
**1 Surface Thermal Heterogeneities and the
2 Atmospheric Boundary Layer: the Relevance of
3 Dispersive Fluxes**

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8 Abstract While the increasing availability of computational power is en-
9 abling finer grid resolutions in numerical-weather-prediction models, repre-
10 senting land-atmosphere exchange processes remains challenging. This par-
11 tially results from the fact that land-surface heterogeneity exists at all spatial
12 scales, and its variability does not necessarily ‘average’ out with decreasing
13 size. The work presented here uses large-eddy simulations and the concept
14 of dispersive fluxes to quantify the effects of a surface that is thermally in-
15 homogeneous (with scales that are approximately 10% of the height of the
16 atmospheric boundary layer), but for uniformly rough. These near-canonical
17 cases describe inhomogeneous scalar transport over a broad range of unstable
18 atmospheric flows. Results illustrate the existence of a regime where the mean
19 flow is mostly driven by the surface thermal heterogeneities. In this regime,
20 the contribution of the dispersive fluxes can account for more than 40% of the
21 total sensible heat flux at 100 m above the ground and about 5–10% near the
22 surface. This result is independent of the spatial distribution of the thermal
23 heterogeneities and weakly dependent on the averaging time used to define the
24 dispersive fluxes. Additionally, an alternative regime exists where the effects
25 of the surface thermal heterogeneities are quickly blended and the dispersive
26 fluxes match those obtained over an equivalent homogeneous surface. Results
27 further illustrate the existence of a new cospectral scaling for the dispersive
28 sensible heat fluxes that differs from the traditional turbulence cospectral scal-

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29 ing. We believe that these results might elucidate pathways for developing new
 30 parametrizations for the non-canonical atmospheric surface layer.

31 **Keywords** Large-eddy simulation, Scalar transport, Sensible heat flux,
 32 Surface temperature

33 1 Introduction

34 While advances in computation (and computing) are enabling finer grid res-
 35 olutions in numerical-weather-prediction (NWP) models, representing land-
 36 atmosphere exchange processes as a lower boundary condition remains a chal-
 37 lenge (regardless of the numerical resolution but not independent from it).
 38 Because land-surface heterogeneity exists at all spatial scales and does not
 39 necessarily ‘average out’ with decreasing size, its variability is not rapidly
 40 blended away from the boundary. Thus, it greatly affects the atmospheric sur-
 41 face layer (ASL). The induced mixing and forcing processes are characterized
 42 by short time scales ($\mathcal{O}(1 \text{ h})$) and limited spatial extent ($\mathcal{O}(100 \text{ m})$) that fall
 43 within the ‘terra-incognita’ range (Wyngaard 2004; Beare 2014), making it
 44 difficult for traditional NWP models to capture them.

45 Figure 1 presents a conceptual schematic that illustrates the relationship
 46 between the different atmospheric flow scales, the land-surface heterogeneity
 47 scales, and the available numerical resolution, which has been used to guide the
 48 development of the present study. In Fig. 1, the abscissa represents the length
 49 scale of the surface thermal heterogeneities (l_h) and the ordinate the length
 50 scale of the flow dynamics (l_d). Four relevant length scales are represented on
 51 both coordinate axis, $\mathcal{O}(100 \text{ m})$ represents the smallest flow scales resolved in
 52 near-future high-resolution mesoscale models ($\Delta_{meso}^{H,resol}$), and $\mathcal{O}(100 \text{ km})$ the
 53 horizontal domain size (L_{meso}). Additionally, $\mathcal{O}(10 \text{ m})$ and $\mathcal{O}(1 \text{ km})$, repre-
 54 sent the same corresponding length scales for high-resolution large-eddy simu-
 55 lations (LES) ($\Delta_{LES}^{H,resol}$ and L_{LES}). Moreover, the flow scales are identified as
 56 either numerically resolved (region shaded in blue) or parametrized for high-
 57 resolution mesoscale models (region shaded in yellow). An arc is placed at
 58 scales $\mathcal{O}(100 \text{ m})$. Within this arc and above the diagonal are regions where
 59 unresolved (in mesoscale models) fluid motions are much larger than the scales
 60 of heterogeneity ($l_d/l_h \gg 1$). Here, the fluid flow senses the surface as ho-
 61 mogeneous and traditional ASL formulations should be valid. Within the arc
 62 and below the diagonal, unresolved fluid motions are much smaller than the
 63 scales of heterogeneity ($l_d/l_h \ll 1$) and again the turbulence feels the surface
 64 as homogeneous and traditional ASL formulations should be valid. Within an
 65 uncertain zone along the diagonal, represented between the grey lines, the fluid
 66 motion scales are of the same order as the scales of heterogeneity (i.e., $l_d \sim l_h$).
 67 It is precisely this heterogeneous numerically unresolved zone that frames the
 68 scope of this work (region shaded in red). Much of this zone overlaps with
 69 anticipated increases in NWP resolution as well as the ‘terra-incognita’ (Wyn-
 70 gaard 2004; Beare 2014). It is also the zone that is most problematic for ASL
 71 similarity relations (Patton et al. 2005; Li and Bou-Zeid 2013) and has been

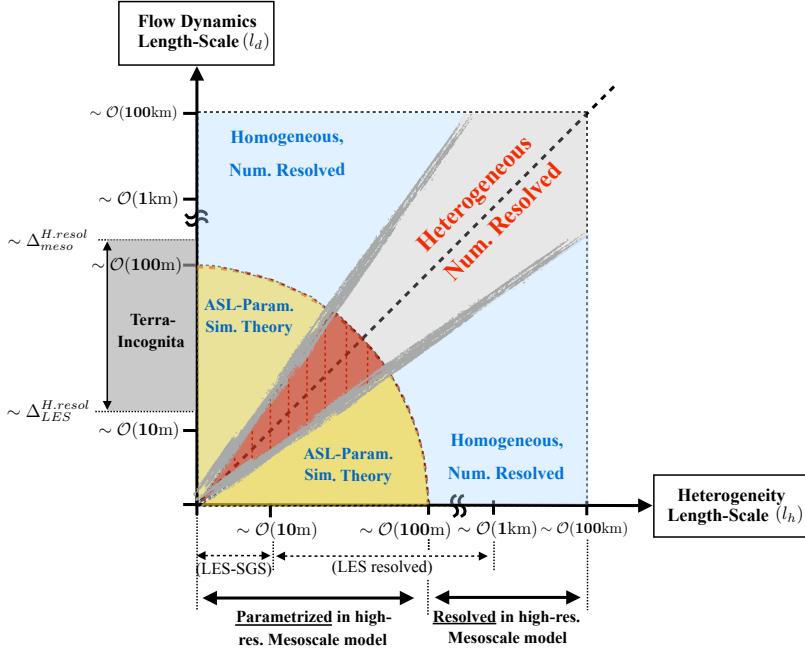


Fig. 1 Schematic of the interaction between turbulent eddies and heterogeneous surface patches of different scales illustrating the need for new ASL parametrizations in mesoscale modelling. The shaded regions represent different categories of turbulent scales: in blue, the scales are numerically resolved, and in yellow, the scales are parametrized for high-resolution mesoscale models. Between the grey lines is a region where eddy scales are of the same order as scales of heterogeneity (i.e., $l_e \sim l_h$). The region shaded in red represents the scope of this work

known as a source of error that is introduced by the indirect homogenization of surface spatial heterogeneities. This error depends on the numerical-model resolution, the characteristic length scale of the surface heterogeneities, and the parametrization used for the near-surface conditions (Patton et al. 2005; Li and Bou-Zeid 2013).

Because the flux-gradient similarity relationships were developed under the assumption of spatial homogeneity and statistical stationarity (Monin and Obukhov 1954; Stull 1988), prior efforts have attempted to overcome this inherent limitation using more or less sophisticated approaches to provide adjustments to the parametrization of the Reynolds-flux component of the sensible heat flux. Some examples of these strategies include the effective surface parameter approach (Wieringa 1986; Taylor 1987; Mason 1988; Claussen 1990; Wood and Mason 1991; Beljaars and Holtslag 1991; Bou-Zeid et al. 2004), the statistical-dynamical approach (Avissar 1991, 1992), and the mosaic and tile approaches (Avissar and Pielke 1989; Claussen 1991; Ament and Simmer 2006) or any modification of the ones above such as the extended tile (Blyth et al. 1993; Blyth 1995; Arola 1999). These methods are currently used in

most NWP and climate models to parametrize surface energy fluxes over heterogeneous terrain because they are easy to implement and use, and due to the current lack of better alternatives. Further, the framework under which the tile and mosaic approaches should be used will become more severely challenged as numerical resolution increases because these methods were developed to be applied above the blending height. As additional compelling evidence of the limitations of these approaches in most heterogeneous cases, both methods lead to consistent overestimation of fluxes when used together with the flux-profile method (Stoll and Porté-Agel 2009).

As a first step towards developing new ASL parametrizations for high-resolution NWP models that overcome the problems presented above, the contribution of the dispersive fluxes is evaluated on flows over idealized thermally-heterogeneous surfaces with uniform roughness across a wide range of unstable stratification. The dispersive fluxes are representative of the spatial-averaging operation that explicitly represents the critical processes dependent on heterogeneity (Wilson and Shaw 1977; Raupach and Shaw 1982; Finnigan 1985; Raupach et al. 1986). They arise from the spatial averaging operator applied over a multiply-connected domain across solid–fluid interfaces (e.g. canopies) or they can be associated with the spatial variability of time-averaged quantities. The latter case is explored herein, where it is hypothesized that the thermal spatial heterogeneity of the land surface is generating important dispersive fluxes. A similar approach to the contribution of the time- and space-averaged fluxes was used in Kanda et al. (2004), Patton et al. (2005), Inagaki et al. (2006), Mauder et al. (2008), and Zhou et al. (2018) to analyze the closure problem of the energy balance, where it was found that part of the imbalance was due to the transport of heat by the mean vertical motion.

In Sect. 2, the derivation and theory related to the dispersive fluxes is reviewed; in Sect. 3, the numerical platform used and study cases are presented; Sect. 4 identifies the contribution of the dispersive fluxes and evaluates the work hypothesis indicated above; and Sect. 5 presents a spectral analysis of the fluxes. Finally, an extended discussion of the results, implications, and limitations of this work is provided in Sect. 6, with conclusions in Sect. 7.

