

# Optimizing the Rigid or Compliant Behavior of a Novel Parallel-Actuated Architecture for Exoskeleton Robot Applications

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## Abstract

The purpose of this work is to optimize the rigid or compliant behavior of a new type of parallel-actuated robot architecture developed for exoskeleton robot applications. This is done in an effort to provide those that utilize the architecture with the means to maximize, minimize, or simply adjust its stiffness property so as to optimize it for particular tasks, such as augmented lifting or impact absorption. This research even provides the means to produce non-homogeneous stiffness properties for applications that may require non-homogeneous dynamic behavior. In this work, the new architecture is demonstrated in the form of a shoulder exoskeleton. An analytical stiffness model for the shoulder exoskeleton is created and validated experimentally. The model is then used, along with a method of bounded nonlinear multi-objective optimization to configure the parallel substructures for desired rigidity, compliance or nonhomogeneous stiffness behavior. The stiffness model and its optimization can be applied beyond the shoulder to any embodiment of the new parallel architecture, including hip, wrist and ankle robot applications. In order to exemplify this, we present the rigidity optimization for a theoretical hip exoskeleton.

## 1. Introduction

In the field of exoskeleton robotics, parallel actuation can offer many advantages over more commonly used serial actuation. Despite having complex kinematics and a typically small workspace, parallel actuation has numerous useful properties including low end-effector inertia, high acceleration, high position accuracy, and the potential for high stiffness (Li and Bone, 2001; Merlet, 2012; Taghirad, 2013). Furthermore, certain types of parallel architectures, such as the 3-SPS (spherical-prismatic-spherical) (Alici and Shirinzadeh, 2004), 3-RRR (revolute- revolute-revolute) (Wu et al., 2011) and 3-UPU (universal-prismatic-universal) (Di Gregorio, 2003), can operate without occupying the center of rotation, which is

42 particularly useful when interfacing with multiple degrees-of-freedom (DoF) biological joints such as the  
43 ankle, hip, shoulder and wrist.

44 Parallel actuation has been utilized for a number of exoskeleton applications. These include devices for  
45 the wrist, ankle, hip and shoulder. The wrist exoskeleton RiceWrist (Gupta et al., 2008), uses a 3-RPS  
46 (revolute-prismatic-spherical) architecture with an additional serial revolute joint to generate 4-DoF. These  
47 DoF include the rotation of the forearm, wrist height and 2-DoF in rotation of the end-effector platform.  
48 Since the introduction of the RiceWrist, several other exoskeleton research prototypes have adopted the 3-  
49 RPS architecture (Fan and Yin, 2009; Nurahmi et al., 2017). The ankle exoskeleton Anklebot (Roy et al.,  
50 2009) uses a 2-SPS-1S (spherical-prismatic-spherical, spherical) manipulator in conjunction with the ankle  
51 joint to achieve semi-spherical motion. The shoulder exoskeleton BONES (Klein et al., 2010) uses a RRPS  
52 (revolute-revolute-prismatic-spherical) manipulator to achieve spherical motion. Because all of these  
53 architectures, along with the previously mentioned 3-SPS, 3-RRR and 3-UPU, generate spherical motion  
54 through parallel actuation, they can further be categorized as spherical parallel manipulators

55 Spherical parallel manipulators (SPMs) are the most popular choice for exoskeleton applications,  
56 primarily because they offer a greater workspace than parallel architectures with a high degree of actuation,  
57 like the Stewart-Gough Platform (Stewart, 1965). This is a result of SPMs typically having two to three  
58 actuated substructures instead of the four, five or six of typical of higher DoF parallel manipulators. This  
59 means that SPMs have less mechanical interference between substructures. However, fewer active DoF  
60 also means that SPMs typically have lower stiffness performance than higher active DoF parallel  
61 manipulators (Gosselin and Angeles, 1989; Jiang and Gosselin, 2009; Walter et al., 2009). This can be  
62 problematic, particularly for augmentative exoskeleton systems that require high rigidity.

