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# BEARING-ONLY LOCALIZATION OF A QUASI-STATIC SOUND SOURCE WITH A BINAURAL MICROPHONE ARRAY

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## ABSTRACT

*Sound source localization is the ability to successfully understand the bearing and distance of a sound in space. The challenge of sound source localization has been a major are of research for engineers, especially those studying robotics, for decades. One of the main topics of focus is the ability for robots to track objects, human voices, or other robots robustly and accurately. Common ways to accomplish this goal may use large arrays, computationally intensive machine learning methods, or known dynamic models of a system which may not always be available. We seek to simplify this problem using a minimal amount of inexpensive equipment alongside a Bayesian estimator, capable of localizing an emitter using easily available a-priori information and timing data received from a prototype binaural sensor. We perform an experiment in a full anechoic chamber with a sound source moving at a constant speed; this experimental environment provides a space that allows us to isolate the performance of the sensor. We find that, while our current system isn't perfect, it is able to track the general motion of a sound source and the path to even more accurate tracking in the future is clear.*

## NOMENCLATURE

SSL sound source localization
DOA direction of arrival
TDOA time difference of arrival
EKF extended kalman filter
MMSE minimum mean squared error

## INTRODUCTION

Sound source localization (SSL) is the process of using passive transducers to estimate the location of an emitting source in space. This ability has interested biologists for centuries, originating from their interest in human and echolocating animal audition [1], [2], [3]. For engineers working in the field of robotics, harnessing this ability allows for tracking of acoustic objects, inter-robot communication, and mapping of acoustic environments, see for example [4]. A recent review by Rascon and Meza [5] illustrates many more of the current solutions and challenges in the field while also providing a good introduction to the topic of SSL in robots.

For SSL to be successful, two main questions must be answered. Which direction is the sound coming from? And how far away is it? While robotic audition is a mature field of research, there are still many challenges regarding these two objectives, especially when it comes to distance estimation with minimal hardware. Direction of arrival (DOA) estimation is most popularly undertaken using two main methods: generalized cross-correlation with phase transform weighting [6] (GCC-PHAT) and multiple signal classification [7] (MUSIC). These techniques are employed for single and multiple DOA estimation respectively and have proven to be robust in practical situations. The confounding factor of DOA estimation comes in the form of the so called 'cone of confusion', an area arising due to geometric symmetry of transducer arrays where received sound has identical phase delays. Breaking the symmetry using a rotating array or artificial head in conjunction with a head related transfer function can resolve this issue [8], [9], [10] though it does add more complication to SSL algorithms.

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The challenge of distance estimation has been tackled using a number of solutions including: triangulation from time difference of arrival (TDOA) using arrays of three or more microphones, direct-to-reverberant ratio estimation (DRR) techniques, and learning based approaches [5]. While shown to work successfully, these techniques are not always robust and may require large amounts of equipment, *a-priori* information about the operating environment and dynamics of emitter if motion is involved, or require large amounts of computing power not allowing real-time operation.

While SSL alone is a robotics challenge, its functionality becomes apparent when repeated over time, enabling the tracking of a dynamic emitter or other robot. Two popular estimation techniques used when tracking are Kalman filtering, particularly extended Kalman filtering (EKF) in nonlinear cases, and particle filtering. For instance [11] tracks a sound source assuming direct measurement of its location and use a multiple mode Kalman filter to reduce the effect of noise. A mixture Kalman filter is implemented in [12] to track an intermittent sound source with known state space dynamics. Recent work presented in [13] uses EKF with TDOA estimation to show that, with a minimum of 4 distributed agents connected in a network, the localization process is generically observable with each agent being able to localize a target accurately. Particle filters have the same general objectives as Kalman filters but do not require that the system and measurement models be linear or Gaussian and instead estimate probability density functions of the system state. Work investigating the localization of a moving sound source and sensor motion that reduces error is presented in [14].

This paper seeks to be an early investigation into using a previously simulated algorithm for performing SSL with minimal *a-priori* information [15] using a prototype physical system. Assuming knowledge of the sound source that one may have if they were implementing an echolocating robot, such as robot speed or sound patterns, it may be possible to localize positions based on passive sonar alone. Using an emitter attached to a track to ensure constant speed and a predictable path, we use a prototype binaural microphone array to gather data online. We then perform localization with post processed data and EKF offline using the cited algorithm above. We are able to address the challenge of SSL using inexpensive equipment and minimal *a-priori* knowledge while still maintaining acceptable accuracy in measurements. This paper presents the first step in bringing a simulation-tested algorithm into a physical laboratory setting.