2 Dispersive Fluxes

When the temporally-averaged equations of motion are spatially averaged, dispersive fluxes appear as an additional term that accounts for the effect of the spatial fluctuations of the time-averaged variables. As a result, dispersive fluxes quantify the spatial correlations of temporally-averaged quantities. Initially introduced by Wilson and Shaw (1977) and Raupach and Shaw (1982), dispersive fluxes can arise when a time-averaged variable (\bar{u}_i , with the overline representing time averaging) is represented as the sum of a time- and spatially-averaged variable ($\langle \bar{u}_i \rangle$, with the $\langle \cdot \rangle$ representing horizontal spatial averaging) with a time-averaged spatial deviation (\bar{u}_i''), for example,

$$\bar{u}_i = \langle \bar{u}_i \rangle + \bar{u}_i''. \quad (1)$$

131 Hence, by using this decomposition, which is the result of the spatial averaging
 132 of time-averaged variables, new covariances arise. These are the result of the
 133 spatial correlation of quantities averaged in time but varying with position.
 134 For example, the case for the heat fluxes may be written as

$$\langle \bar{w}''\bar{\theta}'' \rangle = \langle \bar{w}\bar{\theta} \rangle - \langle \bar{w} \rangle \langle \bar{\theta} \rangle. \quad (2)$$

135 Initially applied in atmospheric flows for the study of vegetated canopies
 136 (Wilson and Shaw 1977; Raupach and Thom 1981; Raupach and Shaw 1982;
 137 Raupach 1994; Finnigan 2000), dispersive fluxes have been shown to poten-
 138 tially play an important role in the description of spatially-averaged flow statis-
 139 tics (Poggi et al. 2004; Poggi and Katul 2008). Early work in this area indicated
 140 that dispersive fluxes are negligible in the mean flow outside canopies (Rau-
 141 pach et al. 1986; Cheng and Castro 2002). In contrast, Poggi et al. (2004)
 142 showed, using a flume study, that the contribution of the dispersive flux to the
 143 momentum fluxes could be larger than 10% in sparse canopies. Smaller dis-
 144 persive fluxes of about 6% were also measured by Mignot et al. (2009) in flow
 145 over a gravel bed. However, Bailey and Stoll (2013) showed that, in sparse,
 146 row-oriented canopies, the dispersive flux was more than 20% of the magnitude
 147 of the turbulent flux. Similarly, a recent study demonstrated that dispersive
 148 fluxes are generated around canopy edges and that they can be large in the
 149 entry region of the canopy (Moltchanov et al. 2015). In addition, in urban-
 150 canopy studies, the contribution of dispersive fluxes has also been shown to
 151 be non-negligible (Martilli and Santiago 2007). For example, in LES studies
 152 of flow over random urban-like obstacles, the magnitude of the dispersive flux
 153 represents 15% of the magnitude of the turbulent flux, and the peak of the dis-
 154 persive flux is located at the top of the canopy (Xie et al. 2008). These results
 155 have been confirmed in flow over realistic urban surfaces by Giometto et al.
 156 (2016). Similarly, studies of flow inside wind farms have shown that dispersive
 157 fluxes constitute a non-negligible fraction of the total vertical momentum flux
 158 (Calaf et al. 2010).

159 In the present study, the concept of dispersive fluxes is extended and ap-
 160 plied to the sensible heat flux with two objectives in mind. First, as a means of
 161 accounting for the missing contribution to the surface energy budget resulting
 162 from persistent spatial thermal heterogeneities present in the time-averaged ve-
 163 locity and temperature fields. This is similar to the approach used by Kanda
 164 et al. (2004), Patton et al. (2005), Inagaki et al. (2006), Mauder et al. (2008),
 165 Zhou et al. (2018), and De Roo and Mauder (2018) to analyze the closure of
 166 the surface energy balance, where it was found that part of the energy im-
 167 balance was due to heat transport by the mean vertical motion. Second, as
 168 a potential means of developing new parametrizations for NWP models that
 169 inherently account for the effects of the unresolved thermal heterogeneities.

170 3 Numerical Simulations and Study Cases

171 The LES approach and details of the different study cases are briefly presented.
 172 A more detailed overview of the LES method can be found in Moeng and
 173 Sullivan (2015), and more details on the procedure used here can also be
 174 found in Bou-Zeid et al. (2004, 2005), Calaf et al. (2011), and Margairaz et al.
 175 (2018).

176 3.1 Large-Eddy Simulations Framework

177 In the LES method, the turbulent flow is separated into resolved and modelled
 178 scales. The resolved flow is obtained by numerically integrating the filtered
 179 incompressible Navier–Stokes equations. In the LES implementation used here,
 180 these equations are written in rotational form to ensure adequate conservation
 181 of energy by the inertial terms (Kravchenko and Moin 1997). Additionally, the
 182 momentum is coupled to the advection–diffusion equation of heat using the
 183 Boussinesq approximation. The dimensional form of the governing equations
 184 is therefore

$$185 \partial_i \tilde{u}_i = 0, \quad (3)$$

$$186 \partial_t \tilde{u}_i + \tilde{u}_j (\partial_j \tilde{u}_i - \partial_i \tilde{u}_j) = -\partial_i \tilde{p}^* - \partial_j \tau_{ij}^{\Delta, d} + g \left(\frac{\tilde{\theta} - \langle \tilde{\theta} \rangle_{xy}}{\langle \tilde{\theta} \rangle_{xy}} \right) \delta_{i3} + \tilde{f}_i, \quad (4)$$

$$187 \partial_t \tilde{\theta}_i + \tilde{u}_j \partial_j \tilde{\theta} = \partial_j \pi_j^{\Delta} \quad (5)$$

188 where \tilde{u}_i ($i = 1, 2, 3$) refer to the filtered velocity components in the three
 189 Cartesian directions (horizontal: x, y , and vertical: z), $\tilde{\theta}$ represents the fil-
 190 tered potential temperature, and \tilde{p}^* denotes the dynamic modified pressure
 191 field. This is defined as $\tilde{p}^* = \tilde{p} + \frac{1}{3} \tau_{kk}^{\Delta} + \frac{1}{2} \tilde{u}_j \tilde{u}_j$, where the first term is the
 192 kinematic pressure, the second term is the trace of the subgrid scale (SGS)
 193 momentum flux and the last term derives from the rotational form of the
 194 convective term. In Eq. 4, the coupling term $g \left(\frac{\tilde{\theta} - \langle \tilde{\theta} \rangle_{xy}}{\langle \tilde{\theta} \rangle_{xy}} \right) \delta_{i3}$ results from the
 195 Boussinesq approximation where $\langle \rangle_{xy}$ represents a horizontal average, and δ_{ij}
 196 is the Kronecker-delta operator. The flow is driven by a geostrophic forcing,
 197 imposed using the body force term $\tilde{f}_i = (\tilde{u}_2 - V_G) \nu_G \delta_{i1} - (\tilde{u}_1 - U_G) \nu_G \delta_{i2}$, where
 198 (U_G, V_G) are the horizontal geostrophic velocity components, and $\nu_G = 10^{-4}$
 199 Hz is the geostrophic frequency at a latitude of 43.3° N.

200 The deviatoric part of the SGS momentum flux is written using the eddy-
 201 viscosity approach and may be written as $\tau_{ij}^{\Delta, d} = \tau_{ij}^{\Delta} - \frac{1}{3} \tau_{kk}^{\Delta} \delta_{ij} = -2\nu_T \tilde{S}_{ij}$,
 202 where $\nu_T = (C_S \Delta)^2 |\tilde{S}|$ is the turbulent eddy viscosity, $\tilde{S}_{ij} = \frac{1}{2} (\partial_j \tilde{u}_i + \partial_i \tilde{u}_j)$ is
 203 the resolved strain rate tensor, and C_S is the Smagorinsky coefficient (Smagorin-
 204 sky 1963; Lilly 1967). This coefficient is computed dynamically using the La-
 205 grangian scale-dependent dynamic model of Bou-Zeid et al. (2005). Similarly
 206 for temperature, the SGS temperature diffusion is given by $\pi_j^{\Delta} = \nu_T / Pr_{sgs} \partial_j \tilde{\theta} =$
 207 $(D_S \Delta)^2 |\tilde{S}| \partial_j \tilde{\theta}$, where the coefficient D_S is computed dynamically using a La-
 208 grangian scale-dependent dynamic model for scalars (Calaf et al. 2011).

208 The numerical implementation is based on Albertson and Parlange (1999),
 209 later modified by Bou-Zeid et al. (2005), Calaf et al. (2011), and Margairaz
 210 et al. (2018). This pseudo-spectral code treats the horizontal derivatives in
 211 Fourier space, the vertical derivatives are computed using second-order finite
 212 differences on the vertically staggered grid, and the second-order Adam–
 213 Bashforth scheme is used for time integration. The lateral boundary conditions
 214 are, as a result, periodic. The top boundary conditions are prescribed using a
 215 stress-free-lid condition for the horizontal velocity ($\partial_z \tilde{u}_i = 0, i = 1, 2$) and a
 216 constant temperature gradient corresponding to the initial strength of the cap-
 217 ping inversion ($\partial_z \tilde{\theta} = cst$). The non-penetration condition ($\tilde{w} = 0$) is imposed
 218 on the vertical velocity at the top and bottom of the domain. Monin–Obukhov
 219 similarity theory (MOST) (Monin and Obukhov 1954) is used for the bottom
 220 boundary condition for the horizontal velocity and temperature. The latter
 221 gives a formulation for the surface shear stress and vertical heat flux. The
 222 drag from the underlying surface is entirely modelled through the logarithmic
 223 wind profile for rough surfaces (von Kármán 1931; Prandtl 1932) corrected for
 224 atmospheric stability through MOST. The surface friction velocity u_* , related
 225 to the shear stress, is given by

$$u_*^2 = \left[\frac{\kappa}{\ln \left(\frac{\Delta z/2}{z_0} \right) + \psi_m \left(\frac{\Delta z/2}{L} \right)} \right]^2 \left(\hat{u}_1^2(x, y, \Delta z/2) + \hat{u}_2^2(x, y, \Delta z/2) \right), \quad (6)$$

226 where \hat{u}_i is the velocity field filtered a second time at $2\Delta_{LES}$ and sampled at
 227 $\Delta z/2$ where $\Delta_{LES} = \sqrt{\Delta x \Delta y}$ is the horizontal grid size and Δz is the vertical
 228 grid size. The aerodynamic roughness length is denoted by z_0 and $\kappa = 0.4$ is
 229 the von Kármán constant. The stability correction function of momentum ψ_m
 230 is based on Brutsaert (2005) and depends on atmospheric stability assessed by
 231 the local Obukhov length $L = -\frac{u_*^3 \theta_S}{\kappa g q_s}$. Here, θ_S is the surface temperature, g is
 232 the acceleration due to gravity, and q_s denotes the kinematic surface heat flux
 233 (Brutsaert 1982). The wall shear stress is then dynamically projected over the
 234 horizontal directions using the unit direction vector of the horizontal velocity
 235 sampled at $z = \Delta z/2$ and filtered at $2\Delta_{LES}$ (Bou-Zeid et al. 2004; Hultmark
 236 et al. 2013).

237 Similarly, the vertical kinematic sensible heat flux is computed as

$$q_s = \frac{\left[\theta_s(x, y) - \tilde{\theta}(x, y, \Delta z/2) \right]}{\left[\ln \left(\frac{\Delta z/2}{z_{0s}} \right) + \psi_s \left(\frac{\Delta z/2}{L} \right) \right]} \kappa u_*, \quad (7)$$

238 where ψ_s is the stability correction function for the temperature and z_{0s} is the
 239 thermal roughness length, which is specified as $z_{0s} = 0.1 z_0$. The corresponding
 240 vertical derivatives of the horizontal velocities and temperature are imposed
 241 at the first grid point of the vertically staggered grid following Albertson and

242 Parlange (1999). This framework was developed for idealized homogeneous
 243 surfaces. We are aware of the limitations of the surface parametrization (Basu
 244 and Lacser 2017), however, it is also known that this approach provides ac-
 245 ceptable results in the case of non-homogeneous conditions (Bou-Zeid et al.
 246 2004; Stoll and Porté-Agel 2006).

