mathbb{E}[Z]$. The indicator function of a statement $\mathfrak{S}$ is defined as follows:

$$\mathbb{I}(\mathfrak{S}) \stackrel{\text{def}}{=} \begin{cases} 1 & \text{if } \mathfrak{S} \text{ is true} \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

We also adopt the following convention.

1) Consider the set $\mathbb{W} \stackrel{\text{def}}{=} \{1, 2, \ldots, N\}$ and a function $\mathcal{F} : \mathbb{W} \to \mathbb{R}$ are given. If $\overline{\mathbb{W}}$ is the subset of elements that maximize $\mathcal{F}$, then $\arg\max_{\alpha \in \mathbb{W}} \mathcal{F}(\alpha)$ is defined as the smallest number in $\overline{\mathbb{W}}$.

II. Problem Statement

A. Basic Definitions

Consider a system with two sensor–estimator pairs and one energy-harvesting scheduler. All the subsequent results hold for an arbitrary number of sensor–estimator pairs, a fact that will be formally stated in IX-A. Therefore, the focus on two sensor–estimator pairs is without loss of generality.

The system operates sequentially over a finite time horizon $T \in \mathbb{N}$. The role of the scheduler is to mediate the communication between the sensors and estimators such that, at any given time step, at most, one sensor–estimator pair is allowed to communicate. We proceed to define the stochastic processes observed at the sensors. Let $X_t^i \in \mathbb{R}^{n_i}$ denote the random vector observed at the ith sensor, $t \in \{1, \ldots, T\}$, $i \in \{1, 2\}$. Let $n_1 + n_2 = n$. We shall refer to X_t^i, $i \in \{1, 2\}$, as outputs of information sources at time t. Throughout the article, we assume that the sources are i.i.d. in time. Moreover, the random variables X_t^i admit a pdf π_i for all $i \in \{1, 2\}$ and $t \in \{1, \ldots, T\}$. We assume that the stochastic processes $\{X_t^1, t \geq 1\}$ and $\{X_t^2, t \geq 1\}$ are independent.

The scheduler operates with a battery of finite capacity denoted by $B \in \mathbb{N}$ such that $B < T$. Let the state of the battery E_t be defined as the number of energy units available at time step t. At each time t, the scheduler makes a decision $U_t \in \{0, 1, 2\}$, where $U_t = 0$ denotes that no transmissions are scheduled, $U_t = 1$ denotes that the scheduler transmits X_t^1, and $U_t = 2$ denotes that the scheduler transmits X_t^2. Each transmission depletes the battery by one energy unit and only no transmissions can be scheduled if the battery is empty, i.e., if $E_t = 0$. Thus, the scheduling decision $U_t \in \mathbb{U}(E_t)$, where

$$\mathbb{U}(E_t) \stackrel{\text{def}}{=} \begin{cases} \{0, 1, 2\} & \text{if } E_t > 0 \\ \{0\} & \text{if } E_t = 0. \end{cases} \tag{2}$$

At time t, the scheduler harvests Z_t units of energy from the environment. The random variable Z_t is i.i.d. in time according to a probability mass function $p_Z(z)$, $z \in \mathbb{Z}_{\geq 0}$, and is independent of the information source processes. The state of the battery evolves according to the following equation:

$$E_{t+1} = \mathcal{F}(E_t, U_t, Z_t), \quad t \in \{1, \ldots, T-1\} \tag{3}$$

where

$$\mathcal{F}(E_t, U_t, Z_t) \stackrel{\text{def}}{=} \min\{E_t - \mathbb{I}(U_t \neq 0) + Z_t, B\} \tag{4}$$

and initial energy $E_1 = B$.

We will assume that the communication between the scheduler and the estimators occurs over a so-called *unicast network*, where only the intended estimator receives the transmitted packet. For $i \in \{1, 2\}$, the observation of the estimator $\mathcal{E}^i$ at time t is denoted by Y_t^i, which is determined according to $Y_t^i = h^i(X_t^i, U_t)$, where

$$h^i(X_t^i, U_t) \stackrel{\text{def}}{=} \begin{cases} X_t^i & \text{if } U_t = i \\ \varnothing & \text{if } U_t \neq i. \end{cases} \tag{5}$$

Remark 1: One way to think about the unicast network model is that there are independent point-to-point links between different sensor and estimator pairs. At each time instant, the scheduler chooses at most one of these links to be active, and the others remain idle.

B. Information Sets and Strategies

Let $\mathbf{X}_t \stackrel{\text{def}}{=} (X_t^1, X_t^2)$ and $\mathbf{Y}_t \stackrel{\text{def}}{=} (Y_t^1, Y_t^2)$. The scheduler decides what to transmit based on its available information at time t, which is $\mathcal{I}_t^{\mathcal{S}} \stackrel{\text{def}}{=} \{\mathbf{X}_{1:t}, E_{1:t}, \mathbf{Y}_{1:t-1}\}$. The decision variable U_t is computed according to a function f_t as follows:

$$U_t = f_t(\mathbf{X}_{1:t}, E_{1:t}, \mathbf{Y}_{1:t-1}). \tag{6}$$

We refer to the collection $\mathbf{f} \stackrel{\text{def}}{=} \{f_1, \ldots, f_T\}$ as the *scheduling strategy* of the scheduler.

Let $i \in \{1, 2\}$. The estimator $\mathcal{E}^i$ computes the state estimate based on the entire history of its observations, $\mathcal{I}_t^{\mathcal{E}^i} \stackrel{\text{def}}{=} \{Y_{1:t}^i\}$, according to a function g_t^i as follows:

$$\hat{X}_t^i = g_t^i(Y_{1:t}^i). \tag{7}$$

We refer to the collection $\mathbf{g}^i \stackrel{\text{def}}{=} \{g_1^i, \ldots, g_T^i\}$ as the *estimation strategy* of estimator $\mathcal{E}^i$.

Remark 2: From now on, we assume that f_t, g_t^1, and g_t^2, $t \in \{1, \ldots, T\}$, are measurable functions with respect to the appropriate sigma-algebras.

C. Cost

We consider a performance index that penalizes the mean-squared estimation error and a communication cost for every transmission made by the scheduler.

The cost functional and optimization problem are defined as follows:

$$\mathcal{J}(\mathbf{f}, \mathbf{g}^1, \mathbf{g}^2) \stackrel{\text{def}}{=} \sum_{t=1}^{T} \mathbb{E}\left[\sum_{i \in \{1,2\}} \|X_t^i - \hat{X}_t^i\|^2 + c\mathbb{I}(U_t \neq 0)\right]. \tag{8}$$

Problem 1: For the model described in this section, given the statistics of the sensor's observations, the statistics of the energy-harvesting process, the battery storage limit B, communication cost c, and the horizon T, find scheduling and estimation strategies $\mathbf{f}$, $\mathbf{g}^1$, and $\mathbf{g}^2$ that jointly minimize the cost $\mathcal{J}(\mathbf{f}, \mathbf{g}^1, \mathbf{g}^2)$ in 8.

D. Signaling

In problems of decentralized control and estimation with non-classical information structures, the optimal solutions typically involve a form of implicit communication known as *signaling*. Signaling is the effect of conveying information through actions [29], and it is the reason why problems within this class are difficult to solve, e.g., [30].

In order to illustrate the fundamental difficulty imposed by signaling, consider the instance of 1 with two zero-mean independent scalar sources, $c = 0$ and $T = 1$. Here, we will show how the coupling between scheduling and estimation leads to nonconvex optimization problems. First, consider a fixed scheduling function $f_1 : \mathbb{R}^2 \to \{1, 2\}$. Let $i, j \in \{1, 2\}$ such that $i \neq j$. Since the cost is the mean-squared error between the observations and the estimates, the optimal estimator is the conditional mean, i.e.,

$$g_1^{i\star}(y) = \mathbb{E}\left[X_1^i \mid Y_1^i = y\right], \quad i \in \{1, 2\}. \tag{9}$$

When $y = (i, x_1^i)$, we have

$$g_1^{i\star}(i, x_1^i) = x_1^i, \quad i \in \{1, 2\}. \tag{10}$$

However, when $y = \varnothing$, we have

$$g_1^{i\star}(\varnothing) = \mathbb{E}[X_i \mid f_1(X_1^1, X_1^2) \neq i] \tag{11}$$

from which two important points can be drawn: 1) The estimate $g_1^{i\star}(\varnothing)$ is an implicit function of the scheduling function f_1; 2) the event that X_i was not transmitted always carries some implicit information about X_i. It means that even no-transmission symbols received over the network can be used as side information for estimation. Therefore, solving the resulting optimization problem for the scheduling function f_1, which seeks to minimize the cost functional

$$\mathcal{J}(f_1) = \sum_{(i,j):i\neq j} \int_{\mathbb{R}^2} \left(x_1^i - g_1^{i\star}(\varnothing)\right)^2$$
$$\mathbb{I}(f_1(x_1^1, x_1^2) = j)\pi_1(x_1^1)\pi_2(x_1^2)dx_1^1 dx_1^2 \tag{12}$$

where $g_1^{i\star}(\varnothing)$ is given by 11, for arbitrary pdfs π_1 and π_2, is intractable.

