OPTIMAL COIL DESIGN FOR WIRELESS POWERING OF BIOMEDICAL IMPLANTS CONSIDERING SAFETY CONSTRAINTS

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Abstract— **This paper presents an investigation on the design of a transmit coil for a wireless power transfer system. The transmitter is optimized subject to an imposed safety limit for magnetic field exposure. Three cases are considered as follows: unconstrained, geometrically constrained and current constrained. Equations for determining design parameters for an optimal solenoid wireless power transmitter are found given either system size or current constraints. If a certain magnetic field strength is required, equations for the size and current of the transmitter are found that will allow the necessary fields without violating the safety constraint.**

Keywords—Wireless Power Transfer, Transmit Antenna Design, Optimization, Biomedical Implants

I. INTRODUCTION

Wireless power transfer offers an attractive way to power implantable medical devices thus reducing or eliminating the requirement for large energy storage units such as batteries or super capacitors. However, safety standards limit the magnetic fields to which humans may be subjected. For example, the IEEE standard [1] limits exposure to below 200 µT in the frequency range 3 kHz to 100 kHz. Although previous work has investigated the optimal coil design to maximize a magnetic field [2], [3] at a distance, or optimizing the power and thermal efficiency of solenoid electromagnets [4-6], these works did not examine how the optimal design changes when a safety limit is introduced. This paper presents an analysis of the optimal antenna design for a wireless power transfer system that seeks to maximize the magnetic field at some distance while staying below a prescribed magnetic field safety limit. It is essential to maximize the magnetic field since the power achieved at the receiver is proportional to the squared magnetic field strength. For low-power systems such as medical implantable devices, the central objective is to maximize the delivered power rather than to optimize the transmission efficiency. In the case of utilizing a small magnetoelectric (ME) transducer as a receiver, the coupling between the transmitter and receiver is relatively low. Therefore, the reflected impedance of the ME generator on the transmitting coil is neglected.

Models of the magnetic field produced by a thick solenoid coil were created [7]. An optimization problem was constructed to maximize the B-field at a chosen distance within a person, called Z_{implant}, while keeping magnetic exposure under the safety limit of 200 µT at all distance within the person [1]. Solutions to this optimization problem are heavily dependent on the imposed constraints. This paper examines three different scenarios: (i) unconstrained, (ii) geometry constrained, (iii) and applied current constrained.

II. TRANSMIT COIL OPTIMIZATION

The magnetic field, B, at some distance in the human body, z, is a function of the coil's current density, J, thickness, t, inner and outer radii, r_1 and r_2 respectively, and the air gap between the coil and the human body, y, as outlined in Figure 1. The optimization problem can be stated as

$$\begin{aligned} &\max B(r1, r2, t, y, z, J) \; at \; z = Z_{implant} \\ &s.t. \quad B(r1, r2, t, y, z, J) < B_{safe} \; \forall \; z \in \left[0, Z_{implant}\right]. \end{aligned} \tag{1}$$

where Z_{implant} is the depth of the implanted medical device. The parameters of the optimization problem can be split into three main groups: (a) the geometry of the coil, r_1, r_2, and t, (b) the current density, J, and (c) the air gap, y.

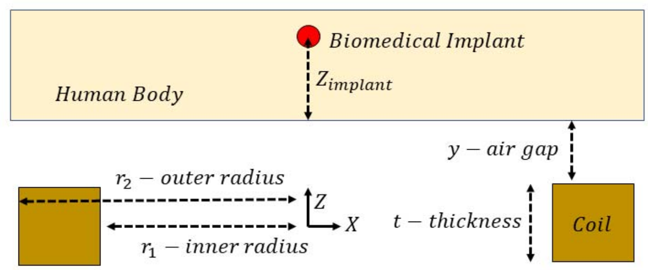

Figure 1 Cross-sectional view of the solenoid coil with the optimization parameters. The biomedical implant is located at a distance of Z_{crit} into the human body.