247 **3.2 Study Cases**

248 The unstable nature of the boundary layer is established by initially setting
 249 the air temperature 5 K lower than the mean surface temperature. The simu-
 250 lations are separated into two sets to study the differences between heteroge-
 251 neous and homogeneous surfaces. In the first set, a total of seven configura-
 252 tions are considered, all with a homogeneous surface temperature fixed at a value
 253 of $\theta_S = 290$ K, and for which the geostrophic wind speed has been increased
 254 from 1 to 15 m s⁻¹ (i.e., $U_g = 1, 2, 3, 4, 6, 9, 15$ m s⁻¹). Hereafter, these ho-
 255 mogeneous cases are referred to as HM-X, where X indicates the geostrophic
 256 wind speed corresponding case (see Table 1). In the second set, the surface
 257 temperature is distributed amongst square patches, where the temperature of
 258 each patch is determined by sampling a Gaussian distribution with a mean
 259 temperature of 290 K and a standard deviation of 5 K. In this case, three
 260 different patch sizes were considered (i.e., $l_h = 800, 400$, and 200 m). The sizes
 261 of the heterogeneities were chosen to be of similar size ($l_h/l_d \approx 1$), half the
 262 size ($l_h/l_d \approx 1/2$), and about a quarter of the size ($l_h/l_d \approx 1/4$) of the largest
 263 flow motions within the represented thermal boundary layer, assuming that
 264 this is of the order of the boundary-layer height ($l_d \sim z_i$). Further, these het-
 265 erogeneities are typically not resolved in NWP models. These cases have also
 266 been studied for the different geostrophic wind speeds indicated above, and
 267 hereafter are referred to as HT-X-sYYY, where X indicates the corresponding
 268 geostrophic wind speed, and sYYY refers to the size of the patches (e.g., HT-
 269 1-s800 would be the heterogeneous case with patches of 800 m, and forced with
 270 $U_g = 1$ m s⁻¹). Additionally, for the case with larger patches, three different
 271 random distributions of the patches have been considered to evaluate the po-
 272 tential effect of a given surface distribution for all geostrophic wind speeds.
 273 This is further indicated with the indicator v_i with $i = 1, 2, 3$. Therefore, in
 274 this second set of study cases a total of 35 different configurations have been
 275 considered (see Table 1). Figure 2 shows the surface temperature distributions
 276 used for the different heterogeneous surface conditions. These temperature dis-
 277 tributions emulate the surface thermal conditions observed in Morrison et al.
 278 (2017), where measurements of the surface temperature were taken with a
 279 thermal camera at the SLTEST site of the US Army Dugway Proving Ground
 280 in Utah, USA. This is an ideal site with uniform roughness and a large unper-
 281 turbed fetch, where surface thermal heterogeneities are naturally created by
 282 differences in surface salinity.

283 In all studied cases, the surface roughness is assumed homogeneous, with
 284 $z_0 = 0.1$ m, and representative of a surface with sparse forest or farmland

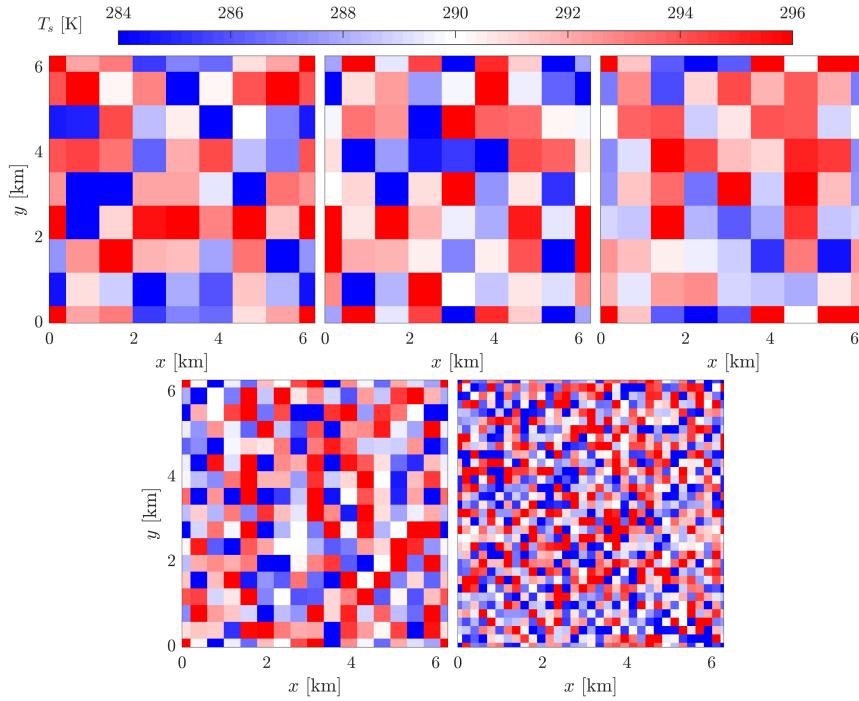


Fig. 2 Surface temperature distributions for the different patches cases. On top: the three 800 m patches (HT-X-s800.vi). On bottom, left to right: 400 m patches (HT-X-s400), and 200 m patches (HT-X-s200)

285 with many hedges (Brutsaert 1982; Stull 1988). The initial boundary-layer
 286 height is set to $z_i = 1000$ m. The temperature profile is initialized with a
 287 mean air temperature of 285 K. At the top of the initial boundary layer,
 288 a capping inversion of 1000 m is used to limit its growth. The strength of
 289 this inversion is fixed at $\Gamma = 0.012 \text{ K m}^{-1}$. The atmospheric boundary layer
 290 (ABL) is considered dry and the latent heat flux is neglected in all cases.
 291 Further, in all simulations, the surface heat flux is computed using MOST, as
 292 explained in Sect. 3.1, where the surface temperature is kept constant in time
 293 throughout the simulations. Thus, there is no feedback from the atmosphere
 294 to the surface as the surface temperature does not cool down or warm up
 295 with local changes in velocities. As a consequence, the ABL gradually warms
 296 up as the simulations progress, and hence becomes less convective over time.
 297 However, the runs are not long enough for this to be significant. In addition,
 298 to ensure a degree of homogeneity within each patch and a certain degree of
 299 validity of MOST, note that even for the heterogeneous cases with the fewest
 300 amount of grid points per patch, a minimum of eight grid points is granted in
 301 each horizontal direction. The domain size is set to $(L_x, L_y, L_z) = (2\pi, 2\pi, 2)$
 302 km at a grid size of $(N_x, N_y, N_z) = (256, 256, 256)$ resulting in a horizontal
 303 resolution of $\Delta x = \Delta y = 24.5$ m and a vertical grid spacing of $\Delta z = 7.8$ m. A

Table 1 Summary of the study cases and the corresponding most relevant simulation statistics for 30-min averages. The homogeneous cases are referenced as HM, and cases with heterogeneous surfaces are referred to as HT. In the case of the 800 m patches, the statistics for all three cases are very similar. Thus, HT-X-s800. v_i is presented as representative of all three cases. The properties presented in the table are the geostrophic forcing velocity (U_g), the boundary-layer height (z_i), the Obukhov length (L), the stability factor ($-z_i/L$), the friction velocity (u_*), the temperature scale (θ_*), the planar-averaged surface heat flux ($\langle \overline{(w'\theta')}_S \rangle$), and the convective velocity scale (w_*). The boundary-layer height and the Obukhov length have been rounded up to the nearest m. The stability factor has been rounded up to the nearest integer

| Name | U_g m s ⁻¹ | z_i m | L m | $-z_i/L$ - | u_* m s ⁻¹ | θ_* K | $\langle \overline{(w'\theta')}_S \rangle$ m K s ⁻¹ | w_* m s ⁻¹ | w_*/u_* |
|-------------------|----------------------------|------------|----------|---------------|----------------------------|-----------------|---|----------------------------|-----------|
| HM-1 | 1 | 1313 | -4 | 332 | 0.17 | 0.52 | 0.09 | 1.44 | 8.58 |
| HM-2 | 2 | 1305 | -6 | 223 | 0.19 | 0.45 | 0.09 | 1.43 | 7.54 |
| HM-3 | 3 | 1289 | -11 | 114 | 0.23 | 0.36 | 0.08 | 1.42 | 6.05 |
| HM-4 | 4 | 1281 | -20 | 65 | 0.28 | 0.29 | 0.08 | 1.41 | 5.02 |
| HM-6 | 6 | 1273 | -44 | 27 | 0.37 | 0.23 | 0.08 | 1.42 | 3.83 |
| HM-9 | 9 | 1297 | -97 | 13 | 0.49 | 0.18 | 0.09 | 1.44 | 2.96 |
| HM-15 | 15 | 1328 | -266 | 5 | 0.71 | 0.14 | 0.10 | 1.49 | 2.11 |
| HT-1-s800. v_i | 1 | 1422 | -2 | 715 | 0.16 | 0.97 | 0.16 | 1.75 | 10.80 |
| HT-2-s800. v_i | 2 | 1383 | -3 | 431 | 0.19 | 0.81 | 0.15 | 1.73 | 9.20 |
| HT-3-s800. v_i | 3 | 1359 | -6 | 232 | 0.23 | 0.64 | 0.14 | 1.70 | 7.53 |
| HT-4-s800. v_i | 4 | 1344 | -10 | 133 | 0.27 | 0.51 | 0.14 | 1.67 | 6.28 |
| HT-6-s800. v_i | 6 | 1336 | -24 | 56 | 0.35 | 0.37 | 0.13 | 1.63 | 4.72 |
| HT-9-s800. v_i | 9 | 1344 | -67 | 20 | 0.49 | 0.26 | 0.13 | 1.62 | 3.34 |
| HT-15-s800. v_i | 15 | 1359 | -196 | 7 | 0.69 | 0.18 | 0.13 | 1.62 | 2.34 |
| HT-1-s400 | 1 | 1487 | -1 | 3433 | 0.10 | 1.57 | 0.15 | 1.01 | 10.52 |
| HT-2-s400 | 2 | 1463 | -2 | 897 | 0.15 | 1.06 | 0.16 | 0.99 | 6.47 |
| HT-3-s400 | 3 | 1452 | -4 | 397 | 0.20 | 0.79 | 0.16 | 0.94 | 4.76 |
| HT-4-s400 | 4 | 1434 | -8 | 170 | 0.26 | 0.58 | 0.15 | 0.90 | 3.50 |
| HT-6-s400 | 6 | 1406 | -22 | 63 | 0.35 | 0.40 | 0.14 | 0.88 | 2.51 |
| HT-9-s400 | 9 | 1390 | -62 | 22 | 0.48 | 0.27 | 0.13 | 0.83 | 1.73 |
| HT-15-s400 | 15 | 1404 | -197 | 7 | 0.70 | 0.18 | 0.13 | 0.76 | 1.08 |
| HT-1-s200 | 1 | 1439 | -1 | 1741 | 0.12 | 1.28 | 0.15 | 0.93 | 7.74 |
| HT-2-s200 | 2 | 1434 | -2 | 933 | 0.15 | 1.04 | 0.15 | 0.89 | 6.02 |
| HT-3-s200 | 3 | 1430 | -4 | 377 | 0.20 | 0.77 | 0.15 | 0.83 | 4.17 |
| HT-4-s200 | 4 | 1434 | -8 | 180 | 0.25 | 0.59 | 0.15 | 0.79 | 3.12 |
| HT-6-s200 | 6 | 1402 | -23 | 61 | 0.35 | 0.40 | 0.14 | 0.75 | 2.13 |
| HT-9-s200 | 9 | 1374 | -65 | 21 | 0.48 | 0.26 | 0.13 | 0.70 | 1.45 |
| HT-15-s200 | 15 | 1383 | -207 | 7 | 0.70 | 0.18 | 0.12 | 0.63 | 0.89 |

304 timestep of $\Delta t = 0.1$ s is used to ensure the stability of the time integration.
 305 This set-up is very similar to that used by Salesky et al. (2017).

306 The two sets of simulations span a large range of geostrophic forcing con-
 307 ditions, allowing the study of the effect on the structure of the convective
 308 boundary layer (CBL) above a patchy surface compared to a homogeneous
 309 surface. The range of U_g covers values between 1 m s^{-1} and 15 m s^{-1} . The
 310 procedure used to spin up the simulations is the following: a spin-up phase of
 311 four hours of real time is used to achieve converged turbulent statistics, which
 312 is then followed by an evaluation phase. During the latter, running averages
 313 are computed for the next hour of real time. Statistics have been computed
 314 for averaging times of 5 min to 1 h, showing statistical convergence at 30-min
 315 averages with negligible changes between the 30-min and the 60-min averages.

