Constraining liberated supergravity

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(Received 24 October 2020; accepted 22 December 2020; published 12 January 2021)

Fermionic terms in a class of locally supersymmetric theories called "liberated supergravity" are nonrenormalizable interactions proportional to inverse powers of the supersymmetry breaking scale and Planck mass $M_{\rm pl}$. This property defines an intrinsic cutoff for liberated supergravities, which are therefore effective theories valid only below energies that never exceed the cutoff. Requiring that the cutoff exceeds current theoretical and observational bounds shows that the new scalar potential terms allowed by liberated supergravity can neither change the cosmological constant predicted by supergravity by any observable amount, nor give measurable contributions to particle masses. We show that it is nevertheless possible to define a simple liberated supergravity model of slow roll inflation valid up to energy scales that are well above the Hubble parameter during inflation and exceeds observable limits after inflation. The key to constructing a viable model is to change the supersymmetry breaking scale, from a Planck-scale value during inflation, to TeV-scale after inflation.

DOI: 10.1103/PhysRevD.103.025008

I. INTRODUCTION

Supergravity strongly constrains the form of the scalar potential [1], hence it makes the construction of specific models of elementary interactions or inflation challenging. The literature on supergravity inflationary potentials is vast and somewhat unnecessary for this paper, which is devoted instead to the study of a new model of supergravity, called "liberated supergravity" [2]. As its name suggests, the scalar potential of liberated supergravity need not be of the special form given in [1]; it is in fact a completely general function of the scalar fields.

As any other gravity theory, liberated supergravity contains nonrenormalizable interaction, so it is by construction an effective theory valid only up to a finite cutoff that cannot exceed the (reduced) Planck scale $\Lambda_{\rm cut} \lesssim 1/\sqrt{8\pi G} \equiv M_{\rm pl}$. Differently from models where supersymmetry is nonlinearly realized this scale can be parametrically larger than the supersymmetry (SUSY) breaking scale M_S. This is also different from the models described in [3–6], where the cutoff is also only required to be $\mathcal{O}(M_S)$. In liberated supergravity, instead, the cutoff can be parametrically larger than M_S, because the latter is defined through the F- and/or D-term expectation values while the former depends on both the new terms present in the "liberated" scalar potential $\mathcal{U}$ and the expectation values of the auxiliary fields.

For a given value of $\Lambda_{\rm cut}$ the corrections to the scalar potential due to the new terms allowed by liberated supergravity are severely constrained by our Eq. (30), which refine and make quantitative Eqs. (3.30)–(3.31) of Ref. [2]. These constraints are the first main result of our paper, because they show that even in the least restrictive case $\Lambda_{\rm cut} \sim M_S$ the liberated corrections to the vacuum energy are completely negligible. The same is true for liberated corrections to particle masses.

Another question worth asking is whether liberated supergravity can nevertheless produce a slow-roll inflationary potential with a cutoff well above the Hubble scale (the latter condition is necessary for the self-consistency of an effective theory of inflation). We answer in the affirmative by exhibiting a toy model with a simple, explicit potential that can satisfy all constraints. The key feature that makes it possible to describe Hubble scale physics in our model is a change occurring in the supersymmetry breaking potential from one that gives a fixed high value $M_S \gg H$ during slow roll to a no-scale potential after the end of inflation.

Section II succinctly describes the construction of liberated supergravity in the superconformal tensor calculus formalism. Section III describes the construction of its fermionic terms and derives the key inequalities and constraints that any such theory must obey. Section IV presents a toy model of slow-roll inflation in liberated supergravity.

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II. LIBERATED $\mathcal{N}=1$ SUPERGRAVITY IN SUPERCONFORMAL TENSOR CALCULUS

In this section, we summarize the construction of liberated $\mathcal{N}=1$ supergravity [2] using superconformal tensor calculus [7,8] that will be discussed in more details in [9]. Liberated supergravity was described in [2] using the superspace formalism, where a Kähler transformation is introduced to remove the variation of the action under a super-Weyl rescaling, while a super-Weyl-Kähler transformation is promoted to an Abelian gauge symmetry to produce the liberated supergravity. In the superconformal formalism one instead introduces a conformal compensator multiplet, called S_0, which eliminates the variation while keeping the Kähler potential invariant under superconformal symmetry. Hence, differently from the superspace case, we need to define such a gauge transformation independently of superconformal symmetry.

To do this, it is useful to write the invariant actions in the two different formalisms [10]:

$$[\mathcal{V}]_D = 2\int d^4\theta E\mathcal{V}, \qquad [\mathcal{S}]_F = \int d^2\theta\mathcal{E}\mathcal{S} + \int d^2\bar{\theta}\,\bar{\mathcal{E}}\,\bar{\mathcal{S}}, \tag{1}$$

where $\mathcal{V}$ is a composite superconformal real multiplet with the Weyl/chiral weights (2, 0), $\mathcal{S}$ is a composite superconformal chiral multiplet with Weyl/chiral weights (3, 3), while E and $\mathcal{E}$ are the corresponding D/F-term measure densities [7]. The action must be invariant under a super-Weyl-Kähler transformation that shifts the superspace densities as $E\to Ee^{2\Sigma+2\bar{\Sigma}}$ and $\mathcal{E}\to\mathcal{E}e^{6\Sigma}$. This requires that the corresponding superconformal multiplets transform as

$$\mathcal{V}\to\mathcal{V}e^{-2\Sigma-2\bar{\Sigma}}, \qquad \mathcal{S}\to\mathcal{S}e^{-6\Sigma}. \tag{2}$$

To describe the liberated supergravity in the superconformal formalism, we therefore promote the super-Weyl-Kähler transformation to a gauge symmetry under which the compensator is inert, so that the transformation rules are given by

$$\begin{aligned} &K\to K+6\Sigma+6\bar{\Sigma}, \qquad W\to We^{-6\Sigma}, \qquad \bar{W}\to\bar{W}e^{-6\bar{\Sigma}},\\ &T\to e^{-4\Sigma+2\bar{\Sigma}}T, \qquad \bar{T}\to e^{2\Sigma-4\bar{\Sigma}}\bar{T}, \qquad S_0\to S_0,\\ &\mathcal{W}_\alpha\to e^{-3\Sigma}\mathcal{W}_\alpha, \qquad T(\bar{\mathcal{W}}^2)\to T(\bar{\mathcal{W}}^2)e^{-4\Sigma-4\bar{\Sigma}}, \end{aligned} \tag{3}$$

where Σ is a chiral superfield, W is the superpotential, $\mathcal{W}_\alpha$ is the field strength of a real vector multiplet, and T is the chiral projection. With these rules, a Lagrangian of the liberated $\mathcal{N}=1$ supergravity equivalent to that of Ref. [2] is

$$\mathcal{L}_{\rm NEW}\equiv\left[\mathcal{Y}^{-2}\frac{\mathcal{W}^2(K)\bar{\mathcal{W}}^2(K)}{T(\bar{w}^2(K))\bar{T}(w^2(K))}\mathcal{U}(Z^I,\bar{Z}^{\bar{I}})\right]_D. \tag{4}$$