Solving this optimization requires using the full magnetic field model produced by a thick coil [7]. For any point in space, this can only be solved through numerical integration of elliptical integrals. Due to this nature, the problem was then solved numerically using gradient-based optimization algorithms [8]. However, during the optimization, it became apparent that the ideal thick coil or solenoid was one with its radius much greater than its thickness; and the optimal positioning of the coil is always with the biomedical implant directly along its centerline. Therefore, the optimal design of a thick coil closely resembles that of a simple current loop [9], and the governing equation of the magnetic field can be simplified as

$$B = \frac{\mu}{4\pi} \frac{2\pi R^2 I}{(z^2 + R^2)^{\frac{3}{2}}} \tag{2}$$

where B is the B-field produced along the centerline some distance z away by a current loop with a radius R, and current I, operating in some space with magnetic permeability μ.

A. Unconstrained

When both the current and the coil size are unconstrained, the optimum solution is to produce a nearly-uniform B-field (below the safety limit) at the receiver, which can be done by utilizing a large coil with high current. The air gap between the transmitter and the body, in this case, is much larger than the critical distance, $y \gg Z_{\text{implant}}$. An advantage of this method is that the uniform B-field can power multiple receivers even when their positions are unknown. However, the large coils and large currents required in this case are most likely impractical to implement for real-world application.

B. Geometry Constrained

If constraints are applied to the coil's geometry and air gap, the uniform magnetic field is no longer achievable. The optimization results show that the optimal coil radius should be at its limit. The current in this optimal design is equal to

$$I = 2\frac{B_{safe} * R}{\mu}, \tag{3}$$

where R is the optimal radius, B_{safe} is the safety limit in T, and μ is the magnetic permeability of the material. For air and water, which has similar dielectric properties to human fat tissue, $\mu = \mu_0$. Equation (3) results from solving the current loop equation for with the depth, z, equal to zero. In other words, given the radius of the optimal coil, solving for the current that would make the magnetic field at the skin equal to just under the safety limit.

With the coil radius equal to the limit and operating at the optimal current, the maximum achievable B-field can be reduced to a function of the size constraint. Dividing the B-field produced by an optimally designed coil for a depth of $Z_{implant}$ by the B-field produced by the same optimal coil at the skin (i.e., which is equal to the safety limit), yields

$$B_{limit} = \frac{R^3}{(z^2 + R^2)^{\frac{3}{2}}}. \tag{4}$$

where B_{limit} is the fraction (i.e., 0.75, 0.86, etc.) of the magnetic field safety limit that can be created with the optimal coil. Furthermore, if the radius of the coil is defined by a size constraint multiplied by the depth of the medical implant,

$$R = a * Z_{implant} \tag{5}$$

where a is the size constraint (i.e., 2x the implant depth, 4x the implant depth, etc.), then Eq. (4) can be further reduced to

$$B_{limit} = \frac{a^3}{(1 + a^2)^{\frac{3}{2}}}. \tag{6}$$

This relation defines the maximum achievable magnetic field for any implant depth which is solely a function of the size constraint. If a certain field strength is required at the biomedical implant located a distance $Z_{implant}$ inside the body, Eq. (6) can be used to determine the necessary size constraint. Figure 2 is a plot of Eq. (6) which shows the maximal achievable B field for size constraints up to 10x.

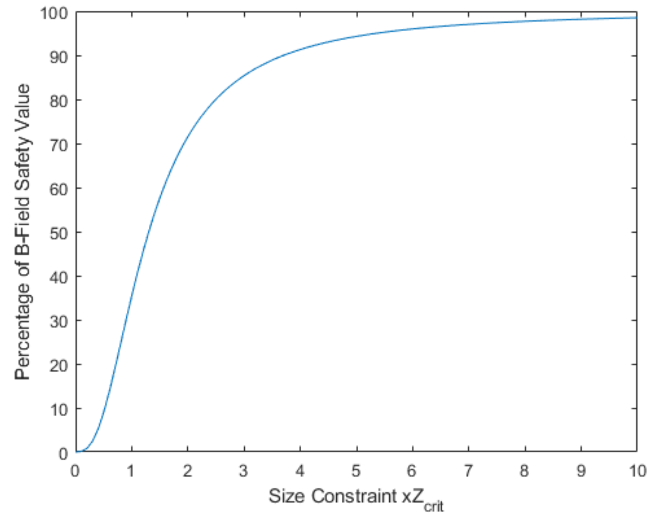

Figure 2 Size constraint effects on maximum achievable B-field. The larger the allowable size of the coil, the greater the achievable B-field at $Z_{implant}$.