If on the other hand, we fix the estimation functions g_1^1 and g_1^2, such that the following identities are satisfied:

$$g_1^i(y) = \begin{cases} x_1^i & \text{if } y = (i, x_1^i) \\ \eta_1^i & \text{if } y = \varnothing \end{cases} \tag{13}$$

where $\eta_1^i \in \mathbb{R}$, the optimal scheduler is determined by the following inequality:

$$f_1^\star(\mathbf{x}_1) = 1 \Leftrightarrow |x_1^2 - \eta_1^2| < |x_1^1 - \eta_1^1| \tag{14}$$

which leads to the following nonconvex objective function:

$$\mathcal{J}\left(g_1^1, g_1^2\right) = \mathbb{E}\left[\min\left\{\left(X_1^1 - \eta_1^1\right)^2, \left(X_1^2 - \eta_1^2\right)^2\right\}\right]. \tag{15}$$

In both cases, the globally optimal solution to 1 is nontrivial for arbitrary pdfs π_1 and π_2 due to the coupling between f_1, g_1^2, and g_2^1.

In this article, we attempt to solve the more general problem statement for arbitrary $T \geq 1$ and $c \geq 0$ assuming that the pdfs π_1 and π_2 satisfy certain properties.

III. Main Result

The following definition will be used to state our main result.

Definition 1 (Symmetric and unimodal pdfs): Let $\pi : \mathbb{R}^n \to \mathbb{R}$ be a pdf. The pdf π is symmetric and unimodal around $a \in \mathbb{R}^n$ if it satisfies the following property:

$$\|x - a\| \leq \|y - a\| \Rightarrow \pi(x) \geq \pi(y), \quad x, y \in \mathbb{R}^n. \tag{16}$$

Theorem 1: Provided that π_1 and π_2 are symmetric and unimodal around $a^1 \in \mathbb{R}^{n_1}$ and $a^2 \in \mathbb{R}^{n_2}$, respectively, the following scheduling and estimation strategies are globally optimal for 1:

$$f_t^\star(\mathbf{x}, e) \stackrel{\text{def}}{=} \begin{cases} 0, & \text{if } \max_{i\in\{1,2\}} \left\{\|x^i - a^i\|\right\} \leq \tau_t^\star(e) \\ \arg\max_{i\in\{1,2\}} \|x^i - a^i\|, & \text{otherwise} \end{cases} \tag{17}$$

where $\tau_t^\star : \mathbb{Z} \to \mathbb{R}$ is a threshold, and

$$g_t^{i\star}(y^i) \stackrel{\text{def}}{=} \begin{cases} x^i & \text{if } y^i = x^i \\ a^i & \text{if } y^i = \varnothing \end{cases} \tag{18}$$

for $t \in \{1, \ldots, T\}$.

IV. Information Structures

Problem 1 can be understood as a sequential stochastic team with three decision makers: The scheduler and the two estimators. One key aspect to note is that Problem 1 has a nonclassical information structure. Such team problems are usually nonconvex, and their solutions are found on a case-by-case basis. Our analysis relies on the *common information approach* [28], where the idea is to transform the decentralized problem into an equivalent centralized one where the information for decision making is the common information among all the decision makers in the decentralized system.

We begin by establishing a structural result for the optimal scheduling strategy. The following lemma states that the scheduler may ignore the past state observations at each sensor without any loss of optimality.

Lemma 1: Without loss of optimality, the scheduler can be restricted to strategies of the form

$$U_t = f_t\left(\mathbf{X}_t, E_{1:t}, \mathbf{Y}_{1:t-1}\right). \tag{19}$$

Proof: Let the strategy profile of the estimators $\mathbf{g}^1$ and $\mathbf{g}^2$ be arbitrarily fixed. The problem of selecting the best scheduling policy (for the fixed estimation strategy profiles $\mathbf{g}^1$ and $\mathbf{g}^2$) simplifies to a Markov decision process (MDP), whose state is defined as $S_t \stackrel{\text{def}}{=} (\mathbf{X}_t, E_{1:t}, \mathbf{Y}_{1:t-1})$. Using simple arguments involving conditional probabilities and the basic definitions of II-A, we can show that the state process $\{S_t, t \geq 1\}$ is a controlled Markov chain, i.e.,

$$\mathbb{P}(S_{t+1} \mid S_{1:t}, U_{1:t}) = \mathbb{P}(S_{t+1} \mid S_t, U_t). \tag{20}$$

The cost incurred at time t of the equivalent MDP is

$$\rho(S_t, U_t) \stackrel{\text{def}}{=} \sum_{i\in\{1,2\}} \left\|X_t^i - \hat{X}_t^i\right\|^2 + c\mathbb{I}(U_t \neq 0) \tag{21}$$

$$\stackrel{(a)}{=} \sum_{i\in\{1,2\}} \left\|X_t^i - g_t^i(Y_{1:t}^i)\right\|^2 + c\mathbb{I}(U_t \neq 0) \tag{22}$$

$$\stackrel{(b)}{=} \sum_{i\in\{1,2\}} \left\|X_t^i - g_t^i\left(Y_{1:t-1}^i, h^i(X_t^i, U_t)\right)\right\|^2$$
$$+ c\mathbb{I}(U_t \neq 0) \tag{23}$$

where (a) follows from 7 and (b) follows from 5.

Thus, the problem of finding the optimal scheduling strategy to minimize the cost $\mathcal{J}(\mathbf{f}, \mathbf{g}^1, \mathbf{g}^2)$ becomes equivalent to finding the optimal decision strategy for an MDP with state process S_t and instantaneous cost $\rho(S_t, U_t)$. Standard results for MDPs [31] imply that there exists an optimal scheduling strategy of the form in lemma. Since this is true for any arbitrary $\mathbf{g}^1$ and $\mathbf{g}^2$, it is also true for the globally optimal $\mathbf{g}^{1\star}$ and $\mathbf{g}^{2\star}$. ■

Under the structural result in 1, the information sets available at the scheduler and estimators can be reduced to

$$\mathcal{I}_t^{\mathcal{S}} \stackrel{\text{def}}{=} \{\mathbf{X}_t, E_{1:t}, \mathbf{Y}_{1:t-1}\} \tag{24}$$

$$\mathcal{I}_t^{\mathcal{E}^i} \stackrel{\text{def}}{=} \left\{Y_{1:t}^i\right\}, \quad i \in \{1, 2\} \tag{25}$$

without any loss of optimality. However, the information structure described by (24) and (25) do not share any common information. In other words, the information sets $\mathcal{I}_t^{\mathcal{S}}$, $\mathcal{I}_t^{\mathcal{E}^1}$, and $\mathcal{I}_t^{\mathcal{E}^2}$ have no common random variables, a fact that limits the utility of the common information approach. We resort to a technique which consists of judiciously expanding the information available at the decision makers such that the common information approach can be more profitably employed.

A. Information Structure Expansion

We expand the estimators' information sets to the following:

$$\bar{\mathcal{I}}_t^{\mathcal{E}^1} \stackrel{\text{def}}{=} \left\{E_{1:t}, \mathbf{Y}_{1:t-1}, Y_t^1\right\} \tag{26}$$

$$\bar{\mathcal{I}}_t^{\mathcal{E}^2} \stackrel{\text{def}}{=} \left\{E_{1:t}, \mathbf{Y}_{1:t-1}, Y_t^2\right\}. \tag{27}$$

The optimal cost for Problem 1 under an expanded information structure is at least as good as the optimal cost under the original information structure. (Having more information at each estimator cannot worsen its performance.) Moreover, if the optimal solution under the expanded information structure is adapted to the original information structure, then this solution is also optimal under the original information structure [5, Proposition 3.5.1].