In Eq (4), we have introduced the notations[1] $\mathcal{Y}\equiv(S_0\bar{S}_0e^{-K(Z^I,\bar{Z}^{\bar{I}})/3})$, $w^2(K)\equiv\mathcal{W}^2(K)/\mathcal{Y}^2$, $\bar{w}^2(K)\equiv\bar{\mathcal{W}}^2(K)/\mathcal{Y}^2$; we denoted with $\mathcal{U}(Z^I,\bar{Z}^{\bar{I}})$ a generic function of the matter chiral multiplets Z^I's and with $K(Z^I,\bar{Z}^{\bar{I}})$ the supergravity Kähler potential. We also call $\mathcal{W}_\alpha(K)$ the field strength multiplet corresponding to the Kähler potential. By denoting with (w,c) the Weyl/chiral weights of a multiplet we have the following assignment: (1, 1) for S_0, (2, 0) for $\mathcal{Y}$, $(3/2,3/2)$ for $\mathcal{W}_\alpha(K)$, $(-1,3)$ for $w^2(K)$, and (0, 0) for $Z, K(Z^I,\bar{Z}^{\bar{I}}), T(\bar{w}^2(K)), \bar{T}(w^2(K)), \mathcal{U}(Z^I,\bar{Z}^{\bar{I}})$. By assuming that $\mathcal{U}(z^I,\bar{z}^{\bar{I}})$ is inert under the gauge symmetry and using $w^2\to w^2e^{-2\Sigma+4\bar{\Sigma}}$, $\bar{T}(w^2)\to\bar{T}(w^2)$, we see that the whole Lagrangian is invariant under the gauge symmetry, so that it reproduces liberated supergravity.

Next we find the bosonic contribution to the scalar potential. We define a composite superconformal multiplet $\mathbb{N}$ made of the superconformal chiral multiplets $Z^i\equiv(z^i,P_L\chi^i,F^i)$ $(i=0,I,W,T)$, which can be $(z^0\equiv s_0, P_L\chi^0,F^0)$, (z^I,P_L,χ^I), $(z^W\equiv W,P_L\chi^W,F^W)$,[2] $(z^T\equiv T(\bar{w}^2), P_L\chi^T,F^T)$. The lowest component N of $\mathbb{N}$, is given by

$$N\equiv(s_0s_0^*e^{-K(z^I,\bar{z}^{\bar{I}})/3})^{-2}\frac{W\bar{W}}{T(\bar{w}^2)\bar{T}(w^2)}\mathcal{U}(z^I,\bar{z}^{\bar{I}}), \tag{5}$$

and we get the component Lagrangian by using the D-term formula. It reads as follows:

$$\begin{aligned}\mathcal{L}_{\rm NEW}\equiv[\mathbb{N}]_De^{-1}=&N_{i\bar{j}}\left(-\mathcal{D}_\mu z^i\mathcal{D}^\mu\bar{z}^{\bar{j}}-\frac{1}{2}\bar{\chi}^i\not{D}\chi^{\bar{j}}-\frac{1}{2}\bar{\chi}^{\bar{j}}\not{D}\chi^i+F^i\bar{F}^{\bar{j}}\right)+\frac{1}{2}[N_{ij\bar{k}}(-\bar{\chi}^i\chi^j\bar{F}^{\bar{k}}+\bar{\chi}^i(\not{D}z^j)\chi^{\bar{k}})+\text{H.c.}]\\&+\frac{1}{4}N_{ij\bar{k}\bar{l}}\bar{\chi}^i\chi^j\bar{\chi}^{\bar{k}}\chi^{\bar{l}}+\left[\frac{1}{2\sqrt{2}}\bar{\psi}\cdot\gamma\left(N_{i\bar{j}}F^i\chi^{\bar{j}}-N_{i\bar{j}}\not{D}\bar{z}^{\bar{j}}\chi^i-\frac{1}{2}N_{ij\bar{k}}\chi^{\bar{k}}\bar{\chi}^i\chi^j\right)+\frac{1}{8}i\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\gamma_\nu\psi_\rho\left(N_i\mathcal{D}_\sigma z^i+\frac{1}{2}N_{i\bar{j}}\bar{\chi}^i\gamma_\sigma\chi^{\bar{j}}\right.\right.\\&\left.\left.+\frac{1}{\sqrt{2}}N_i\bar{\psi}_\sigma\chi^i\right)+\text{H.c.}\right]+\frac{1}{6}N\left(-R(\omega)+\frac{1}{2}\bar{\psi}_\mu\gamma^{\mu\nu\rho}R'_{\nu\rho}(Q)\right)-\frac{1}{6\sqrt{2}}(N_i\bar{\chi}^i+N_{\bar{i}}\bar{\chi}^{\bar{i}})\gamma^{\mu\nu}R'_{\mu\nu}(Q),\end{aligned} \tag{6}$$

where $N_{i\bar{j}}$, $N_{ij\bar{k}}$, and $N_{ij\bar{k}\bar{l}}$ are the derivatives with respect to $z^i, z^{\bar{i}}$ for $i,j=0,I,W,T$: $N_i\equiv\partial N/\partial z^i$ etc.. The gravitino is denoted by ψ and other details will be given in [7]. As for the detailed structure of the fermions $\chi^i\equiv P_L\chi^i$, we find [9]

[1] In superconformally invariant theories there are no dimensionless parameters and the Planck scale is introduced by the super-Weyl gauge choice $\Upsilon\equiv s_0s_0^*e^{-K/3}=M_{\rm pl}^2$, where Υ is by definition the lowest component of $\mathcal{Y}$.