In addition, the optimization results reveal that the air gap, *y*, should always be as close to zero as possible, or that it is not beneficial to achieve a stronger magnetic field by using a transmit coil with a higher current located some distance away from the skin. Figure 3 shows the B-field of 3 different optimally designed transmitters with three different specified radii, as well as a graph of the slope of the B-field, which can be found by taking the derivative of Eq. (2) with respect to the depth, *z,* yielding

$$\frac{dB}{dz} = -\frac{\mu}{4\pi}\frac{6\pi R^2 I * z}{(z^2 + R^2)^{\frac{5}{2}}}. \tag{7}$$

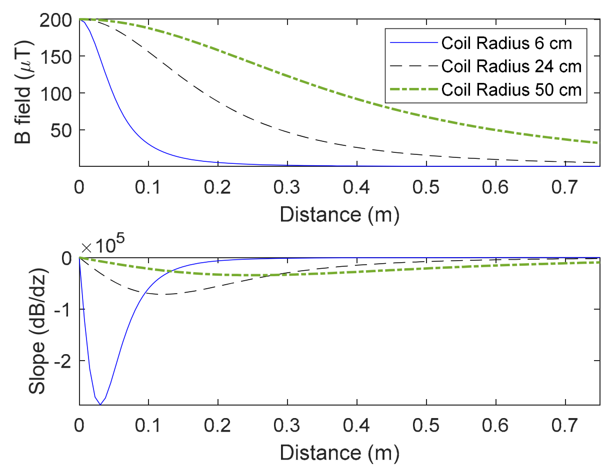

Figure 3 B-field vs. distance and $\frac{dB}{dz}$ *vs. distance for three optimally design transmitters with different radii. The shape of the B-field and its derivative is similar in all three cases. The slope of the B-field is always negative, and always initially becomes more negative.*

While the shape of the B-field produced by these three coils varies, the slope of the B-field is always negative, but flattens out and becomes near zero for distances where $z >> R$. The magnitude of this decrease depends on the optimal coil design. Furthermore, it is never beneficial to increase the air gap between the transmitter and the skin in order to operate at a higher current. Trying to operate in the region where the slope of the coils is near zero, where $z >> R$, could produce a nearly uniform magnetic field which is ideal but represents an unrealistic scenario because the required currents and power would be enormous. As seen in Fig. 3, the actual B-field at these distances approaches zero when their respective slopes flatten out.

C. Current Constrained

Similar to the geometric constraint case, the optimal design for a current only constrained system always occurs when the current constraint is active. The radius of the corresponding optimal coil design becomes dependent on the current constraint, and the depth of the biomedical implant. If the depth of the implant, $Z_{implant}$, becomes greater than a critical distance, Z_{crit},

$$Z_{implant} \geq Z_{crit}, \quad (8)$$

where Z_{crit} is defined as

$$Z_{crit} = \frac{\frac{1}{2}\mu I}{B_{safe}\sqrt{2}}, \quad (9)$$

I is the current limit, μ is the magnetic permeability, and B_{safe} is the safety limit of the B-field. The optimal radius of the transmitter is

$$R_{opt} = \sqrt{2} Z_{implant}. \quad (10)$$

If the inequality in Eq. (8) does not hold, meaning $Z_{implant}$ is at a shallower depth than Z_{crit}, then the optimal radius of the transmit coil is

$$R_{opt} = \frac{\mu I}{2 B_{safe}}. \quad (11)$$

The two different optimal radius conditions stem from the magnetic field safety limit being active or not at the skin. If the safety constraint is not active at the skin, by extension, it won't be active anywhere in the body due to the nature of the B-field produced by a single coil. If the safety constraint is not active, Eq. (1) can be solved for the optimal radius subject only to the given a current constraint which yields Eq. (10). This situation occurs when either the current constraint is so low that its generated B-field is below the safety limit, or the implant is deep enough that the optimal radius given by Eq. (9) becomes large enough that at the transmit coil's center the B-field safety limit is not violated. It is interesting to note that once the implant depth exceeds Z_{crit}, the optimal radius is not explicitly dependent on any parameter besides the depth of the biomedical implant. Note that, Z_{crit} itself is dependent on the current constraint and magnetic field safety limit.

