

Dimensions of Variation in Group Work within the “Same” Multi-Section Undergraduate Course

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This paper reports a qualitative study of how small group problem solving was enacted differently across sections of a multi-section undergraduate introduction to proof course. Common course materials, common guidelines for instruction, and weekly instructor meetings led by a faculty course coordinator supported similar instruction across sections, including an emphasis on in-class group work. But within that shared structure, classroom observations revealed important differences in how group work was introduced, organized, and managed. Our results focus on differences in the time allotted to group work, the rationale for group work, the selection and organization of groups, and aspects of student activity and participation. We suggest that these differences shaped different opportunities to learn proof writing in small groups. These results have implications for the design and teaching of collegiate mathematics courses where group work is a regular element of classroom work.

Keywords: Proof and proving, Group work, Undergraduate

Broad support now exists for using “active learning” methods in collegiate mathematics and science courses, where lecture has previously been the dominant element of instruction (Braun, Bremser, Duval, Lockwood, & White, 2017; Ernest, Hodge, & Yoshinobu, 2017; Freeman, et al., 2014; Laursen & Rasmussen, 2019). Where “active learning” has generally been described in inclusive terms—embracing a wide range of instructional activities (e.g., Braun, et al., 2017), most presentations include small group problem solving work as a central element. These presentations have emphasized mathematically challenging tasks (“problems,” not “exercises”) and peer collaboration in solving them. Prior publications focusing on cooperative learning in undergraduate mathematics have provided the rationale for small group work as well as practical advice on implementation in classrooms (Ahmadi, 2002; Dubinsky, Mathews, & Reynolds, 1997; Mathematical Association of America, 2017; Rogers, Reynolds, Davidson, & Thomas, 2001). But research has not yet examined how implementation can differ within multi-section courses that are common in collegiate STEM programs, much less the impact of those differences on students’ learning.

The paper reports differences in the enactment of group work in four sections of one multi-section undergraduate mathematics course. Designed to introduce students to the nature of proof and proof writing, the course committed substantial class time to small group work on proof tasks. The paper first summarizes the common elements of content and pedagogy that united the four sections. Efforts to align teaching and learning across the sections had substantial effect. It is within that commonality that differences in how group work was introduced, undertaken, and managed were observed. Our data do not allow us to make the claim that differences in the enactment of group work had clear and definite impact on either students’ views of group work

or their growing competence with mathematical proof. But the results do show that a different set of affordances and constraints for working productively in groups on challenging proof tasks existed in the four sections. We hope this paper becomes a contribution to scholarship that identifies the issues that instructors and course coordinators of multi-section undergraduate courses should attend to, discuss, and manage in their efforts to maximize the positive impact of group work in undergraduate mathematics.

The Course (as Research Setting)

The study was part of a larger research project that has examined the nature of students' experience as they adjust from computation to proof in collegiate mathematics, with particular attention to the development of their mathematical agency and autonomy in challenging proof and proving work. One locus of the study is one mathematics department's introduction to proof (ITP) course. The course is required for mathematics majors and minors, is taken after students complete at least one semester of calculus, and is populated by majors and minors in about equal numbers. Minors typically major in some other STEM discipline. The department routinely offers 13 sections of the course each year (of three semesters), each enrolling between 20 and 25 students. ITP is a four credit-hour course; sections meet either three or four times each week.

The course introduces students to proof as the central process for establishing the truth of mathematical statements; to different proof methods (direct, cases, induction, contradiction, and contrapositive); and to concepts and statements in set theory, real analysis, and number theory as "content." Homework—assigned, graded, and returned weekly—includes short-answer tasks, informal arguments, and full proofs. As the semester moves along, full proofs become the dominant task. In spring and fall semesters, four exams are given, and students' grades depend primarily on exam and homework performance. Outside of class, students have access to a course-specific on-line *Piazza* forum—where they can ask questions (e.g., about homework tasks) and read suggestions from instructors and other students—and to the department's Math Learning Center (MLC) which provides a separate room and staffing for the ITP course.

In the semester of the study, an instructor and a teaching assistant were assigned to each section. The faculty member who coordinated the course taught one section, which was not observed. The other instructors were mathematics graduate students; all had prior experience as teaching assistants in the course. The teaching assistants were also mathematics graduate students. The instructional team (coordinator, instructors, and assistants) met weekly on Mondays to plan/review activities for the week and address emerging issues.