[2] Here, $W(K)$ denotes the lowest component of the field strength multiplet $\mathcal{W}^2(K)$. It is *not* the usual superpotential W.

$$P_L\chi^i = \begin{cases} P_L\chi^0, \\ P_L\chi^I, \\ P_L\chi^W = 4\tilde{\mathcal{F}}K_{\bar{\imath}J}[(\partial\!\!\!/ z^J)P_R\chi^{\bar{\imath}} - \bar{F}^{\bar{\imath}}P_L\chi^J] + \cdots + 7\,\text{fermion terms}, \\ P_L\chi^T = \frac{1}{\Upsilon^2}\Big[4(\partial\!\!\!/\tilde{\mathcal{F}})K_{\bar{\imath}J}[(\partial\!\!\!/ z^J)P_R\chi^{\bar{\imath}} - \bar{F}^{\bar{\imath}}P_L\chi^J] - \Big(\frac{2}{s_0^*}\partial\!\!\!/ s_0^* - \frac{2}{3}K_{\bar{K}}\partial\!\!\!/\bar{z}^{\bar{K}} - 2\gamma^\mu(b_\mu + iA_\mu)\Big) \\ \times 4\tilde{\mathcal{F}}K_{\bar{\imath}J}[(\partial\!\!\!/ z^J)P_R\chi^{\bar{\imath}} - \bar{F}^{\bar{\imath}}P_L\chi^J]\Big] + \cdots + 9\,\text{fermion terms}, \end{cases} \tag{7}$$

where $\tilde{\mathcal{F}} = 2K_{I\bar{J}}(-\partial_\mu z^I\partial^\mu\bar{z}^{\bar{J}} + F^I\bar{F}^{\bar{J}})$ and

$$\partial\!\!\!/\tilde{\mathcal{F}} = 2(\partial\!\!\!/ K_{I\bar{J}})(-\partial_\mu z^I\partial^\mu\bar{z}^{\bar{J}} + F^I\bar{F}^{\bar{J}}) + 2K_{I\bar{J}}(-(\partial\!\!\!/\partial_\mu z^I)\partial^\mu\bar{z}^{\bar{J}} - \partial_\mu z^I(\partial\!\!\!/\partial^\mu\bar{z}^{\bar{J}}) + (\partial\!\!\!/ F^I)\bar{F}^{\bar{J}} + F^I(\partial\!\!\!/\bar{F}^{\bar{J}})). \tag{8}$$

Notice that χ^i contains not only the fundamental fermions χ^0 and χ^I but also two composite chiral fermions χ^W and χ^T. A straightforward use[3] of the superconformal tensor calculus then gives the scalar potential in the form

$$V_{\text{NEW}} = (s_0 s_0^* e^{-K/3})^2\mathcal{U}(z^I, \bar{z}^{\bar{\imath}}). \tag{9}$$

The super-Weyl gauge choice $\Upsilon = s_0 s_0^* e^{-K/3} = M_{\text{pl}}^2 \equiv 1$ puts the action in the Einstein frame and reproduces the scalar potential of Ref. [2]

$$V_{\text{NEW}} = \mathcal{U}(z^I, \bar{z}^{\bar{\imath}}). \tag{10}$$

This then implies that the total scalar potential generically is

$$V = V_D + V_F + V_{\text{NEW}}, \tag{11}$$

where V_D, V_F are the usual D/F-term potentials. The additional contribution to the scalar potential, V_{NEW}, is an arbitrary function of the z^Is; since it does not obey any constraint it fully justifies the name "liberated" for the new class of supergravities introduced in [2].

III. FERMIONIC TERMS IN LIBERATED $\mathcal{N}=1$ SUPERGRAVITY

In this section, we investigate the fermionic terms in liberated $\mathcal{N}=1$ supergravity in the superconformal formalism.[4] First of all, focusing only on matter couplings, i.e., looking at terms independent of ψ, we read the following terms from Eq. (6):

$$\mathcal{L}_{F1} \equiv -N_{i\bar{\jmath}}\mathcal{D}_\mu z^i\mathcal{D}^\mu\bar{z}^{\bar{\jmath}}|_{\psi=0}, \qquad \mathcal{L}_{F2} \equiv -\frac{1}{2}N_{i\bar{\jmath}}\bar{\chi}^i\not{\mathcal{D}}\chi^{\bar{\jmath}}|_{\psi=0},$$
$$\mathcal{L}_{F3} \equiv -N_{i\bar{\jmath}}F^i\bar{F}^{\bar{\jmath}}|_{\psi=0}, \qquad \mathcal{L}_{F4} \equiv -\frac{1}{2}N_{ij\bar{k}}\bar{\chi}^i\chi^j\bar{F}^{\bar{k}}|_{\psi=0},$$
$$\mathcal{L}_{F5} \equiv \frac{1}{2}N_{ij\bar{k}}\bar{\chi}^i(\not{\mathcal{D}}z^j)\chi^{\bar{k}}|_{\psi=0}, \qquad \mathcal{L}_{F6} \equiv \frac{1}{4}N_{ij\bar{k}\bar{l}}\bar{\chi}^i\chi^j\bar{\chi}^{\bar{k}}\chi^{\bar{l}}|_{\psi=0},$$
$$\mathcal{L}_{F7} \equiv -\frac{N}{6}R(\omega)|_{\psi=0}. \tag{12}$$

Here, we observe that the fermionic terms in the effective Lagrangian contain couplings to the function $\mathcal{U}$ and its derivatives since $N \propto \mathcal{U}$.

The general structure of the fermionic terms can be found as a power series in derivatives of the composite multiplet N (i.e., $N_i, N_{i\bar{\jmath}}, N_{ij\bar{k}}$, and $N_{ij\bar{k}\bar{l}}$). The rth derivative of N, denoted with $N^{(r)}_{i\ldots\bar{l}}$ has the following generic form:

$$N^{(r)}_{i\ldots\bar{l}} = N^{(r=q+p+m+k)}_{q,p,m,k} = (\partial_0^q\partial_I^{(k-n)}\Upsilon^{-2})\Upsilon^{4+2m}\frac{\mathcal{U}^{(n)}}{\tilde{\mathcal{F}}^{4+2m}}W^{1-p_1}\bar{W}^{1-p_2}$$
$$= ((-1)^{q_1+q_2}(q_1+1)!(q_2+1)!s_0^{-(2+q_1)}s_0^{*-(2+q_2)}(\partial_I^{(k-n)}e^{2K/3}))\Upsilon^{4+2m}\frac{\mathcal{U}^{(n)}}{\tilde{\mathcal{F}}^{4+2m}}W^{1-p_1}\bar{W}^{1-p_2}, \tag{13}$$

where

$$W = -2K_{\bar{\imath}J}[\bar{\chi}^J(\overline{\not{\mathcal{D}}\bar{z}^{\bar{\imath}}}) - \bar{F}^{\bar{\imath}}\bar{\chi}^J]K_{\bar{\imath}'J'}[(\not{\mathcal{D}}\bar{z}^{\bar{\imath}'})\chi^{J'} - F^{J'}\chi^{\bar{\imath}'}] - K_{\bar{\imath}J}[\bar{\chi}^J(\overline{\not{\mathcal{D}}\bar{z}^{\bar{\imath}}}) - \bar{F}^{\bar{\imath}}\bar{\chi}^J]K_{\bar{\imath}'\bar{\jmath}'K'}[\chi^{K'}\bar{\chi}^{\bar{\imath}'}\chi^{\bar{\jmath}'}]$$
$$- K_{\bar{\imath}\bar{\jmath}K}[\bar{\chi}^{\bar{\jmath}}\chi^{\bar{\imath}}\bar{\chi}^K]K_{\bar{\imath}'J'}[(\not{\mathcal{D}}\bar{z}^{\bar{\imath}'})\chi^{J'} - F^{J'}\chi^{\bar{\imath}'}] - \frac{1}{2}K_{\bar{\imath}\bar{\jmath}K}[\bar{\chi}^{\bar{\jmath}}\chi^{\bar{\imath}}\bar{\chi}^K]K_{\bar{\imath}'\bar{\jmath}'K'}[\chi^{K'}\bar{\chi}^{\bar{\imath}'}\chi^{\bar{\jmath}'}], \tag{14}$$