316 Table 1 presents a summary of the simulation statistics for the homoge-
 317 neous and heterogeneous surface cases. The values of the Obukhov length

318 $L = -\frac{u_*^3 \theta_S}{\kappa g (w' \theta')_S}$, the stability parameter $-z_i/L$, the convective velocity $w_* =$
 319 $\left[\frac{g}{\theta_S} z_i (w' \theta')_S \right]^{\frac{1}{3}}$, and the temperature scale $\theta_* = \left[\frac{g}{\theta_S} z_i \right]^{-\frac{1}{3}} \left[(w' \theta')_S \right]^{\frac{2}{3}}$ have
 320 been obtained using the planar averages of the 30-min averages of the friction
 321 velocity u_* (from Eq. 6), the sensible heat flux $(w' \theta')_S$ (from Eq. 7) and the
 322 height of the boundary layer z_i . The simulations cover a wide range of atmo-
 323 spheric stability regimes ranging from $-z_i/L < 5$ to $-z_i/L > 700$, and hence
 324 spanning from near neutral to highly convective scenarios.

325 4 Quantification of Surface Thermal Heterogeneity Effects on the 326 Atmospheric Boundary Layer

327 As a first step of this investigation, Sect. 4.1 presents a quantitative analy-
 328 sis of the effect of the surface thermal heterogeneities on the ABL flow field,
 329 considering the impact of increasing geostrophic forcing. Section 4.2 quantifies
 330 the contribution of the dispersive flux over the entire ABL and evaluates the
 331 dependence with the corresponding averaging time used to compute the dis-
 332 persive fluxes. Sect. 4.3 examines the contribution of dispersive fluxes in the
 333 surface layer, and evaluates its dependence on the size of the averaging area
 334 at a specific height.

335 4.1 Impact of Surface Thermal Heterogeneities on the Flow Field

336 As a first step of this analysis, Fig. 3 presents instantaneous spatial fluctua-
 337 tions of the vertical velocity and temperature at a height of 100 m ($\approx 1/10 z_i$).
 338 The instantaneous snapshots for the homogeneous cases (Figs. 3a, b) are useful
 339 to understand the effect of increasing geostrophic forcing. In the case corre-
 340 sponding to a low wind speed ($U_g = 1 \text{ m s}^{-1}$, Fig. 3a), the expected convective

341 cells can be observed, both in the velocity and temperature fields. These con-
 342 vective open cells have a diameter of 2–4 km, corresponding to observations
 343 made in the CBL (Konrad 1970; Weckwerth et al. 1999; Bennett et al. 2010).
 344 As the geostrophic wind speed is increased ($U_g = 15 \text{ m s}^{-1}$, Fig. 3b), the
 345 structure of the flow is transformed from a convective-cell geometry into a
 346 roll structure. The transition from cell to roll structure is related to the in-
 347 crease in shear stress, which destroys the original convective-cell structure.
 348 To quantify this transition, Salesky et al. (2017) developed a metric based on
 349 the two-point correlation function of the vertical velocity in cylindrical coor-
 350 dinates $R_{ww}(r_\eta, r_\phi, z)$, where r_η is the radial lag, r_ϕ is the angular lag, and z
 351 is the height above the ground. This method detects the angular dependencies
 352 present in the roll-type convection using the statistical range of the two-point
 353 correlation function. The statistical range, defined as

$$R(r_\eta) = \max_{r_\phi} [R_{ww}(r_\eta, r_\phi, z)] - \min_{r_\phi} [R_{ww}(r_\eta, r_\phi, z)], \quad (8)$$

354 will be large for the roll-type convection and small for the cell-type convection.
 355 Following this observation, the roll factor can be defined as

$$\mathcal{R} = \max_{r_\eta} [R(r_\eta) | r_\eta/z_i \leq 0.5], \quad (9)$$

356 where the radial lag is cut at $z/z_i = 0.5$ because only large, convective struc-
 357 tures are of interest. Using this metric, the cases presented in Fig. 3 have a roll
 358 factor of $\mathcal{R} \approx 0.1$ for $U_g = 1 \text{ m s}^{-1}$ and $\mathcal{R} \approx 0.3$ for $U_g = 15 \text{ m s}^{-1}$, matching
 359 the conceptual structure defined in Salesky et al. (2017).

360 Interestingly, the original convective-cell structure is also modified when
 361 the flow is overlaid on a thermally-heterogeneous surface. In this case (Fig. 3c),
 362 the convective-cell structure adjusts to the thermal patches at the surface. The
 363 instantaneous snapshot of the flow field illustrates how cells merge into larger
 364 cells, or break into much smaller cells or updrafts that concentrate forming
 365 boarders that surround the larger cells. In this case, the characteristic length
 366 scale of the larger fluid motions (l_d) is related to the length scale of the surface
 367 thermal heterogeneities (l_h) or larger, if there are nearby patches with similar
 368 surface temperature. For smaller updrafts, the characteristic length scale is of
 369 the order of 200 m to 400 m, compatible with the observations made by Bennett
 370 et al. (2010) over Oklahoma. This length scale is smaller than the individual
 371 surface temperature patch sizes (l_h). Subsequently, when the geostrophic wind
 372 speed increases ($U_g = 9 \text{ m s}^{-1}$), the convective-cell structure is destroyed,
 373 similar to the homogeneous case. Furthermore, the footprint of the surface
 374 temperature patches is no longer evident in the temperature and velocity fields
 375 (Fig. 3d). In this case, the roll factor is larger than 0.25, very similar to what
 376 was obtained in the case of a homogeneous surface temperature. It is therefore
 377 clear that under conditions of moderate geostrophic forcing the impact of the
 378 surface thermal heterogeneities is blended, and its corresponding impact on
 379 the flow structure is reduced.

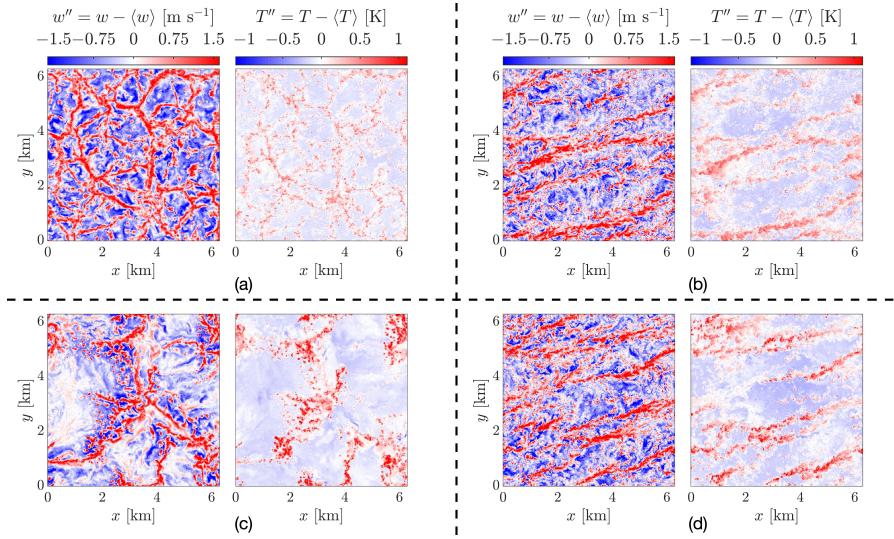


Fig. 3 Instantaneous spatial fluctuations of the two-dimensional-horizontal ($x - y$) fields at $z = 100$ m and $t = 4$ h of the vertical velocity $w'' = w(x, y, z, t) - \langle w \rangle_{x,y}(z, t)$ and temperature $T'' = T(x, y, z, t) - \langle T \rangle_{x,y}(z, t)$. The top subfigures (a, b) correspond to cases over a homogeneous surface with a geostrophic wind speed of (a) $U_g = 1 \text{ m s}^{-1}$ (HM-1) and (b) $U_g = 15 \text{ m s}^{-1}$ (HM-15). The bottom subfigures (c, d) correspond to the corresponding heterogeneous cases for the 800-m patches with a geostrophic wind speed of (c) $U_g = 1 \text{ m s}^{-1}$ (HT-1-s800.v1) and (d) $U_g = 15 \text{ m s}^{-1}$ (HT-15-s800.v1)

380 4.2 Dispersive Fluxes in the Atmospheric Boundary Layer

381 The progressive blending of thermal heterogeneities with increasing geostrophic
 382 wind speed can be better observed through the magnitude of the dispersive
 383 fluxes computed for the study cases introduced in Sect. 3.2. To quantify the
 384 relative importance of the dispersive flux with respect to the total sensible
 385 heat flux, the following metric is introduced:

$$\frac{\int_0^{z_i} Q_{disp}(\xi) d\xi}{\int_0^{z_i} [Q_{Reynolds}(\xi) + Q_{SGS}(\xi) + Q_{disp}(\xi)] d\xi} = \frac{\int_0^{z_i} Q_{disp}(\xi) d\xi}{\int_0^{z_i} Q_{tot}(\xi) d\xi}, \quad (10)$$

386 where Q_{disp} represents the dispersive flux, $Q_{Reynolds}$ is the planar-averaged
 387 resolved turbulent sensible heat flux, and Q_{SGS} is the planar-averaged SGS
 388 contribution of the sensible heat flux. Because the fluxes are integrated over
 389 the full ABL column, this metric represents the integral fraction of the total
 390 sensible heat flux that arises from the dispersive fluxes. Note that the
 391 boundary-layer height is taken as the height where the total sensible heat
 392 flux crosses the zero value before the capping inversion. Figure 4 illustrates
 393 these results for values of fluxes computed for a 30-min time period. In this
 394 figure, the contribution of the dispersive fluxes for the heterogeneous surface
 395 cases progressively decreases as the geostrophic wind speed increases (which
 396 is a result of the increased blending as illustrated above) until the effect of

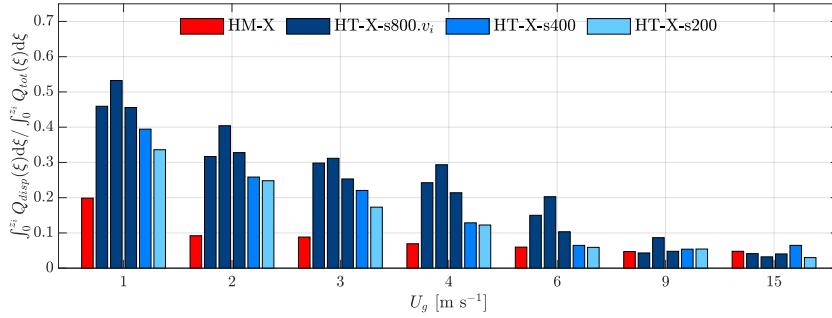


Fig. 4 Integral fraction of sensible heat flux accounted for by the dispersive fluxes as a function of geostrophic wind speed when averaged over a 30-min time period. (red) indicates the homogeneous cases; (dark-blue) heterogeneous cases with 800-m patches; (blue) heterogeneous cases with 400-m patches; (light-blue) heterogeneous cases with 200-m patches

397 the surface thermal heterogeneities does not have an impact on the disper-
 398 sive fluxes. These results seem to indicate the existence of two differentiated
 399 regimes. In the first regime, the contribution of the dispersive fluxes is intrinsic
 400 to the surface thermal heterogeneities, and hence represent a measure of the
 401 impact of the surface on the flow. In the second regime, the dispersive fluxes
 402 are fully due to the turbulent coherent structure, or consistency in time related
 403 to the surface induced shear. For the homogeneous cases, the dispersive fluxes
 404 arise from the coherent structures that remain in similar locations for the av-
 405 eraging period and their contributions decrease with increasing geostrophic
 406 wind speed. At low wind speeds, the dispersive fluxes arise from the persistent
 407 Rayleigh–Bénard convective cells. In the second regime, the dispersive fluxes
 408 are the result of coherent, turbulent structures induced by the shear. In both
 409 cases, in the limit of very long time averages, these would become zero for the
 410 homogeneous surface conditions. It is also interesting to see that the transi-
 411 tion between the convective regime and the second regime occurs faster for
 412 the homogeneous cases.