We proceed by defining another problem identical to Problem 1 but with expanded information sets at the estimators.

Problem 2: Consider the model of II with the expanded information sets of (26) and (27) at the estimators $\mathcal{E}^1$ and $\mathcal{E}^2$, respectively.

Given the statistics of the sensors' observations, the statistics of the energy harvested at each time, the battery storage limit B, communication cost c, and the horizon T find the scheduling and estimation strategies $\mathbf{f}, \mathbf{g}^1$, and $\mathbf{g}^2$ that jointly minimize the cost $\mathcal{J}(\mathbf{f}, \mathbf{g}^1, \mathbf{g}^2)$ in 8.

Under the expanded information structure, the common information among the decision makers is

$$\mathcal{I}_t^{\text{com}} \stackrel{\text{def}}{=} \{E_{1:t}, \mathbf{Y}_{1:t-1}\}. \tag{28}$$

Note that the common information contains several variables that were not initially available to the estimators. However, we will eventually show at the end of VI that the optimal estimation strategy for 2 does not depend on this additional information. To show this independence, we first establish the following lemma, which provides a structural result for the estimation strategies under the expanded information sets.

Lemma 2: Without loss of optimality, the search for optimal strategies for estimator $\mathcal{E}^i$ can be restricted to functions of the form

$$g_t^i(E_{1:t}, \mathbf{Y}_{1:t-1}, Y_t^i) = \begin{cases} X_t^i & \text{if } Y_t^i = X_t^i \\ \tilde{g}_t^i(E_{1:t}, \mathbf{Y}_{1:t-1}) & \text{otherwise.} \end{cases} \tag{29}$$

Proof: Let the strategy of the scheduler be fixed to some arbitrary $\mathbf{f}$. We can view 2 from the perspective of the estimator $\mathcal{E}^i$ at time t as follows:

$$\inf_{g_t^i} \ \mathbb{E}\left[\|X_t^i - \hat{X}_t^i\|^2\right] + \tilde{\mathcal{J}} \tag{30}$$

where

$$\tilde{\mathcal{J}} \stackrel{\text{def}}{=} \mathbb{E}\left[\sum_{k=1}^{T} c\mathbb{I}(U_k \neq 0) + \sum_{k=1}^{T}\sum_{j\neq i} \|X_k^j - \hat{X}_k^j\|^2 + \sum_{k\neq t} \|X_k^i - \hat{X}_k^i\|^2\right]. \tag{31}$$

Note that the estimation function g_t^i only affects the value of the estimate $\hat{X}_t^i$, i.e.,

$$\hat{X}_t^i = g_t^i(\bar{\mathcal{I}}_t^{\mathcal{E}^i}) \tag{32}$$

which does not appear in 31. Since g_t^i does not affect $\tilde{\mathcal{J}}$, the optimal estimate can be computed by solving

$$\inf_{g_t^i} \ \mathbb{E}\left[\|X_t^i - \hat{X}_t^i\|^2\right]. \tag{33}$$

This is the standard minimum mean-square error estimation problem whose solution is the conditional mean, i.e.,

$$\hat{X}_t^i = \mathbb{E}\left[X_t^i \mid \bar{\mathcal{I}}_t^{\mathcal{E}^i}\right]. \tag{34}$$

Therefore, the optimal estimation strategy is of the form

$$g_t^{i\star}(\bar{\mathcal{I}}_t^{\mathcal{E}^i}) = \begin{cases} X_t^i & \text{if } Y_t^i = X_t^i \\ \mathbb{E}\left[X_t^i \mid E_{1:t}, \mathbf{Y}_{1:t-1}, Y_t^i = \varnothing\right] & \text{otherwise.} \end{cases} \tag{35}$$

Note that $(E_{1:t}, \mathbf{Y}_{1:t-1})$ is known to $\mathcal{E}^i$ in Problem 2. Thus

$$\tilde{g}_t^i(E_{1:t}, \mathbf{Y}_{1:t-1}) \stackrel{\text{def}}{=} \mathbb{E}\left[X_t^i \mid E_{1:t}, \mathbf{Y}_{1:t-1}, Y_t^i = \varnothing\right]. \tag{36}$$

Since 35 holds for any $\mathbf{f}$, it also holds for the globally optimal scheduling strategy $\mathbf{f}^\star$. Therefore, the optimal estimate is of the form given in the lemma. ■

V. An Equivalent Problem with a Coordinator

In this section, we will formulate a problem that will be used to solve 2. We consider the model of II and introduce a fictitious decision maker referred to as the *coordinator*, which has access to the common information $\mathcal{I}_t^{\text{com}}$. The coordinator is the only decision maker in the new problem. The scheduler and the estimators act as "passive decision makers" to which strategies chosen by the coordinator are prescribed.

The equivalent system operates as follows: Let n_1 and n_2 denote the dimensions of the observation made by sensors 1 and 2, respectively. At each time t, based on $\mathcal{I}_t^{\text{com}}$, the coordinator chooses a map $\Gamma_t : \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \to \{0,1,2\}$ for the scheduler, and a vector $\tilde{X}_t^i \in \mathbb{R}^{n_i}$ for each estimator $\mathcal{E}^i$, $i \in \{1,2\}$. The function Γ_t and vectors $\tilde{X}_t^1$ and $\tilde{X}_t^2$ are referred to as the scheduling and estimation *prescriptions*. The scheduler uses its prescription to evaluate U_t according to

$$U_t = \Gamma_t(\mathbf{X}_t). \tag{37}$$

The estimator $\mathcal{E}^i$ uses its prescription to compute the estimate $\hat{X}_t^i$ according to

$$\hat{X}_t^i = \begin{cases} X_t^i & \text{if } Y_t^i = X_t^i \\ \tilde{X}_t^i & \text{otherwise.} \end{cases} \tag{38}$$

The coordinator selects its prescriptions for the scheduler and the estimators using strategies d_t, ℓ_t^1, and ℓ_t^2 as follows:

$$\Gamma_t = d_t(E_{1:t}, \mathbf{Y}_{1:t-1}) \tag{39}$$

and

$$\tilde{X}_t^i = \ell_t^i(E_{1:t}, \mathbf{Y}_{1:t-1}), \;\; i \in \{1,2\}. \tag{40}$$

We refer to the collections $\mathbf{d} \stackrel{\text{def}}{=} \{d_1, \ldots, d_T\}$ and $\boldsymbol{\ell}^i \stackrel{\text{def}}{=} \{\ell_t^i, \ldots, \ell_T^i\}$ as the prescription strategies for the scheduler and the estimator $\mathcal{E}^i$, respectively. The strategies $\boldsymbol{\ell}^1$ and $\boldsymbol{\ell}^2$ must be a valid estimation strategies in Problem 2. The strategy $\boldsymbol{d}$ must be such that

$$f_t(X_t, E_{1:t}, Y_{1:t-1}) \stackrel{\text{def}}{=} \left[d_t(E_{1:t}, Y_{1:t-1})\right](X_t) \tag{41}$$

is a valid scheduling strategy in Problem 2. The cost incurred by the prescription strategies $\mathbf{d}, \boldsymbol{\ell}^1$, and $\boldsymbol{\ell}^2$ is identical as in 8, i.e.,

$$\hat{\mathcal{J}}(\mathbf{d}, \boldsymbol{\ell}^1, \boldsymbol{\ell}^2) = \sum_{t=1}^{T} \mathbb{E}\left[c\mathbb{I}(U_t \neq 0) + \sum_{i \in \{1,2\}} \|X_t^i - \hat{X}_t^i\|^2\right]. \tag{42}$$

Problem 3: Find prescription strategies $\mathbf{d}, \boldsymbol{\ell}^1$, and $\boldsymbol{\ell}^2$ that jointly minimize $\hat{\mathcal{J}}(\mathbf{d}, \boldsymbol{\ell}^1, \boldsymbol{\ell}^2)$.

Problem 3 is equivalent to Problem 2 in the sense that for every scheduling strategy $\mathbf{f}$ and estimation strategies $\mathbf{g}^1, \mathbf{g}^2$ in Problem 2, there exist prescription strategies $\mathbf{d}, \boldsymbol{\ell}^1$, and $\boldsymbol{\ell}^2$ such that $\mathcal{J}(\mathbf{f}, \mathbf{g}^1, \mathbf{g}^2) = \hat{\mathcal{J}}(\mathbf{d}, \boldsymbol{\ell}^1, \boldsymbol{\ell}^2)$ and vice versa. Thus, solving Problem 3 allows us to obtain optimal $\mathbf{f}^\star, \mathbf{g}^{1\star}$, and $\mathbf{g}^{2\star}$ for Problem 2. The same technique is used in [20] to prove a similar equivalence in a problem involving a single sensor–estimator pair.

