

Feasibility of Gait Entrainment to Hip Mechanical Perturbation for Locomotor Rehabilitation*

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Abstract— While rehabilitation of upper-limb motor function with human-interactive robots has been met with success, robot-aided locomotor rehabilitation has proven challenging. To inform more effective approaches to robotic gait therapy, it is important to understand neuro-mechanical dynamics and control of unimpaired locomotion. Our previous studies reported that human gait entrained to periodic mechanical perturbations at the ankle when the perturbation period was close to preferred walking cadence. Moreover, entrainment was accompanied by synchronizing the perturbations to a constant gait phase, the same for all subjects, where they provided mechanical assistance. To test the generality of entrainment-based assistance, the present study evaluated the behavior of five unimpaired subjects who walked overground while wearing a hip exoskeleton robot. Periodic torque pulses were applied to the subjects' hips, with a period different from, but close to, their preferred stride cadence. Results indicated that unimpaired subjects entrained their gait to periodic mechanical perturbations at the hip. Convergence of relative phase between gait and perturbations was observed, but clustered around two distinct gait phases, in contrast to the single converged phase observed in entrainment to periodic ankle torques. These entrainment studies quantify important aspects of the nonlinear neuro-mechanical dynamics underlying the control of walking, which will inform the development of effective approaches to robotic walking therapy.

I. INTRODUCTION

Between 2017 and 2050, the portion of the global population over the age of 60 is expected to nearly double from 13% to 21.5% [1]. With this rise in the aged population and age-related disorders comes an increasing need for effective locomotor assistance and rehabilitation. Robot-aided therapy has emerged as a promising method to meet the enormous demand. Despite the proven success of upper-extremity robotic rehabilitation, lower-extremity robotic therapy has shown limited efficacy [2]–[7].

Conventional therapeutic robots for walking impose pre-planned repetitive kinematic trajectories on patients using high-gain trajectory tracking controllers [8], [9]. This approach may have discouraged active engagement of patients in making movements [7] and interfered with the natural oscillatory dynamics of walking [10], [11]. More

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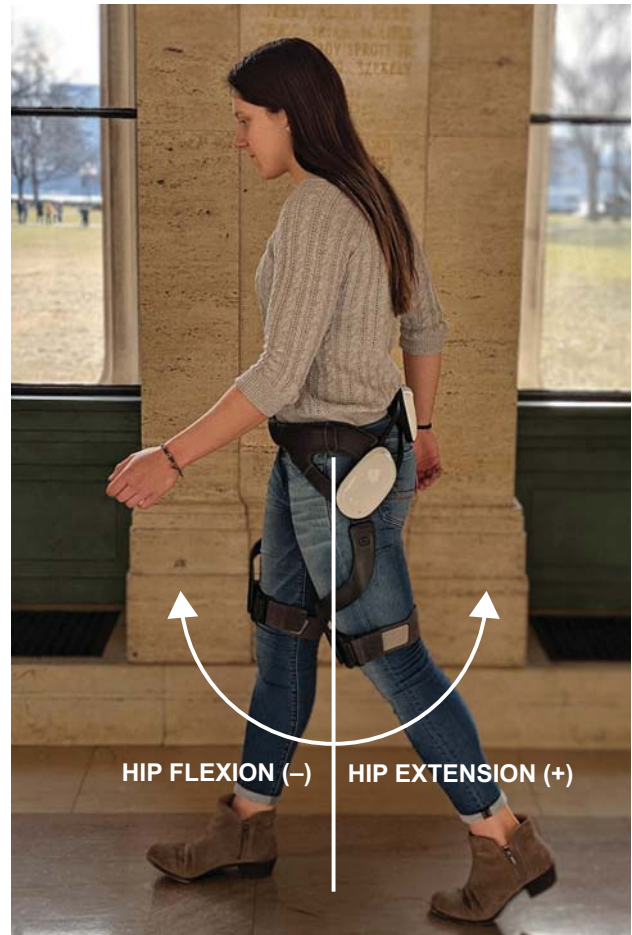


Fig. 1. Experimental Setup. Subjects were instructed to walk comfortably as the Samsung GEMS-H exoskeleton applied periodic torque pulses to both legs (flexion torque about right hip and extension torque about the left hip).

recently, lower limb exoskeletons have been used to apply biologically-consistent torque patterns to a wearer in an adaptive manner, resulting in a remarkable reduction in metabolic cost [12]–[15] or muscle effort [16], [17]. Even though these methods may provide effective assistance in daily life, their efficacy for rehabilitation is as yet unclear.

