INNOVATING SCAFFOLDED PROTOTYPING FOR DESIGN EDUCATION: TOWARD A CONCEPTUAL FRAMEWORK DERIVED FROM MATHEMATICS PEDAGOGY

Hadi Ali	Barbara Kinach, Ed.D.	Micah Lande, Ph.D.
Arizona State University	Arizona State University	South Dakota School of Mines & Technology
hwali@asu.edu	barbara.kinach@asu.edu	micah.lande@gmail.com

The practice of prototyping is challenging to novice designers as they underutilize insights that prototyping offers to solving design problem. Central to this challenge is the abstract nature of design concepts like idea representation, iteration, and problem solution-space exploration. A unique opportunity from mathematics education presents itself for design educators and facilitators; that is, teaching with manipulatives. We seek to transfer such practices in mathematics education to design education and practice. Challenges exist for design researchers to carefully craft activities in design education mainly because of the open-endedness of problems, decision-making that takes place while designing, and the inherent uncertainties in the design problems. Ultimately, the goal is to develop students' ability to flexibly transfer expertise to other contexts and new design challenges.

Keywords: Prototyping, manipulatives, design education, mathematics education.

Objective: Navigating the Prototyping Challenge in Design Education

Engineers build and create with materials ("stuff") to fashion new designs. The practice of prototyping in the design process can be challenging for novice designers however. Students learning to become engineers often underutilize prototyping as a step for their design process (Ali & Lande, 2018). The abstract and nebulous nature of what and how to prototype is central to this challenge. We propose to navigate the prototyping challenge within design education with insights from mathematics education.

Theoretical Perspective: Prototyping in Design Education Against a New Horizon

Drawing parallels for design education from the manipulative "prototyping" literature within mathematics education we glean insight into the learning and teaching of abstract notions and processes. Borrowing teaching with manipulatives as a pedagogical approach from mathematics education presents a unique opportunity for design educators and facilitators. In mathematics education, modeling mathematical ideas with manipulatives has been demonstrated to be effective in teaching students abstract concepts and processes of mathematics, like addition and subtraction (Carpenter, Fennema, Franke, Levi & Empson, 2015), fractions (Cramer, 2002, 2003) and bases of ten (Dienes, 1960), and function (Arcavi, Tirosh & Nachmias, 1989). Established theories of mathematics learning (Dienes, 1960; Bruner, 1963) support the practical development of activities to introduce abstract mathematical ideas to students by means of manipulating physical objects while reasoning about the represented mathematical abstraction (Thompson & Thompson, 1990; Arcavi, 2003). Students are able to complete sophisticated mathematical processes with understanding through this visual externalization of their "thinking with" tangible items (Carpenter et al., 2015). We map such practices in mathematics education to transfer to design education and practice. Specifically, we aim to develop examples of scaffolded prototyping in manipulatives as a way to move from concrete representations of prototypes to abstract understanding of design.

There are parallels between the conceptualizations of prototyping within the “design” and “mathematics” education literature. Within each domain, there is a specific subject-matter content to be applied: mathematical ideas and algebraic symbols, built and tangible creations applying scientific and engineering principles. The phenomena of thinking moving from abstract to concrete though is similar. Both design artifacts and manipulative representations provide conceptual affordances otherwise difficult to access for learners.

Within engineering design, a prototype provides a means for feedback (Jørgensen, 1984), between the designer and the material (Lim, Stolterman & Tenenberg, 2008), amongst the design team (Hartmann, 2009; (Hollan, Hutchins & Kirsh, 2000; Hutchins, 1995; Star & Griesemer, 1989), or between the team and the user (Buchenau & Suri, 2000; Kelley & Littman, 2006; Schrage, 2013). Similarly, the function of the “prototype” physical embodiment (manipulative representation) of an abstract mathematical idea is to feature certain aspects of the mathematical notion, ideally in a one-to-one correspondence between objects, processes, and relations. Representational theories of mathematics learning (Dienes, 1960; Bruner, 1963) posit that humans learn abstract ideas (no matter the field) not from the abstraction or symbolization itself but through generalizing the abstract idea from multiple concrete representations of it. No single “prototype” (aka manipulative representation or concrete embodiment) will ever fully depict a mathematical abstraction. But from each prototypical manipulative representation, the learner gleans an aspect of the abstraction not captured in the same way by the other prototypical manipulative representations. From this collection of concrete (prototypical) representations, the learner generalizes the abstract mathematical concept. There are clear connections to a design artifact and mathematical education research provides language to more precisely develop mindful design learning experiences and expectations for physical prototyping.