Four sections (A, B, C, and D) were observed in the semester of study. In the shared instructional plan, content was presented on Mondays in all sections. In sections A, C, and D, group work was the focus on Tuesdays and Thursdays. In section B, content was presented on Mondays and Wednesdays, with group work on Tuesdays and Fridays. In addition to lecture and group work, two other instructional activities were review for exams and interactive presentation of proofs of particular statements. In the latter, instructors presented a proof but also solicit suggestions (and questions) from students as part of the presentation.

In place of a textbook, work in the course was supported by a lengthy "Examples document" that had been compiled and revised over many prior semesters. All students were given a physical copy and on-line access to a digital copy. The digital version provided students with model solutions to each task. The Examples document provided the tasks for group work; instructors typically initiated segments of group work by writing a list of those tasks on the board. In addition, the course coordinator provided six tasks to introduce group work in the first two weeks. But for all but one, those tasks did not involve proof.

Data Collection

Part of the data collected for the larger project were classroom observations; these became the main data for this study. Observations were conducted roughly twice each week in the first five weeks to see how patterns of activity, including group work, were established. Thereafter, frequency decreased to once per week. The observations were “passive,” but focused; observers did not engage in classroom activity. The objective was to document the nature and duration of the instructional activities, including but not limited to group work. Early in the semester, the focus was the class as a whole (students and instructors). Later in the semester, we focused on how study participants (3 to 5 per section) engaged in class activities, particularly group work. The observers were a faculty member with expertise in mathematics education and three graduate students in a mathematics education doctoral program. All four are authors.

Observations were completed on a common template used to characterize different segments of the class sessions (e.g., nature and duration). The template also included common focal questions that observers addressed at the end of each session. All observations were posted to a central project repository. Observations in all four sections included some “lecture” days but more “group work” days. The timing of observations was constrained by observers’ schedules. For example, though we recognized that the first few class meetings were particularly important to observe, we were not always able to do so. Table 1 summarizes the number and timing of the observations carried out in the four sections.

Table 1: Summary of Observations in the four sections A, B, C, and D

	A	B	C	D
Class Meetings	M,T,Th @ 80 mins each	M,W,F @ 50 mins; T @ 80 mins	M,T,Th @ 80 mins each	M,T,Th @ 80 mins each
# of observations	12*	12	12	14
# of obs. by day	4 (M), 2 (T), 6 (Th)	3 (M), 9 (T)	5 (M), 7 (Th)	4 (M), 10 (T)
# of obs. in 1st 2 weeks	2	2	3	4
1st class observed?	Yes	No	Yes	Yes

* Two of the 12 observations in Section A were exam days

Whole-class activities (e.g., lecture and interactive presentation) were relatively unproblematic to document. In contrast, interactions involving students during group work were sometimes difficult to observe and document due to challenges of distance and audibility. Nevertheless, some important aspects of interaction and activity were observable in segments of group work. These included: (1) the physical location of groups, (2) students’ physical positions and gaze as indicators of engagement, (3) the distribution of vocal participation in the group, (4) the actions of instructors and teaching assistants, and (5) often, the groups’ methods for completing assigned tasks (e.g., working together on each task versus distributing tasks among group members [i.e., division of labor]).

Data Analysis

Attention to differences in how group work was enacted across sections arose in project meetings in the semester of study. Preliminary work identified some initial dimensions of contrast such as timing and duration and how groups were formed. After the semester ended, the

first author read each summary (N = 50; each five to nine pages of single-spaced text within the template) and wrote summaries that characterized the group work segments of each observed session. Those summaries in turn supported the (a) development of a framework of seven dimensions used to characterize the enactment of group work (Table 2 below) and (b) analysis to address the question: *How did the enactment of group work differ across the four sections?*

Table 2: Framework for Analyzing Differences

Dimension	Some Relevant Foci
Frequency & Duration	How often and for how long did group work happen?
Rationale	What rationale was given, orally and in writing, about the purpose of group work?
Selection/Organization	How were groups selected/formed? How were adjustments to groups made?
Location & Integrity	Where were groups located in the classroom? Once formed, did groups stay together?
Content	What tasks were assigned to groups? Did instructors bring in additional tasks?
Student Activity & Participation	Did groups have access to shared work space, and did they use it? What were the patterns of participation in the groups?
Instructor Assistance	What did instructors do while groups were working?
Feedback & Evaluation	Did instructors seek and report back students' views of group work?