The purpose of the present study was to better understand the underlying neuro-mechanical dynamics and control of healthy locomotion to inform the development of effective rehabilitation methods. Prior studies have shown that human gait entrains to periodic mechanical perturbations from an ankle exoskeleton robot, in both treadmill and overground walking [10], [11]. Gait entrainment occurred when the

perturbation period was close to subjects' preferred walking cadence. Phase locking was also observed such that ankle plantar-flexion torque pulses occurred at the end of double stance, where they provided mechanical assistance. These results strongly suggest the presence of a nonlinear neuro-mechanical oscillator which governs human locomotion. Moreover, they pointed to a new, minimally-encumbering method of altering gait period for neurologically-impaired subjects [18].

The present study evaluated whether humans similarly synchronize their gait to periodic torque perturbation applied to the hip using the Samsung Gait Enhancing and Motivating Systems for hip (GEMS-H exoskeleton; Fig. 1). Importantly, subjects performed the experiment overground to avoid any speed constraints imposed by a treadmill [10]. Consistent with prior results from the aforementioned ankle entrainment studies, we hypothesized that subjects would entrain to the periodic perturbations applied to the hip joint. More specifically, we tested the following predictions:

- Stride period will converge to the torque pulse period.
- The relative phase between gait and the perturbations will converge to a constant value within each trial.
- The converged relative phase will be consistent across all trials and subjects.

This paper is organized as follows. Section II provides experimental design and statistical methods to evaluate gait entrainment. Section III presents results which are discussed in Section IV. Concluding remarks follow in Section V.

II. METHODS

A. Subjects

Five healthy, young adults (gender: 3 females, 2 males; age: 20.2 ± 1.30 years) participated in this study. None had any prior experience wearing the hip exoskeleton or participating in similar gait entrainment experiments. All subjects gave informed written consent before the experiment. The experimental protocol was reviewed and approved by the Institutional Review Board of the Massachusetts Institute of Technology.

B. GEMS-H Exoskeleton

The GEMS-H exoskeleton used in this study was developed by Samsung Advanced Institute of Technology (Suwon, South Korea) (Fig. 1). This lightweight (2.1 kg) robot is worn around the waist and fastened to the thighs. It has a pair of actuators, one at each hip joint, that are each capable of applying torque in the sagittal plane (flexion/extension; Fig. 1). Two specially-designed thigh frames connect the output of the actuators to the thighs to apply torque about the hip joints. Actuator torque is estimated and controlled by sensing electrical current in the motor. Encoders embedded in the actuator modules measure hip joint angles, and the device is equipped with an inertial measurement unit centered at the back. It also has passive hinges which allow hip abduction and adduction motion in the frontal plane. All electronics, actuators, and power sources are located onboard the device, allowing for autonomous operation.

C. Experimental Design

Each subject performed a total of 10 walking trials wearing the GEMS-H exoskeleton, and each trial consisted of 120 strides. Trials were performed in a long corridor (approximately 250 m) with low foot traffic to simulate real-world walking conditions. Throughout all trials, subjects listened to white noise through wireless, over-the-ear headphones to mask the sound of the exoskeleton robot. The whole experiment lasted approximately one hour.

1) *Baseline Trials*: In the first two trials, the hip exoskeleton was inactive (i.e., motor current was zero), and subjects were instructed to walk at a comfortable pace. Preferred stride period was quantified as the average stride period from the middle 30 strides in the second baseline trial.