## MODELING

This experiment tackles the problem of bearing-only sensing of a quasi-static emitter, with known constant speed $v$ and sound emission interval $\Delta$ . Using a binaural microphone array, it is possible to calculate the bearing angle of the incoming sound and localize its distance with respect to the sensor using an estimation algorithm built from these assumed values and unknown but calculable dynamics.

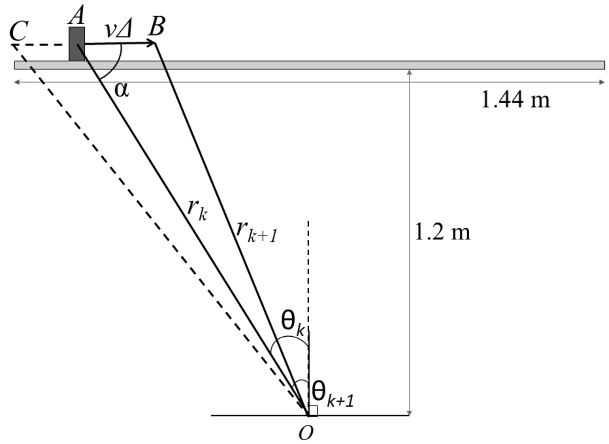


**FIGURE 1**. A schematic of the emitter moving along the track with a constant speed. The emitter is represented as point $A$ at time step $k$, with $B$ and $C$ being potential positions for the emitter at time step $k+1$. The center of the microphone array is indicated as point $O$. Dimensions of the experimental setup are also shown in meters.

A similar problem is studied in [15], where the time interval of pulse arrival of the sound to the binaural microphone array is used to build a dynamic model for the sound source. However, we may not need the time interval of arrival if we know that the speed of the emitter is much less than the speed of sound. In that case, the emitter is quasi-static and we can assume that the dynamic equation of the sound source obeys the following:

$$r_{k+1} = r_k + \nu_k, \tag{1}$$

where $r_k$ is the range of the emitter to the microphone array at time step $k$ and $\nu_k$ is the process noise, which compensates for the unknown slight movement of the sound source between two pulses as well as random noise in the motion of the sound source.

A schematic shown in Figure 1 represents the layout of the experiment. At time step $k$, the sound source located at point $A$ is emitting a pulse and moves on the track with constant speed $v$ for time $\Delta$ until it reaches point $B$, where it sends the second pulse. Application of the rule of cosines in triangle $OAB$ yields,

$$(v\Delta)^2 = r_{k+1}^2 + r_k^2 - 2r_{k+1}r_k\cos(\theta_{k+1} - \theta_k). \tag{2}$$

where $\theta_k$ is the bearing of the sound source with respect to the heading direction of the sensor at time step $k$. We can manipulate this equation and explicitly solve for $\theta_{k+1}$ and then disturb the result with a zero-mean independent noise $\omega_k$ to find the measurement model as

$$\theta_{k+1} = \theta_k \pm \cos^{-1}\left(\frac{r_{k+1}^2 + r_k^2 - v^2\Delta^2}{2r_{k+1}r_k}\right) + \omega_k. \quad (3)$$

The resulting sign uncertainty in (3) can be resolved by choosing the sign that minimizes the error between the outcome of the measurement model (3) and the measured bearing angle at time step $k+1$.

The dynamic and measurement update equations presented respectively in (1) and (3) make it possible to estimate the range of the sound source using model-based Bayesian estimation algorithms. To this end, we employ the estimation algorithm presented in [15] based on the linear minimum mean squared error (MMSE) estimator. This algorithm is summarized in the following section.