[3] $\mathcal{L}_B \supset N_{i\bar{\jmath}}F^i\bar{F}^{\bar{\jmath}} \sim N_{W\bar{W}}F^W\bar{F}^{\bar{W}} = \Upsilon^{-2}\frac{1}{\mathcal{C}_T\mathcal{C}_{\bar{T}}}\mathcal{U}F^W\bar{F}^{\bar{W}} = \Upsilon^{-2}\frac{\Upsilon^2\Upsilon^2}{\bar{F}^{\bar{W}}F^W}\mathcal{U}F^W\bar{F}^{\bar{W}} = \Upsilon^2\mathcal{U} \equiv V_{\text{NEW}}$ where we have used $\mathcal{C}_T = -\frac{1}{2}\mathcal{K}_{\bar{w}} \sim -\frac{1}{2}\frac{\mathcal{K}_{\bar{W}}}{\Upsilon^2} = -\frac{1}{2}\frac{(-2\bar{F}^{\bar{W}})}{\Upsilon^2} = \frac{\bar{F}^{\bar{W}}}{\Upsilon^2}$ and $\mathcal{C}_{\bar{T}} = \frac{F^W}{\Upsilon^2}$, and $\mathcal{C}_T$ is the lowest component of the superconformal multiplet of T.

[4] The detailed derivation will be done in [9].

$$\bar{W} = (W)^*. \tag{15}$$

$\tilde{\mathcal{F}} \equiv 2K_{I\bar{J}}(-\partial_\mu z^I \partial^\mu \bar{z}^{\bar{J}} + F^I \bar{F}^{\bar{J}})$; $\mathcal{U}^{(n)}$ $(0 \le n \le 4)$ is the nth derivative of the function $\mathcal{U}(z^I, \bar{z}^{\bar{I}})$ with respect to $z^I, \bar{z}^I$, which are the lowest components of the matter chiral multiplets; $q = q_1 + q_2$ where q_1 (q_2) is the order of the derivative with respect to the compensator scalar s_0 (s_0^*); $p = p_1 + p_2$ where p_1 (p_2) is the order of the derivative with respect to the field strength multiplet scalar W $(\bar{W})$; $m = m_1 + m_2$ where m_1 (m_2) is the order of the derivative with respect to the chiral projection multiplet scalar $T(\bar{w}^2)$ $(\bar{T}(w^2))$; k is the order of the derivative with respect to the matter multiplet scalar z^I; n is the order of the derivative acting on the new term $\mathcal{U}$ with respect to the matter multiplet; and q is the total order of derivatives with respect to the compensator scalars s_0 and s_0^*.

To find restrictions on V_{NEW} coming from fermionic terms, we have to identify the most singular terms in the Lagrangian. These terms can be found using the fact that powers of $\tilde{\mathcal{F}}$ in the denominator may lead to a singularity which gets stronger when m increases by taking more derivatives with respect to the lowest component of the multiplet $T(\bar{w}^2)$ as seen from Eq. (13). Hence, we will investigate the fermionic terms containing only derivatives with respect to the chiral projection and matter scalar indices, i.e., T and I, in order to find the terms coupled to $\mathcal{U}^{(n)}$ that contain the maximal inverse powers of $\tilde{\mathcal{F}}$. They are those with $q = p = 0$ and $k = n$. We note in particular that if our theory has a single chiral matter multiplet then the most singular terms are found to be the couplings to the derivatives proportional to $N_{T\bar{T}}$, $N_{WT\bar{T}}$, $N_{W\bar{W}T\bar{T}}$ while for two or more chiral matter multiplets they are $N_{TT\bar{T}\bar{T}}$. The latter terms vanish identically for a single multiplet because of Fermi statistics.

First of all, let us examine the single matter chiral multiplet case. Due to Fermi statistics, the possible fermionic terms are proportional only to three tems, $\mathcal{U}^{(0)}, \mathcal{U}^{(1)}$, and $\mathcal{U}^{(2)}$, so that the maximal order of the derivative with respect to the chiral projection that can appear in the Lagrangians scalar is two and appears in the terms proportional to $N_{T\bar{T}}$, $N_{WT\bar{T}}$, and $N_{W\bar{W}T\bar{T}}$. To show that such terms do not vanish consider

$$\mathcal{L}_{F2}|_{q=p=0,k=n} \supset \Upsilon^{2+2m} \frac{\mathcal{U}^{(n)}}{\tilde{\mathcal{F}}^{4+2m}} W\bar{W}\bigg[-\frac{1}{2}((\bar{\chi}^I)^{n_1}(\Upsilon^{-2}4(\not{\partial}\tilde{\mathcal{F}})K_{\bar{\imath}J}(\not{\partial}z^J)\bar{\chi}^{\bar{\imath}}P_R)^{m_1}) \times ((\not{D}\chi^I)^{n_2}(\not{D}(\Upsilon^{-2}4(\not{\partial}\tilde{\mathcal{F}})K_{\bar{\imath}J}(\not{\partial}z^J)P_R\chi^{\bar{\imath}}))^{m_2})\bigg]_{\psi=0}, \tag{16}$$

where $m = m_1 + m_2$, $n = n_1 + n_2$, and $2 = m + n$. Restoring the mass dimensions by fixing the super-Weyl gauge[5] (i.e., $\Upsilon = M_{\mathrm{pl}}^2$, $s_0 = s_0^* = M_{\mathrm{pl}}e^{K/6}$, $P_L\chi^0 = \frac{1}{3}s_0 K_I P_L \chi^I = \frac{1}{3}M_{\mathrm{pl}}e^{K/6}K_I P_L\chi^I$, and $b_\mu = 0$), we obtain