2) *Experimental Trials*: The remaining eight trials proceeded as follows. During the first 20 strides, the hip exoskeleton was inactive. Over the subsequent 80 strides, the exoskeleton delivered periodic torque pulses to both legs simultaneously. The torque pulses were trapezoidal in shape with a peak magnitude of 6 Nm and a duration of 200 ms. The torque pulse applied to the right hip was always in the flexion direction and always oppositely in the extension direction to the left hip (Fig. 1). The pulse period was set to be 25 ms faster than the preferred stride period of each subject. During the remaining 20 strides, the hip exoskeleton was again inactive. Subjects were instructed to walk in whatever way they found most comfortable. They were not informed how the exoskeleton would act.

D. Data Processing

All signals measured by the device were resampled at 1kHz resolution using the `interp1` function in MATLAB (The Mathworks, Natick, MA, USA) before calculating dependent measures.

1) *Gait Cycle*: The gait cycle was estimated using the extrema of the left hip angle profile. The left hip angle profile was time-normalized from 0% to 100% to define a gait phase for each stride. 0% phase was defined as left leg toe-off, which coincided with maximum extension of the left hip joint [19]. From 0% to 50% of gait cycle, the left hip was flexing (i.e., negative joint velocity) while the right hip was extending (i.e., positive joint velocity). From 50% to 100% of gait cycle, the left hip was extending while the right hip was flexing.

2) *Trial Segments*: In each experimental trial, the dependent measures were calculated in each of the following four trial segments, as appropriate:

- *pre-pulses*: the last ten strides before the onset of torque pulses,
- *initial-pulses*: the first ten strides after the onset of torque pulses,
- *terminal-pulses*: the last ten strides before the offset of torque pulses, and
- *post-pulses*: the first ten strides after the offset of torque pulses.

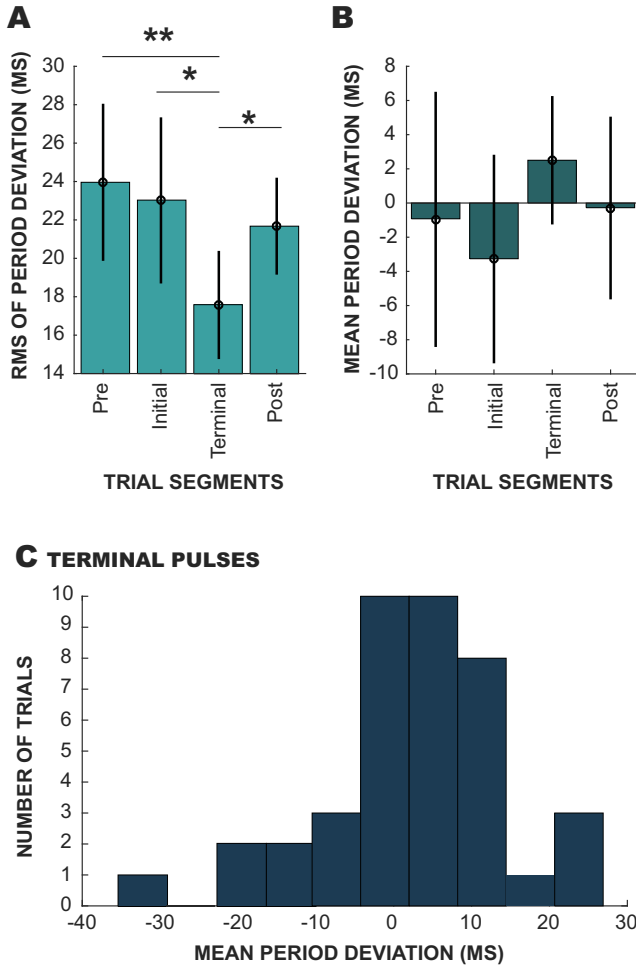


Fig. 2. Period Deviation Results. Plot of (A) RMS and (B) mean of period deviation for each trial segment. Circles represent the mean of all trials in each trial segment, and error bars represent ± 2 standard errors of the mean. * indicates $p < 0.05$ and ** indicates $p < 0.01$. (C) Histogram of mean period deviations during the *terminal-pulses* segment for all trials.