## METHODS

### Linear MMSE Estimation Algorithm

The dynamic update equation (1) and the measurement update equation (3) can be written in the following general format,

$$x_{k+1} = f(x_k) + v_k, \quad (4)$$

$$z_{k+1} = h(x_{k+1}, x_k; z_k) + \omega_k, \quad (5)$$

where $x_k$, $z_k$, $v_k$ and $\omega_k$ are the state vector (the current predicted Cartesian position of the emitter), the measurement vector (the measured TDOA), and process and measurement noises at time step $k$, respectively. Here, $f(.)$ and $h(.)$ represent the process and measurement models, respectively. Although the dynamic equation is linear and in the common format of Kalman filtering, the measurement equation is not in that standard format. Therefore, we cannot simply apply EKF equations to this problem. However, we can implement the linear MMSE to design an algorithm to estimate the state vector as suggested in [15]. This algorithm is summarized in table 1 and the derivation steps are explained in [15] for zero-mean uncorrelated Gaussian process and measurement noises. In this table, $\bar{x}_k$ and $\bar{P}_{xx}$ are respectively the predicted state at time step $k$ and its covariance calculated only by using dynamic model, $\hat{x}_k$ and $P_{xx}$ are the estimated state at time step $k$ and its covariance after making correction by using sensor measurement, $Q$ is the process noise covariance and $R$ is the measurement noise covariance.

### Binaural Microphone Array

A prototype binaural array was used to calculate and collect TDOA data from the emitter position. The array was built using a

**TABLE 1.** An iteration of the estimation algorithm

**Inputs:** $\hat{x}_k, P_{xx}, z_k, z_{k+1}$

**Outputs:** $\hat{x}_{k+1}, P_{xx}$

**Prediction:**

$$\bar{x}_{k+1} = f(\hat{x}_k)$$

$$F_k = \left.\frac{\partial f}{\partial x_k}\right|_{(\hat{x}_k, v_k=0)}, \Gamma_k = \left.\frac{\partial f}{\partial v_k}\right|_{(\hat{x}_k, v_k=0)}$$

$$\bar{P}_{xx} = F_k P_{xx} F_k^T + \Gamma_k Q \Gamma_k^T$$

**Correction:**

$$\bar{z}_{k+1} = h(\bar{x}_{k+1}, \hat{x}_k; z_k)$$

$$H_k = \left.\frac{\partial h}{\partial x_k}\right|_{(\bar{x}_{k+1}, \hat{x}_k)}, \bar{H}_k = \left.\frac{\partial h}{\partial x_{k+1}}\right|_{(\bar{x}_{k+1}, \hat{x}_k)}$$

$$\bar{P}_{xz} = \bar{P}_{xx}\bar{H}_k^T + F_k P_{xx} H_k^T.$$

$$\bar{P}_{zz} = \bar{H}_k \bar{P}_{xx} \bar{H}_k^T + \bar{H}_k F_k P_{xx} H_k^T + H_k P_{xx} \bar{H}_k^T F_k^T + H_k P_{xx} H_k^T + R$$

$$\hat{x}_{k+1} = \bar{x}_{k+1} + \bar{P}_{xz}\bar{P}_{zz}^{-1}\left(z_{k+1}^c - \bar{z}_{k+1}^c\right)$$

$$P_{xx} = \bar{P}_{xx} - \bar{P}_{xz}\bar{P}_{zz}^{-1}\bar{P}_{xz}^T$$

$k = k+1$ and repeat

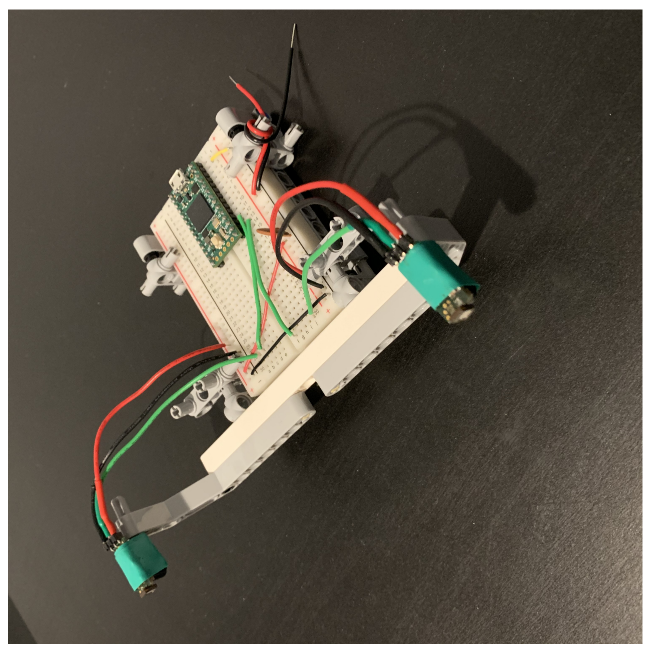

**FIGURE 2.** The prototype sensor above is made up of a Teensy 4 microcontroller (attached to the bread board), with a binaural array of analog ultrasonic microphones (attached to the front of the mount). The capacitor on the breadboard is a bypass capacitor, used to reduce high frequency noise coming from the power supply.