$$\mathcal{L}_{F2}|_{q=p=0,k=n} \supset M_{\mathrm{pl}}^{2(2+2m)} \frac{\mathcal{U}^{(n)}}{\tilde{\mathcal{F}}^{4+2m}} W\bar{W}\bigg[-\frac{1}{2}((\bar{\chi}^I)^{n_1}(M_{\mathrm{pl}}^{-4}4(\not{\partial}\tilde{\mathcal{F}})K_{\bar{\imath}J}(\not{\partial}z^J)\bar{\chi}^{\bar{\imath}}P_R)^{m_1}) \times ((\not{D}\chi^I)^{n_2}(M_{\mathrm{pl}}^{-4}(4\not{D}(\not{\partial}\tilde{\mathcal{F}})K_{\bar{\imath}J}(\not{\partial}z^J)P_R\chi^{\bar{\imath}}))^{m_2})\bigg]_{\psi=0} \approx M_{\mathrm{pl}}^4 \frac{\mathcal{U}^{(n)}}{\tilde{\mathcal{F}}^{4+2m}} \mathcal{O}_F^{(\delta)}, \tag{17}$$

where we require $m_a + n_a = 1$ for $a = 1, 2$ since we are studying the second derivative term $N_{i\bar{j}}$ coupled to $\bar{\chi}^i$ and $\not{D}\chi^{\bar{j}}$. Redefining $\tilde{\mathcal{F}}$ to be dimensionless by $\tilde{\mathcal{F}} \to M_{\mathrm{pl}}^2\tilde{\mathcal{F}}$, we obtain

$$\mathcal{L}_{F2}|_{q=p=0,k=n} \supset M_{\mathrm{pl}}^{-4-2m} \frac{\mathcal{U}^{(n)}}{\tilde{\mathcal{F}}^{4+2m}} \mathcal{O'}_F^{(\delta)}. \tag{18}$$

We then find $\delta = 8 + 4m$ by trivial dimensional analysis because the Lagrangian has mass dimension 4. Then, since $2 = m + n$, we find that the most singular term is

$$\mathcal{L}_{F2}|_{q=p=0,k=n} \supset M_{\mathrm{pl}}^{2(n-4)} \frac{\mathcal{U}^{(n)}}{\tilde{\mathcal{F}}^{2(4-n)}} \mathcal{O'}_F^{(2(6-n))}. \tag{19}$$

Next we consider the general case with several multiplets. We shall focus on the fourth derivative term denoted by $N_{ij\bar{k}\bar{l}}$, which gives a four-fermion term. Also, we have to consider the four-fermion product made only of the chiral fermions with $i = 0, I, T$ because they do not contribute one power of the F-term $\tilde{\mathcal{F}}$ in the numerator, which would reduce the number of inverse power of the F-term $\tilde{\mathcal{F}}$. This is

[5]Here, we use the convention of the superconformal formalism that all physical bosonic and fermionic matter fields have dimensions 0 and 1/2 respectively and $\mathcal{F}$ has dimension 2 while the compensator s_0 has dimension 1 [7]. Through dimensional analysis, we find $[\mathcal{D}_\mu] = 1$, $[z^i] \equiv 0 + [i]$, $[\chi^i] \equiv \frac{1}{2} + [i]$, $[F^i] \equiv 1 + [i]$ where $i = 0, I, W, T$ and $[0] = 1$, $[I] = 0$, $[W] = 3$, $[T] = 0$.

because the overall factor of χ^W contains such linear dependence. The effective fermionic Lagrangian (12) reads then as follows:

$$\begin{aligned}\mathcal{L}_{F6}|_{q=p=0,k=n} \\ \supset \Upsilon^{2+2m}\frac{\mathcal{U}^{(n)}}{\tilde{\mathcal{F}}^{4+2m}}W\bar{W} \\ \times\left(\frac{1}{4}(\chi^I)^n(4\Upsilon^{-2}(\not{\partial}\tilde{\mathcal{F}})K_{\bar{\imath}J}(\not{\partial}z^J)P_R\chi^{\bar{\imath}})^m\right).\end{aligned} \tag{20}$$

After the super-Weyl gauge fixing we obtain

$$\begin{aligned}&\mathcal{L}_{F6}|_{q=p=0,k=n} \\ &\supset M_{\rm pl}^{2(2+2m)}\frac{\mathcal{U}^{(n)}}{\tilde{\mathcal{F}}^{4+2m}}W\bar{W} \\ &\times\left(\frac{1}{4}(\chi^I)\right)^n(4M_{\rm pl}^{-4}(\not{\partial}\tilde{\mathcal{F}})K_{\bar{\imath}J}(\not{\partial}z^J)P_R\chi^{\bar{\imath}})^m) \\ &\approx cM_{\rm pl}^4\frac{\mathcal{U}^{(n)}}{\tilde{\mathcal{F}}^{4+2m}}\mathcal{O}_F^{(\delta)},\end{aligned} \tag{21}$$

where $\mathcal{O}(1)\lesssim c\lesssim\mathcal{O}(10^3)$. After doing the same dimensional analysis as in the single-multiplet case, we obtain $\delta=8+4m$ and

$$\mathcal{L}_{F6}|_{q=p=0,k=n}\supset cM_{\rm pl}^{-4-2m}\frac{\mathcal{U}^{(n)}}{\tilde{\mathcal{F}}^{4+2m}}\mathcal{O}_F'^{(8+2m)}. \tag{22}$$

Then, since $4=m+n$, we can write the most singular terms as

$$\mathcal{L}_{F6}|_{q=p=0,k=n}\supset cM_{\rm pl}^{2(n-6)}\frac{\mathcal{U}^{(n)}}{\tilde{\mathcal{F}}^{2(6-n)}}\mathcal{O}_F'^{(2(8-n))}. \tag{23}$$

Since we are going to use liberated supergravity to describe time-dependent backgrounds such as slow-roll inflation we need to look more closely at the structure of $\tilde{\mathcal{F}}$. From its definition $\tilde{\mathcal{F}}\equiv 2K_{I\bar{J}}(-\partial_\mu z^I\partial^\mu\bar{z}^{\bar{J}}+F^I\bar{F}^{\bar{J}})$, we find $\tilde{\mathcal{F}}\equiv 2K_{I\bar{J}}(\dot{z}^I\dot{\bar{z}}^{\bar{J}}+F^I\bar{F}^{\bar{J}})>0$ whenever spatial gradients can be neglected. We see that the most singular behaviors of the fermionic terms arises when $\dot{z}^I=0$. By expanding $\tilde{F}$ around a static vacuum and reserving the notation $\mathcal{F}$ for the expectation value $< K_{I\bar{J}}F^I\bar{F}^{\bar{J}} >$, the effective Lagrangian can finally be rewritten as follows:

(i) For the single chiral matter multiplet case,

$$\mathcal{L}_{F2}|_{q=p=0,k=n}\supset M_{\rm pl}^{2(n-4)}\frac{\mathcal{U}^{(n)}}{\mathcal{F}^{2(4-n)}}\mathcal{O}_F'^{(2(6-n))}. \tag{24}$$

(ii) For two or more chiral matter multiplets,

$$\mathcal{L}_{F6}|_{q=p=0,k=n}\supset c'M_{\rm pl}^{2(n-6)}\frac{\mathcal{U}^{(n)}}{\mathcal{F}^{2(6-n)}}\mathcal{O}_F'^{(2(8-n))} \tag{25}$$

where $\mathcal{O}(10^{-2})\lesssim c'\lesssim\mathcal{O}(1)$.