Like aspects of prototypical engineering thinking, algebraic thinking involves the practice of abstraction; generalizing functions from the relationship between quantities; modifiability; and multiple representations and generalizations (Kieran, 2006). These are practices that are transferrable to the abstract representation of ideas in design prototyping and defining the relationships between shapes and functions in prototypes. One key misconception in the teaching and learning of design is that students approach prototyping as a final product, as opposed to intermediate steps to learn about the design problem (Cross, 2018), gain insight (Lawson & Dorst, 2009), and reflectively learn design (Beckman & Barry, 2007). Moreover, the focus, in teaching design currently, is not on simplifying the approach to the solution, nor attempting to find the most efficient one; rather to breakdown the process and make it visible, with the ultimate goal for students to take this learning to other contexts. Thus, just as the literature on mathematical knowledge for teaching recognizes that being able to “do” math is different from being able to “teach” math (Ball & Bass, 2002), design instructors would do well to recognize the same for engineering and engineering design. Similar to the expectations for a mathematics instructor, the design instructor should be able to carefully plan the lesson, creating a range of possible, yet expected, paths that the students may take.

Modes of Inquiry: Results on the Horizon

Design educators do well on encouraging students to be creative, but they do not take this to the next step of actually teaching design, harnessing this creativity, and guiding the process. There is a gap, beyond students sharing their ideas and methods in the class, to actually learn design. The questioning approach to teaching is very much needed (Boaler & Humphreys, 2008). A design educator should build on these teaching practices, as intermediate milestones, asking questions, and using the artifact to elicit understanding (Cramer, 2003). There is an opportunity to use case studies to capture such moments in design education, which is often beyond simply characterizing students' behavior or observing current practices in teaching.

The overarching research question for this research design is as follows: *How can the utilization of object representation of mathematical concepts in mathematics education be transferred to object representation in scaffolded prototyping in design education?*

Preliminary Research Design

Overview and Participant Characteristics

In order to conduct the research study, our strategy involves creating a case for building prototypes in a controlled observation environment. The research design emphasizes producing relevant and meaningful results through substantiated rationale and supported by evidence from the collected data. In the preliminary stage of the research, participants of four students from four different disciplines (design, engineering, business and sustainability) will be recruited to complete a structured design task. Because different disciplines emphasize different epistemologies and practices, the setting will evoke the unique utilization of manipulatives as boundary spanning objects between the different participants. The research design will aim for case creation and comparisons across multiple groups of participants. Ideally, design projects span multiple months; however, the controlled task setting in a laboratory setting will allow a focused laboratory-based study.

Characterizing the Design Task

The participants will be provided an open prompt accompanied by a carefully selected set of manipulatives. Observations of group interactions will be video- and audio-recorded to allow for careful analysis. Control for the objects that participants will have access to should allow for augmentation of the underlying cognition and the corresponding exhibited behavior by participants. In order to select the manipulatives that participants will have access to, key considerations should be provided for the following issues surrounding prototyping (Lewrick, Link & Leifer, 2018):

- What are the basic functions for the user?
- What hasn't the user taken into consideration at all yet?
- How has nobody ever done it before?

The manipulatives should allow physical modeling to represent products, spaces and environment, in a way that provides an executable version of the design but with only the most necessary functions. In addition, the observatory lab setting should allow the reproduction of specific situations with team members doing physical acting.

Example. In an attempt to pilot the study to understand students' approaches to prototyping, we asked students to create three different prototypes of "an exercise machine that saves time and space." In this project, the idea was to push the students beyond the machine itself, thinking about larger contexts of exercising and healthy living—a readily available machine in a dorm room, for example, can save time for the students not needing to walk for the gym if it is designed in a way not to take much space as well.

Contextualizing the Design Task

Generally, the prompt that will be provided to the students should be planned to cross the boundaries between their different areas of expertise. More specifically, it should be framed to push the students to construct a collective, shared understanding in a very direct and visual manner. The *directness* and *visibility* of the idea representations should be a center in the design activity, so that playful insights are attained across multiple interactions with the objects. The goal is to provide a deeper, shared understanding of the solution vision while prompting collaboration and discussions. The design task tentatively has the following structure, which moves participants in a series of divergent and convergent modes of thinking:

- Given a set of provided initial solution scenarios, identify which one you would like to test. In the process, you should ponder which functions are absolutely critical
- Think about which variant should be built
- At this point, an intervention on how to allow the "objects to talk" should be perceived
- The built prototypes should allow feedback, moving to the next phase of essential features and solutions in a scaffolded manner. Breaking the groups of four into pairs should allow some variety for testing ideas
- Finally, revised new variants of prototypes should be conceived.

Example. In our pilot study of the "exercise machine," students were expected to combine analytical thinking with creative synthesis. Looking ahead at the implementation of the study, a promising opportunity exists because the learning objectives of the design course are aligned with the mathematical and physical knowledge that the students have acquired at this stage. For example, students are expected to utilize springs as means to achieve the desired functionality in their proposed design. This presents an opportunity to develop course modules, using manipulative, to present the different behavior of springs to the students. Once they understand and are able to mathematically analyze the behavior of the spring, they can start using it in their prototype that is directed to fulfill the need of a user. Figure 1 below provides an example of the students' work in the pilot study.

Figure 1: Sample of a students' prototype in the pilot study: "Resistive Core Trainer"

Conclusions and Future Work

Ultimately, the goal is to allow students to flexibly transfer the learned expertise in the proposed activities to other contexts and new design challenges. Building on the understandings for learning from mathematics can have useful implications for the teaching and learning of engineering design. Furthermore, the outcome of this study could guide future bidirectional studies by benefiting not only design education, but also mathematics education, especially at the college-level courses.