63 In order to improve the workspace/stiffness tradeoff of SPMs, the authors introduced a new type of SPM  
64 architecture (Hunt et al., 2017). The architecture utilizes a new design method that the authors refer to as  
65 modular motion coupling (MMC). The method involves coupling multiple DoF of each actuated  
66 substructure in order to maintain a high level of actuation while still maintaining a relatively low number  
67 of substructures. The authors developed a shoulder exoskeleton prototype that utilized this new architecture  
68 and performed a stiffness analysis on it (Hunt et al., 2018). Many approaches to analyzing the stiffness of  
69 parallel manipulators have been proposed over the years. One popular method utilizes the Jacobian matrix  
70 to calculate the stiffness matrix (Gosselin, 1990). While this method provides a reasonable approximation  
71 of stiffness, it does not take into account linkage flexibility, which is critical for an accurate end-effector  
72 stiffness estimate. Another method utilizes strain energy to develop a model of stiffness (Yan et al., 2016).  
73 While promising, this strain energy method is quite new and therefore less proven than other solutions.  
74 Additional methods include a lumped parameter approach (Pashkevich et al., 2009) and a more traditional  
75 FEA approach (El-Khasawneh and Ferreira, 1999). After considering each of these, the authors opted for  
76 a different method that utilized matrix structural analysis techniques that have been used extensively in  
77 civil engineering and have been proven to provide accurate estimates of end-effector stiffness for parallel  
78 manipulators with both passive and active DoF and flexible linkages (Deblaise et al., 2006). The results of  
79 the stiffness analysis identified some non-homogeneous stiffness behavior for certain end-effector  
80 orientations of the MMC design. This was determined to be a result of each substructure not having an  
81 actuated roll DoF. In addition, the MMC architecture was non-backdrivable, which limited its number of  
82 practical applications. Having identified these limitations, the authors developed a second-generation SPM  
83 that resolved these issues (Hunt and Lee, 2018, 2019).

84 The second-generation SPM developed by the authors utilized a system of 4-bar (4B) mechanisms to  
85 rotate a mobile platform about a center point. The advantage of this new 4B-SPM design is that the 4-bar  
86 system achieves similar arc motion to the previous design while utilizing a far more simplistic construction  
87 and maintaining back-drivability. Furthermore, the 4B-SPM utilizes three additional motors to actuate the  
88 roll DoF of each substructure, eliminating the primary issues of the MMC design.

89 An additional property of the 4B-SPM architecture is flexibility of actuator placement. The three  
90 substructures that comprise the device can be placed in any position about a center point. Placement is  
91 critical, as the stiffness of the 4B-SPM will be highly dependent on the configuration chosen. Therefore, a  
92 stiffness model with substructure placement as an input and end effector stiffness as an output would be  
93 useful for achieving desired dynamic behavior. Several examples of this include:

94            1. Maximizing stiffness for applications such as lifting or crush protection.  
 95            2. Maximizing compliance for applications requiring a high degree of unpredictable human-robot  
 96            interaction or collision protection.  
 97            3. Designing custom non-homogeneous stiffness ellipsoids for applications that may require non-  
 98            homogeneous dynamic behavior.

100          With a stiffness model, the 4B-SPM could have widespread application for exoskeleton devices, as it  
 101        has been shown to (1) interface well the shoulder, hip, wrist and ankle, (2) not require any complex  
 102        mechanical components, (3) have very flexible actuator placement, and (4) not require the human joint for  
 103        a singular kinematic solution (Hunt and Lee, 2018). For this reason, a 4B-SPM stiffness model is developed  
 104        and presented in this work. It should be noted that, as previously mentioned, the authors have developed  
 105        stiffness models for past parallel architectures. However, the ability of the 4B-SPM to interface well with  
 106        different biological joints, along with its economic design, makes it a major improvement over past parallel  
 107        architectures development by the authors. Therefore, a separate stiffness analysis of this architecture is  
 108        justified as it would offer other researchers and members of the robotics community a complete and flexible  
 109        parallel actuated solution that could be customized to fit many different exoskeleton design requirements.

110          The rest of this paper presents the steps taken to optimize the rigid or compliant behavior of the 4B-  
 111        SPM for a given workspace. The sections are organized as follows: Section II includes (1) a brief overview  
 112        of the of the 4B-SPM architecture, (2) the model used to characterize stiffness, (3) the experimental setup  
 113        to validate the stiffness model, and (4) the optimization techniques used to maximize the rigid, compliant  
 114        or nonhomogeneous stiffness behavior of the 4B-SPM. Section III details (1) the results of the stiffness  
 115        model validation experiment, (2) the optimal actuator placement for maximum rigid, compliant or  
 116        nonhomogeneous stiffness behavior of a 4B-SPM shoulder exoskeleton embodiment, and (3) the maximum  
 117        rigid stiffness of a 4B-SPM hip exoskeleton embodiment. Finally, Section IV concludes the paper with a  
 118        discussion and summary of the contribution.