E. Dependent Measures

1) *Period Deviation*: Period deviation (ms) was defined as the difference between the period of a given stride and the torque pulse period for the respective subject. For each trial segment, the mean and root-mean-square (RMS) of period deviation of the pulses were calculated.

2) *Pulse Phase*: Pulse phase (%) was defined as the phase at which the onset of the torque pulse occurred within the gait cycle. In each trial, the mean pulse phases during the *initial-pulses* and *terminal-pulses* segments of the trial were calculated.

3) *Pulse Phase Slope*: The slope of pulse phase over pulse number ($\Delta\%$), as determined using linear regression, was calculated over the *initial-pulses* and *terminal-pulses*.

F. Statistical Analyses

1) *Period Convergence*: A linear mixed-effects analysis was conducted to further assess the effect of trial segment on the mean and RMS of period deviation. Trial segment

was included as a fixed effect and intercepts for subject and trial numbers were included as random effects. To test the significance of trial segment, we conducted a likelihood ratio test comparing models with and without the effect of trial segment.¹

To test whether the mean period deviation of the *terminal-pulses* was different from zero, an intercept was included as a fixed effect and intercepts for subject and trial numbers were again included as random effects. To test the significance of fixed effect intercept, we conducted a likelihood ratio test comparing models with and without it.²

Based on the analysis of period convergence, a trial was deemed entrained if the RMS of period deviation was lower for the *terminal-pulses* compared to the *pre-pulses*.

2) *Phase Convergence*: Assessing whether pulse phase converged to a constant value provided a further test of entrainment. Thus, the same linear mixed-effects analyses described above were conducted (1) to assess the effect of trial segment on the magnitude of the pulse phase slope and (2) test whether the pulse phase slope across the *terminal-pulses* was different from zero for entrained trials.

The k -means clustering algorithm [20] was used to assess whether converged pulse phase was consistent across entrained trials. Specifically, the within cluster sum of squares (defined as the sum of the squared deviations from each observation and its respective cluster centroid) for different numbers of clusters ($k = \{1, 2, \dots, 10\}$) was calculated to determine the appropriate number of clusters in the mean pulse phase of the *terminal-pulses*.

All linear mixed-effects analyses were conducted using the lme4 package [21] in R [22], and the significance value was set to $p < 0.05$ for all statistical tests.

III. RESULTS

A. Period Convergence

We predicted that stride period would converge to the torque pulse period. Comparison of the mixed-effects model with and without the effect of trial segment revealed that it had a significant effect on the RMS of period deviation, $\chi(1)^2 = 9.07, p = 0.028$. As illustrated in Fig. 2A, the RMS of period deviation was significantly lower during the *terminal-pulses* compared to the other three trial segments (*pre-pulses* vs. *terminal-pulses*: $\chi(1)^2 = 7.14, p = 0.0071$; *initial-pulses* vs. *terminal-pulses*: $\chi(1)^2 = 5.11, p = 0.024$; *post-pulses* vs. *terminal-pulses*: $\chi(1)^2 = 4.93, p = 0.026$).

There was, however, no significant effect of trial segment on mean period difference, $\chi(1)^2 = 3.14, p = 0.37$ (Fig. 2B). During the *terminal-pulses*, there was also no effect of the intercept on mean period deviation, $\chi(1)^2 = 0.72, p = 0.40$, indicating that the overall mean period deviation was

¹In R-notation, we compared “mean (RMS) period deviation $\sim$ trial segment + (1|subject) + (1|trial number) + ϵ ” versus “mean (RMS) of period deviation $\sim$ (1|subject) + (1|trial number) + ϵ ”.

²In R-notation, we compared “mean period deviation of the *terminal-pulses* $\sim$ 1 + (1|subject) + (1|trial number) + ϵ ” versus “mean (RMS) period deviation of the *terminal-pulses* $\sim$ 0 + (1|subject) + (1|trial number) + ϵ ”.