The effective operators we obtained are generically nonzero even after considering possible cancellations due to Fermi statistics or nonlinear field redefinitions. As an example we can take terms containing χ^i. They are made of two composite chiral multiplets χ^W and χ^T and these produce terms that do not vanish on shell (i.e., imposing $\not{\partial}P_L\chi^I\approx 0$ for matter fermions). For instance, in a theory with only one matter chiral multiplet $(z,P_L\chi,F)$, we have $W=-2K_{\bar{z}z}[\{(\not{\partial}z)^2-F^*\not{\partial}z\}(\bar{\chi}P_R\chi)+\{F^{*2}-F^*\not{\partial}z\}(\bar{\chi}P_L\chi)]+2K_{\bar{z}z}K_{\bar{z}\,\bar{z}\,z}(\not{\partial}z-F^*)(\bar{\chi}P_L\chi)(\bar{\chi}P_R\chi)$, and $\bar{W}=(W)^*$, so that $W\bar{W}=4K_{\bar{z}z}^4(|\not{\partial}z|^2+|F|^2)|\not{\partial}z-F^*|^2(\bar{\chi}P_L\chi)(\bar{\chi}P_R\chi)$. Hence, looking at the possible fermionic terms from $\mathcal{L}_{F1}$, when $i=z,\bar{\jmath}=\bar{z}$ (i.e., $q=p=m=0,k=n=2$), we get

$$\mathcal{L}_{F1}\supset\Upsilon^2\frac{\mathcal{U}^{(2)}}{\tilde{\mathcal{F}}^4}W\bar{W}(\partial_\mu z\partial^\mu\bar{z})=\Upsilon^2\frac{\mathcal{U}^{(2)}}{\tilde{\mathcal{F}}^4}4K_{\bar{z}z}^4(|\not{\partial}z|^2+|F|^2)|\not{\partial}z-F^*|^2(\partial_\mu z\partial^\mu\bar{z})(\bar{\chi}P_L\chi)(\bar{\chi}P_R\chi). \tag{26}$$

It is easy to see that this operator does not vanish on the mass shell of the matter scalars, $\Box z\approx 0$. As another example, from $\mathcal{L}_{F2}$ we get terms containing up to three matter fermions when we consider $q=m=p_1=0$, $k=n=1$, $p_2=p=1$,

$$\mathcal{L}_{F2}\supset\Upsilon^2\frac{\mathcal{U}^{(1)}}{\tilde{\mathcal{F}}^4}\frac{1}{2}W\bar{\chi}\not{\partial}P_R\chi^{\bar{W}}\approx\Upsilon^2\frac{\mathcal{U}^{(1)}}{\tilde{\mathcal{F}}^4}4K_{\bar{z}z}^2(\not{\partial}\tilde{\mathcal{F}})(\not{\partial}z)(\bar{\chi}P_R\chi)P_L(\not{\partial}z-F^*)^2\chi|_{\text{3-fermion terms}}+\cdots. \tag{27}$$

Back to the results in Eqs. (24) and (25), the general effective Lagrangians can be cast in the form

$$\mathcal{L}_F=\Lambda_{\rm cut}^{4-\delta}\mathcal{O}_F'^{(\delta)}=\begin{cases}M_{\rm pl}^{2(n-4)}\frac{\mathcal{U}^{(n)}}{\mathcal{F}^{2(4-n)}}\mathcal{O}_F'^{(\delta=2(6-n))} & \text{for } N_{\rm mat}=1,\\ c'M_{\rm pl}^{2(n-6)}\frac{\mathcal{U}^{(n)}}{\mathcal{F}^{2(6-n)}}\mathcal{O}_F'^{(\delta=2(8-n))} & \text{for } N_{\rm mat}\geq 2,\end{cases} \tag{28}$$

where $\mathcal{O}(10^{-2})\lesssim c'\lesssim\mathcal{O}(1)$; N_{mat} and $\Lambda_{\rm cut}$ are defined to be the number of chiral multiplets of matter, and the cutoff scale of our effective theory, respectively.

If we demand that our effective theory describes physics up to the energy scale Λ_{cut} we obtain the following inequalities:

$$\mathcal{U}^{(n)} \lesssim \begin{cases} \mathcal{F}^{2(4-n)}\left(\frac{M_{\mathrm{pl}}}{\Lambda_{\mathrm{cut}}}\right)^{2(4-n)} & \text{where } 0 \leq n \leq 2 \quad \text{for } N_{\mathrm{mat}} = 1, \\ \mathcal{F}^{2(6-n)}\left(\frac{M_{\mathrm{pl}}}{\Lambda_{\mathrm{cut}}}\right)^{2(6-n)} & \text{where } 0 \leq n \leq 4 \quad \text{for } N_{\mathrm{mat}} \geq 2. \end{cases} \tag{29}$$

A conventional definition of the supersymmetry breaking scale M_S is in terms of F-term expectation value so we define $M_S^4 = M_{\mathrm{pl}}^4 \mathcal{F}$, so the constraints on $\mathcal{U}^{(n)}$ become

$$\mathcal{U}^{(n)} \lesssim \begin{cases} \left(\frac{M_S}{M_{\mathrm{pl}}}\right)^{8(4-n)}\left(\frac{M_{\mathrm{pl}}}{\Lambda_{\mathrm{cut}}}\right)^{2(4-n)} & \text{where } 0 \leq n \leq 2 \quad \text{for } N_{\mathrm{mat}} = 1, \\ \left(\frac{M_S}{M_{\mathrm{pl}}}\right)^{8(6-n)}\left(\frac{M_{\mathrm{pl}}}{\Lambda_{\mathrm{cut}}}\right)^{2(6-n)} & \text{where } 0 \leq n \leq 4 \quad \text{for } N_{\mathrm{mat}} \geq 2. \end{cases} \tag{30}$$

Equation (30) is the crucial one in our paper, as it constrains precisely the new function $\mathcal{U}$ introduced by liberated $\mathcal{N} = 1$ supergravity. The constraint depends on the reduced Planck scale M_{pl}, the supersymmetry breaking scale M_S, Λ_{cut} and the number of chiral multiplets of matter in the theory. Of course, when we push both the cutoff and supersymmetry breaking scales to the reduced Planck scale, i.e., $\Lambda_{\mathrm{cut}} \sim M_S \sim M_{\mathrm{pl}}$, we obtain a model-independent universal constraint

$$\forall\, n : \mathcal{U}^{(n)} \lesssim 1. \tag{31}$$

A model where supersymmetry is broken at the Planck scale is hardly the most interesting. In the more interesting case that $M_S \ll M_{\mathrm{pl}}$ we need the constraints (30) again, so we need to first determine how many matter chiral multiplets we have in our theory. The constraints will then depend only on our choice of Λ_{cut} and M_S.