If, however, the biomedical implant is located at a shallow depth, shallow being any depth less than Z_{crit} defined in Eq. (9), using the optimal coil design solution produced by Eq. (10) will produce a B field that would violate the safety limit at the skin. Equation (9) then represents the depth of an implant at which the optimal radius given in Eq. (10) for a transmitter operating at a given current constraint, would violate the given safety limit at the skin. For depths shallower than this distance, the optimal coil design is dependent upon solving the constrained optimization problem with the safety constraint being active. This solution for the optimal radius is given by Eq. (11). Contrasting with the previous case, Eq. (11) is not a function of $Z_{implant}$. This leads to a transmit antenna design with a single optimal radius for all implant depths up to Z_{crit}. Figure 4, which is a graph of optimal transmit coil radius for different current constraints as a function of $Z_{implant}$, illustrates this principle. For shallow depths, when the biomedical implant depth $Z_{implant}$ is less than Z_{crit}, the optimal coil radius is constant and dependent upon the safety and current constraints according to Eq. (9). When the implant depth exceeds Z_{crit}, the coil radius becomes solely a function of $Z_{implant}$regardless of the current constraint.

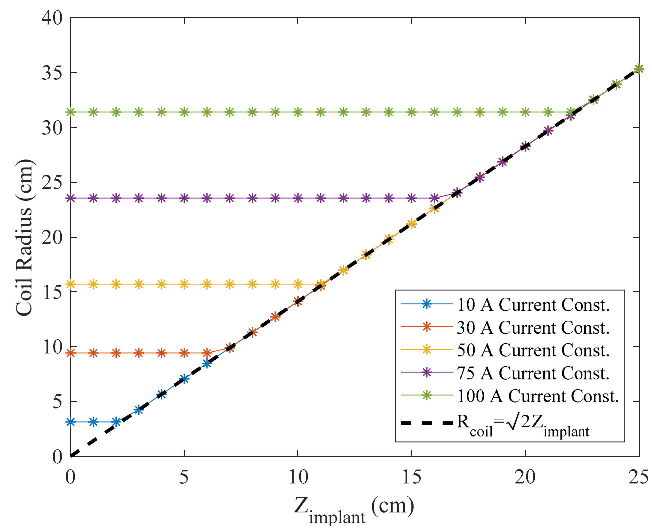

Figure 4 For a given current constraint the optimal radius is constant for $Z_{implant}$ less than Z_{crit}. If the implant depth is moved past Z_{crit}, then the optimal radius of the coil is equal to Eq. (10).

Figure 5 shows how the changes to the optimal transmit coil radius coil with a current constraint changes the B-field within the body depending on the depth of the implant. For example, using Eq. (9) with a current limit of 30 Amps and a safety constraint of 200 µT, yields a Z_{crit} equal to 0.066 m. For implants below this depth, the optimal radius is found to be 0.0942 m using Eq. (11) and the B field produced corresponds to the blue line in Fig 5. After the implant depth passes Z_{crit}, the optimal radius of the wireless power transmitter increases linearly according to Eq. (10). Each non-blue line in Fig 3, represents the B field generated by an optimal transmitter for a given implant depth $Z_{implant}$ greater than Z_{crit}. The red and greens lines on the graph representing the B field produced by an optimal transmitter for a $Z_{implant}$ past Z_{crit}. The distinctly red line represents a the B-field from a coil optimized for a $Z_{implant}$ just past Z_{crit} and while the bright green line represents the B-field from a transmitter optimized for $Z_{implant}$ of 25 cm. The other colored lines represent the B-field from coils optimized for $Z_{implant}$ between these two, with the lightening of the line indicating the B-field of a transmitter optimized for an increasing $Z_{implant}$ (i.e. the greener the line, the greater the depth of $Z_{implant}$ for which the coil was optimized for).