 

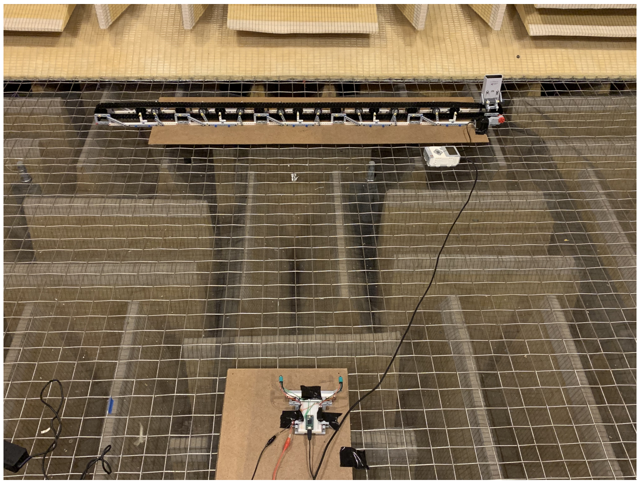

**FIGURE 3**. The experimental setup. The sensor was placed 1.4 m from the track and was reasonably centered. Show near the top of the figure is the track with the emitter secured on top on the far right of the track

Teensy microcontroller (Teesny 4.0, PJRC, USA) in conjunction with two Dodotronic Momimic analog ultrasonic microphones (Momimic Analog Microphone, Dodotronic, Italy) as seen in Figure 2. TDOA estimation begins when a signal is received in one microphone, starting a microsecond timer that continues until the noise is detected in the second microphone. Currently, the microphones use raw amplitude data to detect signals and there are no hardware anti-aliasing filters incorporated in the sensor, making the sensor extremely susceptible to corrupting noise and unusable for experiments outside of an anechoic chamber or quiet environments. Future versions of the sensor will make use of anti-alias filtering and will take advantage of cross-correlation TDOA estimation to increase robustness which is necessary for use in robotic applications.

Our inspiration for the sensor comes from the frequency ranges and shapes of the calls of echolocating bats [16]. The Teensy 4.0's ARM Cortex M7 CPU has enough speed to allow for simultaneous conversion of the onboard analog-to-digital converters at sub 2 $\mu$s speeds, which is well beyond what is necessary to convert calls between 35-80 kHz, the most frequently used range for echolocation.

## Experimental Setup

Due to the prototype nature of the sensor, all experimental data was gathered within a full anechoic chamber on the Virginia Tech campus. The emitter (Ultrasonic Calibrator, Wildlife Acoustics, USA) outputs a 40 kHz tone every 350 ms and microphone readings were taken at a frequency of 200 kHz, with TDOA data only being recorded if the microphones picked up a tone. The emitter was attached to a 1.44 m track created from Lego MINDSTORM EV3 robotics pieces, with the binaural sensor placed approximately 1.2 m away from the track and centered within a few centimeters of error, see Figure 3. Using a set track with a motor allowed for the major assumption of the algorithm, constant speed of the sound source, to be achieved and provided us with a known trajectory to look for in the data. To come up with an estimate of the standard deviation of the sensor's bearing angle measurement noise, we performed 100 repetitions of angle measurement at 6 different positions on the track. For each position, we found the standard deviation of the bearing angle by analyzing a histogram of the data. The standard deviations were then averaged pooling data from all locations and set as the standard deviation of the measurement noise. Data collection consisted of forty repetitions of traveling both left and right across the track, which we felt was adequate with the very low variability in the data due to using the anechoic chamber and systematically repeating the motion.

## Post-processing

We note that data collection and localization were not performed simultaneously and that all results are an outcome of post processing performed using MATLAB. We converted the TDOA data into angles using the far-field assumption and the formula

$$\theta_k = \sin^{-1}\left(\frac{V_{sound} * \tau_o}{f_s * d}\right) \tag{6}$$

where $V_{sound}$ is the speed of sound, here taken as 343 $\frac{m}{s}$, $\tau_0$ is the TDOA of the sound source in samples, $f_s$ is sampling frequency, and $d$ is the inter-microphone distance. The angular time series were then passed through a first-order low-pass Butterworth filter to remove measurement noise.