Problem 3 can be described as a centralized partially observed Markov decision process (POMDP) as follows:

1) *State process:*
The state is $S_t \stackrel{\text{def}}{=} (\mathbf{X}_t, E_t)$.
2) *Action process:*
Let the set $\mathbb{A}(E_t)$ be defined as the collection of all measurable functions from $\mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \to \mathbb{U}(E_t)$, where $\mathbb{U}$ is defined in 2. The coordinator selects the prescription for the network manager, $\Gamma_t \in \mathbb{A}(E_t)$, and the prescriptions for the estimators $\tilde{X}_t^1 \in \mathbb{R}^{n_1}$ and $\tilde{X}_t^2 \in \mathbb{R}^{n_2}$.
3) *Observations:*
After choosing its action at time t, the coordinator observes Y_t and E_{t+1}.
4) *Instantaneous cost:*
Let $\tilde{\mathbf{X}}_t \stackrel{\text{def}}{=} (\tilde{X}_t^1, \tilde{X}_t^2)$. The instantaneous cost incurred is given by

$$\rho(\mathbf{X}_t, \Gamma_t, \tilde{\mathbf{X}}_t) \stackrel{\text{def}}{=} \begin{cases} \sum_{i \in \{1,2\}} \|X_t^i - \tilde{X}_t^i\|^2 & \text{if } \Gamma_t(\mathbf{X}_t) = 0 \\ c + \|X_t^2 - \tilde{X}_t^2\|^2 & \text{if } \Gamma_t(\mathbf{X}_t) = 1 \\ c + \|X_t^1 - \tilde{X}_t^1\|^2 & \text{if } \Gamma_t(\mathbf{X}_t) = 2. \end{cases} \tag{43}$$

5) *Markovian dynamics:*
Since $\mathbf{X}_t$ is an i.i.d process, $\mathbf{X}_{t+1}$ is independent of S_t. The evolution of the energy E_{t+1} is given by

$$E_{t+1} = \min\left\{E_t - \mathbb{I}\left(\gamma_t(\mathbf{X}_t) \neq 0\right) + Z_t, B\right\}. \tag{44}$$

Noting that 44 can be written as a function of the state S_t, action γ_t, and the noise Z_t, the state S_t satisfies 20 and forms a controlled Markov chain.

A. Dynamic Program

Having established that Problem 3 is a POMDP, the optimal prescriptions can be computed by solving a dynamic program whose information state is the belief of the state process given the common information. However, since E_t is perfectly observed, the coordinator only needs to form a belief on $\mathbf{X}_t$. Let $\mathbf{x} = (x^1, x^2)$. We define the belief state at time t as follows:

$$\Pi_t(\mathbf{x}) \stackrel{\text{def}}{=} \mathbb{P}\left(\mathbf{X}_t = \mathbf{x} \mid E_{1:t}, \mathbf{Y}_{1:t-1}\right). \tag{45}$$

Since the sources are i.i.d. and independent of the energy process, we have

$$\Pi_t(\mathbf{x}) = \pi(\mathbf{x}), \;\; t \in \{1, \ldots, T\} \tag{46}$$

where, due to the independence of the sources

$$\pi(\mathbf{x}) = \pi_1(x^1)\pi_2(x^2). \tag{47}$$

Lemma 3: Define the functions $\mathcal{V}_t^\pi : \mathbb{Z} \to \mathbb{R}$ for $t \in \{0, 1, \ldots, T+1\}$ as follows:

$$\mathcal{V}_{T+1}^\pi(e) \stackrel{\text{def}}{=} 0, \;\; e \in \{0, 1, \ldots, B\} \tag{48}$$

and

$$\mathcal{V}_t^\pi(e) \stackrel{\text{def}}{=} \inf_{\tilde{\mathbf{x}}_t, \gamma_t} \mathbb{E}\left[\rho(\mathbf{X}_t, \gamma_t, \tilde{\mathbf{x}}_t) + \mathcal{V}_{t+1}^\pi\left(\mathcal{F}\left(e, \gamma_t(\mathbf{X}_t), Z_t\right)\right)\right] \tag{49}$$

where $\tilde{\mathbf{x}}_t \in \mathbb{R}^n$, $\gamma_t \in \mathbb{A}(e)$.

If the infimum in 49 is achieved, then at each time $t \in \{1, \ldots, T\}$ and for each $e \in \{0, 1, \ldots, B\}$, the minimizing γ_t and $\tilde{\mathbf{x}}_t$ in 49 determines the optimal prescriptions for the network manager and the estimators, respectively. Furthermore, $\mathcal{V}_1(B)$ is the optimal cost for Problem 3.

Proof: This result follows from standard dynamic programming arguments for POMDPs. ■

VI. Solving the Dynamic Program

In this section, we will find the optimal prescriptions using the dynamic program in 3. For the remainder of this section, without loss of generality, we will assume that π_1 and π_2 are symmetric and unimodal around 0. The same arguments apply for general $a^i \in \mathbb{R}^{n_i}$, $i \in \{1,2\}$.

Note that each step of the dynamic program in 49 is an optimization problem with respect to $\tilde{\mathbf{x}}_t$ and γ_t. This is an infinite-dimensional optimization problem since γ_t is a mapping which lies in $\mathbb{A}(E_t)$. The next lemma will describe the structure of the optimal prescription for the scheduler and show that the infinite-dimensional optimization in 49 can be reduced to a finite-dimensional problem with respect to the vector $\tilde{\mathbf{x}}_t$. For that purpose, we define the functions $\mathcal{C}^0_{t+1}, \mathcal{C}^1_{t+1} : \mathbb{Z} \to \mathbb{R}$ as follows:

$$\mathcal{C}^0_{t+1}(e) \stackrel{\text{def}}{=} \mathbb{E}\left[\mathcal{V}^\pi_{t+1}\left(\min\{e+Z_t, B\}\right)\right] \tag{50}$$

$$\mathcal{C}^1_{t+1}(e) \stackrel{\text{def}}{=} c + \mathbb{E}\left[\mathcal{V}^\pi_{t+1}\left(\min\{e-1+Z_t, B\}\right)\right]. \tag{51}$$

Lemma 4: Suppose the prescription to the estimators are $\tilde{x}^1_t, \tilde{x}^2_t$ at time t. Then, the optimal prescription to the scheduler has the following form when $e > 0$:

$$\gamma^\star_t(\mathbf{x}_t) \stackrel{\text{def}}{=} \begin{cases} 0, & \text{if } \max_{i\in\{1,2\}}\left\{\|x^i_t - \tilde{x}^i_t\|\right\} \le \tau^\star_t(e) \\ \arg\max_{i\in\{1,2\}}\left\{\|x^i_t - \tilde{x}^i_t\|\right\}, & \text{otherwise} \end{cases} \tag{52}$$

where $\tau^\star_t(e) \stackrel{\text{def}}{=} \sqrt{\mathcal{C}^1_{t+1}(e) - \mathcal{C}^0_{t+1}(e)}$.[1] Moreover, the value function $\mathcal{V}^\pi_t$ of 3 can be obtained by solving the finite-dimensional optimization in the following equation (53) shown at the bottom of this page.

Proof: If $e = 0$, there is only one feasible scheduling policy

$$\gamma^\star_t(\mathbf{x}_t) = 0, \quad \mathbf{x}_t \in \mathbb{R}^n. \tag{54}$$

Therefore

$$\mathcal{V}^\pi_t(0) = \inf_{\tilde{\mathbf{x}}_t} \mathbb{E}\left[\sum_{i\in\{1,2\}} \|X^i_t - \tilde{x}^i_t\|^2\right] + \mathcal{C}^0_{t+1}(0). \tag{55}$$

If $e > 0$, the value function in 49 can be written as in the following equation:

$$\mathcal{V}^\pi_t(e) = \inf_{\tilde{\mathbf{x}}_t}\left\{\inf_{\gamma_t}\int\left[\left(\sum_{i\in\{1,2\}}\|x^i_t-\tilde{x}^i_t\|^2 + \mathcal{C}^0_{t+1}(e)\right)\mathbb{I}(\gamma_t(\mathbf{x}_t)=0) + \left(\|x^2_t-\tilde{x}^2_t\|^2+\mathcal{C}^1_{t+1}(e)\right)\mathbb{I}(\gamma_t(\mathbf{x}_t)=1) + \left(\|x^1_t-\tilde{x}^1_t\|^2+\mathcal{C}^1_{t+1}(e)\right)\mathbb{I}(\gamma_t(\mathbf{x}_t)=2)\right]\pi(\mathbf{x}_t)d\mathbf{x}_t\right\}. \tag{56}$$

For any fixed $\tilde{x}^i_t \in \mathbb{R}^{n_i}$, $i \in \{1,2\}$, the scheduling prescription that achieves the minimum in the inner optimization problem in 56 is determined as follows.