413 In situations where the dispersive fluxes are related to the surface ther-
 414 mal heterogeneities, the dispersive fluxes can account for more than 40% of
 415 the total sensible heat flux for the HT-1-s800.v_i cases, about 40% for HT-1-
 416 s400, and about 33% for the HT-1-s200 case when integrated over the full
 417 ABL depth. Also, from the results presented in Fig. 4, it can be observed that
 418 different surface spatial arrangements of patches with the same size (l_h) and
 419 standard deviation (cases HT-1-s800.v_i) lead to very similar dispersive flux
 420 contributions. This is an important result because it indicates that the disper-
 421 sive flux fraction is independent of any specific spatial distribution of thermal
 422 heterogeneities.

423 Because the definition of the dispersive fluxes is dependent on its time-
 424 averaging operation (as explained in Sect. 2), it is important to evaluate the
 425 dependence of the dispersive flux fraction on the time-averaging operation.
 426 Hence, Fig. 5 illustrates the integral fraction of sensible heat flux accounted for

427 by the dispersive fluxes as a function of time-averaging period, and geostrophic
428 wind speed. In Figs. 5a and b, the geostrophic forcing is weak and hence the
429 contribution of the dispersive fluxes is related to the surface thermal hetero-
430 geneities. There is a maximum decrease in dispersive flux contribution with
431 increasing time-averaging period from 5–30 min ranging between 30% and
432 50%. Note that this decrease is much smaller (< 15%) for averaging periods
433 greater than 20 min. Furthermore, the contribution of the dispersive fluxes
434 remains relevant even at 60-min averages, ranging between 30% and 45% for
435 the weakest geostrophic wind speed ($U_g = 1 \text{ m s}^{-1}$) and between 10% and
436 30% for a moderate geostrophic wind speed ($U_g = 3 \text{ m s}^{-1}$). Energy balance
437 closure studies have demonstrated similar dependence on the averaging time
438 (Foken 2006; Charuchittipan et al. 2014).

439 Further analysis of the contributions of the dispersive fluxes in the homoge-
440 neous cases indicate that the dispersive fluxes are appreciable at all geostrophic
441 wind speeds for short averaging times. In this case, the dispersive fluxes are
442 interpreted as being generated by coherent spatial distribution of turbulent
443 flow that are persistent over the short averaging times. When the geostrophic
444 forcing is strong and blends the surface heterogeneity, the dispersive fluxes
445 account for the structure of the time-resilient and shear-dominated turbulent
446 flow. This is illustrated well in Fig. 5c, cases where $U_g = 9 \text{ m s}^{-1}$ and the
447 effect of the surface heterogeneities is mostly blended. In these cases, the cor-
448 responding values of the dispersive fluxes found for large averaging times are
449 small, but not negligible.

450 To further explore this direct relationship between the dispersive fluxes and
451 the surface imposed thermal heterogeneities, Fig. 6 is a scatter plot comparing
452 the contribution of the dispersive flux for the homogeneous case with the het-
453 erogeneous cases (Fig. 6a), as well as the corresponding correlation between the
454 surface-temperature distribution and the air temperature at $z/z_i = 0.05$ (Fig.
455 6b). It can be observed that the correlation is maximum for lowest geostrophic
456 wind speed and decreases with increasing wind speed. In particular, the cases
457 with the largest correlations between the surface and air temperature corre-
458 spond well with the cases in which the integral contribution of the dispersive
459 fluxes is much larger than those found over the homogeneous surfaces. This
460 is illustrated in Fig. 6b. This important relationship between the surface and
461 air temperature at $z/z_i = 0.05$ is interpreted as being responsible for the
462 relevant contribution of the dispersive flux to the overall heat flux (see Fig.
463 6a). Further, we believe that this result could be exploited to develop simpli-
464 fied parametrizations of the dispersive fluxes based on remotely sensed surface
465 temperature measurements. In the case of the largest geostrophic wind speed,
466 this correlation is reduced to a minimum (< 50%), and the dispersive fluxes
467 match well with those measured over homogeneous surfaces indicating that
468 they are unrelated to the surface thermal patchiness.

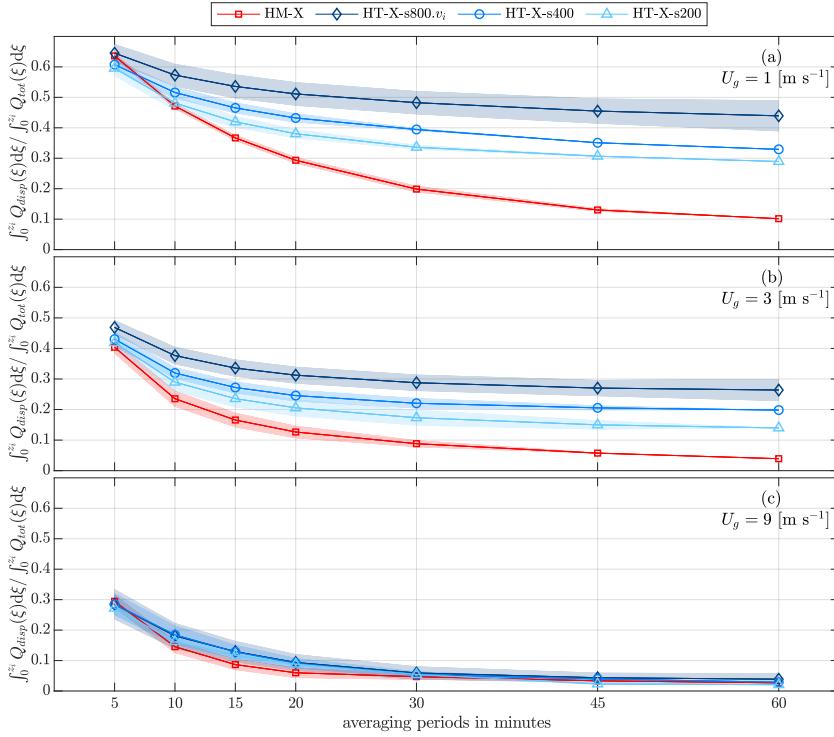


Fig. 5 Integral fraction of sensible heat flux accounted by the dispersive fluxes as a function of averaging time, and geostrophic wind intensity (top $U_g = 1 \text{ m s}^{-1}$, middle $U_g = 3 \text{ m s}^{-1}$, and bottom $U_g = 9 \text{ m s}^{-1}$). (red) indicates the homogeneous cases; (dark-blue) heterogeneous cases with 800-m patches; (blue) heterogeneous cases with 400-m patches; (light-blue) heterogeneous cases with 200-m patches

469 4.3 Dispersive Fluxes in the Surface Layer

470 Figure 7 provides additional information on the vertical distribution of the
 471 dispersive fluxes throughout the boundary layer. Specifically, Fig. 7 shows
 472 vertical profiles of the integral fraction of the dispersive flux with respect to
 473 the total sensible heat flux for 30-min time averages as a function of height,

$$\frac{\int_0^z Q_{disp}(\xi) d\xi}{\int_0^z [Q_{Reynolds}(\xi) + Q_{SGS}(\xi) + Q_{disp}(\xi)] d\xi} = \frac{\int_0^z Q_{disp}(\xi) d\xi}{\int_0^z Q_{tot}(\xi) d\xi}. \quad (11)$$

474 Note that in this case, both the numerator and the denominator represent the
 475 integral up to a given height z . The ratio of the dispersive flux over the total
 476 sensible turbulent heat flux represents the averaged cumulative contribution
 477 of the dispersive flux up to a given height. Hence, based on this definition, the
 478 dispersive flux contribution is zero at the surface and increases quickly with
 479 height until reaching saturation at around $z/z_i \approx 0.3\text{--}0.5$, depending on the

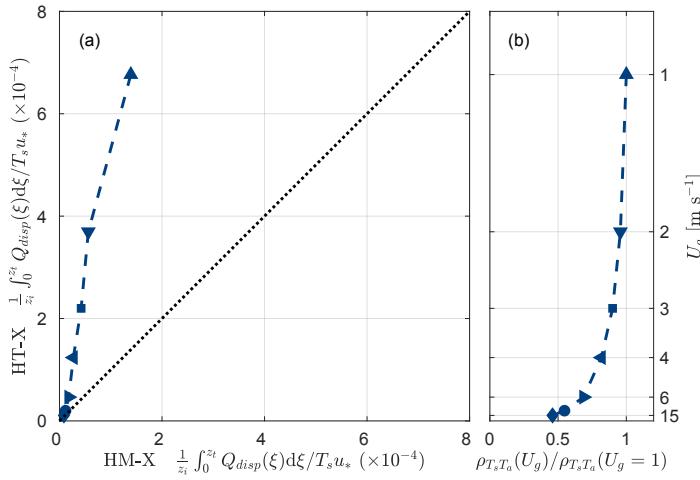


Fig. 6 (a) Comparison between the contribution of the dispersive fluxes over the full ABL for the cases with a heterogeneous thermal surface (averaged over the different patch sizes) and the case with a homogeneous thermal surface. (b) Correlation between the surface temperature and air temperature at $z/z_i = 0.05$ as a function of the geostrophic wind speed. Each point corresponds to the geostrophic forcing reported on the right vertical axis as represented by the different style of markers

480 geostrophic wind speed. In this figure, the horizontal black line illustrates the
 481 height above which the contribution of the dispersive fluxes does not change
 482 by more than 10%. From the profiles, it can be observed that close to the
 483 surface ($z/z_i \approx 0.02$) the contribution of the dispersive fluxes can be as much
 484 as 5–10% for a spatial average spanning the full domain. Similar to what had
 485 been observed in Fig. 4, the net contribution to the total sensible heat flux
 486 by the dispersive fluxes for the different surface conditions diminishes with
 487 increasing geostrophic forcing until there is no difference between the homo-
 488 geneous and heterogeneous cases (i.e. all vertical profiles overlap). This result
 489 is also in line with the two regimes discussed above, one in which the dis-
 490 persive fluxes are directly correlated with the surface thermal heterogeneities,
 491 and another one in which they are related to the surface shear-induced turbu-
 492 lent structure. These two regimes can be related to the LES study of Inagaki
 493 et al. (2006), which showed fluxes originating from two different processes:
 494 thermally-induced circulations and turbulent organized structures. Addi-
 495 tionally, Fig. 7 shows the ratio of the contribution of the dispersive fluxes to that
 496 of the turbulent contribution (dotted lines). This complementary illustration
 497 of the results further illustrates that dispersive fluxes are most important for
 498 low geostrophic wind speed cases, potentially being of equal or larger value
 499 than the turbulent fluxes. In fact, for weak geostrophic forcing, the ratio of
 500 the dispersive flux to the turbulent flux can be up to three times bigger for
 501 heterogeneous cases than the homogeneous case. In contrast, under strong
 502 geostrophic forcing, this difference between homogeneous and heterogeneous

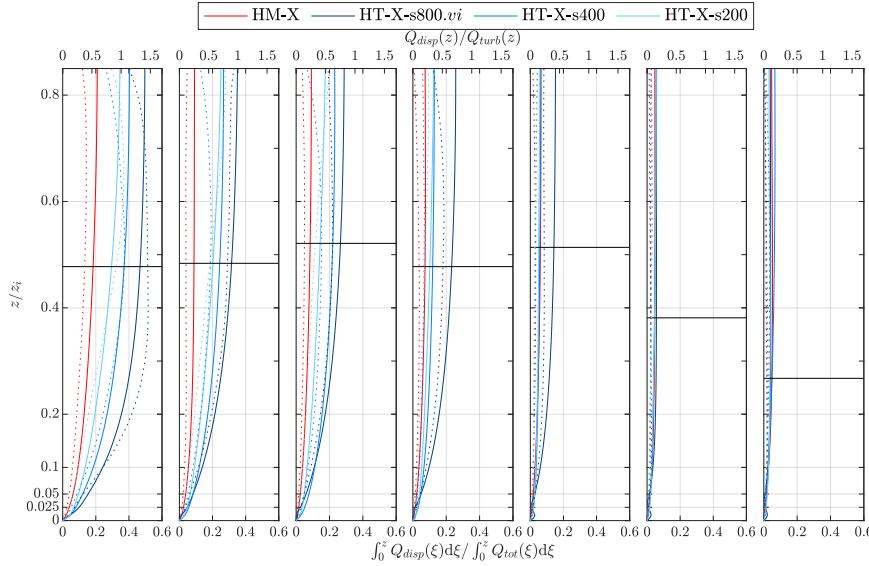


Fig. 7 Continuous lines represent the vertical profiles of the integral fraction of sensible heat flux accounted for by the dispersive fluxes for 30-min averages as a function of geostrophic wind speed (from left to right: $U_g = 1, 2, 3, 4, 6, 9, 15 \text{ m s}^{-1}$). The horizontal black line illustrates the height above which the contribution of the dispersive fluxes does not change by more than 10%. The dotted lines illustrate the local ratio of the dispersive flux to the total sensible heat flux at a given height z (with respect to the top axes)

503 cases falls below 15% compared to weaker forcing. In addition, even close to
 504 the surface ($z/z_i \approx 0.02$) the dispersive-flux contribution is non-negligible. The
 505 existence of large-scale structures close to the surface has been demonstrated
 506 through field measurements by Eder et al. (2015). These structures might be
 507 responsible for the behaviour of the fluxes observed here.