In the rest of this section we will examine the constraints in two cases. The first is the true, postinflationary vacuum of the theory. To make a supergravity theory meaningful we want it to be valid at least up to energies $\Lambda_{\mathrm{cut}} \gtrsim M_S$. The second is slow-roll inflation. In this case we must have $\Lambda_{\mathrm{cut}} \gtrsim H$, with H the Hubble constant during inflation.

For the postinflationary vacuum the interesting regime is when M_S is relatively small, say $M_S \sim 10$ TeV $\approx 10^{-15} M_{\mathrm{pl}}$ and the effective theory is valid up to an energy scale not smaller than M_S, i.e., $\Lambda_{\mathrm{cut}} \gtrsim M_S$. If $\Lambda_{\mathrm{cut}} < M_S$ liberated supergravity would be a useless complication, since in its domain of validity supersymmetry would be always nonlinearly realized. In the postinflationary vacuum, for the single matter chiral multiplet case, the constraints (30) thus give for

$$\mathcal{U}^{(0)} \lesssim \left(\frac{M_S}{M_{\mathrm{pl}}}\right)^{32}\left(\frac{M_{\mathrm{pl}}}{M_S}\right)^{8} \Rightarrow \mathcal{U}^{(0)} \sim 10^{-360}, \tag{32}$$

$$\mathcal{U}^{(1)} \lesssim \left(\frac{M_S}{M_{\mathrm{pl}}}\right)^{24}\left(\frac{M_{\mathrm{pl}}}{M_S}\right)^{6} \Rightarrow \mathcal{U}^{(1)} \sim 10^{-270}, \tag{33}$$

$$\mathcal{U}^{(2)} \lesssim \left(\frac{M_S}{M_{\mathrm{pl}}}\right)^{16}\left(\frac{M_{\mathrm{pl}}}{M_S}\right)^{4} \Rightarrow \mathcal{U}^{(2)} \sim 10^{-180}. \tag{34}$$

Notice that $\mathcal{U}^{(3)}, \mathcal{U}^{(4)}$ are not restricted. For two or more matter chiral multiplets, the constraints are given by

$$\mathcal{U}^{(0)} \lesssim \left(\frac{M_S}{M_{\mathrm{pl}}}\right)^{48}\left(\frac{M_{\mathrm{pl}}}{M_S}\right)^{12} \Rightarrow \mathcal{U}^{(0)} \sim 10^{-540}, \tag{35}$$

$$\mathcal{U}^{(1)} \lesssim \left(\frac{M_S}{M_{\mathrm{pl}}}\right)^{40}\left(\frac{M_{\mathrm{pl}}}{M_S}\right)^{10} \Rightarrow \mathcal{U}^{(1)} \sim 10^{-450}, \tag{36}$$

$$\mathcal{U}^{(2)} \lesssim \left(\frac{M_S}{M_{\mathrm{pl}}}\right)^{32}\left(\frac{M_{\mathrm{pl}}}{M_S}\right)^{8} \Rightarrow \mathcal{U}^{(2)} \sim 10^{-360}. \tag{37}$$

From the constraints on $\mathcal{U}^{(0)}$ and $\mathcal{U}^{(2)}$, we find that the liberated scalar potential contributes only a negligibly small cosmological constant and negligibly small corrections to the mass terms of the chiral multiplet scalars. For the single chiral multiplet case, restoring dimensions we get a vacuum energy density

$$\mathcal{U}^{(0)} \lesssim 10^{-360} M_{\mathrm{pl}}^4 \tag{38}$$

and scalar masses

$$M_z \lesssim M_{\mathrm{pl}} \sqrt{|\mathcal{U}^{(2)}|} = 10^{-90} M_{\mathrm{pl}}. \tag{39}$$

These constraints become even tighter if the theory contains more than one chiral multiplet, but the ones we obtained are already so stringent as to rule out any observable contribution to the cosmological constant and scalar masses from the new terms made possible by liberated supergravity. We can say that Eqs. (38) and (39) already send liberated supergravity back to prison after the end of inflation.

The constraints during inflation instead can be easily satisfied if during inflation the supersymmetry breaking scale is very high, say $M_S = M_{\rm pl}$. In that case, $\mathcal{U}^{(0)} \lesssim \mathcal{O}(1)$. After inflation the "worst case scenario" constraints coming from Eq. (30) with $N_{\rm mat} \geq 2$ and $M_S = 10^{-15} M_{\rm pl}$ are

$$\forall\, n\colon\ \mathcal{U}^{(n)} \lesssim 10^{-120(6-n)}. \qquad (40)$$

A simple way to satisfy all these constraints is to choose a no scale structure for the supersymmetric part of the scalar potential. This ensures the vanishing of the F-term contribution to the potential independently of the magnitude of the F-terms,

$$V_F = e^G (G_I G^{I\bar{J}} G_{\bar{J}} - 3) = 0, \qquad (41)$$

The total scalar potential is then given by $V = V_D + V_{\rm NEW}$. Thanks to the no-scale structure, we can have both $M_S \sim M_{\rm pl}$ and $\mathcal{U}^{(0)} \sim H^2 \sim 10^{-10}$ during inflation.

Our scenario has $M_S = M_{\rm pl}$ during inflation and $M_S = 10^{-15} M_{\rm pl}$ at the true vacuum in the postinflation phase, so we see that to satisfy all constraints a transition between the two different epochs must occur, for which the scale of the composite F-term $\mathcal{F}$ changes from $\mathcal{O}(M_{\rm pl})$ to $\mathcal{O}(10^{-15} M_{\rm pl})$.

IV. A MINIMAL MODEL OF SINGLE-FIELD AND SLOW-ROLL INFLATION IN LIBERATED $\mathcal{N} = 1$ SUPERGRAVITY

In the previous sections we have argued that liberated $\mathcal{N} = 1$ supergravity can be an effective field theory for describing the inflationary dynamics while at the same time satisfying all the constraints if a transition that changes the supersymmetry breaking scale at the end of inflation is allowed. Note that due to the no-scale structure, the scalar potential is given only by an eventual D-term supersymmetric potential V_D and the "liberated" term $\mathcal{U}$. In this section we present an explicit minimal model of single-field, slow-roll inflation in liberated $\mathcal{N} = 1$ supergravity which obeys the inequality $H \ll \Lambda_{\rm cut} = M_{\rm pl} = 1$.