The observations produced clearer and more complete data on some dimensions than others. For example, data on the frequency and duration of group work was nearly always reported, and the time stamps provided a strong basis for comparison across sections. Similarly, because many observed class sessions included group work, we had extensive data on how instructors selected and organized their groups. By contrast, data on student activity and participation was less complete and robust due to the number of groups and the distance between observers and those groups. Also, we were unable to observe section B in the first week of the semester, limiting what we can say below about that instructor's rationale for group work.

Results

We present our results to highlight that important differences in the enactment of group work across the four sections existed *within* the inter-section commonalities that justified the sections as different instantiations of "the same course." As outlined above, these commonalities resulted from sustained work by the course coordinator to align content and pedagogy within the course. Important common elements included: (1) a single course syllabus and schedule of sessions, (2) a shared macro-level approach to instruction that devoted substantial time to group work, (3) a shared commitment to groups of three to four students, (4) shared roles for instructors and teaching assistant during group work (i.e., circulate, listen, question, and/or direct/correct; engage all groups), and (5) a common source of group work tasks (the Examples document). Relative to the framework outlined in Table 2, one other commonality united the four sections: The course syllabus did not state the purpose of group work in the course (Rationale).

Differences in the Enactment of Group Work

We focus on four dimensions—frequency & duration, rationale, selection/organization, and student activity & participation—where we observed marked differences in how group work was implemented. We also found differences in location & integrity and content, but we judged these of lesser impact, as differences in location & integrity correlated to differences along the four dimensions. Issues related to student activity & participation are very complex, and our presentation below addresses only some aspects of that dimension of group work.

Frequency & duration. Despite the shared commitment to group work on two days each week, the sections could still differ in the time allotted to group work. Table 3 summarizes the time devoted to group work on instructional days when that activity was observed. Row three entries present the number of observations that reported some group work. Entries in rows four through six present measures of central tendency and range for total group work time.

Table 3: Frequency and duration of group work

	A	B	C	D
# of Overall Observations	12	12	12	14
Observations of Group Work	10	9	8	11
Mean Duration (min)	37	58	32	48
Median Duration (min)	31	56	29	52
Range (min)	23-52	36-80	18-55	24-80

The entries in Table 3 indicate substantial differences in time allotted to group work; Sections B and D devoted substantially more time to that activity than Sections A and C. But the amount of time devoted to group work is independent of its character and quality. As the paragraphs below reveal, more minutes devoted to group work do not necessarily map onto more productive opportunities for student interaction (see Selection/Organization below).

Rationale. We found no direct reference to why group work was a central activity in the course, either in the syllabus or the observation data. But observations of sections A, C, and D in the first two weeks showed that instructors introduced group work differently—suggesting quite different purposes for group work in learning mathematics. (Recall that section B was not observed during the first week of the semester.)

The instructor in section A posed three questions on the first day of class—*why do group work, what does good group look like, and what does bad group work look like?* In the ensuing discussion, that instructor emphasized three points to students: (1) don't judge people as smart or not, (2) think about your body language in group interactions, (3) don't separate yourself from the group. The instructor did not, however, present any personal experience with or stance toward group work. In contrast, the instructor in section C stated that most people do not like group work and he himself "hated it," before adding that everyone recognizes that students should be good at it. Shortly thereafter, that instructor sent students an optional survey to learn their views of group work. The instructor in section D said nothing about group work generally in the first week, indicating only that the goal of the introductory group tasks was for students to get to know each other. These introductions expressed quite different messages about the purpose/rationale for group work: Group work can work for you (A), group work is a pain—if somewhat useful (C), and no opinion/not worth discussing (D). These introductory rationales suggest caution in interpreting the entries in Table 3 with a simple "more minutes are more productive" lens.

Selection/Organization. An important set of implementation decisions involve the composition of groups: How large groups will be, how will they be formed, and how will instructors address “problems” (e.g., late and absent students, students sitting alone, group non-interaction)? Though our data showed that all instructors initially formed groups of three to four students, their adherence to that principle varied widely. Their methods for composing groups also varied, as did the consistency in their application of those methods. These variations had direct consequences for whom students interacted with, daily and through the semester.

Group size. In section A, the instructor created and maintained groups of four students for the entire semester, using the same method (see below). Early in the semester, groups of four were sometimes broken into two pairs and paired work was observed, but that practice was limited to the first three weeks. The instructors in the other sections initially composed groups of four, but at different points in the semester, stopped attending to group size. The instructors in section B and D stopped focusing on group size by weeks 2 and 4 respectively, and the size of their groups varied widely thereafter (from one to five students). The instructor in section C maintained groups of four longer but then permitted smaller groups, including pairs and individuals, and intervened only when group size got large (e.g., six students).

