An Improved Approximation Algorithm for Maximin Shares

JUGAL GARG, University of Illinois at Urbana-Champaign
SETAREH TAKI, University of Illinois at Urbana-Champaign

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Additional Key Words and Phrases: fair division, maximin shares, strongly polynomial algorithm

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We study the problem of fair allocation of m indivisible items among n agents with additive valuations using the popular notion of *maximin share* (MMS) as our measure of fairness. An MMS allocation provides each agent a bundle worth at least her maximin share. While it is known that such an allocation need not exist [5, 7], a series of remarkable work [1–3, 6, 7] provided 2/3 approximation algorithms in which each agent receives a bundle worth at least 2/3 times her maximin share. More recently, [4] showed the existence of 3/4 MMS allocations and a PTAS to find a $3/4 - \epsilon$ MMS allocation. Most of the previous works utilize intricate algorithms and require agents' approximate MMS values, which are computationally expensive to obtain.

In this paper, we develop a new approach that gives a simple algorithm for showing the existence of a $\frac{3}{4}$-MMS allocation. Furthermore, our approach is powerful enough to be easily extended in two directions: First, we get a strongly polynomial time algorithm to find a $\frac{3}{4}$-MMS allocation, where we do not need to approximate the MMS values at all. Second, we show that there always exists a $(\frac{3}{4} + \frac{1}{12n})$-MMS allocation. This considerably improves the approximation guarantee for small n. We note that there are works, e.g., [1, 4], exploring better approximation factors for a small number of agents. We note that 3/4 was the best factor known for $n > 4$.

Our algorithms are extremely simple. First, we describe the basic algorithm that shows the existence of $\frac{3}{4}$-MMS allocations. Since MMS values are scale-invariant, we scale valuations to make each agent's MMS value 1. Then, we assign *high-value items* to agents, who value them at least $\frac{3}{4}$, with a simple greedy approach based on the pigeonhole principle, and remove the assigned items and the agents receiving these items from further consideration. This reduces the number of high-value items to be at most $2n'$, where n' is the number of remaining agents. These greedy assignments massively simplify allocation of high-value items, which was the most challenging part of previous algorithms. Next, we prepare n' bags, one for each remaining agent, and put at most two high-value items in each bag. Then, we add low-value items on top of each of these bags one by one using a bag-filling procedure until the value of bag for some agent is at least $\frac{3}{4}$. The main technical challenge here is to show that there are enough low-value items to give every agent a bag they value at least $\frac{3}{4}$.

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Next, we extend the basic algorithm to compute a $\frac{3}{4}$-MMS allocation without any need to compute the actual MMS values. Here, we define a notion of *tentative assignments* and a novel way for updating the MMS upper bound. For each agent, we use the average value, that is the value of all items divided by the number of agents, as an upper bound of her MMS value. The only change from the basic algorithm is that some of the greedy assignments are tentative, i.e., they are valid only if the current upper bound of the MMS values is tight enough. We show that this can be checked by using the total valuation of low-value items. If the upper bounds are not tight enough for some agents, then we update the MMS value of such an agent and repeat. We show that we do not need to update the MMS upper bounds more than $O(n^3)$ times before we have a good upper bound on all MMS values. Then, we show that the same bag filling procedure, as in the basic algorithm, satisfy every remaining agent. The running time of the entire algorithm is $O(nm(n^4 + \log m))$.

We also extend the basic algorithm to show the existence of $(\frac{3}{4} + \frac{1}{12n})$-MMS allocations. The entire algorithm remains exactly the same but with an involved analysis. The analysis is tricky in this case, so we add a set of *dummy items* to make proofs easy and more intuitive. We use these items to make up for the extra loss for the remaining agents due to the additional factor, and, of course, these items are not assigned to any agent in the algorithm.

The full version of this paper can be accessed at: https://arxiv.org/abs/1903.00029.

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REFERENCES

[1] Georgios Amanatidis, Evangelos Markakis, Afshin Nikzad, and Amin Saberi. 2017. Approximation algorithms for computing maximin share allocations. *ACM Transactions on Algorithms (TALG)* 13, 4 (2017), 52.

[2] Siddharth Barman and Sanath Kumar Krishna Murthy. 2017. Approximation algorithms for maximin fair division. In *Proceedings of the 2017 ACM Conference on Economics and Computation*. ACM, 647–664.

[3] Jugal Garg, Peter McGlaughlin, and Setareh Taki. 2018. Approximating Maximin Share Allocations. In *2nd Symposium on Simplicity in Algorithms (SOSA 2019)*.

[4] Mohammad Ghodsi, MohammadTaghi HajiAghayi, Masoud Seddighin, Saeed Seddighin, and Hadi Yami. 2018. Fair allocation of indivisible goods: Improvement and generalization. In *EC*. Available on arXiv:1704.00222 since April 2017.

[5] David Kurokawa, Ariel D Procaccia, and Junxing Wang. 2016. When can the maximin share guarantee be guaranteed?. In *AAAI*, Vol. 16. 523–529.

[6] David Kurokawa, Ariel D. Procaccia, and Junxing Wang. 2018. Fair Enough: Guaranteeing Approximate Maximin Shares. *J. ACM* 65, 2 (2018), 8:1–8:27.

[7] Ariel D Procaccia and Junxing Wang. 2014. Fair enough: Guaranteeing approximate maximin shares. In *Proceedings of the fifteenth ACM conference on Economics and computation*. ACM, 675–692.