## 119          2. Methods

### 120          A. 4B-SPM Design Overview

121          The previously developed 4B-SPM architecture is presented in Fig. 1 (Hunt and Lee, 2018). The 4B-  
 122        SPM uses three parallelogram 4-bar substructures. Each substructure has two actuated DoF: pitch and roll.  
 123        The roll DoF axis of each substructure intersects with the others at a singular point which represents the  
 124        virtual center of a spherical workspace. The top linkage in each 4-bar substructure is extended to reach a  
 125        mobile platform that moves tangential to the spherical workspace. Each top linkage is coupled to the mobile  
 126        platform using a spherical joint. Shown in Fig. 2 are four different embodiments of the 4B-SPM architecture  
 127        that the authors have developed forward and inverse kinematic models for (Hunt and Lee, 2018). In  
 128        preparation for the dynamic analysis performed in this work, the authors developed a shoulder exoskeleton  
 129        prototype of the 4B-SPM architecture (Hunt and Lee, 2019). This prototype is shown in Fig. 3. A video of  
 130        the shoulder exoskeleton is included as an attachment to this work.

### 132          B. 4B-SPM Stiffness Model

133          For the purpose of determining end effector stiffness of the 4B-SPM for different substructure  
 134        configurations, an analytical model was created. The model is based off of a matrix structural analysis  
 135        method commonly used for calculating stiffness of complex truss networks typically found in bridges. The  
 136        concept of applying this method to parallel manipulators was first introduced by Dominique Deblaise . For  
 137        brevity, the reader will be referred back to Deblaise's prior work for some of the more derivative or  
 138        expansive steps required in the development of this model. With the model, it is possible to generate the  
 139        end effector rotational stiffness ellipsoids that will govern how the 4B-SPM responds to externally applied  
 140        torques.

141          To start, each actuated substructure  $k$  ( $k = 1, 2, 3$ ) is represented by a nodal system that corresponds to  
 142        characteristic points. Shown in Fig. 4 are the node locations for each substructure. It should be noted that a

143 simplification has been made to the nodal diagram with regards to the 4-bar mechanism. In the prototype  
 144 shown in Fig. 3, there are actually four parallel vertical bars connecting the top and bottom linkage of the  
 145 4-bar mechanism, whereas the nodal diagram shown in Fig. 4 reduces this down to two. This is done to  
 146 simplify the analysis and is justified by the fact that only one of the four parallel vertical bars is actually  
 147 connected to the servo motor and therefore grounded, similar to Fig. 4. Thus, pitch and roll stiffness of the  
 148 substructure will not be affected by this simplification. The yaw may be slightly affected, although it is not  
 149 considered to be of the same contributing magnitude to the overall stiffness model as pitch and roll.  
 150 Nevertheless, to mitigate this error, the authors make an adjustment to the geometric properties of the two  
 151 vertical bars within the model to more accurately reflect the actual prototype.

152 The nodes shown in Fig. 4 are coupled by either a flexible beam or passive revolute joint. Each beam  
 153  $n$  is fixed at its ends by one or two nodes, depending on if the beam is considered rigidly fixed at one end.  
 154 Therefore, each beam is represented by either a  $6 \times 6$  or the  $12 \times 12$  beam stiffness matrix  $\mathbf{K}_{n,k}$  as defined in  
 155 Euler–Bernoulli beam theory. Each of these beam stiffness matrices must be oriented through  
 156 multiplication of matrix  $\mathbf{P}_{n,k}$  comprised of rotational submatrices  $\mathbf{R}_{n,k}$  along its diagonal. The rotated  
 157 beam stiffness matrix  $\mathbf{K}'_{n,k}$  can be expressed as:

$$\mathbf{K}'_{n,k} = \mathbf{P}_{n,k}^{-1} \mathbf{K}_{n,k} \mathbf{P}_{n,k} \quad (1)$$

161 Where rotation matrix  $\mathbf{P}_{n,k}$  can be determined by:  
 162

$$\mathbf{P}_{n,k} = \begin{bmatrix} \mathbf{R}_{n,k} & 0 & \cdots \\ 0 & \mathbf{R}_{n,k} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

164 The  $n$  number of rotated beam stiffness matrices  $\mathbf{K}'_{n,k}$  can then be assemble into a singular substructure  
 165 stiffness matrix  $\mathbf{K}_{T,k}$ . This assembly can be done using recognized stiffness matrix assembly methods .  
 166

167 The substructure stiffness matrix  $\mathbf{K}_{T,k}$  represents substructure stiffness before the addition of passive  
 168 joints shown in Fig. 4. Each passive joint will be defined by a kinematic relationship matrix  $\mathbf{A}_{n,k}$ , which  
 169 can be expressed as:  
 170

$$\mathbf{A}_{n,k} = \begin{bmatrix} \mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{2 \times 3} & r_{n,k} \end{bmatrix} \quad (2)$$

