Journal of Sound and Vibration 500 (2021) 116015

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsv

Rapid Communications

Near-elimination of small oscillations of articulated flexible-robot systems

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ARTICLE INFO

Article history:
Received 22 January 2021
Revised 3 February 2021
Accepted 6 February 2021
Available online 14 February 2021

Keywords:
Articulated flexible robots
Hybrid actuation
Joint/deformation actuation
Fully-constrained inverse dynamics
Vibration suppression

ABSTRACT

This paper proposes a new hybrid actuation system for near-elimination of the small oscillations of *articulated flexible-robot systems* (AFRS). The hybrid actuation control forces are obtained by solving a fully-constrained inverse-dynamics (FCID) problem of relatively stiff robots experiencing small deformations, which lead to deterioration of their performance and precision. The FCID procedure is based on the *floating frame of reference* (FFR) formulation that allows for systematically determining the actuation forces associated with deformation modes. By using the resulting FCID algebraic equations, numerical integration of the equations of motion to determine the AFRS actuation forces can be avoided. This paper summarizes the new *motion/shape control* strategy to be used and defines problems to be addressed in more detailed future investigations.

1. Introduction

High precision is required for all manufacturing robots to ensure safety as well as high-quality and low-cost products. Recognizing the significant economic impact of manufacturing robots, the European Union launched an important initiative, the multi-organization COMET (**Co**mponents and **Met**hods for Adaptive Control of Industrial Robots) project with the goal of exploring new manufacturing robot designs to replace the more-costly and less-versatile traditional machine tools. While robots, in general, are much cheaper than machine tools, the main challenge is the robot accuracy which is in the 1 mm range, while milling machines, for example, have 0.001 mm accuracy range. The COMET researchers were successful in making improvement in the robot accuracy (0.05 mm) without using sophisticated virtual prototyping techniques that allow experimenting with and optimizing different design configurations to achieve a precision superior to the 0.001 mm accuracy of the conventional machine tools.

Undesired oscillations of flexible robot systems can be broadly classified under two main categories: *inverse-dynamics-actuation oscillations* (IDAO) and *disturbance-produced oscillations* (DPO). The IDAO type is the result of using incomplete or improper definitions of the actuation control forces and moments, which are the basis for the open-loop control system. Based on the specified motion trajectories with no disturbance, the actuation forces and moments can be determined using an inverse dynamics procedure. If there are no disturbances, the actuation forces and moments predicted using the inverse

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https://doi.org/10.1016/j.jsv.2021.116015
0022-460X/

dynamics problem are assumed sufficient to achieve the desired motion trajectories. The DPA type, on the other hand, is the result of unexpected disturbances which are not taken into consideration in the design of the actuation system based solely on the specified motion trajectories and the solution of the inverse dynamics problem. For the elimination of this type of oscillations, a feedback control system is required.

The lack of virtual prototyping approaches, applicable to constrained flexible multibody systems (MBS), represents a serious challenge in achieving a higher degree of precision, which cannot be achieved by more reliance on costly physical prototyping and experimentation. This paper addresses this challenge by proposing a new hybrid-actuation approach for the near-elimination of small oscillations of relatively stiff *articulated flexible robot systems* (AFRS) as those used in manufacturing applications. The paper is concerned with the IDAO type and presents preliminary numerical results that demonstrate the significance of accounting for the deformation degrees of freedom when determining the actuation forces and moments based on the solution of the inverse problem. In the proposed new motion/shape control approach, the FFR formulation is used to define the driving control forces associated with the joint coordinates and deformation modes. Two different actuation types are used; the first is conventional motors and/or actuators placed at the robot articulated joints, while the second is a deformation actuation system designed to control the fundamental vibration modes that can contribute to deterioration in the robot overall performance and precision. The deformation actuation can be achieved using, for example, piezoelectric actuators whose number is equal to the number of deformation modes to be controlled [1–5]. The FCID procedure proposed in this paper leads to a system of algebraic equations that can be efficiently solved for the driving control forces associated with the articulated joint and deformation degrees of freedom.