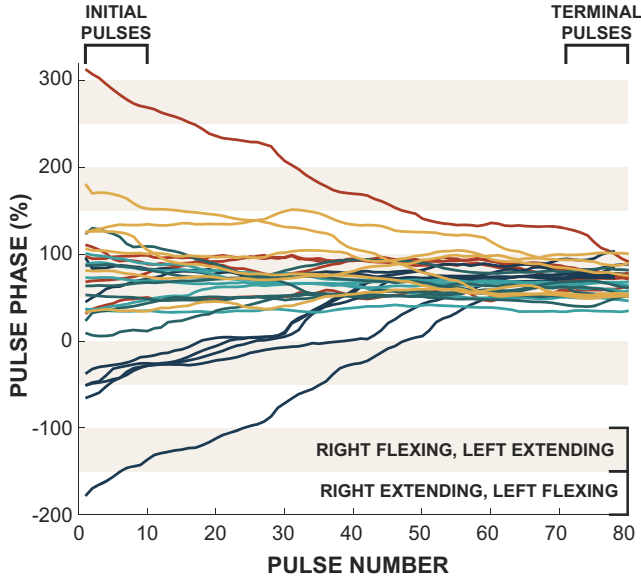


Fig. 3. Plot of pulse phase over pulse number for all entrained trials. Individual subjects are represented by color.

not significantly different from zero ($M = 2.5\text{ms}$, $SD = 11.88\text{ms}$; Fig. 2C).

In 29 of the 40 trials, the RMS of period deviation was lower for the *terminal-pulses* compared to the *pre-pulses*. Hence, 72.5% of trials were deemed entrained.

B. Phase Convergence

We also predicted that in the entrained trials, the pulse phase would converge to a constant value by the *terminal-pulses* and that this value would be consistent across subjects and trials. Fig. 3 shows pulse phase as a function of pulse number for entrained trials.

There was a significant effect of trial segment on the magnitude of pulse phase slope, $\chi(1)^2 = 12.007$, $p = 0.00053$. The magnitude of pulse phase slope was higher during the *initial-pulses* compared to the *terminal-pulses* (Fig. 4A-B).

During the *terminal-pulses*, there was no effect of the intercept on pulse phase slope, $\chi(1)^2 = 0.29$, $p = 0.59$. Thus, the overall pulse phase slope across *terminal-pulses* was not found to be significantly different from zero ($M = -0.09\%$, $SD = 0.95\%$; Fig. 4C).

As seen in Fig. 5A, phases of *initial-pulses* varied widely across the entire gait cycle, whereas the vast majority (86%) of the phases during *terminal-pulses* fell between 50% to 100% of the gait cycle ($M = 68.80\%$, $SD = 15.96\%$; Fig. 5B).

Fig. 5C shows a comparison of the within cluster sum of squares (WC-SS) resulting from the k -means analysis with different cluster numbers (k). Lower values of WC-SS indicate a better fit. The change in WC-SS declines after $k = 2$, suggesting that there are at least 2 clusters (with centroids at 52% and 79%; Fig. 5B) in the distribution of phases of the *terminal-pulses* of entrained trials.