172 Where  $r_{n,k}$  is comprised of the rotation matrix vectors orthogonal to the rotation axis unit vector of the  
 173 passive joint. One of these rotation matrix vectors should also be parallel to the adjacent beam. The  $\mathbf{A}_{n,k}$   
 174 matrices can then be assembled into a singular substructure kinematic matrix  $\mathbf{A}_{T,k}$ , similar to  $\mathbf{K}_{T,k}$ . The  
 175 kinematically adjusted substructure stiffness matrix, with the inclusion of passive joints, is derived using  
 176 the minimum total potential energy principle (Deblaise et al., 2006). It can be expressed as:  
 177

$$\mathbf{K}_{G,k} = \begin{bmatrix} \mathbf{K}_{T,k} & \mathbf{A}_{T,k}^T \\ \mathbf{A}_{T,k} & \mathbf{0} \end{bmatrix} \quad (3)$$

180 At this point, it is necessary to permute  $\mathbf{K}_{G,k}$  in order to move the last node submatrix to the end of the  
 181  $\mathbf{K}_{G,k}$  so that it can be redefined as the endpoint substructure stiffness matrix  $\mathbf{K}_{eq,k}$ .  
 182

183 In order to determine the global stiffness of the 4B-SPM architecture, the substructure end point  
 184 stiffness matrices  $\mathbf{K}_{eq,k=1,2,3}$  must be assembled to the end effector node 7 shown in Fig 7. The shoulder  
 185 plate that connects  $\mathbf{K}_{eq,k=1,2,3}$  is considered rigid and therefore cannot be modelled using Euler–Bernoulli  
 186 beam theory. Instead, it will be modeled as series of rigid beams with infinite stiffness. This rigid beam  
 187 model will be defined by the kinematic relationship matrix  $\mathbf{B}_n$ , which can be expressed as:

$$\mathbf{B}_n = \begin{bmatrix} \mathbf{0}_{3 \times 3} & \mathbf{I}_{3 \times 3} \\ \mathbf{I}_{3 \times 3} & \hat{\mathbf{L}}_{W_n} \end{bmatrix} \quad (4)$$

190 Where  $\hat{\mathbf{L}}_{W_n}$  is the symmetric skew matrix defined by the rigid beam direction vector  $W_n = [L_x \ L_y \ L_z]_n^T$ .  
 191 With the kinematic relationship matrix  $\mathbf{B}_n$  defined, the kinematic relation matrix  $\mathbf{A}_T$  of the shoulder plate  
 192 can be constructed in a similar manner to  $\mathbf{A}_{T,k}$ . The shoulder plate stiffness matrix  $\mathbf{K}_T$ . Can also be  
 193 constructed in a similar to  $\mathbf{K}_{T,k}$ . The kinematically adjusted shoulder plate stiffness matrix, with the  
 194 inclusion of passive joints and rigid beams, is once again derived using the minimum total potential energy  
 195 principle:

$$\mathbf{K}_{eq,T} = \begin{bmatrix} \mathbf{K}_T & \mathbf{A}_T^T \\ \mathbf{A}_T & \mathbf{0} \end{bmatrix} \quad (5)$$

$$\mathbf{K}_{ee} = \begin{bmatrix} \mathbf{K}_{xx} & \mathbf{K}_{xy} \\ \mathbf{K}_{yx} & \mathbf{K}_{yy} \end{bmatrix} \quad (6)$$

Then  $\mathbf{K}_s$  and  $\mathbf{K}_a$  can be written as:

$$\mathbf{K}_s = \begin{bmatrix} \mathbf{K}_{xx} & \frac{\mathbf{K}_{xy} + \mathbf{K}_{yx}}{2} \\ \frac{\mathbf{K}_{yx} + \mathbf{K}_{xy}}{2} & \mathbf{K}_{yy} \end{bmatrix} \quad (7)$$

$$\mathbf{K}_a = \begin{bmatrix} 0 & \frac{\mathbf{K}_{xy} - \mathbf{K}_{yx}}{2} \\ \frac{\mathbf{K}_{yx} - \mathbf{K}_{xy}}{2} & 0 \end{bmatrix} \quad (8)$$

where  $\mathbf{K}_{ee} = \mathbf{K}_s + \mathbf{K}_a$ . The first three eigenvalues and eigenvectors of  $\mathbf{K}_s$  represent the direction and magnitude of the three pairwise perpendicular axes of symmetry for the translational stiffness matrices. The last three correspond to the perpendicular axes of symmetry of the rotational stiffness ellipsoid.