As we mentioned before, this algorithm is derived from linear MMSE estimation based on zero-mean uncorrelated Gaussian process and measurement noise. While this assumption is valid for the measurement noise in our problem, it is not exactly true for the process noise, since the dynamic update model assumes the sound source to be stationary, since it is moving with a speed significantly less than speed of the sound. Therefore, the process noise is not zero mean and has an unknown distribution that is not Gaussian. Even the mean value of the distribution is unknown. However, we can still assume the noise distribution to be Gaussian knowing that it will affect the performance of the filter. To implement the estimation algorithm, we need to choose a covariance for the process noise. Since we know the speed of the sound source $v$ and the pulse interval $\Delta$, the change in range of the sound source due to its motion can be estimated by $dr = v\Delta cos(\alpha)$, where $\alpha$ is the angle between the path of the sound source and the line segment from sound source to sensor at time step $k$ (see figure 1). If the sound source performs a random walk, i.e. $\alpha$ is sampled from a uniform distribution, then it can be shown that the random parameter $dr$ will be a sample from a zero-mean distribution with standard deviation $v\Delta/\sqrt{2}$. This value can be used as a clue to tune the process noise covariance. Since the sound source in practical problems is usually following a smooth path rather than a random walk, we chose $v\Delta/3\sqrt{2}$ as the standard deviation of the process noise.

If the change in bearing angle in equation (3) is small comparing to the noise, the measurement is dominated by the random noise value and its ability to correct dynamic model prediction may negatively effect the stability of the filter. As suggested in [15], we can use the following metric, similar to signal to noise ratio (SNR), to detect poor bearing angle measurement:

$$\rho = 20\log\frac{|d\theta|}{\sigma_\theta}. \tag{7}$$

When the value of $\rho$ is less than a threshold, we ignore the measurement update state of the algorithm and only use the prediction of the dynamic model as an estimate. To find an appropriate threshold, we calculated the value of $\rho$ for the measured data and plot it versus time as it is shown in Figure 4b. Based on this figure, we set the value of this threshold to be -10dB.

## RESULTS

The emitter is moving on the track with constant speed of $v = 0.086$ m/s and emits a 40 kHz 80 ms tone with a pulse interval of 350 ms. The inter-microphone distance of the mount is 143.39 mm with the microphones placed to be facing the track, Figure 3. The process and measurement noise covariances are calculated as explained before to be $Q = 5.0334\times10^{-05}$ m$^2$ and $R = 4.3865\times10^{-04}$ rad$^2$. The initial range of the emitter for the algorithm is tuned to be 1.8 m.

The motion of the emitter is illustrated in Figure 4a where 0° corresponds to the position directly in front of the sensor and positive angles correspond to the right side of the sensor and vice-versa for the left side. Figure 4b shows the SNR over a period of 100 s; based on this figure, we set the threshold for $\rho$ discussed above to be -10 dB. At first, the estimation is inaccurate due to the initial guess for range, a necessary part of Kalman Filtering, which happens to be incorrect. Over time, this estimate converges to be close to the value measured for the experiment setup, as shown in Figure 4c. The convergence of the estimation algorithm can be confirmed by looking at the covariance of the estimated distance in Figure 4d, which shows the algorithm becoming more confident.

A plot of the estimated location of the emitter is shown in Figure 5. While the initial estimate is rather far off of the track, shown by a dashed red line, the estimation converges to be reasonably accurate within traveling one length of the track. We note that the indicated position of the track was obtained from physical measurements and may not be exactly illustrative of the true location.

## DISCUSSION

From these results, we find the algorithm is accurately measuring the sensor position in general, as verified by how close it is to the measured track position shown in Figure 5. After converging, the estimated path begins to form a slight curve at the extremes of the track. This can be expected due to the low signal to noise ratio at the end sections, in which the algorithm only relies on the dynamic model which assumes the emitter range is constant resulting in a circular path. This curve also explains why our distance measurements in Figure 4c do not exhibit a clear sinusoidal shape which would be expected if the model calculated that the sound source was moving back in forth in a straight line. It warrants mentioning that it was apparent from raw measurement data that our binaural array was slightly turned to the right skewing TDOA measurements.