1) $\gamma^\star_t(\mathbf{x}_t) = 0$ if and only if

$$\|x^i_t - \tilde{x}^i_t\|^2 \le \mathcal{C}^1_{t+1}(e) - \mathcal{C}^0_{t+1}(e), \quad i \in \{1,2\}. \tag{57}$$

2) $\gamma^\star_t(\mathbf{x}_t) = 1$ if and only if

$$\|x^1_t - \tilde{x}^1_t\|^2 > \mathcal{C}^1_{t+1}(e) - \mathcal{C}^0_{t+1}(e) \tag{58}$$

and

$$\|x^1_t - \tilde{x}^1_t\| \ge \|x^2_t - \tilde{x}^2_t\|. \tag{59}$$

3) $\gamma^\star_t(\mathbf{x}_t) = 2$ if and only if

$$\|x^2_t - \tilde{x}^2_t\|^2 > \mathcal{C}^1_{t+1}(e) - \mathcal{C}^0_{t+1}(e) \tag{60}$$

and

$$\|x^2_t - \tilde{x}^2_t\| > \|x^1_t - \tilde{x}^1_t\|. \tag{61}$$

Therefore

$$\gamma^\star_t(\mathbf{x}_t) \stackrel{\text{def}}{=} \begin{cases} 0, & \text{if } \max_{i\in\{1,2\}}\left\{\|x^i_t - \tilde{x}^i_t\|^2\right\} \le \mathcal{C}^1_{t+1}(e) - \mathcal{C}^0_{t+1}(e) \\ \arg\max_{i\in\{1,2\}}\left\{\|x^i_t - \tilde{x}^i_t\|\right\}, & \text{otherwise.} \end{cases} \tag{62}$$

Using the optimal scheduling prescription in 62, the value function becomes

$$\mathcal{V}_t(e) = \inf_{\tilde{\mathbf{x}}_t} \mathbb{E}\left[\min\left\{\|X^1_t - \tilde{x}^1_t\|^2 + \|X^2_t - \tilde{x}^2_t\|^2 + \mathcal{C}^0_{t+1}(e), \|X^2_t - \tilde{x}^2_t\|^2 + \mathcal{C}^1_{t+1}(e), \|X^1_t - \tilde{x}^1_t\|^2 + \mathcal{C}^1_{t+1}(e)\right\}\right]. \tag{63}$$

■

Lemma 4 implies that the optimal solution to 3 can be found by solving the finite-dimensional optimization problem in 53. We will show that 53 admits a globally optimal solution under certain conditions on the probabilistic structure of the problem.

Lemma 5: Let X^1_t and X^2_t be independent continuous random vectors with pdfs π_1 and π_2. Provided that π_1 and π_2 are symmetric and unimodal around zero[2] then $\tilde{\mathbf{x}}^\star_t = 0$ is a global minimizer in 53 for all $e \in \{0, 1, \ldots, B\}$.

Proof: The proof is in Appendix B. ■

We are now ready to provide the proof of 1.

Proof of Theorem 1: We will first show that $(\mathbf{f}^\star, \mathbf{g}^{1\star}, \mathbf{g}^{2\star})$ as defined in 1 is globally optimal for 2.

[1]The function $\mathcal{C}^1_{t+1}(e)$ is larger than $\mathcal{C}^0_{t+1}(e)$. Therefore, the threshold $\tau^\star_t(e)$ is a real number for all $e \in \{1, \ldots, B\}$ and $t \in \{1, \ldots, T\}$.

[2]This assumption is without loss of generality. The same result holds for pdfs symmetric and unimodal around arbitrary $a^i \in \mathbb{R}^{n_i}$, $i \in \{1,2\}$, with $\tilde{\mathbf{x}}^\star_t = (a^1, a^2)$ instead of $\tilde{\mathbf{x}}^\star_t = 0$.

$$\mathcal{V}^\pi_t(e) = \begin{cases} \inf_{\tilde{\mathbf{x}}_t} \mathbb{E}\left[\sum_{i\in\{1,2\}}\|X^i_t - \tilde{x}^i_t\|^2\right] + \mathcal{C}^0_{t+1}(e) & \text{if } e = 0 \\ \inf_{\tilde{\mathbf{x}}_t} \mathbb{E}\left[\min\left\{\sum_{i\in\{1,2\}}\|X^i_t - \tilde{x}^i_t\|^2 + \mathcal{C}^0_{t+1}(e), \|X^2_t - \tilde{x}^2_t\|^2 + \mathcal{C}^1_{t+1}(e), \|X^1_t - \tilde{x}^1_t\|^2 + \mathcal{C}^1_{t+1}(e)\right\}\right] & \text{if } e > 0 \end{cases} \tag{53}$$

The optimal prescriptions for 3 are obtained using 4 5. The optimal prescription for the scheduler is given by

$$\gamma_t^\star(\mathbf{x}_t) \stackrel{\text{def}}{=} \begin{cases} 0, & \text{if } \max_{i\in\{1,2\}} \{\|x_t^i\|\} < \tau_t^\star(e) \\ \arg\max_{i\in\{1,2\}} \{\|x_t^i\|\}, & \text{otherwise} \end{cases} \quad (64)$$

whose threshold functions $\tau_t^\star(e)$ can be computed recursively (see VII); and the optimal prescription for the estimators are

$$\tilde{x}_t^{i\star} = 0, \quad i \in \{1,2\}. \quad (65)$$

Therefore, using the equivalence between 2 and 3, the optimal strategy profiles for 2 are

$$f_t^\star(\mathbf{x}_t, e_t) \stackrel{\text{def}}{=} \begin{cases} 0, & \text{if } \max_{i\in\{1,2\}} \{\|x_t^i\|\} < \tau_t^\star(e_t) \\ \arg\max_{i\in\{1,2\}} \|x_t^i\|, & \text{otherwise} \end{cases} \quad (66)$$

and

$$g_t^{i\star}\left(y_t^i\right) \stackrel{\text{def}}{=} \begin{cases} x_t^i & \text{if } y_t^i = x_t^i \\ 0 & \text{if } y_t^i = \varnothing. \end{cases}, \quad i \in \{1,2\} \quad (67)$$

Moreover, since the solution to 2, $(\mathbf{f}^\star, \mathbf{g}^{1\star}, \mathbf{g}^{2\star})$ does not depend on the additional information provided to the estimators and is adapted to the original information structure of the estimators in 1, it is also a globally optimal strategy profile for 1. ■

VII. Computation of Optimal Thresholds

Once the structural result in 1 is established, the optimal scheduling strategy is completely specified by the sequence of optimal threshold functions $\tau_t^\star$, $t \in \{1,\ldots,T\}$. The thresholds $\tau_t^\star(e)$ are obtained using the functions $\mathcal{C}_{t+1}^0(e), \mathcal{C}_{t+1}^1(e)$ in (50) and (51). The functions $\mathcal{C}_t^0(\cdot), \mathcal{C}_t^1(\cdot)$ can be computed by computing the value functions $\mathcal{V}_t^\pi$ via a backward inductive procedure. Note that we can simplify the expression for the value function using 5 and 53 to

$$\mathcal{V}_t^\pi(0) = \mathbb{E}\left[\|X_t^1\|^2 + \|X_t^2\|^2 + \mathcal{V}_{t+1}^\pi\left(\min\{Z_t, B\}\right)\right] \quad (68)$$

and

$$\mathcal{V}_t^\pi(e) = \mathbb{E}\left[\min\left\{\|X_t^1\|^2 + \|X_t^2\|^2 + \mathcal{C}_{t+1}^0(e), \right.\right.$$
$$\left.\left. \|X_t^2\|^2 + \mathcal{C}_{t+1}^1(e), \|X_t^1\|^2 + \mathcal{C}_{t+1}^1(e)\right\}\right] \quad \text{if } e > 0. \quad (69)$$

The following algorithm outlines the recursive computation of the threshold function $\tau_t^\star$:

Remark 3: The expectations in the algorithm are taken with respect to the random vectors X_t^1 and X_t^2. Computing these expectations for high-dimensional random vectors may be computationally intensive for some source distributions, but, in practice, they can be approximated using Monte Carlo methods. The remaining operations in the algorithm admit efficient implementations.