### C. Stiffness Model Testing

An experiment was performed to test the validity of the stiffness model through a comparison of the theoretical 4B-SPM stiffness to that of the prototype. The shoulder exoskeleton was oriented at 90° flexion and coupled to one end of a 6-axis force/torque sensor (Delta IP65, ATI, NC). To provide an accurate displacement of the load cell, a 7-DoF research robotic arm (LBR iiwa R820, KUKA, Germany) was connected to the other end of the sensor. This robot was chosen for its ability to perform these sensitive experiments. In addition to a rated payload that exceeds the forces exerted during these tests, the device has highly repeatable position control ( $\pm 0.015$  mm), which is necessary for accurate stiffness estimates (KUKA Robot Group, 2015). The 7-DoF robotic arm was in turn bolted to a steel structural support column. The experimental setup is shown in Fig. 5.

228 The roll ( $\psi$ ), pitch ( $\theta$ ) and yaw ( $\phi$ ) angles of the shoulder exoskeleton were perturbed  $\pm 3^\circ$  by the 7-DoF  
229 robotic arm. A sinusoidal perturbation profile commanded over 3000 ms was used. The corresponding  
230 forces were recorded by the 6-axis load cell at 1 kHz. All the collected measurements were filtered using a  
231 zero-phase 2<sup>nd</sup> order Butterworth filter with a 20 Hz cutoff frequency. With measurements of corresponding  
232 displacement  $\Delta\theta$  and force  $F$ , it is possible to calculate the stiffness  $k$  of the prototype using  $F_\theta = k\Delta\theta$ . Peak displacement and the corresponding force were used for calculating stiffness. It should be noted  
233 that the theoretical stiffness model is a function of the kinematic relationship matrix  $\mathbf{A}_T$  and stiffness matrix  
234  $\mathbf{K}_T$ . These matrices are sensitive to change, so if it were incorrect, then significant differences from the  
235 theoretical stiffness model and prototype would be expected.

236 For the simulation, all flexible beams were modeled as 1045 carbon steel, except for the top linkage that  
237 was modeled as 2024 aluminum. This is representative of the materials used for the prototype. All critical  
238 dimensions used in the simulation match those of the prototype. The only exception to this was the flexible  
239 beam connecting nodes 4 and 5 of the 4-bar mechanism shown in Fig. 4. For the reasons mentioned in the  
240 beginning of this Section, the cross-sectional area of this beam was doubled to more accurately reflect the  
241 dual beam design used in the prototype.

#### 242 *D. Stiffness Optimization*

243 In order to maximize overall rigidity, compliance, or nonhomogeneous stiffness behavior for a given  
244 workspace, the placement of each substructure (i.e., XYZ mounting locations of each actuator) needs to be  
245 optimized. There are a couple of parameters applied to this optimization. First, solutions for each  
246 substructure location must be bounded to a practical region were mechanical interference between robot-  
247 robot and human-robot cannot occur. After considering the geometry of the human model shown in Fig. 4  
248 and the approximate workspace of the human shoulder, the regions  $[-0.3 < x_t < 0.1, -0.4 < y_t < 0,$   
249  $0 < z_t < 0.3]$  m,  $[-0.3 < x_m < 0.1, -0.4 < y_m < 0, -0.3 < z_m < 0.1]$  m, and  $[-0.4 < x_b < 0.1,$   
250  $-0.4 < y_b < 0, -0.4 < z_b < -0.2]$  m were selected for the top, middle and bottom substructure,  
251 respectively. As is convention, the coordinates  $x$ -y,  $y$ -z and  $z$ -x used here represent the transverse, sagittal  
252 and coronal planes, respectively. Second, in order to optimize the rigidity or compliance of the 4B-SPM,  
253 the stiffness ellipsoid volume equation  $O = (4\pi/3)k_a k_b k_c$  was chosen as the objective function to  
254 maximize or minimize, here  $k_a$ ,  $k_b$  and  $k_c$  are the orthogonal axes of the ellipsoid. These two parameters  
255 make the problem a bounded nonlinear multi-objective (roll, pitch and yaw axes) optimization problem.  
256 Because of the multiple parameters, a genetic algorithm was chosen as the optimization method for  
257 determining substructure placement. The genetic algorithm attempts to minimize the objective function, so  
258 in order to maximize rigidity and compliance,  $O = -(4\pi/3)k_a k_b k_c$  and  $O = (4\pi/3)k_a k_b k_c$  were used,  
259 respectively. For maximizing nonhomogeneous stiffness, the objective function  $O = -(k_a - k_b - k_c)$  was  
260 used, which drives  $k_a \rightarrow \infty$ ,  $k_b \rightarrow 0$  and  $k_c \rightarrow 0$  as the objective function is minimized. In this case,  
261 maximizing  $k_a$  and minimizing  $k_b$  and  $k_c$  is the arbitrarily chosen nonhomogeneous behavior.  
262 Alternatively,  $k_b$  or  $k_c$  could also be maximized if desired.