508 The results presented to this point have been horizontally averaged over
 509 the full domain. Therefore, while providing a measure of the overall contribu-
 510 tion of the dispersive fluxes in the bulk ABL transport processes as a function
 511 of differential geostrophic and surface forcings, this approach does not provide
 512 an answer to the a priori opening hypotheses of this work, namely: 1. Can
 513 dispersive fluxes provide a means of capturing the effect induced by the unre-
 514 solved surface heterogeneities in near-future NWP resolutions ($\sim 100 \text{ m}$)? 2.
 515 Can dispersive fluxes explain the non-closure of the surface energy budget?

516 To address these questions, a wide range of different horizontal-averaging
 517 length scales (Δ_{CS}) are used in the averaging operator $\langle \cdot \rangle$ that defines the dis-
 518 persive fluxes. Figure 8 provides a measure of the contribution of the disper-
 519 sive fluxes versus the turbulent fluxes as a function of the horizontal-averaging
 520 length scale. In this figure, the horizontal-averaging length scale is normalized
 521 by a ‘dispersive’ integral length scale. Results are shown at the study height of
 522 32 m. This height corresponds to the fourth grid point in the LES domain in
 523 the vertical direction, in which the potential numerical artifacts introduced by

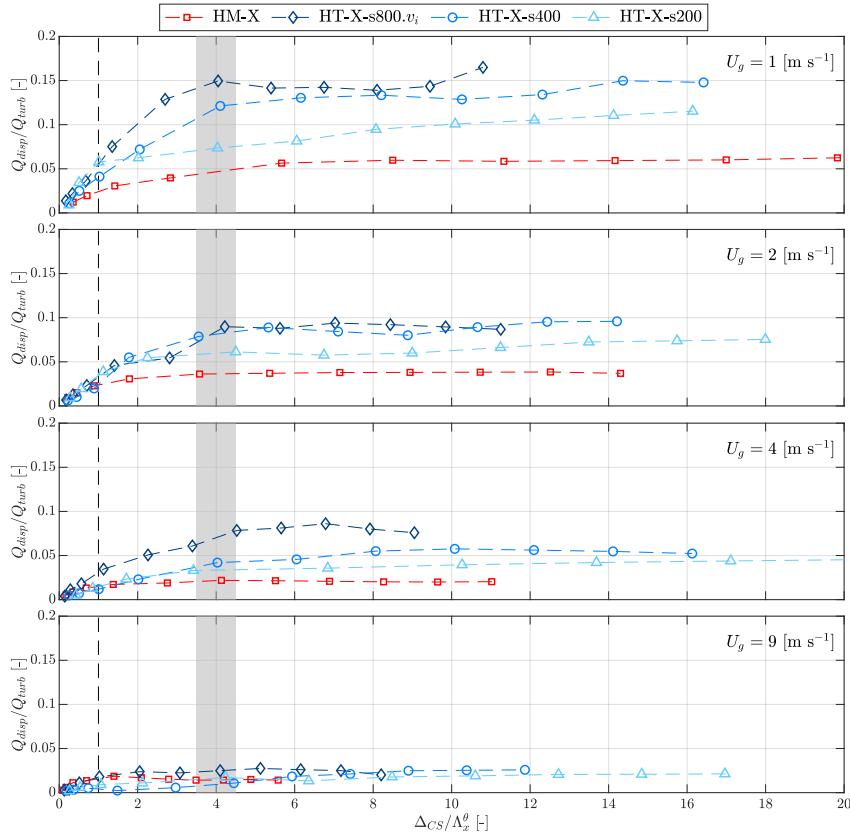


Fig. 8 Ratio of the dispersive fluxes versus the turbulent fluxes as a function of averaging surface defined by the Control Surface area Δ_{CS} and the dispersive integral length scale for temperature Λ_x^θ at $z = 32$ m. From top to bottom, the geostrophic forcing increases. The dashed line illustrates where $\Delta_{CS}/\Lambda_x^\theta = 1$, and the grey-shaded bar indicates the averaging area wherein the contribution from the dispersive fluxes and the turbulent fluxes reach equilibrium

524 the LES wall-boundary conditions have been diffused, and also approximately
 525 corresponds to the height of the first grid point in NWP models. Therefore,
 526 this is where surface parametrizations would be applied in NWP models.

527 The dispersive integral length scale (Λ_x^θ , dispersive integral length scale for
 528 temperature in the x -direction) characterizes the footprint of the surface heterogeneities on the time-averaged flow. This length scale is computed through
 529 the correlation of the spatial fluctuations of the time-averaged temperature
 530 field ($\bar{\theta}''$) in the x -direction, similar to what traditionally is done to compute
 531 the turbulent integral length scale (Pope 2000). This dispersive length
 532 scale (Λ_x^θ) is assumed to have a magnitude similar to the surface-heterogeneity
 533 length scale l_h shown in Fig. 1. Results indicate that when the ratio of the av-
 534 eraging control surface characteristic length scale Δ_{CS} to Λ_x^θ is approximately
 535

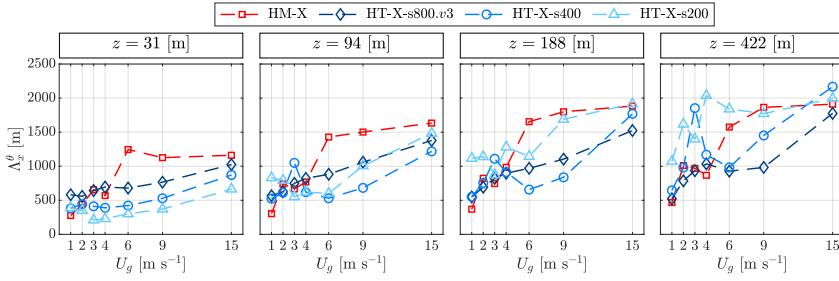


Fig. 9 Values of the dispersive integral length scale for temperature in the streamwise direction Λ_x^θ as a function of increasing height (left to right) and increasing geostrophic wind speed

536 4, one obtains the full contribution of the dispersive fluxes, with only minor
 537 changes to the fluxes as Δ_{CS} is increased.

538 The dispersive integral length scale is of similar magnitude to the size of
 539 the surface thermal patches when the geostrophic forcing is weak, as indicated
 540 in Fig. 9. This measure remains smaller than the physical size of the numerical
 541 domain ($2\pi z_i$) even for the largest patches. Therefore, an immediate conse-
 542 quence of these results is that the findings presented in Sect. 4 remain valid,
 543 despite having been averaged over the full LES domain, since the asymptotic
 544 behaviour is reached before needing to average over the whole domain. Also,
 545 from Fig. 8, it is worth noting that the dispersive-flux contribution to tends to
 546 zero when the averaging control surface is small in comparison to the size of the
 547 thermal heterogeneity length scale, which is equivalent to the hypothesis illus-
 548 trated in Fig. 1 and described in Sect. 1 when $l_d \gg l_h$ (i.e. the flow field feels
 549 the surface as homogeneous). Alternatively, when averaging over the correct
 550 length scales, the dispersive fluxes can contribute between 3 and 15% of the
 551 turbulent fluxes at moderate to low wind speeds. This result, although from a
 552 highly idealized scenario, already provides an initial preliminary response to
 553 the earlier hypothesis of whether dispersive fluxes could account for the 5–10%
 554 of energy that is traditionally missing when computing surface energy budgets
 555 (Foken 2008; Stoy et al. 2013). This result requires further confirmation either
 556 using experimental data or more realistic numerical simulations.

557 5 Spectral Structure of the Turbulent Flow

558 We further investigate the relationship between the flow structure (modulated
 559 by the surface heterogeneities and the mean flow) and the dispersive fluxes with
 560 the goal of aiding the future development of ASL parametrizations. First, we
 561 interrogate the turbulent fluctuations of vertical velocity and temperature, as
 562 well as their covariance for weak and strong geostrophic forcing, respectively.
 563 To describe the dominant structure in the flow, spectrograms are used, fol-
 564 lowing Jacob and Anderson (2017) and Salesky and Anderson (2018). This
 565 representation of turbulent pre-multiplied two-dimensional spectra provides a

566 good description of the structure of the flow. This method allows us to analyze
 567 the interaction between the different scales of motion and the resolved
 568 turbulent heat flux, noting that it is agnostic to the subgrid component of the
 569 heat flux.

570 The fluctuations used to compute the spectra are based on 30 instantaneous
 571 snapshots taken every 2 min with the 30-min averages subtracted.
 572 The two-dimensional spectra are computed by binning over shells of constant
 573 wavenumber $k_h = \sqrt{k_x^2 + k_y^2}$ and smoothed by averaging over the 30 snapshots
 574 (Wyngaard 2010).

575 Figure 10 presents spectrograms of turbulent fluctuations for the heterogeneous
 576 cases under weak and strong geostrophic forcing. Under weak forcing,
 577 the main peak of the vertical velocity fluctuations is located in the upper part
 578 of the ABL at scales between $\lambda_h = 1000$ m and $\lambda_h = 4000$ m. In addition, the
 579 small eddies ($\lambda_h < 1000$ m) close to the surface ($z < 100$ m) are less energetic
 580 than under stronger forcing. Similar spectrograms were reported by Salesky
 581 and Anderson (2018) and correspond to cell-type convection. The different
 582 surface conditions are modulating the small eddies in the first 100 m of the
 583 ABL. It can be observed that the smaller patches tend to generate smaller and
 584 more energetic eddies closer to the surface. The increasing geostrophic forcing
 585 stretches these coherent structures and redistributes the energy between
 586 different scales. Although the height of the main peak remains approximately
 587 the same, its location shifts towards larger scales. At the same time, the small
 588 eddies close to the surface are more energetic than for the weaker geostrophic
 589 wind speed. Also, no modulation from the surface heterogeneities is observed.
 590 A similar behaviour was also observed for the homogeneous surface cases al-
 591 though it is not represented here in Fig. 10 for the sake of clarity.