Algorithm 1: Computing the Optimal Threshold Functions $\tau_t^\star$.

Initialization:
$t \leftarrow T$
Set $\mathcal{V}_{T+1}^\pi(e) \leftarrow 0$ for $e \in \{0,\ldots,B\}$
while $t \geq 1$ **do**
  Compute $\mathcal{C}_{t+1}^0(e)$ and $\mathcal{C}_{t+1}^1(e)$ using (50) and (51) for $e \in \{1,\ldots,B\}$
  Set $\tau_t^\star(e) \leftarrow \sqrt{\mathcal{C}_{t+1}^1(e) - \mathcal{C}_{t+1}^0(e)}$ for $e \in \{1,\ldots,B\}$
  Compute $\mathcal{V}_t^\pi(e)$ using 68 and 69 for $e \in \{0,\ldots,B\}$
  $t \leftarrow t-1$
end while

VIII. Illustrative Examples

A. Optimal Blind Scheduling

Before we provide a few numerical examples, it is useful to introduce a scheduling strategy which is based exclusively on the statistics of the sources and not on the observations. Consider the following *blind* scheduling strategy: If the battery is not empty, transmit the source whose variance is the largest, i.e.,

$$f_t^{\text{blind}}(e_t) \stackrel{\text{def}}{=} \begin{cases} 0 & \text{if } e_t = 0 \\ \arg\max_{i\in\{1,2\}} \left\{\mathbb{E}\left[\|X_t^i - \mathbb{E}[X_t^i]\|^2\right]\right\} & \text{otherwise.} \end{cases} \quad (70)$$

The estimation strategies associated with blind scheduling are

$$g_t^{\text{blind}\ i}\left(y_t^i\right) \stackrel{\text{def}}{=} \begin{cases} x_t^i & \text{if } y_t^i = x_t^i \\ \mathbb{E}[X_t^i] & \text{if } y_t^i = \varnothing. \end{cases}, \quad i \in \{1,2\} \quad (71)$$

The performance of the blind scheduling and the estimation strategies is given by

$$\mathcal{J}^{\text{blind}}(B) \stackrel{\text{def}}{=} \sum_{t=1}^{T} \left[\mathbb{P}(E_t = 0) \sum_{i\in\{1,2\}} \mathbb{E}\left[\|X_t^i - \mathbb{E}[X_t^i]\|^2\right] \right.$$
$$\left. + (1 - \mathbb{P}(E_t = 0)) \min_{i\in\{1,2\}} \left\{\mathbb{E}\left[\|X_t^i - \mathbb{E}[X_t^i]\|^2\right]\right\}\right] \quad (72)$$

where the probabilities $\{\mathbb{P}(E_t = 0), t \in \{1,\ldots,T\}\}$ are computed recursively using 3 and 4 and assuming $E_1 = B > 0$ with probability 1.

Example 1 (Limited number of transmissions): Consider the scheduling of two i.i.d. zero-mean scalar Gaussian sources with variances $\sigma_1^2 = \sigma_2^2 = 1$. Assume that the total system deployment time is T and that during that time, the scheduler is only allowed to transmit $B < T$ times. Furthermore, assume that during that time, there is no energy being harvested, i.e., $Z_t = 0$ with probability 1, and there are no additional communication costs, i.e., $c = 0$.

The algorithm outlined in VII is used to compute the optimal thresholds, which are functions of the time index and the energy level at the battery. Fig 2a displays the optimal thresholds computed for this example with $T = 100$ and $B = 30$.

Note that when the energy level is greater than the remaining deployment time, the optimal threshold is zero, that is, the observation with the largest magnitude is always transmitted. On the other hand, if the power level is below the remaining

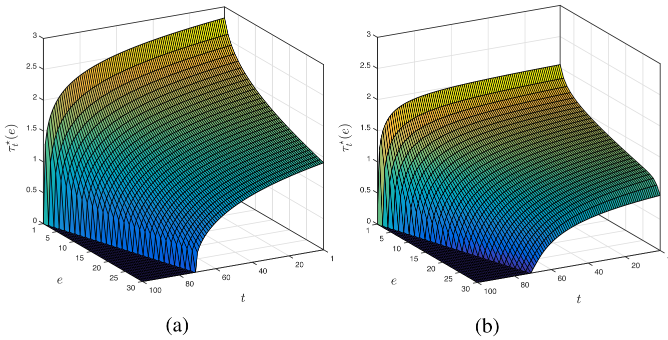

Fig. 2. Optimal threshold function for the scheduling of two i.i.d. standard Gaussian sources. The threshold is a function of the energy level and time. (a) No energy harvesting. (b) Energy harvesting with p_Z^1.

deployment time, the optimal threshold is strictly positive, and it increases as the power level decreases. It means that as the battery depletes, the scheduler will only transmit observations whose magnitudes are increasingly larger.

Example 2 (Energy-harvesting scheduler): Consider a setup identical to that in 1 with $T = 100$, but, in addition, assume that the energy-harvesting process Z_t is distributed according to two possible probability mass functions

$$p_Z^1(z) = \begin{cases} 0.85 & z = 0 \\ 0.1 & z = 1 \\ 0.05 & z = 2 \end{cases} \text{ or } p_Z^2(z) = \begin{cases} 0.7 & z = 0 \\ 0.2 & z = 1 \\ 0.1 & z = 2 \end{cases} \quad (73)$$

yielding on average 0.2 and 0.4 energy units per time step, respectively.

The optimal thresholds obtained for the energy-harvesting system under p_Z^1 are shown in Fig. 2(b), and they are uniformly smaller than the ones of the system without harvesting. We also note a change in the "curvature" of the threshold function for a fixed t.

Fig 3 shows the performance of the optimal strategy and the blind scheduling scheme as a function of the battery capacity B for the three systems: No harvesting, harvesting with p_Z^1 and p_Z^2. The optimal scheme proposed in this article leads to a significant improvement upon the blind scheduling strategy of 70. For $B = 10$, without energy harvesting, the optimal performance is $\mathcal{J}^\star \approx 147.37$. However, in order to achieve a comparable performance using blind scheduling, a battery of capacity equal to 53 energy units would be required. Therefore, the energy savings in this case is of approximately 81.13%.

Finally, Fig. 4 illustrates the performance of the systems with and without harvesting for the scheduling of two standard Gaussian sources over a horizon $T = 100$ and a battery of fixed size $B = 30$ as a function of the communication cost c.

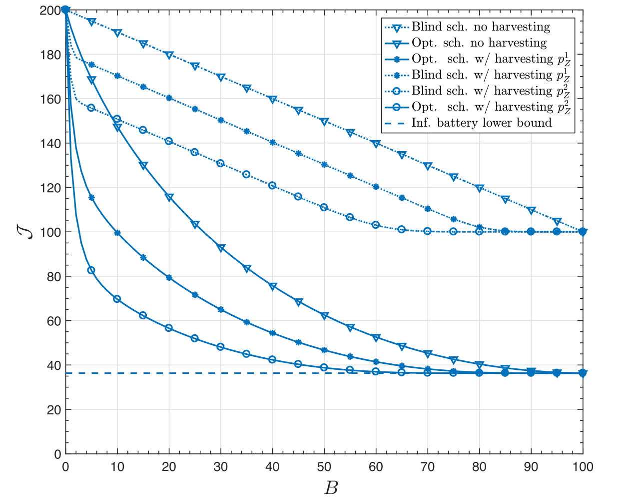

Fig. 3. Comparison between the performances of the optimal open-loop and closed-loop strategies as a function of the battery capacity B. The relative gap between these two curves is defined as the value of information.

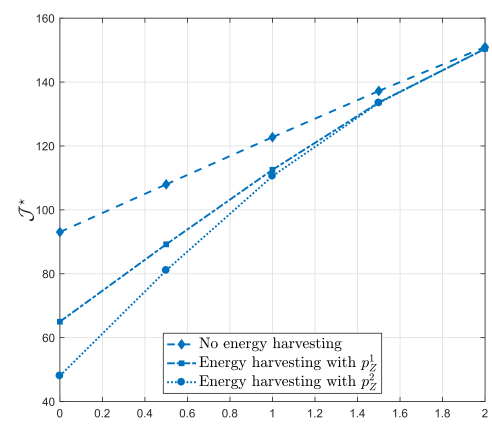

Fig. 4. Optimal performance $\mathcal{J}^\star$ of the systems with and without harvesting of Examples 1 and 2 as a function of the communication cost c.