As shown in figure 4b, the value of signal to noise ratio drops about every 20 seconds when the emitter is located at the far left or far right of the track. This could be due to smaller change in bearing angle measurement as well as changing direction of motion of the emitter. In fact, at those points, the emitter needs to slow down before changing direction which also violates our assumption of constant speed. Therefore, the measurement recorded at these locations does not contain useful information since it is either dominated by noise or is not a good representation of the system's state.

When the algorithm converges, the estimation covariance is about 0.01 m$^2$, which corresponds to a confidence interval of $\pm10$ cm, which is not far away from the estimation error which is $\approx 10$ cm as shown in Figure 4c. However, the EKF algorithm is well known to have consistency problem showing overconfidence in its estimation [17].

The performance of a Kalman filter depends of the accuracy and confidence of the initial state. If the initial guess is too far off, the filter will, at best, converge to a wrong value. We tuned this value to be an arbitrary number within a range that the sensor shows acceptable signal to noise ratio. A more rigorous way to set the initial value is to use an smoothing algorithm to estimate the initial state [17].

The implementation of a post processing low-pass filter is currently essential because sudden changes in measured angle would represent instantaneous changes in direction which are not consistent with reality. In addition, it may affect the decision of the algorithm on the true decision of the next predicted angle. This approach may not be appropriate in real time applications, where a hypothesis testing algorithm using more than one angle measurement leads to better performance, which will be implemented in future incarnations of this work.