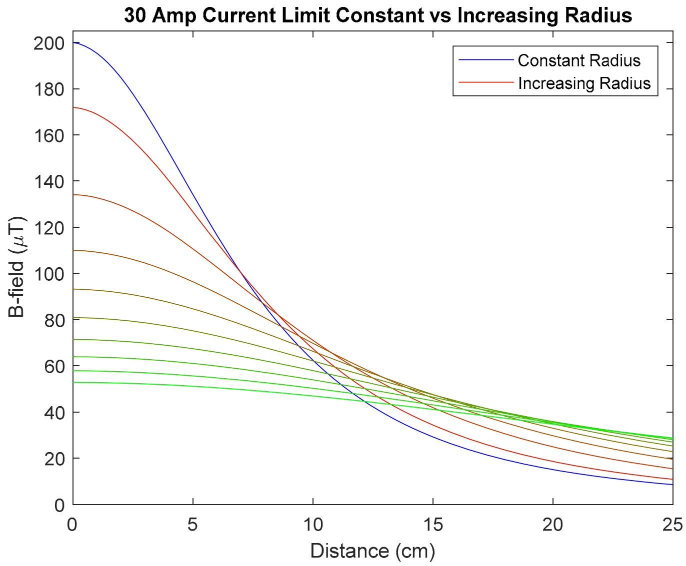

Figure 5 For shallow depths, the optimal radius is constant, and the optimal B field is represented by the blue line which is just under the safety limit at the skin. As $Z_{implant}$ becomes greater than Z_{crit} the radius of the transmit coil increases, and the B-field becomes flatter and flatter. The red line represents the B-field for a $Z_{implant}$ just past Z_{crit}. As $Z_{implant}$ increases past Z_{crit}, the corresponding B-field (represented by a lighter line for deeper depths) becomes flatter and flatter.

III. Generalized Design Considerations

The two previous sections dealt with determining the radius, placement, and current of an optimal transmitter given a set of either geometric or current constraints. Of course, there are many other factors that should be considered in designing a wireless power transmitter. Copper wire naturally has some resistance which produces heat when current is applied. The generally recommend current density to avoid overheating of copper wire is between 5 - 6 $\frac{A}{mm^2}$. Once the optimal radius and current of the transmitter are found, the thickness of the coil should be adjusted such that the current density does not exceed this limit. Additionally, the required current could be lowered by using smaller wire gages with an increasing number of terms.

Although the optimal air gap between the transmit coil and the skin should be zero, in reality, there will always be some small gap due to insulation, clothing, or other factors. This surface boundary between the air and the skin can cause some reflections of the electromagnetic wave produced by the transmitter. However, at low frequencies, 100 KHz and lower, the wavelengths are so large (3 kilometers for 100 KHz) compared with the small air gap that the reflections will be extremely small. If operating at slightly higher frequencies is required, the current in the transmit coil can be easily adjusted to account for the lost magnetic fields due to reflections since the magnetic field produced by the transmitter has a linear relationship to its current.

IV. Conclusion

The optimal design for a transmit coil to power biomedical implants while staying below a magnetic field safety limit depends heavily on the system constraints and the depth of the biomedical device. If size constraints are the only limiting factor in the system, the optimal transmit coil's radius will be at the size constraint and the corresponding current can be calculated to maximize the magnetic field at the biomedical implant. The strength of the maximum achievable magnetic field depends solely on the size constraint. The equations hold for the reverse situation as well, given a minimum needed magnetic field, the necessary coil size and current can be found. Additionally, it was found that the ideal solenoid wireless power transmitter resembles a current loop whose radius is much greater than its thickness. Also, there is no benefit to operating with any air gap between the transmitter and the human body due to the nature of the magnetic fields produced by a current loop.

The design rules for an optimal wireless power transmitter given a current constraint differs from the geometrically constrained case. Like the previous case, the maximum magnetic field occurs when the current constraint is active. The optimal radius of the transmitter is dependent upon the depth of the medical implant, and the magnetic field safety limit as well as the current constraint. A critical distance, Z_{crit}, was introduced and for any implant depths less than Z_{crit}, the optimal radius is constant and is a function of the current constraint and safety limit. For an implant depth greater than Z_{crit}, the optimal radius becomes a linear function of the implant depth.

The current and radius of the coil are the two decisive parameters that have the largest impacts on the system. The thickness of the coils should be adequate, so the current density does not exceed the safety limit for a given coil material. Reflections due to the boundary condition between air and the human body can be minimized by operating at low frequencies and accounted for by a linear increase of the current to the transmitter.

Acknowledgment

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