2. Equipollent systems of forces

In the FFR formulation for flexible bodies, the global position vector of an arbitrary point on a flexible body i can be written as $\mathbf{r}^i = \mathbf{R}^i + \mathbf{A}^i(\bar{\mathbf{u}}_o^i + \bar{\mathbf{u}}_f^i) = \mathbf{R}^i + \mathbf{A}^i(\bar{\mathbf{u}}_o^i + \mathbf{S}^i\mathbf{q}_f^i)$, where $\mathbf{R}^i$ is the global position of the body reference point, $\mathbf{A}^i$ is the transformation matrix of the body coordinate system, $\bar{\mathbf{u}}_o^i = \mathbf{x} = [x_1 \quad x_2 \quad x_3]^T$ is the position of the point before deformation, $\bar{\mathbf{u}}_f^i = \mathbf{S}^i\mathbf{q}_f^i$ is the deformation vector, $\mathbf{S}^i = \mathbf{S}^i(\mathbf{x})$ is a shape function matrix, and $\mathbf{q}_f^i$ is the vector of elastic coordinates that define the body deformations. A virtual change in $\mathbf{r}^i$ leads to $\delta\mathbf{r}^i = \delta\mathbf{R}^i - \mathbf{A}^i\tilde{\bar{\mathbf{u}}}^i\bar{\mathbf{G}}^i\delta\boldsymbol{\theta}^i + \mathbf{A}^i\mathbf{S}^i\delta\mathbf{q}_f^i = \mathbf{L}^i\delta\mathbf{q}^i$, where $\tilde{\bar{\mathbf{u}}}^i$ is a skew symmetric matrix associated with vector $\bar{\mathbf{u}}^i = \bar{\mathbf{u}}_o^i + \bar{\mathbf{u}}_f^i$, $\bar{\mathbf{G}}^i$ is the matrix that relates the angular velocity vector $\bar{\boldsymbol{\omega}}^i$, defined in the body coordinate system, to the time derivatives of the orientation parameters $\boldsymbol{\theta}^i$, that is $\bar{\boldsymbol{\omega}}^i = \bar{\mathbf{G}}^i\dot{\boldsymbol{\theta}}^i$, $\mathbf{L}^i = [\mathbf{I} - \mathbf{A}^i\tilde{\bar{\mathbf{u}}}^i\bar{\mathbf{G}}^i \quad \mathbf{A}^i\mathbf{S}^i]$, and $\mathbf{q}^i = [\mathbf{R}^{iT} \quad \boldsymbol{\theta}^{iT} \quad \mathbf{q}_f^{iT}]^T$ [6].

In the case of flexible bodies, a force acting at a point is equipollent to a system of forces at another point that consists of the same force, a deformation-dependent moment, and a set of generalized forces associated with the deformation degrees of freedom [7–8]. For example, using the virtual displacement $\delta\mathbf{r}^i$, the virtual work of a force vector $\mathbf{F}^i$ acting at a point can be written as $\delta W_e^i = \mathbf{F}^{iT}\delta\mathbf{r}^i = \mathbf{F}^{iT}\delta\mathbf{R}^i - \mathbf{F}^{iT}\mathbf{A}^i\tilde{\bar{\mathbf{u}}}^i\bar{\mathbf{G}}^i\delta\boldsymbol{\theta}^i + \mathbf{F}^{iT}\mathbf{A}^i\mathbf{S}^i\delta\mathbf{q}_f^i$, which can be written as $\delta W_e^i = \mathbf{Q}_R^{iT}\delta\mathbf{R}^i + \mathbf{Q}_\theta^{iT}\delta\boldsymbol{\theta}^i + \mathbf{Q}_f^{iT}\delta\mathbf{q}_f^i$, where $\mathbf{Q}_R^i = \mathbf{F}^i$, $\mathbf{Q}_\theta^i = \bar{\mathbf{G}}^{iT}\tilde{\bar{\mathbf{u}}}^{iT}\mathbf{A}^{iT}\mathbf{F}^i$, and $\mathbf{Q}_f^i = \mathbf{S}^{iT}\mathbf{A}^{iT}\mathbf{F}^i$. This concept of equipollent systems of forces in flexible-body dynamics is fundamental in defining the hybrid actuation for the near-elimination of small oscillations of stiff robot manipulators.