592 In contrast, the temperature fluctuations show the presence of a double
 593 hump in the lower 100 m of the ABL for all the heterogeneous surface cases.
 594 This feature is particularly evident under strong geostrophic forcing. It is
 595 worth noting that the valley in between has a similar scale as the size of the
 596 patches. This characteristic feature is not present in the homogeneous cases
 597 (not shown). Instead, these cases only have a small hump for short wave-
 598 lengths. This observation seems to indicate that the double hump observed
 599 can be explained by a combination of surface fluctuations induced by the flow
 600 ($\lambda_h \lesssim \Delta$) and fluctuations produced by the surface patchiness ($\lambda_h \gtrsim \Delta$). In
 601 addition, the lower hump shifts towards smaller scales under strong forcing,
 602 increasing the valley between the two humps.

603 In comparison, the structure of the covariance does not seem to be signif-
 604 icantly affected by the surface thermal heterogeneity. The weak geostrophic
 605 forcing cases exhibit very similar structures among the different surface condi-
 606 tions, where the dominant peak can be found between $z = 100$ m and $z = 1000$
 607 m. As with the vertical velocity fluctuations, increasing geostrophic forcing
 608 seems to shift the dominant structure towards larger scales that are higher in
 609 the ABL. In this case, the peak is found around $z = 500$ m and at a wavelength
 610 of $\lambda_h > 4$ km. A similar observation can also be made on the homogeneous

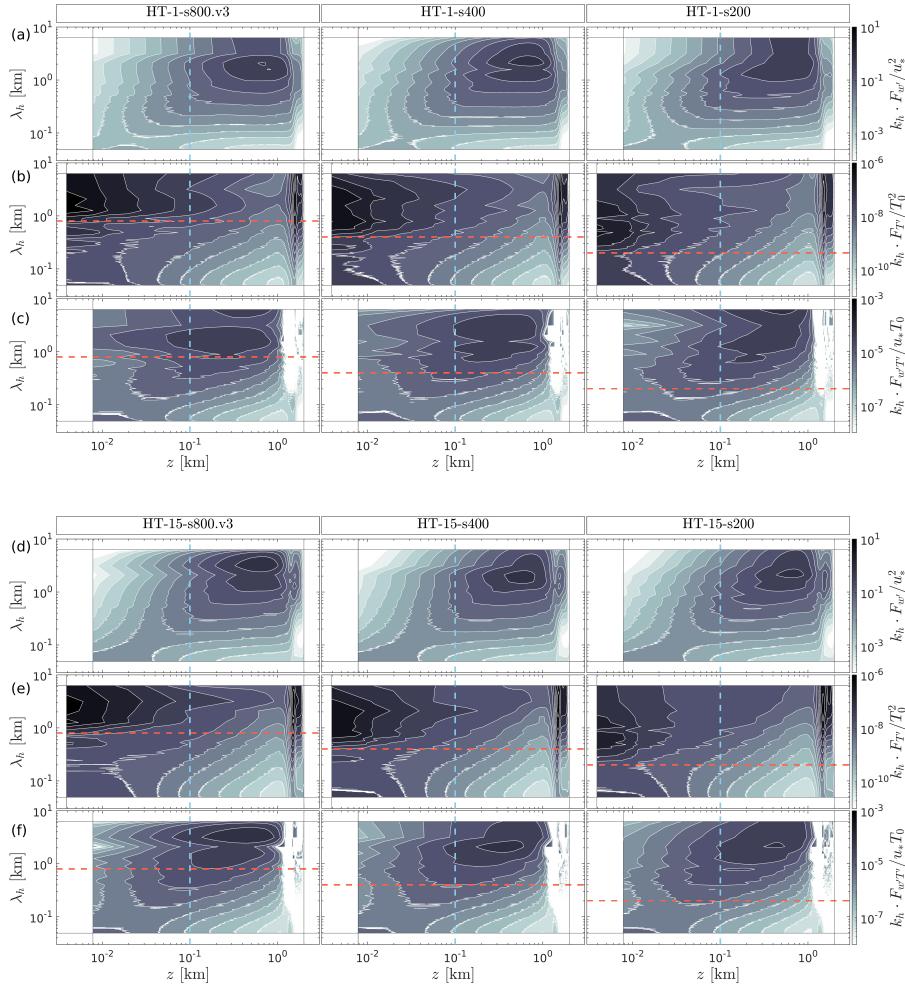


Fig. 10 Spectrograms of turbulent fluctuations for different patch sizes at $U_g = 1 \text{ m s}^{-1}$ (a, b, c) and $U_g = 15 \text{ m s}^{-1}$ (d, e, f). The x -axis represents the height z in km; the y -axis represents the streamwise wavelength $\lambda_h = 2\pi/k_h$ in km. The lines show the limits of the domain in the vertical and horizontal directions. The vertical dashed line (blue) shows the 100 m height corresponding to the top of the ASL. The horizontal dashed line (red) corresponds to the patch size $\lambda_h = \Delta$. For each geostrophic forcing, the panels are: (a, d) vertical velocity $F_{w'}$, (b, e) potential temperature $F_{T'}$, (c, f) cospectra of the vertical velocity and potential temperature $F_{w'T'}$. The spectra are averaged over 30 instantaneous snapshots

611 case (not shown). Hence, the spectral representation of the turbulent heat flux
 612 does not appear to be significantly modulated by the surface patchiness. Al-
 613 though some of the subgrid contributions might be missing in these figures,
 614 the imprint of the thermal surface heterogeneity is imposed on the flow and
 615 the heat exchanges mechanisms are clearly observed.

616 Overall, the turbulent spectrograms reveal that similar features are present
 617 in both the resolved flow field (w') and the resolved turbulent heat flux ($w'T'$)
 618 with minor modulations due to the surface conditions. However, the tem-
 619 perature spectrograms show the footprint of surface heterogeneities on the
 620 turbulent fluctuations with a second structure in the large scales. It is worth
 621 noting that the turbulent fluctuations of temperature do not appear in the
 622 turbulence kinetic energy equation but only appear in the heat flux budget
 623 equation in the buoyancy term (Stull 1988). Hence, the surface heterogeneities
 624 have a limited impact on the production of turbulence kinetic energy and the
 625 balance between shear and buoyant production.

626 Next, we investigate the structure of dispersive fluctuations. Following Rau-
 627 pack and Shaw (1982), the dispersive fluctuations are defined as $\bar{w}'' = \bar{w} - \langle \bar{w} \rangle$
 628 with \bar{w} representing time averaging and $\langle \bar{w} \rangle$ representing horizontal spatial
 629 averaging. Figure 11 illustrates the spectrograms of vertical velocity disper-
 630 sive fluctuations ($\bar{w}'' = \bar{w} - \langle \bar{w} \rangle$), temperature dispersive fluctuations ($\bar{T}'' =$
 631 $\bar{T} - \langle \bar{T} \rangle$), and their corresponding covariance ($\bar{w}''\bar{T}''$).

632 The spectrograms of the vertical velocity dispersive fluctuations show that
 633 the flow is significantly influenced by the geostrophic forcing. Under weak
 634 forcing, the dominant structures of the vertical velocity lie between $z = 300$ m
 635 and $z = 1000$ m, and span between $\lambda_x = 1$ km and $\lambda_x = 6$ km, independent
 636 of the surface thermal patch sizes. In contrast, under strong geostrophic wind
 637 speeds, the spatial fluctuations of the time-averaged velocity are small, as
 638 traditionally expected. The dominant structures are located between $\lambda_h = 1$
 639 km and $\lambda_h = 2$ km, and are mostly observed in the upper region of the ABL.

640 In contrast, the spectrograms of the dispersive temperature fluctuations
 641 are affected both by the size of the patches and by the geostrophic forcing.
 642 First, the size of the underlying patches correlates well with the dominant
 643 spectral length scales. Although these structures seem to be mainly concen-
 644 trated in the ASL, they can reach up to $z = 600$ m in height, spanning most
 645 of the CBL. As the geostrophic wind speed increases, strong mixing signifi-
 646 cantly reduces the span in height of the dominant structures. As expected,
 647 the corresponding dispersive cospectra exhibit a behaviour resulting from the
 648 combined responses observed in the vertical velocity and temperature disper-
 649 sive fluctuations. Therefore, the overlapping in wavelength of the structures
 650 present at weak geostrophic forcing leads to considerable dispersive fluxes. On
 651 the contrary, for high geostrophic wind speeds, the is no region of overlap
 652 between the characteristic wavelengths of vertical velocity and temperature
 653 dispersive fluctuations. Hence, the cospectra have small values under high
 654 wind speed conditions. These observations explain the contribution of the dis-
 655 persive fluxes reported in Fig. 4, because the integration of the cospectra over
 656 the wavelengths at a given height yields the dispersive contribution at that
 657 height. A similar integration can be made for the resolved turbulent flux.

658 To further understand how the cospectra of the turbulent heat flux and
 659 the dispersive flux are modified by the different surface thermal conditions and
 660 mean wind speeds, the scaling of the corresponding turbulent and dispersive
 661 cospectra is analyzed next. As expected from turbulence theory (Kaimal and

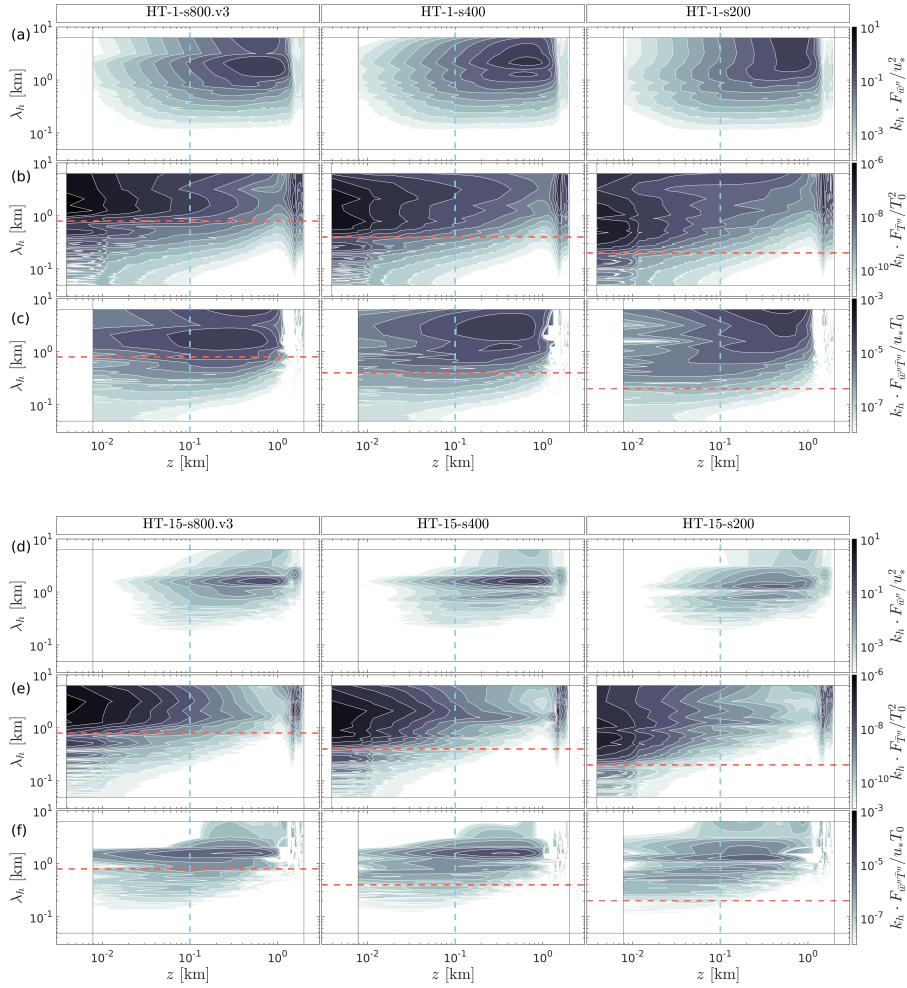


Fig. 11 Spectrograms of the time-averaged spatial fluctuations for the different patch sizes at $U_g = 1 \text{ m s}^{-1}$ (a, b, c) and $U_g = 15 \text{ m s}^{-1}$ (d, e, f). The x -axis represents the height z in km; the y -axis represents the streamwise wavelength $\lambda_h = 2\pi/k_h$ in km. The lines show the limits of the domain in the vertical and horizontal directions. The vertical dashed line (blue) shows the 100 m height corresponding to the top of the ASL. The horizontal dashed line (red) corresponds to the patch size $\lambda_h = \Delta$. For each geostrophic forcing, the panels are: (a, d) vertical velocity $F_{\bar{w}''}$, (b, e) potential temperature $F_{\bar{T}''}$, (c, f) cospectra of the vertical velocity and potential temperature $F_{\bar{w}''\bar{T}''}$. The spectra are averaged over 7 30-min periods

662 Finnigan 1994; Pope 2000; Li et al. 2015) the cospectra of the turbulent fluctuations scale with $k^{-7/3}$. This can be observed in Fig. 12a where the cospectrum corresponding to the homogeneous configuration is presented.