IX. Extensions

A. The N Sensor Case

Theorem 1 holds for any number of sensors ($N \geq 2$). Let $\mathbf{x}_t = (x_t^1, x_t^2, \dots, x_t^N)$, where $x_t^i \in \mathbb{R}^{n_i}$ is the observation at the ith sensor. Provided that the observations are mutually independent and their pdfs are symmetric and unimodal around $a^1, a^2, \dots, a^N$, where $a_i \in \mathbb{R}^{n_i}$, $i \in \{1, 2, \dots, N\}$, the jointly optimal scheduling and estimation strategies are

$$f_t^\star(\mathbf{x}_t, e_t) \stackrel{\text{def}}{=} \begin{cases} 0, & \text{if } \max_{i\in\{1,\dots,N\}} \left\{\|x_t^i - a^i\|\right\} \leq \tau_t^\star(e_t) \\ \arg\max_{i\in\{1,\dots,N\}} \left\{\|x_t^i - a^i\|\right\}, & \text{otherwise} \end{cases} \quad (74)$$

and

$$g_t^{i\star}(y^i) \stackrel{\text{def}}{=} \begin{cases} x_t^i & \text{if } y_t^i = x_t^i \\ a^i & \text{if } y_t^i = \varnothing. \end{cases}, \quad i \in \{1, \dots, N\} \quad (75)$$

B. Unequal Weights and Communication Costs

In specific applications, each sensor may be assigned a different weight in the expected distortion metric. This new metric is used to emphasize the importance of the observations made by one sensor relative to another. Additionally, different sensors may also have different communication costs, which may reflect the dimension of the measurements or used to preserve the battery power, for instance. These cases are captured by the following cost functional:

$$\mathcal{J}\left(\mathbf{f},\mathbf{g}^1,\mathbf{g}^2\right) \stackrel{\text{def}}{=} \sum_{t=1}^{T} \mathbb{E}\left[\sum_{i\in\{1,2\}} w_i\|X_t^i - \hat{X}_t^i\|^2 + c_i\mathbb{I}(U_t=i)\right]. \tag{76}$$

The globally optimal scheduling and estimation strategies for the more general cost functional in 76 are given by the following equations:

$$g_t^{i\star}\left(y_t^i\right) \stackrel{\text{def}}{=} \begin{cases} x_t^i & \text{if } y_t^i = x_t^i \\ a^i & \text{if } y_t^i = \varnothing \end{cases}, \quad i\in\{1,2\} \tag{77}$$

where the thresholds τ_t^1 and τ_t^2 are computed by modified version of Algorithm 1, described in Appendix C. Eqn. (78) is shown at the bottom of this page.

X. Conclusion

This article studied the problem of optimal scheduling in a sequential remote estimation system where noncollocated sensors and estimators communicate over a shared medium. The access to the communication resources was granted by an energy-harvesting scheduler, which implements an observation-driven medium access control scheme to avoid packet collisions. The underlying assumption is that the sensors make measurements that are i.i.d. in time, but the energy level at the scheduler has a stochastic dynamics, which couples the decision-making process in time. The optimal solutions to such remote estimation problems are typically challenging to find due to the presence of signaling between the scheduler and estimators.

The main result herein is to establish, under certain assumptions on the probabilistic model of the sources, the joint optimality of a pair of scheduling and estimation strategies. More important, the globally optimal solution is obtained despite the lack of convexity in the objective function being introduced by signaling. The overarching proof consists of a judicious expansion of the information sets at the estimators, which enables the use of the common information approach to solving a single dynamic program from the perspective of a fictitious coordinator. Finally, by noting that the optimal solution to this “relaxed” problem does not depend on the additional information introduced in the expansion, it is also shown to be optimal for the original optimization problem. As a byproduct, our proof technique also applies to more general settings with an arbitrary number of sensors, unequal weights, and communication costs. Future work in this problem includes the scheduling of correlated sources, but independent in time, independent Gauss–Markov sources (some progress in this area was reported in [32]), and networks prone to packet drops.

Appendix A
Auxiliary Results

The following two definitions and theorem can be found in [33] and [34].

Definition 2 (Symmetric rearrangement): Let $\mathbb{A}$ be a measurable set of finite volume in $\mathbb{R}^n$. Its symmetric rearrangement $\mathbb{A}^*$ is defined as the open ball centered at $\mathbf{0}_n$ whose volume agrees with $\mathbb{A}$.

Definition 3 (Symmetric decreasing rearrangement): Let $f:\mathbb{R}^n \to \mathbb{R}$ be a non-negative measurable function that vanishes at infinity. The symmetric decreasing rearrangement $f^{\downarrow}$ of f is

$$f^{\downarrow}(x) \stackrel{\text{def}}{=} \int_0^{\infty} \mathbb{I}\left(x \in \{\xi\in\mathbb{R}^n \mid f(\xi) > t\}^*\right) dt. \tag{79}$$

Theorem 2 (Hardy–Littlewood inequality): If f and g are two non-negative measurable functions defined on $\mathbb{R}^n$ which vanish at infinity, then the following holds:

$$\int_{\mathbb{R}^n} f(x)g(x)dx \leq \int_{\mathbb{R}^n} f^{\downarrow}(x)g^{\downarrow}(x)dx \tag{80}$$

where $f^{\downarrow}$ and $g^{\downarrow}$ are the symmetric decreasing rearrangements of f and g, respectively.

Appendix B
Proof of 5

A. Empty Battery

Let $e=0$. The value function in 53 is given by

$$\mathcal{V}_t^{\pi}(0) = \inf_{\tilde{\mathbf{x}}_t} \mathbb{E}\left[\sum_{i\in\{1,2\}} \|X_t^i - \tilde{x}_t^i\|^2\right] + \mathcal{C}_{t+1}^0(0). \tag{81}$$

The infimum in the expression above is achieved by

$$\tilde{\mathbf{x}}_t^{\star} = \left(\mathbb{E}[X_t^1], \mathbb{E}[X_t^2]\right). \tag{82}$$

Since π_1 and π_2 are symmetric around 0

$$\tilde{\mathbf{x}}_t^{\star} = 0. \tag{83}$$

Therefore, if $e=0$, the infimum in 53 is achieved by

$$\tilde{\mathbf{x}}_t^{\star} = 0, \quad i\in\{1,2\}. \tag{84}$$

B. Nonempty Battery

Let $e>0$. The value function in 53 is given by

$$\mathcal{V}_t^{\pi}(e) = \inf_{\tilde{\mathbf{x}}_t} \mathbb{E}\left[\min\left\{\|X_t^1 - \tilde{x}_t^1\|^2 + \|X_t^2 - \tilde{x}_t^2\|^2 + \mathcal{C}_{t+1}^0(e),\right.\right.$$
$$\left.\left.\|X_t^2 - \tilde{x}_t^2\|^2 + \mathcal{C}_{t+1}^1(e), \|X_t^1 - \tilde{x}_t^1\|^2 + \mathcal{C}_{t+1}^1(e)\right\}\right]. \tag{85}$$

$$f_t^{\star}(\mathbf{x}_t, e_t) = \begin{cases} 0, & \text{if } \|x_t^1 - a^1\| \leq \tau_t^1(e_t),\ \|x_t^2 - a^2\| \leq \tau_t^2(e_t) \\ 1, & \text{if } \|x_t^1 - a^1\| > \tau_t^1(e_t),\ w_1\|x_t^1 - a^1\|^2 - w_2\|x_t^2 - a^2\|^2 \geq w_1\left(\tau_t^1(e_t)\right)^2 - w_2\left(\tau_t^2(e_t)\right)^2 \\ 2, & \text{otherwise} \end{cases} \tag{78}$$

The optimization problem in 85 is equivalent to

$$\inf_{\tilde{\mathbf{x}}_t} \mathbb{E}\left[\min\left\{\|X_t^1-\tilde{x}_t^1\|^2+\|X_t^2-\tilde{x}_t^2\|^2,\right.\right.$$
$$\left.\left.\|X_t^2-\tilde{x}_t^2\|^2+\kappa_t(e), \|X_t^1-\tilde{x}_t^1\|^2+\kappa_t(e)\right\}\right] \quad (86)$$

where

$$\kappa_t(e) \stackrel{\text{def}}{=} \mathcal{C}_{t+1}^1(e)-\mathcal{C}_{t+1}^0(e). \quad (87)$$

Consider the auxiliary cost function $\mathcal{J}_t^e : \mathbb{R}^{n_1}\times\mathbb{R}^{n_2}\to\mathbb{R}$ defined as follows:

$$\mathcal{J}_t^e(\tilde{\mathbf{x}}_t) \stackrel{\text{def}}{=} \mathbb{E}\left[\min\left\{\|X_t^1-\tilde{x}_t^1\|^2+\|X_t^2-\tilde{x}_t^2\|^2,\right.\right.$$
$$\left.\left.\|X_t^2-\tilde{x}_t^2\|^2+\kappa_t(e), \|X_t^1-\tilde{x}_t^1\|^2+\kappa_t(e)\right\}\right] \quad (88)$$

where the expectation is taken with respect to the random vectors X_t^1 and X_t^2.