263 For executing the genetic algorithm, Matlab's Optimization Toolbox (Mathworks, MA, USA) was used.  
264 The genetic algorithm function (ga) was given the boundary conditions and objective functions stated,  
265 along with the stiffness model with shoulder plate orientation as an input and the stiffness ellipsoid as an  
266 output. The shoulder plate orientation was varied in  $10^\circ$  along the pitch and yaw Euler angles and bounded  
267 by the octant  $(+x, +y, -z)$ . At each orientation, the genetic algorithm was executed and the optimal  
268 substructure mounting points were found. The approach generates a point cloud of best solutions for each  
269 substructure mounting location. The mean of these point clouds is taken as the generalized best solution.

270 In addition to maximum, minimum and nonhomogeneous stiffness models developed for the shoulder,  
271 a fourth model is developed for the hip joint. This is done in an effort to demonstrate the versatility of the  
272 4B-SPM architecture and the stiffness analysis used. In this fourth model, the maximum stiffness ellipsoid  
273 is determined along with the corresponding mounting point positions. This model was developed in the  
274 same manner as the shoulder model. Each mounting point solution was restricted to the following geometric  
275 volumes in order to produce a viable solution that interfaces well with the hip:  $[-0.5 < x_t < -0.2, -0.1 <$   
276  $y_t < 0.2, 0.2 < z_t < 0.4]$  m,  $[-0.2 < x_m < 0.2, -0.3 < y_m < -0.1, 0.2 < z_m < 0.4]$  m, and  $[0.1 <$

278  $x_b < 0.4, -0.1 < y_b < 0.1, 0.2 < z_b < 0.4$  m, The workspace was bounded by the following three thigh  
279 orientations: 90° flexion, 45° adduction and at rest.

### 280 281 282 3. Results

#### 283 A. Stiffness Model Testing

284 A comparison of the theoretical and mean measured stiffness is shown in Fig. 6 for the shoulder plate  
285 orientated at 90° flexion. The mean error along roll-pitch-yaw is 11.8% with a standard deviation of 8.4.  
286 While error does exist, it should be noted that the size and shape of the theoretical model demonstrates a  
287 reasonable approximation of stiffness based on the global axis measurements taken.

288 Several causes for the error have been identified by the authors: (1) Imperfect intersection of the roll  
289 axes for the three substructures. This misalignment could produce increased resistance to applied torque  
290 that may contribute to differing stiffness results. This could be corrected with higher manufacturing  
291 tolerances. (2) Backlash in the servo motors. This could potentially cause play in the shoulder plate that  
292 could affect the stiffness measurements. It should be noted that efforts to minimize backlash were taken by  
293 applying minor tension of the three substructures against the shoulder plate equal to the measured backlash  
294 of the servos. This minimizes backlash without changing the kinematic solution. (3) Imperfect modeling  
295 of the prototype's geometric and material properties. Measurements taken from the prototype and materials  
296 utilized vary within tolerance. These tolerances are not accounted for by the theoretical model and are  
297 therefore a potential source for minor error. (4) Simplification 4-bar mechanism nodal diagram, as  
298 described in Section 2. B.