3. FCID procedure

The new hybrid-actuation procedure proposed in this study is based on using a flexible-body inverse-dynamics formulation in which the number of specified motion-trajectory constraints is equal to the number of the system coordinates. This procedure is referred to as the *fully-constrained inverse-dynamics* (FCID) procedure to distinguish it from the inverse problem in which the number of constraint equations is less than the number of coordinates. The FCID procedure can be used for both *motion and shape controls*.

By enforcing trajectory constraints that ensure that the amplitudes of the fundamental modes remain equal to zero in the FCID algorithm, one obtains a set of algebraic equations that can be solved for the driving joint forces as well as the driving actuation forces that ensure that the deformation-mode amplitudes remain equal to zero. These joint and deformation actuation forces can be used to define the articulated joint motors and actuators as well as the piezoelectric deformation actuators, required to achieve a high accuracy by producing the desired motion trajectories and near-elimination of the small oscillations. The uncontrolled high-frequency modes have high stiffness and complex shapes that make encountering spill-over problems is unlikely. However, this important issue, among others, will be addressed in future investigations. The FFR equations of motion, subjected to the constraint equations $\mathbf{C}(\mathbf{q}, t) = \mathbf{0}$, can be written as $\mathbf{M}\ddot{\mathbf{q}} = \mathbf{Q}_e + \mathbf{Q}_c$, where $\mathbf{M}$ is the system mass matrix, $\mathbf{Q}_e$ is the vector of applied and quadratic-velocity inertia forces, and $\mathbf{Q}_c$ is the vector of constraint forces written in terms of Lagrange multipliers as $\mathbf{Q}_c = -\mathbf{C}_\mathbf{q}^T\boldsymbol{\lambda}$. In this equation, $\mathbf{C}_\mathbf{q} = \partial\mathbf{C}/\partial\mathbf{q}$ is the constraint Jacobian matrix; and $\boldsymbol{\lambda}$ is the vector of Lagrange multipliers. By specifying the motion trajectories, including the deformation amplitudes, one obtains a system of algebraic equations that can be solved for the driving actuation forces associated with the articulated joints and deformation modes. This algebraic system of equations can be written as

$$\begin{bmatrix} \mathbf{M} & \mathbf{C}_{\mathbf{q}}^T \\ \mathbf{C}_{\mathbf{q}} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_e \\ \mathbf{Q}_d \end{bmatrix} \tag{1}$$

In this equation, $\mathbf{Q}_d$ is the vector that appears in the second time derivatives of the constraint functions, that is, $\mathbf{C}_{\mathbf{q}}\ddot{\mathbf{q}} = \mathbf{Q}_d$. Unlike, the *partially-constrained inverse-dynamics* (PCID) problem [9], in the preceding sparse-matrix equations, the number of constraint equations is equal to the number of coordinates, and therefore, the FCID problem does not require numerical integration of differential equations in order to determine the driving forces.