665 The cospectra of the heterogeneous cases also scale with $k^{-7/3}$ through the 666 high wavenumber region of the spectra ($k_h z_i > 10$). In this region, both the 667 homogeneous and the heterogeneous cases are very similar. Hence, within the

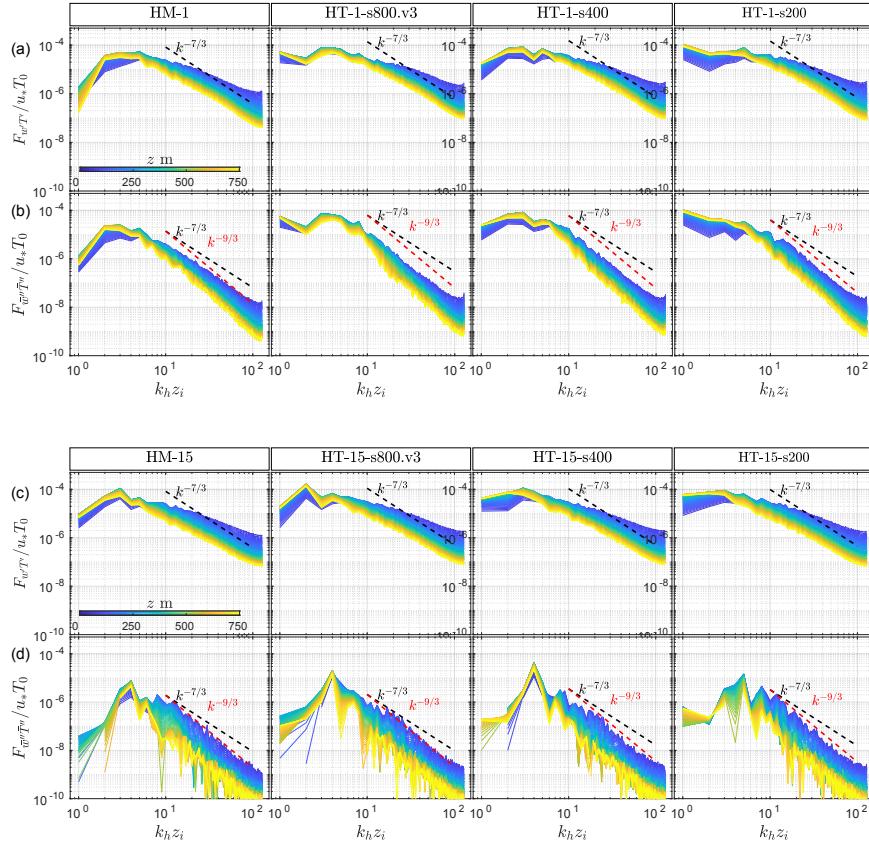


Fig. 12 Cospectra of the vertical velocity and potential temperature for the different patch sizes at $U_g = 1 \text{ m s}^{-1}$ (a, b) and $U_g = 15 \text{ m s}^{-1}$ (c, d). Black-dashed line represent the traditional $k^{-7/3}$ scaling of the turbulent flux cospectre, red-dashed line shows the $k^{-9/3}$ scaling observed in the dispersive flux cospectra. The colours represent the height above the ground where blue is the lowest level (50 m) and yellow correspond to a level around 750 m. For each geostrophic forcing, the panels are: (a, c) turbulent fluctuations, (b, d) time-averaged spatial fluctuation

668 mixed layer, the small-scale turbulent fluctuations follow the same dynamics.
 669 However, the low wavenumber part of the cospectra ($k_h z_i < 10$) seems to be
 670 affected by the surface patchiness with the heterogeneous cases having more
 671 energy at these scales.

672 In contrast, the dispersive flux cospectra exhibit notable differences be-
 673 tween the weak and strong forcing. The weak forcing cases show a $k^{-7/3}$ scal-
 674 ing to the surface similar to the resolved turbulent heat flux where the lower
 675 boundary condition dictates the dynamics of the flow. However, the scaling
 676 shifts progressively towards a $k^{-9/3}$ scaling higher up in the ABL. In the case
 677 of $U_g = 1 \text{ m s}^{-1}$, this $k^{-9/3}$ scaling can be observed for $z > 200 \text{ m}$. Also, the
 678 way in which this shift occurs seems to be independent of surface patch size.

679 In comparison, the change between the two different scalings does not seem
680 to occur in the cospectra of the dispersive flux at strong geostrophic forcing
681 where a $k^{-9/3}$ scaling dominates through the whole ABL. Figure 12d shows
682 that the cospectra are similar for all the surface conditions. Also, the smaller
683 amplitudes of the cospectra confirm that the dispersive flux is less significant
684 in the strong geostrophic forcing cases, which agrees well with the results pre-
685 sented earlier in the analysis developed based on flow statistics.

686

687 6 Discussion

688 In the early 1980s, the concept of dispersive fluxes was introduced to ac-
689 count for momentum fluxes arising in vegetated canopies as a result of first
690 time averaging and then spatial averaging the flow field (Raupach and Shaw
691 1982; Finnigan 1985; Raupach et al. 1986). This process can lead to persistent
692 flow heterogeneities in time. In the present study, dispersive fluxes are reinter-
693 preted as arising from the spatial-averaging operation that explicitly represents
694 critical processes dependent on heterogeneity induced in the flow by surface
695 thermal patchiness. This can be easily generalized to any process inducing
696 persistent flow heterogeneities. From the analysis presented, two main results
697 are extracted: the dispersive-flux contribution to the total energy exchange
698 in the ABL can be important, and dispersive fluxes are associated with the
699 topology of the underlying surface heterogeneity, or with the persistent turbu-
700 lent structure of the ABL flow (depending on the geostrophic forcing and the
701 time-averaging operation).

702 Further, results seem to indicate the existence of two regimes where the
703 role of the dispersive fluxes is modulated by different effects. The first effect is
704 one in which dispersive fluxes are driven by surface heterogeneities. The other
705 is a regime where dispersive fluxes are driven by long-lived coherent structures
706 of atmospheric turbulence in high-shear conditions. Therefore, differentiation
707 based on these two regimes could facilitate developing new parametrizations.
708 This is a topic of ongoing research.

709 In addition, spectral analysis revealed that the footprint of surface patch-
710 iness can be clearly found in the time-averaged quantities, especially the air
711 temperature. The presence of this footprint shows the interaction between the
712 ABL flow and the surface patchiness. This is especially relevant under weak
713 geostrophic forcing, where shear does not dominate over buoyancy. Further-
714 more, large dispersive fluxes result from the combination of reduced blending
715 of the mean temperature spatial fluctuations and increased mean vertical ve-
716 locity spatial fluctuations. Regarding the $k^{-9/3}$ scaling slope observed in the
717 cospectra of the dispersive fluctuations, further investigations are underway.
718 These first results indicate that substantial dispersive-flux contributions are
719 observed when the cospectra follow a $k^{-9/3}$ scaling and that the new scal-
720 ing presented in this paper is not linked to turbulence, as it is different than
721 $k^{-7/3}$. However, at this stage it is unclear how the different scaling is linked

722 to the dispersive flux and what mechanism leads to the new power-law ex-
723 ponent. However, we believe that this newly apparent scaling represents an
724 opportunity to model the dispersive fluxes over heterogeneous land surfaces.

725 While the results illustrate new physical interpretation of the interaction
726 between surface thermal heterogeneities and the atmospheric flow, it is also im-
727 portant to realize that the simulations used remain quite canonical, and hence
728 present certain limitations. For example, the simulations are forced through an
729 imposed surface temperature, which eventually leads to different atmospheric
730 stability values for the different study cases. While this could be an important
731 limitation in a study that focused on a one-to-one intercomparison of cases,
732 it does not affect the interpretation of the results. This is because the aim
733 has focused on illustrating that dispersive fluxes can be relevant in realistic
734 ABL conditions, and that these are dependent on geostrophic forcing, hetero-
735 geneity length scale, and time averaging. The reason for forcing the flow with
736 an imposed surface temperature, instead of an imposed surface flux, as done
737 in Salesky et al. (2017), is because the design of the simulations was inspired
738 from recent experimental measurements using a thermal camera over an alka-
739 line palya in Utah’s West Desert at the U.S. Army Dugway Proving Ground
740 (Morrison et al. 2017). Further, coupling of the surface with the atmosphere
741 through a strong one-way coupling limits the potential feedback that the land
742 surface might have on the atmospheric flow. Further, while some previous LES
743 studies have demonstrated that scalar transport due to turbulent organized
744 structures can be measured by the dispersive fluxes even over homogeneous sur-
745 faces (Kanda et al. 2004), other studies have also shown that the residual value
746 measured by the dispersive fluxes can be the result of locking large coherent
747 structures induced by the limited size of the LES domain and the periodicity
748 of the numerical algorithm as indicated in Munters et al. (2016). To check on
749 this potential pitfall, a test simulation with a four times larger domain was
750 also run for the HT-1-s800 study case. In this case, results corresponded well
751 with those obtained with the smaller domain, indicating that in the heteroge-
752 neous cases the LES domain size did not affect the results. It is also important
753 to realize that the numerical cost associated with such large LES configura-
754 tions makes the analysis presented here unfeasible given the large number of
755 required study cases.

756 7 Conclusions

757 We presented an LES study of the influence of surface thermal heterogeneities
758 on the atmospheric boundary-layer flow as a function of geostrophic forcing,
759 and as a function of thermal patch size. For the first time, we propose the use
760 of dispersive fluxes as a measure of the footprint that these surface thermal
761 heterogeneities have on the flow. Results illustrate that under weak geostrophic
762 forcing, dispersive fluxes can account for up to 40% of the total sensible heat
763 flux at about $0.1z_i$, with a value of 5 to 10% near the surface. These disper-
764 sive fluxes provide an indirect measure of the footprint that thermal hetero-

765 geneities have on the flow. Under stronger geostrophic forcing, heterogeneities
766 are blended, changing the structure of the flow, and reducing the dispersive
767 fluxes to approximately 5%. In this latter case, dispersive fluxes provide a
768 measure of the coherent structure of the mean flow induced by the ground
769 surface shear stress.

770 Finally, an innovative spectral analysis of the dispersive contributions has
771 shed light on the influence of both the geostrophic forcing and the different
772 surface conditions on the spatial fluctuations of the vertical velocity and tem-
773 perature. These observations indicate that large dispersive fluxes arise from
774 the combined effect of the limited blending of surface temperatures and large
775 vertical velocity structures created by the buoyancy forces. Finally, the cospec-
776 tral analysis also revealed the existence of a new power-law scaling for the
777 dispersive fluxes under weak shear forcing.

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