#### 299 B. Stiffness Optimization

300 For the octant workspace bounded by the  $+x$ ,  $+y$ , and  $-z$  axes defined in Fig. 7, the 4B-SPM  
301 substructure configurations to achieve optimal rigid, compliant and nonhomogeneous stiffness behavior  
302 were found. The optimal configurations are shown in Fig. 7, along with a point cloud of best solutions for  
303 different shoulder plate orientations. These solutions were found at 10° increments along the pitch and yaw  
304 Euler angles. The optimal substructure configuration for each result is taken to be the mean location of each  
305 substructure point cloud. For optimal rigidity, the virtual center of each point cloud for the top, middle and  
306 bottom substructure, respectively, are  $A_t = [-0.23, -0.16, 0.27]^T$  m,  $A_m = [-0.27, -0.21, 0.02]^T$  m and  
307  $A_b = [-0.21, -0.12, -0.31]^T$  m. For optimal compliance, the virtual center of each point cloud for the top,  
308 middle and bottom substructure, respectively, are  $A_t = [-0.25, -0.16, 0.11]^T$  m,  $A_m = [-0.29, -0.23, 0.01]^T$   
309 m and  $A_b = [-0.28, -0.14, -0.24]^T$  m. For the optimal nonhomogeneous stiffness behavior, the virtual center  
310 of each point cloud for the top, middle and bottom substructure, respectively, are  $A_t = [-0.29, -0.24, 0.29]^T$   
311 m,  $A_m = [-0.29, -0.24, -0.10]^T$  m and  $A_b = [-0.21, -0.14, -0.26]^T$  m. The generalized rotational stiffness  
312 ellipsoid that represents the average stiffness across the entire workspace for each solution is shown in Fig.  
313 7 as well. Included with them is the standard deviation for each solution.

314 The results shown in Fig. 7 help identify a few interesting characteristics of the 4B-SPM. Firstly, a  
315 comparison between maximum rigidity and compliance suggests that stiffness is largely dependent on the  
316 distances between substructures mounts. This is somewhat intuitive, although the extent of dependency  
317 was not clear until now. Another interesting feature identified by these findings is how the rigid and  
318 compliant results show fairly symmetric solutions corresponding to relatively homogeneous stiffness  
319 ellipsoids. In contrast, the nonhomogeneous stiffness results shown in Fig. 7C correspond to a highly  
320 nonsymmetrical substructure mounting point solution. These observations would suggest that symmetry of  
321 the 4B-SPM affects its degree of homogeneous stiffness behavior.

322 The results shown in Fig. 7 also provide the opportunity to compare the stiffness of this new 4B-SPM  
323 architecture to that of the previous motion-coupled SPM architecture developed by the authors for similar  
324 purposes and discussed in the Introduction. In prior work the authors analyzed the rotational stiffness of  
325 this motion-coupled design across the same workspace used in this paper for the 4B-SPM (Hunt et al.,  
326 2018). For a maximum stiffness configuration, the motion-coupled design had a mean stiffness ellipsoid  
327 volume of  $6.22 \cdot 10^6 (Nm/rad)^3$ . In comparison, the 4B-SPM has a mean stiffness ellipsoid volume of

328  $3.24 \cdot 10^7 (Nm/rad)^3$  for the maximum stiffness configuration. This increase in stiffness is likely due to  
329 (1) the addition of the three revolute actuators that control the roll of each 4B-SPM substructure and (2) the  
330 simplified 4-bar design that possess fewer failure modes. Other factors, such as part materials and geometry  
331 may also contribute to the increased stiffness.

332 In addition to the findings presented for the 4B-SPM shoulder exoskeleton, the maximum stiffness  
333 results of a theoretical hip exoskeleton are also presented. These results are shown in Fig. 8. For optimal  
334 rigidity, the virtual center of each point cloud from left (red) to right (blue) are  $A_t = [-0.37, 0.11, 0.38]^T$   
335 m,  $A_m = [-0.07, -0.18, 0.39]^T$  m and  $A_b = [0.22, 0.34, 0.4]^T$  m, respectively. As previously mentioned, this  
336 second embodiment of the 4B-SPM architecture is included here in order to demonstrate the versatility of  
337 the 4B-SPM architecture and the stiffness analysis used. It should be noted that the choice of a hip  
338 exoskeleton was arbitrary. This second embodiment could have just as easily been a 4B-SPM exoskeleton  
339 wrist or ankle alternative.

#### 340 4. Discussion

341 The work performed for this paper was motivated by the need for exoskeleton architectures that are  
342 capable of matching the workspace of a user while exhibiting desired stiffness characteristics. Because of  
343 limitations in the stiffness or workspace of typical serial and parallel actuated architectures, the authors  
344 developed the new 4B-SPM architecture in prior work that was specifically designed for exoskeleton  
345 applications involving complex biological joints like the shoulder, hip, wrist and ankle. Demonstrated in  
346 the form of a shoulder exoskeleton, the authors performed a dynamic analysis on the 4B-SPM in order to  
347 help validate the derived stiffness model. The model was then used to optimize the 4B-SPM configuration  
348 in order to achieve rigid, compliant and nonhomogeneous stiffness behavior.