4. Hybrid actuation

The constraint-function vector $\mathbf{C}$ can be partitioned into 2 different sets of constraint types as $\mathbf{C} = [\mathbf{C}_m^T \quad \mathbf{C}_s^T]^T$, where $\mathbf{C}_m$ and $\mathbf{C}_s$ are, respectively, the constraints associated with the MBS articulated mechanical joints and the constraints associated with the specified motion trajectories. The vector of Lagrange multipliers can be partitioned accordingly as $\boldsymbol{\lambda} = [\boldsymbol{\lambda}_m^T \quad \boldsymbol{\lambda}_s^T]^T$. Using this partitioning, the actuation constraint forces associated with the coordinate vector $\mathbf{q}$ can be written as $-(\partial \mathbf{C}_s/\partial \mathbf{q})^T \boldsymbol{\lambda}_s$. The virtual work of these driving control forces can be written as $\delta W_a = -[(\partial \mathbf{C}_s/\partial \mathbf{q})^T \boldsymbol{\lambda}_s]^T \delta \mathbf{q}$. The virtual change in the coordinate vector $\mathbf{q}$ can be written in terms of the virtual change in the independent coordinates $\mathbf{q}_i$, which include the joint variables and deformation degrees of freedom, as $\delta \mathbf{q} = \mathbf{B}_{di} \delta \mathbf{q}_i$, where $\mathbf{B}_{di}$ is a velocity transformation matrix [6]. Using this velocity transformation, the virtual work of the driving-constraint forces can be written as $\delta W_a = -[(\partial \mathbf{C}_s/\partial \mathbf{q})^T \boldsymbol{\lambda}_s]^T \mathbf{B}_{di} \delta \mathbf{q}_i = \mathbf{Q}_{ij}^T \delta \mathbf{q}_{ij} + \mathbf{Q}_{if}^T \delta \mathbf{q}_{if}$, where $\mathbf{Q}_{ij}$ and $\mathbf{Q}_{if}$, are, respectively, the generalized articulated-joint and deformation-piezoelectric actuation forces. The piezoelectric voltage can be selected to produce the forces $\mathbf{Q}_{if}$ required to suppress the robot small oscillations.

Because the coordinates in the equation $\delta W_a = -[(\partial \mathbf{C}_s/\partial \mathbf{q})^T \boldsymbol{\lambda}_s]^T \mathbf{B}_{di} \delta \mathbf{q}_i = \mathbf{Q}_{ij}^T \delta \mathbf{q}_{ij} + \mathbf{Q}_{if}^T \delta \mathbf{q}_{if}$ are independent, one has

$$\begin{bmatrix} \mathbf{Q}_{ij}^T & \mathbf{Q}_{if}^T \end{bmatrix} = -\left[(\partial \mathbf{C}_s/\partial \mathbf{q})^T \boldsymbol{\lambda}_s \right]^T \mathbf{B}_{di} \tag{2}$$

The vectors of generalized forces $\mathbf{Q}_{ij}$ and $\mathbf{Q}_{if}$ can be expressed in terms of a number of actual actuation forces and moments equal to the number of joint and deformation degrees of freedom as [9]

$$\begin{bmatrix} \mathbf{Q}_{ij} \\ \mathbf{Q}_{if} \end{bmatrix} = \mathbf{B}_{ia} \mathbf{F}_{ia} = -\mathbf{B}_{di}^T \left[(\partial \mathbf{C}_s/\partial \mathbf{q})^T \boldsymbol{\lambda}_s \right] \tag{3}$$

In this equation, $\mathbf{B}_{ia}$ is the square matrix that relates the generalized actuation forces to the actual actuation forces; this is with the assumption that each mode of vibration is controlled by one actuator such as piezoelectric actuators. Use of number of deformation actuators equal to number of deformation modes in the FCID problem ensures that all the degrees of freedom are controlled. The vector $\mathbf{B}_{di}^T [(\partial \mathbf{C}_s/\partial \mathbf{q})^T \boldsymbol{\lambda}_s]$ is known from the solution of the FCID problem. Therefore, Eq. (3) can be solved for the actual joint and deformation actuation forces. The actual deformation actuation