Methods of composition. The instructor in section A used a transparent and random process (common features of playing cards) to compose groups, with new groups generally formed at the beginning of each week. In section B, the teaching assistant, subbing for the instructor, formed the first groups randomly by having students count off by remainder under division by 4. Thereafter, groups were composed by proximity: The instructor asked students sitting near each other to form groups. Once this practice started, the constituency of the groups in Section B changed little, and two students worked on their own for nearly the entire semester. The instructor in section D pursued a similar practice. The first groups of four were composed by proximity, and in week 2 students were asked to remember their groups. Most students did not change their group membership for the rest of the semester (in groups that varied widely in size). The instructor in section C used transparent but non-random methods to compose groups of four in the first two class sessions (ordering students by their height and then their birthdays). But by week 3, students picked their own groups and by week 4 could work in groups or not.

These two issues, when combined, resulted in quite different patterns of “work with others” in the four sections. Students in Section A had no choice but to work with all (or nearly all) of their peers at least once in the semester. Students in the other three sections had much more control over whom they worked with and as a consequence of their choices, much more limited experience of work with others. In these sections, some students’ experience of work with others was limited to single peers; others who worked alone had no experience of work with peers.

Student activity & participation. Group work in the four sections also differed in how much mathematical work was carried out in private versus public space. We use “public” to designate space that all group members can access visually and “private” to designate space used by some group members that affords limited visual access of others. With the commitment to groups of four, greater use of public space affords greater likelihood that groups will generate collaborative solutions. Access to public space in classrooms depends on the number and size of available whiteboards (or blackboards), as these afford wide visual access. By contrast, work carried out in students’ paper notebooks and tablet computers is more private because it limits visual access. So, one aspect of student activity & participation in group work involves the extent of use of public vs. private space, and that access in turn is shaped by the physical properties of classrooms, specifically the amount of board space.

In Section A, the work of all groups was carried out in public space—on whiteboards and blackboards. One classroom for that section had sufficient board space to accommodate the work of six groups of four, where board space was limited in the other—accommodating only four groups of four. The instructor removed that limitation by attaching two transportable whiteboards to an empty wall of that classroom. In section B, sufficient board space was available for three days of the week (M-W-F) but was limited in the Tuesday classroom where the focus was group work. As might be suggested above (methods of composition) much less use of public space was observed in section B—though this may not have been simply a function of existing board space. In sections C and D, public space in the classrooms used on both “group work” days accommodated only four groups. As with section B, the groups’ use of public space in these two sections was spotty. Many “groups” in sections B, C, and D completed their work in private space, sharing with one other student and generally when snags arose.

Discussion

In one semester of one course at one university, we have shown that the group work undergraduate students of mathematics experience differed in significant ways in different sections of the same coordinated course. Where this result could seem obviously true when multi-section courses are taught without much focused attention to coordination across sections, the results take on more importance in this case, because of the sustained efforts to align content, pedagogy, and assessment across sections. We do not interpret our results as evidence that course coordination failed. Instead, we see our results as identifying the challenges inherent in supporting equally productive group work in multi-section courses, even when coordinated.

The core issue explored in this paper—factors that contribute to college students’ experience of and learning in small group work—is understudied at the level of detail we have examined and reported it. Research on the dynamics of group work over time—within the “same” group over time, within a single section, or within a multi-section course—is needed to understand and advance the promise of “active learning” and inquiry-based mathematics. More observation-based descriptive studies of group work enactment are needed, as well as studies that take on the challenge of linking the dynamics of that enactment to students’ thinking and learning—about mathematics and peer interactions. For such linking studies, observations alone will not be sufficient. For both types of studies, we hope that the framework used here is a modest contribution, though we make no claim for its completeness or sufficiency.

With respect to practice, the study raises important questions and challenges for coordinators of multi-section courses who seek to provide all students with roughly equivalent and productive learning experiences. For example, given the inevitable differences in instructors’ orientation to group work, how best to recruit and select them? In the scarce minutes before the start of the semester, how best to discuss instructors’ personal experiences with group work and how to introduce it to students? Given the many issues that call for attention in course meetings, how much time should be allotted to discussing the “health” of groups and strategies for addressing problems in them? This study has raised our awareness of the challenges inherent in coordinating small group work across sections.

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