References

Ali, H., & Lande, M. (2018). Why make it? Understanding undergraduate engineering students' conceptions for the purpose of prototyping in engineering design activities. *IEEE Frontiers in Education Conference.* Salt Lake City, UT.

Arcavi, A. (2003). The role of visual representations in the teaching and learning of mathematics. *Educational Studies in Mathematics, 52*(3), 215–241.

Arcavi, A., Tirosh, D., & Nachmias, R. (1989). Effects of exploring a new representation on prospective mathematics teachers' conceptions of function. In S. Vinner (Ed.). *Proceedings of the 2nd International Jerusalem Convention on Education: Science and Mathematics Education – Interaction Between Research and Practice* (pp. 269-277). Israel: University of Jerusalem.

Ball, D. L., & Bass, H. (2002). Toward a practice-based theory of mathematical knowledge for teaching. In E. Simmt & B. Davis (Eds.) In E. Simmt & B. Davis (Eds.). *Proceedings of the 2002 Annual Meeting of the Canadian Mathematics Education Study Group.* Edmonton, AB: CMESG/GCEDM.

Beckman, S. L., & Barry, M. (2007). Innovation as a learning process: embedding design thinking. *California Management Review, 50*(1), 25–56.

Boaler, J., & Humphreys, C. (2005). *Connecting mathematical ideas: Middle school video cases to support teaching and learning.* Portsmouth, NH: Heinemann.

Bruner, J. (1963). *The process of Education.* New York, NY: Vintage.

Buchenau, M., & Suri, J.F. (2000). Experience prototyping. *Proceedings of the 3rd conference on designing interactive systems: Processes, practices, methods, and techniques* (pp. 424–433).

Cramer, K. (2002). Using manipulative models to build number sense for addition of fractions. In G. Bright & B. Litwiller (Eds.). *Making sense of fractions, ratios, and proportions, 2002 yearbook of the national council of teachers of mathematics* (pp. 49–60). Reston, VA: National Council of Teachers of Mathematics.

Cramer, K. (2003). Using a translation model for curriculum development and classroom instruction. In R. Lesh & H. M. Doerr (Eds.). *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 449–463). Mahwah, NJ: Lawrence Erlbaum Associates.

Cramer, K., Wyber, T., & Leavitt, S. (2008). The role of representation in fraction addition and subtraction. *Mathematics Teaching in the Middle Grades 13*: 490–496.

Cross, N. (2018). *Design thinking*. London, England: Bloomsbury Visual Arts.

Dienes, Z. (1960). *Building up mathematics* (4th edition). London, England: Hutchinson Educational.

Hartmann, B. (2009). *Gaining design insight through interaction prototyping tools.* PhD Dissertation, Stanford University.

Hollan, J., Hutchins, E., & Kirsh, D. (2000). Distributed cognition: toward a new foundation for human-computer interaction research. *ACM Transactions on Computer-Human Interaction*, *7*(2), 174–196.

Hutchins, E. (1995). *Cognition in the wild.* Cambridge, MA: MIT Press.

Jørgensen, A. (1984). On the psychology of prototyping. In *Approaches to prototyping* (pp. 278–289). Berlin, Germany: Springer Verlag.

Kelley, T., & Littman, J. (2006). *The ten faces of innovation: IDEO's strategies for defeating the devil's advocate and driving creativity throughout your organization.* New York, NY: Crown Publishing Group.

Kieran, C. (2006). Research on the learning and teaching of algebra: A broadening of the sources of meaning. In A. Gutiérrez & P. Boero (Eds.), *Handbook of research on the psychology of mathematics education: Past, present and future* (pp. 11–24). Rotterdam, The Netherlands: Sense Publishers.

Lawson, B., & Dorst, K. (2009). *Design expertise.* Oxford, UK: Architectural Press.

Lewrick, M., Link, P., & Leifer, L. (2018). *The design thinking playbook: Mindful digital transformation of teams, products, services, businesses and ecosystems*. Hoboken, NJ: John Wiley & Sons.

Lim, Y., Stolterman, E., & Tenenberg, J. (2008). The anatomy of prototypes. *ACM Transactions on Computer-Human Interaction*, *15*(2), 1-27.

Schrage, M. (2013). *Serious play: How the world's best companies simulate to innovate.* Cambridge, MA: Harvard Business Press.

Star, S., & Griesemer, J. (1989). Institutional ecology, `translations' and boundary objects: Amateurs and professionals in Berkeley's Museum of Vertebrate Zoology, 1907-39. *Social Studies of Science*, *19*(3), 387-420.

Thompson, P., & Thompson, A. (1990). Salient aspects of experience with concrete manipulatives. In G. Booker, P. Cobb & T. de Mendicuti (Eds.). *Proceedings of the 14th Annual Meeting of the International Group for the Psychology of Mathematics Education* (pp. 46–51). Oaxtepec, Mexico.