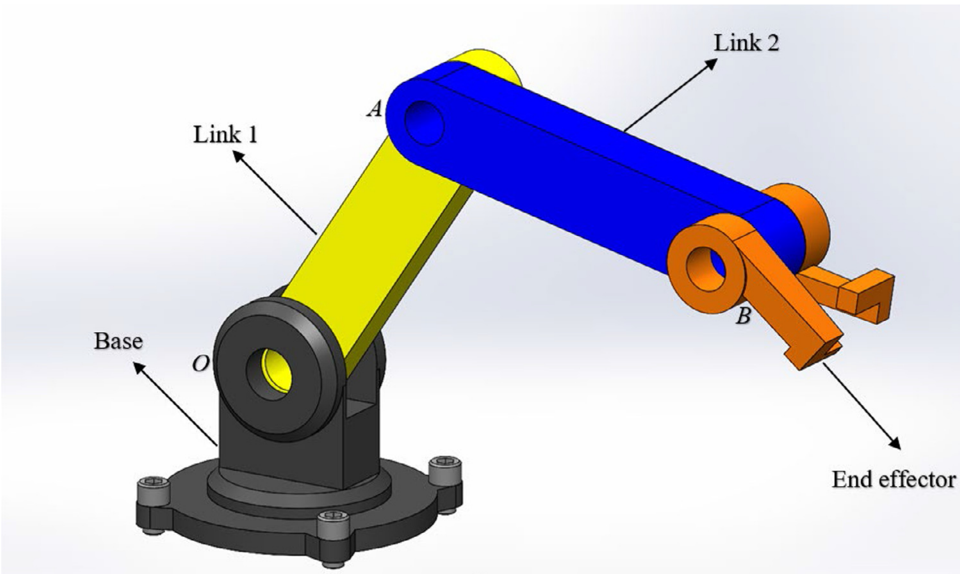

Fig. 1. Flexible robot manipulator.

forces in the vector $\mathbf{F}_{ia}$ can be, for example, used to determine the piezoelectric voltage required to achieve the desired motion [9].

5. Numerical example

In this section, the planar two-link robot example, shown in Fig. 1, is used to demonstrate the significance of the deformation actuation. To this end, the FCID problem is used to define the joint and deformation actuations, which are then applied to two forward-dynamics models; in the first model, both joint and deformation actuations are applied, while in the second model only the articulated joint actuation is applied. The robot example consists of two flexible links; Links 1 and 2; with Link 1 connected to the base by a revolute joint at point O and to Link 2 by a revolute joint at point A. Each link has a length 1 m and 0.025×0.015 m^2 cross section. The two links are made of steel with elastic modulus 2.068427×10^{11} N/m^2 and the material density 7850 kg/m^3. The initial configuration of the two links is assumed to be $\theta_1 = 0$ and $\theta_2 = \pi/3$. The effect of gravity is considered in this example. Twenty Euler-Bernoulli planar beam elements are used in the FE discretization of the two links. Simply supported reference conditions are selected for Link 1, while Link 2 has cantilever reference conditions. Four modes are used for each link in both the inverse- and forward-dynamics problem. The specified motion trajectory of the end effector is assumed $x_d(t) = x_0 \cos(\pi t/4)$ and $y_d(t) = y_0 \cos(\pi t/4)$, where x_o and y_o are the coordinates of the end effector in the reference configuration. The modal coordinates in the FCID problem are constrained to zero; and the two forward-dynamic problems, with and without deformation actuations, are solved. Fig. 2 shows comparison between the end-effector transverse deformations in the two cases. The results presented in this figure clearly demonstrates a high precision achieved by considering the deformation actuations in the case of articulated flexible robots.