349 The results of this paper detail a theoretical stiffness model for the 4B-SPM presented, along with an  
350 experiment to validate the model. An error between the prototype stiffness and theoretical stiffness of  
351 11.8% with a standard deviation of 8.4 was reported. Despite some error, the model still proved to be a  
352 reasonable approximation of stiffness. Possible causes for the error are discussed in Section 3. A.

353 The stiffness model was used in conjunction with a bounded nonlinear multi-objective optimization  
354 method in order determine the optimal placement of the three actuated substructures to achieve certain  
355 dynamic behavior within a given workspace. The workspace was chosen to be one octant of a sphere  
356 defined by the three arm orientations: 90° flexion, 90° abduction, and at rest. For this workspace, the  
357 actuator placements for optimal rigid, compliant and certain nonhomogeneous stiffness behavior were  
358 demonstrated.

359 The main contribution of this work is providing researchers and members of the robotics community  
360 who chose to use the 4B-SPM architecture a means of adjusting its dynamic performance to fit many  
361 different exoskeleton applications. To reiterate, there are many reasons to use the 4B-SPM, the primary  
362 ones being: (1) interfaces well the shoulder, hip, wrist and ankle; (2) does not require any complex  
363 mechanical components; (3) has very flexible actuator placement; and (4) does not require the human joint  
364 for a singular kinematic solution. With the addition of the presented stiffness model, future wearable 4B-  
365 SPM devices could be optimized for a variety of tasks and applications, such as lifting, jumping, running,  
366 crush protection and impact absorption.

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## Figure Captions

Fig. 1: 4-Bar Spherical Parallel Manipulator (4B-SPM) architecture. The 4B-SPM uses three parallelogram 4-bar substructures. Each substructure has two actuated DoF: pitch and roll. The roll DoF axis of each substructure intersects with the others at a singular point which represents the virtual center of a spherical workspace. The top linkage in each 4-bar substructure is extended to reach a mobile platform that moves tangential to the spherical workspace. Each top linkage is coupled to the mobile platform using a spherical joint (Hunt et al., 2017).

Fig. 2: Four embodiments of the 4B-SPM architecture for which the authors have solved the kinematics for include: ankle, shoulder, wrist and hip exoskeletons (Hunt et al., 2017).

Fig. 3: 4B-SPM shoulder exoskeleton prototype mounted to a stationary platform with a human subject in the seated position. The subject is coupled to the device through the use of an upper arm cuff. To maintain good contact between the subject and device, a blood pressure cuff is used at the contact point. The pitch, roll and yaw axes are represented by the orthogonal red, green and blue axes, respectively.

Fig. 4: (Top) 4-bar substructure equivalent nodal diagram, (Bottom) shoulder plate end effector equivalent nodal diagram.

Fig. 5: Experimental setup for evaluating the 4B-SPM prototype stiffness oriented at 90° flexion. (A) 4B-SPM Shoulder exoskeleton, (B) 7-DoF robotic arm (LBR iiwa R820, KUKA, Germany), (C) 6-axis load cell (Delta IP65, ATI, NC). The shoulder exoskeleton was mechanically coupled to the load cell, which was in turn coupled to the 7-DoF robotic arm. The roll, pitch and yaw angles of the shoulder exoskeleton about its center-of-rotation O are represented  $\psi$ ,  $\theta$  and  $\phi$ , respectively.

Fig. 6: Orientation of the shoulder exoskeleton along with projections of the associated theoretical rotational stiffness ellipsoid (Nm/rad) shown in black. The roll, pitch and yaw stiffness measurements are shown in red for contrast. The origin of the frame is at the center-of-rotation of the human shoulder.

Fig. 7: (A) Shown at top is the generalized maximum stiffness configuration for the 4B-SPM shoulder exoskeleton substructures along with point clouds of the best solutions found throughout the workspace. Shown at bottom are projections of the generalized maximum stiffness ellipsoid. (B) Shown at top is the generalized minimum stiffness configuration for the 4B-SPM substructures along with point clouds of the

best solutions found throughout the workspace. Shown at bottom are projections of the generalized minimum stiffness ellipsoid. (C) Shown at top is the generalized maximum desired nonhomogeneous stiffness configuration for the 4B-SPM substructures along with point clouds of the best solutions found throughout the workspace. Shown at bottom are projections of the generalized maximum nonhomogeneous stiffness ellipsoid. For all three figures, the origin of each frame is at the center-of-rotation of the human shoulder.

Fig. 8: Shown at top is the generalized maximum stiffness configuration for the 4B-SPM hip exoskeleton substructures along with point clouds of the best solutions found throughout the workspace. Shown at bottom are projections of the generalized maximum stiffness ellipsoid.