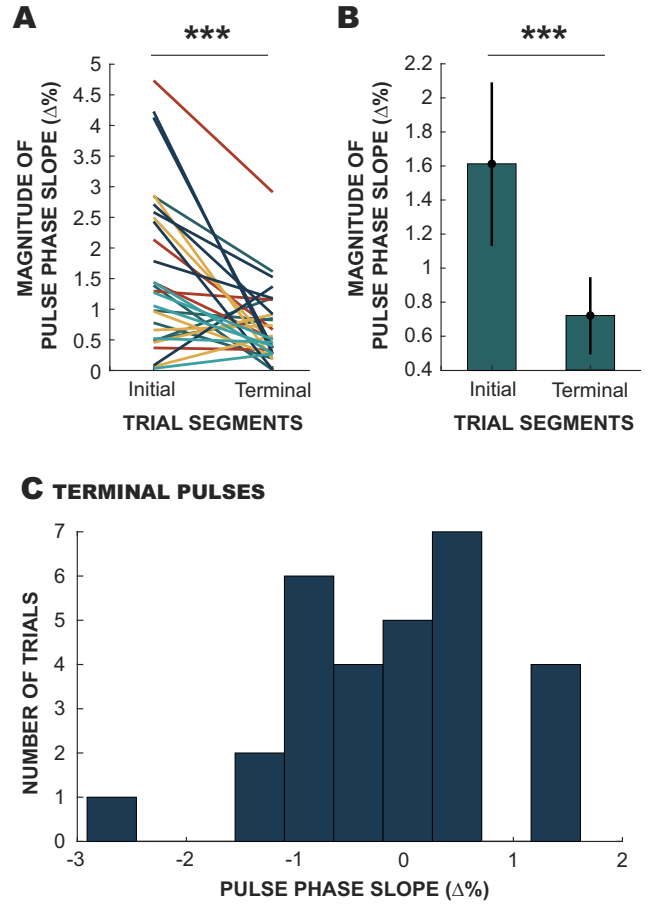


Fig. 4. Pulse Phase Slope Results. Plot of (A) individual and (B) group averaged magnitude of pulse phase slope during *initial-pulses* and *terminal-pulses* for entrained trials. Individual subjects are represented by color. Circles represent the mean of all entrained trials in each trial segment, and error bars represent ± 2 standard errors of the mean. *** indicates $p < 0.001$. (c) Histogram of pulse phase slope during the *terminal-pulses* segment for all entrained trials.

IV. DISCUSSION

The results indicated that subjects entrained their gait to the periodic torque pulses applied to the hip joints using the Samsung GEMS-H exoskeleton. The stride period during *terminal-pulses* was not significantly different from that of the torque pulses, representing 1:1 frequency locking between human gait and the perturbations. Reduction in the variability of stride periods within each trial provided further evidence of entrainment (Fig. 2)

Observation of phase locking in addition to frequency locking provides stronger evidence of entrainment between two oscillators. If relative phase between two oscillators converges to a constant from a random initial distribution, it indicates that they are phase locked [23], [24]. In our experiment, the pulse phase slopes during *terminal-pulses* were not statistically different from zero (Fig. 4). However, for the hip torque profiles used in this study, the converged pulse phases were not the same for all subjects and trials. Instead, they were clustered around two distinct gait phases

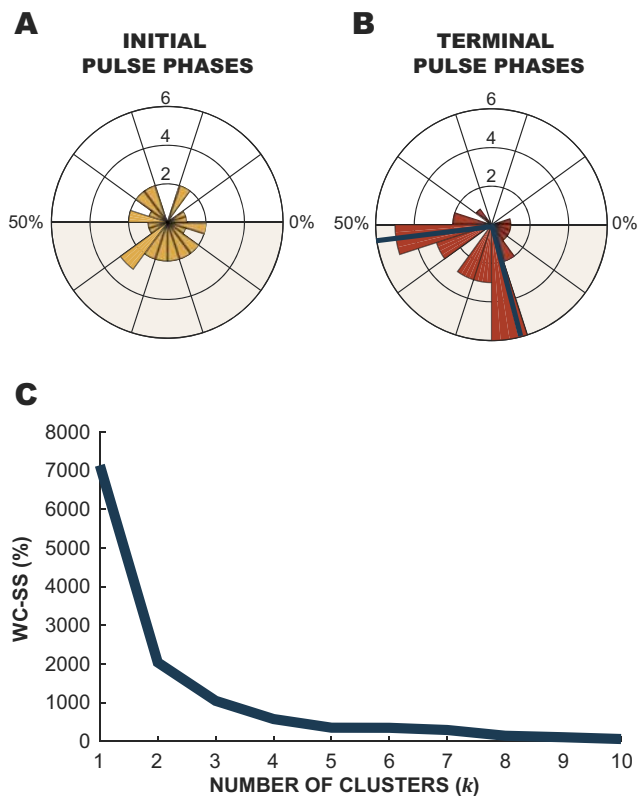


Fig. 5. Pulse Phase Results. Histogram of pulse phases during the (A) *initial-pulses* and (B) *terminal-pulses* from all entrained trials. The thick line represents the cluster centroid phases identified from the k -means analysis with $k = 2$. (C) Plot of within cluster sum of squares (WC-SS) from k -means analysis for different values of k .