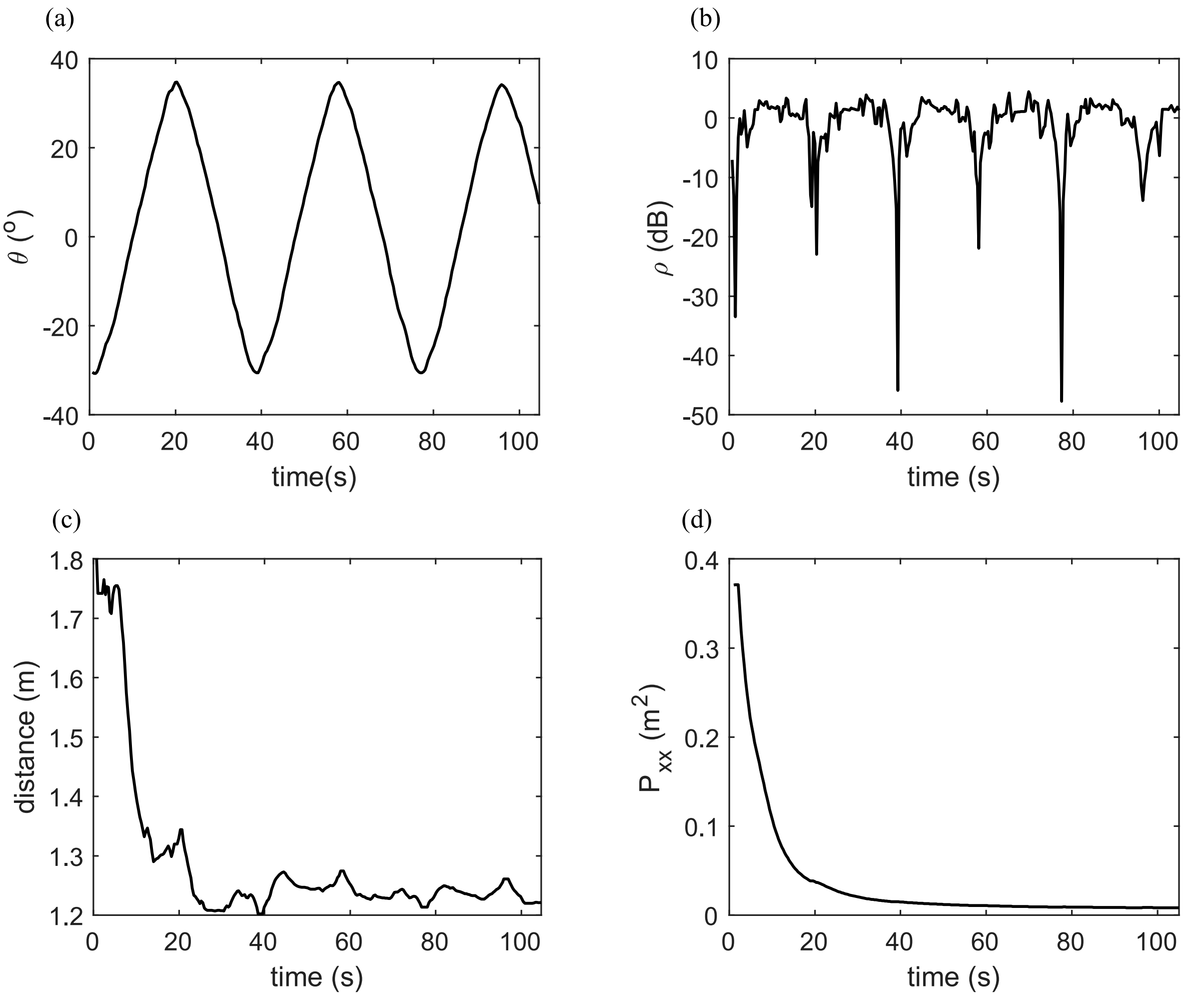


**FIGURE 4**. (a) The measured bearing angle used for estimation after implementing a first order Butterworth low pass filter. (b) Calculated values of signal to noise ratio versus time. (c) The estimated distance of the sound source to the sensor location. (d) The covariance of the estimated distance.

## CONCLUSION AND FUTURE WORK

In this work, we performed an initial investigation into using a binaural array to localize a sound source with unknown dynamics. We sought to solve this challenge using minimal inexpensive hardware and computational power while still maintaining accuracy in the measurements. What we have presented here shows that the goal we are working towards is possible, but requires some tuning of the algorithm and hardware used.

Since the performance of estimation strongly depends on the accuracy and confidence of initial state estimate, one way to improve the performance of the localization is by using a smoothing algorithm to go backward and correct the initial estimate and then implement the filtering algorithm [17]. Another improvement can be made in the performance of the localization by using multiple measurements and a hypothesis testing approach to resolve the sign confusion in (3) to make this decision making less sensitive to noise in bearing angle measurement. We also plan to bring the algorithm onto another microcontroller which will allow for online position estimation and bring us closer to employing the sensor on an actual robotic platform.

We plan to improve the binaural microphone array by using generalized cross correlation (GCC) methods to estimate the TDOA, instead of a timer that may not be accurate for every clock cycle. This will set the resolution of measurements to the known sampling frequency, improving accuracy of repeated measurements. Using GCC and introducing anti-aliasing filters will help protect the system from corrupting noise, making it more robust for practical use. To help validate our binaural data,

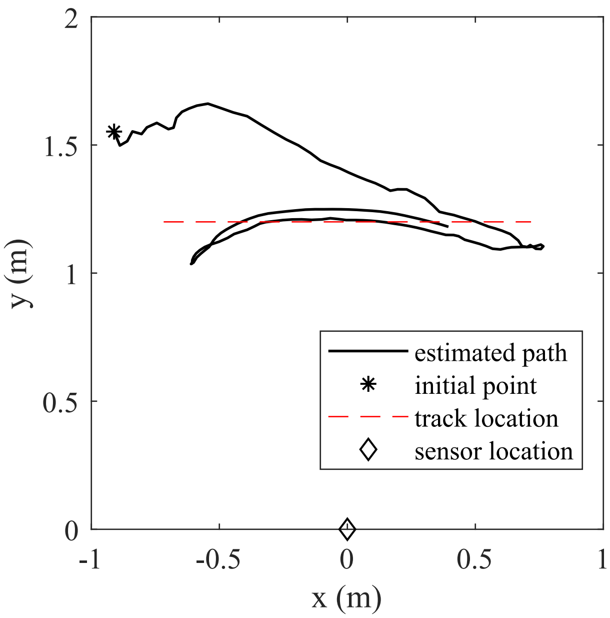


**FIGURE 5**. The estimation of sound source's path using estimated range and measured bearing angle. The sensor is marked by a diamond located at the origin and the asterisk shows the initial estimation of the range. The red dashed line shows the location of the emitter's track as it is measured before the experiment.

we plan to make use of a larger professional microphone array that can record full time series of data and provides a more proven form of localization against which our results can be directly validated. After ensuring the improved sensor is functioning correctly, we will install it on team of homogeneous robotic platforms and use them to run experiments investigating the efficacy of this minimal form of sensing in multi-agent teams.

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