The remainder of the proof consists of solving the following optimization problem:

$$\inf_{\tilde{\mathbf{x}}_t} \mathcal{J}_t^e(\tilde{\mathbf{x}}_t). \quad (89)$$

Define the function $\mathcal{G}:\mathbb{R}^n\times\mathbb{R}^n\to\mathbb{R}$ such that

$$\mathcal{G}_t^e(\tilde{\mathbf{x}}_t;\mathbf{x}_t) \stackrel{\text{def}}{=} \min\left\{\|x_t^1-\tilde{x}_t^1\|^2+\|x_t^2-\tilde{x}_t^2\|^2,\right.$$
$$\left.\|x_t^2-\tilde{x}_t^2\|^2+\kappa_t(e), \|x_t^1-\tilde{x}_t^1\|^2+\kappa_t(e)\right\}. \quad (90)$$

Using the fact that X_t^1 and X_t^2 are independent and the function $\mathcal{G}_t^e$ defined in 90, we can rewrite the function $\mathcal{J}_t^e(\tilde{\mathbf{x}}_t)$ in integral form as follows:

$$\mathcal{J}_t^e(\tilde{\mathbf{x}}_t) = \int_{\mathbb{R}^{n_2}}\left[\int_{\mathbb{R}^{n_1}} \mathcal{G}_t^e(\tilde{\mathbf{x}}_t;\mathbf{x}_t)\pi_1(x_t^1)dx_t^1\right]\pi_2(x_t^2)dx_t^2. \quad (91)$$

The function $\mathcal{G}_t^e$ can be alternatively represented as follows:

$$\mathcal{G}_t^e(\tilde{\mathbf{x}}_t;\mathbf{x}_t) = \min\left\{\|x_t^2-\tilde{x}_t^2\|^2+\kappa_t(e),\right.$$
$$\left.\|x_t^1-\tilde{x}_t^1\|^2+\min\left\{\kappa_t(e), \|x_t^2-\tilde{x}_t^2\|^2\right\}\right\}. \quad (92)$$

Finally, let the function $\mathcal{H}_t^e:\mathbb{R}^n\times\mathbb{R}^n\to\mathbb{R}$ be defined as follows:

$$\mathcal{H}_t^e(\tilde{\mathbf{x}}_t;\mathbf{x}_t) \stackrel{\text{def}}{=} \|x_t^2-\tilde{x}_t^2\|^2+\kappa_t(e)-\mathcal{G}_t^e(\tilde{\mathbf{x}}_t;\mathbf{x}_t). \quad (93)$$

Note that the function $\mathcal{H}_t^e$ vanishes as the norm of x_t^1 tends to infinity, i.e.,

$$\lim_{\|x_t^1\|\to+\infty} \mathcal{H}_t^e(\tilde{\mathbf{x}}_t;\mathbf{x}_t) = 0. \quad (94)$$

From the Hardy–Littlewood inequality (see Appendix A), we have

$$\int_{\mathbb{R}^{n_1}} \mathcal{H}_t^e(\tilde{\mathbf{x}}_t;\mathbf{x}_t)\pi_1(x_t^1)dx_t^1 \le \int_{\mathbb{R}^{n_1}} \mathcal{H}_t^{e\downarrow}(\tilde{\mathbf{x}}_t;\mathbf{x}_t)\pi_1^{\downarrow}(x_t^1)dx_t^1 \quad (95)$$

where $\pi_1^{\downarrow}$ and $\mathcal{H}_t^{e\downarrow}$ denote the symmetric decreasing rearrangements of π_1 and $\mathcal{H}_t^e$, respectively. The following facts hold.

1) Since π_1 is symmetric and unimodal around 0

$$\pi_1^{\downarrow} = \pi_1. \quad (96)$$

2) Since $\mathcal{H}_t^e(\tilde{\mathbf{x}}_t;\mathbf{x}_t)$, as a function of x_t^1, is symmetric and unimodal around $\tilde{x}_t^1$ (a fact that can be verified by inspection), we have

$$\mathcal{H}_t^{e\downarrow}(\tilde{\mathbf{x}}_t;\mathbf{x}_t) = \mathcal{H}_t^e\left(\left(0,\tilde{x}_t^2\right);\mathbf{x}_t\right). \quad (97)$$

Therefore, the Hardy–Littlewood inequality implies that

$$\int_{\mathbb{R}^{n_1}} \mathcal{H}_t^e(\tilde{\mathbf{x}}_t;\mathbf{x}_t)\pi_1(x_t^1)dx_t^1 \le \int_{\mathbb{R}^{n_1}} \mathcal{H}_t^e\left(\left(0,\tilde{x}_t^2\right);\mathbf{x}_t\right)\pi_1(x_t^1)dx_t^1 \quad (98)$$

which is equivalent to

$$\int_{\mathbb{R}^{n_1}} \mathcal{G}_t^e\left(\left(0,\tilde{x}_t^2\right);\mathbf{x}_t\right)\pi_1(x_t^1)dx_t^1 \le \int_{\mathbb{R}^{n_1}} \mathcal{G}_t^e(\tilde{\mathbf{x}}_t;\mathbf{x}_t)\pi_1(x_t^1)dx_t^1. \quad (99)$$

Therefore

$$\tilde{x}_t^{1\star} = 0. \quad (100)$$

Fixing $\tilde{x}_t^{1\star}=0$ and following the same sequence of arguments exchanging the roles of x_t^1 and x_t^2, we show that $\tilde{x}_t^{2\star}=0$. Therefore

$$\tilde{\mathbf{x}}_t^{\star} = 0. \quad (101)$$

Appendix C
Optimal Thresholds for the Asymmetric Case

In the case of asymmetric costs and weights, the modified recursive algorithm is as follows. For $t\in\{1,\ldots,T-1\}$, compute the function $\mathcal{C}_{t+1}^0$ according to (50) and $\mathcal{C}_{t+1}^1$ and $\mathcal{C}_{t+1}^2$ for according to

$$\mathcal{C}_{t+1}^i \stackrel{\text{def}}{=} c_i+\mathbb{E}\left[\mathcal{V}_{t+1}^{\pi}\left(\min\{e-1+Z_t,B\}\right)\right], \quad i\in\{1,2\} \quad (102)$$

where

$$\mathcal{V}_t^{\pi}(0) \stackrel{\text{def}}{=} \mathbb{E}\left[\sum_{i\in\{1,2\}} w_i\|X_t^i-a^i\|^2+\mathcal{V}_{t+1}^{\pi}\left(\min\{Z_t,B\}\right)\right] \quad (103)$$

and

$$\mathcal{V}_t^{\pi}(e) \stackrel{\text{def}}{=} \mathbb{E}\left[\min\left\{\sum_{i\in\{1,2\}} w_i\|X_t^i-a^i\|^2+\mathcal{C}_{t+1}^0(e),\right.\right.$$
$$\left.\left.w_2\|X_t^2-a^2\|^2+\mathcal{C}_{t+1}^1(e), w_1\|X_t^1-a^1\|^2+\mathcal{C}_{t+1}^2(e)\right\}\right]. \quad (104)$$

Finally, the optimal thresholds are given by

$$\tau_t^{i\star}(e) \stackrel{\text{def}}{=} \sqrt{\frac{\mathcal{C}_{t+1}^i(e)-\mathcal{C}_{t+1}^0(e)}{w_i}}, \quad i\in\{1,2\}. \quad (105)$$

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