(Fig. 5).

Overall, entrainment to periodic hip torque perturbations was observed, but the pattern of entrainment was different from that of previous studies with ankle perturbations [10], [11]. In contrast to the present study, entrainment with ankle perturbations was always accompanied by synchronizing the plantar-flexion pulse torques to a constant gait phase at the end of double stance, where they provided mechanical assistance.

Phase locking of two coupled nonlinear oscillators is exhibited when the coupled dynamics have a stable point attractor, a fixed point to which their relative phase converges. The experimental results with ankle perturbations indicated that the end of double stance was a global attractor for phase locking, and a simple model could provide a mechanism to account for this [25]. The experimental results of the present study suggest that at least two equally strong stable attractors for phase locking exist when human gait is coupled to periodic torque pulses at hip. Note that in the previous studies plantar-flexion torque pulses were applied to the ankle joint of a single leg, while we applied flexion torque pulses to the hip joint of the right leg and extension to the left, simultaneously. The two attractors we observed may correspond to periodic pulses coupled to each leg.

The region of attraction in state-space of such fixed-points

may be small. If its size was comparable to the inevitable variability of healthy gait, we would not be able to observe clear evidence of phase locking. While the stable attractor tends to keep dynamics within its neighborhood, stochasticity will ‘push’ the system to escape from it. The dynamics may enter a region where a different attractor dominates, or may come back to the original [26]. The two clusters of phases of *terminal-pulses* may be attributable to this property. It may also give us a plausible explanation why pulse phases in some trials showed alternating epochs of drift and plateau, wrapping-around several times.

Aside from the dynamical point of view, a bio-mechanical interpretation may yield some insight. Almost all pulses occurred between 50% to 100% of the gait cycle during *terminal-pulses*, which roughly corresponds to the phase of right hip flexion (swing) and left hip extension (stance). This also correspond to the phase when the robot is driving the two legs apart, and positive power flows into the human from the robot. Moreover, the most frequent converged phases at 52% and 79% of the gait cycle approximately coincided with activity bursts of the major hip flexor muscles of the right leg and hip extensor muscles of the left leg, respectively [27]. This behavior appears to be consistent with the previous studies showing that humans adapt to ‘exploit’ robot torques such that they provide mechanical assistance [10].

The intervention we implemented in this study investigated the natural oscillatory dynamics of healthy walking and its adaptation to a robotic perturbation. In order to facilitate understanding, we used deliberately simple torque profiles that did not adapt to users’ behavior. While we observed consistent evidence of entrainment, further studies are required to better understand the neuro-mechanics of human interaction with a hip exoskeleton robot, including basin of entrainment, behavioral changes and retention in multi-day trials, and a mathematical account of the source of two attractors for phase locking.

A possible explanation for the ineffectiveness of conventional robotic walking therapy for rehabilitation is the use of human-interactive robots in ways that may inadvertently suppress the natural oscillatory dynamics of walking. A proper robotic therapy may need to encourage a user’s active engagement in generating motion, while minimally encumbering the natural dynamics. Ultimately, we envision that impaired patients could first be entrained to torque pulses at their preferred stride period, which then progressively shortened to attract them toward faster cadence. Together with ankle perturbation studies [18], this may serve as a novel multi-degree-of-freedom locomotor rehabilitation robot therapy.

V. CONCLUSIONS

We tested dynamic entrainment of human walking to periodic hip torque perturbations using the Samsung GEMS-H exoskeleton. Experimental evidence of entrainment was observed in our unimpaired subjects. Entrainment was accompanied by convergence of the relative phase between gait and the perturbations. However, for the hip torque

profiles used in this study, entrainment did not converge to the same phase for different subjects and trials, but clustered around two distinct gait phases. Further experimental studies with different parameter values and longer exposure to the perturbations and mathematical analysis with simplified models may further enhance our understanding of the neuro-mechanical interaction between human gait and an exoskeleton robot. Exploiting the natural oscillatory dynamics of walking to induce entrainment when coupled to an exoskeleton robot may provide a novel approach to locomotor rehabilitation therapy.

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