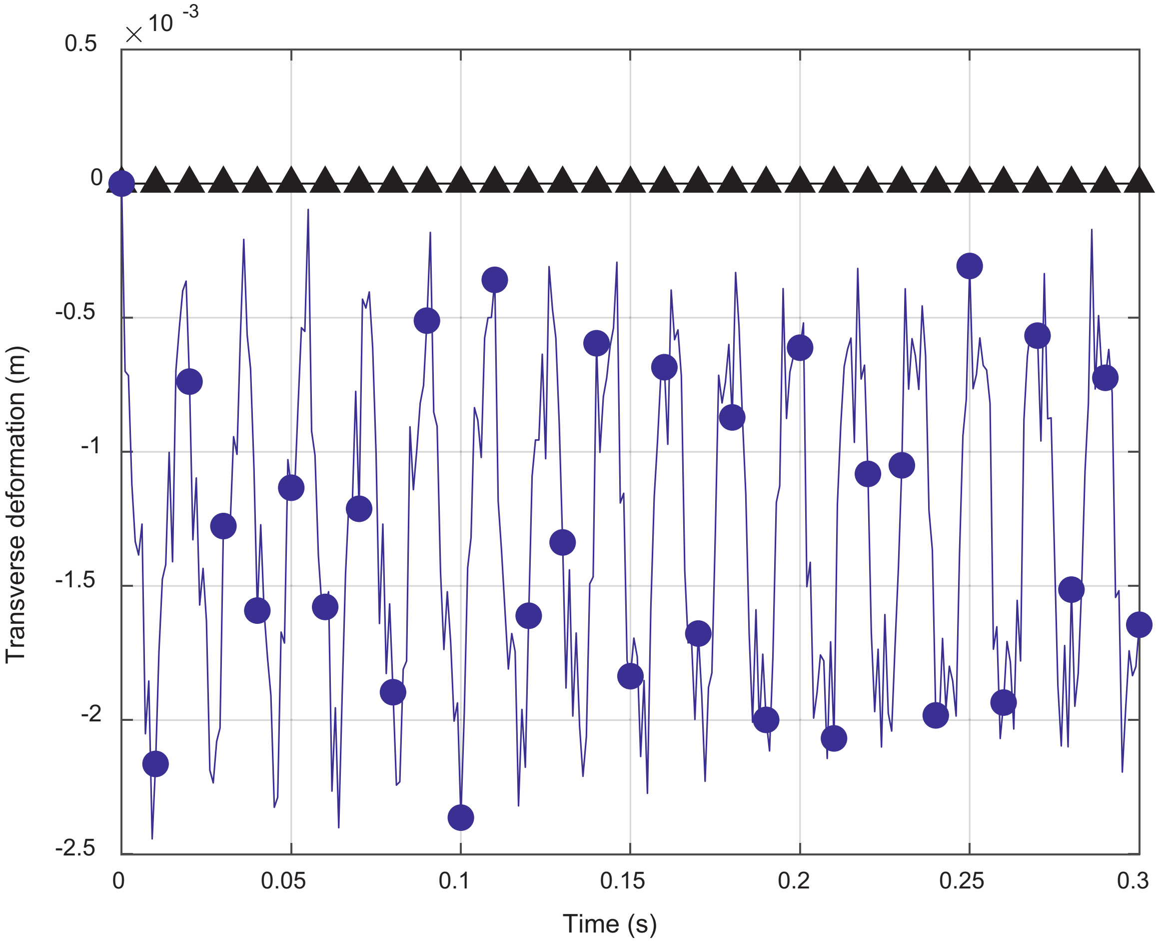

Fig. 2. End-effector forward-dynamics transverse deformation (—▲— With deformation actuation, —●— Without deformation actuation).

6. Summary and future investigations

This paper proposes a new hybrid actuation method for articulated flexible robot systems (AFRS) that can be used to achieve a near-elimination of the AFRS small oscillations. Motion and shape constraints, whose number equal to the number of the system degrees of freedom, are enforced. This leads to a sparse system of algebraic equations that can be efficiently solved for the driving constraint forces. These driving constraint forces can be used to define the articulated joint actuations as well as deformation actuations required to achieve the near-elimination of the AFRS small oscillations. The preliminary results obtained in this paper demonstrate the high precision achieved by considering the hybrid actuation method which controls the flexible link deformations.

Important issues that will be discussed in future investigations are the control spill-over, the number and locations of deformation-piezoelectric actuators, the power consumption required for the near-elimination of the small oscillations, the response of the system to disturbance and the need for feedback control system use of under-actuation [10], and the precision that can be achieved in comparison with what was achieved by the COMET project.

Credit author statement

The first author, Ahmed Shabana, proposed the approach and prepared the first draft of the paper. The second and third authors, Ahmed Eldeeb and Zhengfeng Bai, respectively, have been involved in this research, contributed to further developing the approach and definition of the actuation forces, and developed the two-link robot model presented in the paper.

Declaration of Competing Interest

There are no interests to declare.

Acknowledgments

This research was supported, in part, by the National Science Foundation (Projects # 1852510).

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