To begin with, let us consider a chiral multiplet T with Kähler potential $K(T, \bar{T}) = -3 \ln[T + \bar{T}]$ and a constant superpotential W_0. Then, the supergravity G-function [1] is given by

$$G \equiv K + \ln|W|^2 = -3 \ln[T + \bar{T}] + \ln W_0 + \ln \bar{W}_0. \qquad (42)$$

It automatically produces a no-scale structure in which the F-term potential vanishes identically: $V_F = 0$.

Next, let us find the canonically normalized degrees of freedom of the theory. From the kinetic term corresponding to the G function (42), we read

$$\begin{aligned}\mathcal{L}_K &= \frac{3}{(T+\bar{T})^2} \partial T \partial \bar{T} \\ &= \frac{3}{4({\rm Re}T)^2} (\partial {\rm Re} T)^2 + \frac{3}{4({\rm Re}T)^2} (\partial {\rm Im} T)^2 \\ &= \frac{1}{2} (\partial \chi)^2 + \frac{1}{2} e^{-2\sqrt{2/3}\chi} (\partial \phi)^2, \end{aligned} \qquad (43)$$

where we have used the following field redefinition:

$$T = {\rm Re}T + i {\rm Im}T = \frac{1}{2} e^{\sqrt{2/3}\chi} + i \frac{\phi}{\sqrt{6}}. \qquad (44)$$

Note that the $\mathbb{Z}_2$ symmetry $\chi \to -\chi$ is already explicitly broken by the kinetic Lagrangian, while the symmetry $\phi \to -\phi$ is unbroken. However, even the latter symmetry will be broken by the inflationary potential. The field χ is always canonically normalized while ϕ has a canonical kinetic term only at $\chi = 0$.

The composite F-term is given by $\mathcal{F} = e^G G_T G^{T\bar{T}} G_{\bar{T}}$ after solving the equation of motion for the auxiliary fields F^I. For our G function we obtain an exponentially decreasing function $\mathcal{F} = 3|W_0|^2/(T+\bar{T})^3 = 3|W_0|^2 e^{-3\sqrt{2/3}\chi}$. This is what we want to get a viable supersymmetry breaking mechanism. The reason is that we look for a supersymmetry breaking scale during inflation $M_S^i \sim M_{\rm pl} = 1$, while the final scale should be parametrically lower than the Planck scale—for instance $M_S^f = 10^{-15} M_{\rm pl}$. To achieve this large difference of scales, the vacuum expectation value of the field χ should change during the phase transition. On the other hand, the cutoff scale of our model can remain $\mathcal{O}(M_{\rm pl})$ both before and aft

er the phase transition.

We will achieve this with a potential that changes from $\phi \neq 0, \chi = 0$ during inflation to $\chi \neq 0, \phi = 0$ after inflation. We will also choose ϕ as the inflaton field and χ as the field that controls the supersymmetry breaking scale.

A function $\mathcal{U}$ producing a correct inflationary dynamics is

$$\mathcal{U} \equiv \alpha (1 - e^{-\sqrt{2/3}\phi})^2 \left(1 + \frac{1}{2} \sigma \chi^2\right), \qquad (45)$$

where α, β, γ, σ are arbitrary positive constants.

Next, we assume that the mass m_χ of χ is greater than the Hubble scale H during inflation; this is necessary to describe a single-field slow-roll inflation governed only by the inflaton field ϕ. Hence, we impose that during inflation $m_\chi^2 = \alpha\sigma \gg H^2$. Since $\alpha \sim H^2 \sim 10^{-10}$, the condition reduces to $\sigma \gg 1$.

We must also analyze the vacuum structure of the potential. First of all, we explore the minima with respect to χ. By computing $\frac{\partial \mathcal{U}}{\partial \chi} = 0$ and defining $V_{\rm inf} \equiv \alpha(1 - e^{-\sqrt{2/3}\phi})^2$ we find that during inflation (where $\phi \neq 0$) the equation of motion for χ is given by $\sigma\chi V_{\mathrm{inf}} = 0$ so it gives a unique minimum at $\chi = 0$. On the other hand after inflation we have $\phi = 0$ and $V_{\mathrm{inf}} = 0$, so the equation of motion gives a flat potential in χ. The final position of the field χ is then determined either by the initial conditions on χ or by small corrections to the either the liberated supergravity potential $\mathcal{U}$ or to V_F. We will describe the explicit forms of such corrections in a forthcoming paper [9]. Here we content ourselves with pointing out that the simple potential (45) already achieves the goal of making the final supersymmetry breaking scale different from M_S^i.

Before studying supersymmetry breaking we notice that a deformation of the scalar potential such as $\mathcal{U}$ was obtained using an off-shell *linear* realization of supersymmetry in [8]. Therefore, for the new term to be consistent, supersymmetry must be broken as usual by some nonvanishing auxiliary field belonging to the standard chiral multiplets and moreover the Kähler metric of the scalar manifold must be positive [8]. So, in spite of the presence of the new term $\mathcal{U}$, the analysis of supersymmetry breaking is completely standard. Since the supersymmetry breaking scale M_S comes from the positive potential part V_+, as shown in the Goldstino SUSY transformation $\delta_\epsilon P_L v = \frac{1}{2} V_+ P_L \epsilon$ that is constructed with the fermion shifts with respect to the auxiliary scalar contributions [7], we have

$$V_+ = e^G G_T G^{T\bar{T}} G_{\bar{T}} = \frac{3|W_0|^2}{(T+\bar{T})^3} = 3|W_0|^2 e^{-3\sqrt{2/3}\chi}. \quad (46)$$

During inflation we demand that the initial supersymmetry breaking scale is M_{pl}, so we identify $W_0 \equiv \frac{(M_S^i)^2}{\sqrt{3}}$ and therefore $V_+ = (M_S^i)^4 e^{-3\sqrt{2/3}\chi}$. Because $\chi = 0$ during inflation we indeed have $V_+|_{\chi=0,\phi\neq 0} = (M_S^i)^4 = 1 \gg H^2 = \mathcal{O}(10^{-10} M_{\mathrm{pl}}^2)$.

On the other hand, we want to get a much smaller SUSY breaking scale $M_S^f \approx 10^{-15} M_{\mathrm{pl}}$ around the true vacuum at the end of inflation. Thus, at the true vacuum (i.e., $\chi = C$ and $\phi = 0$) where $\mathcal{U} = 0$, we get $V_+|_{\chi=C,\phi=0} \approx (M_S^i)^4 e^{-3\sqrt{2/3}C} \equiv (M_S^f)^4$. From this, we find where the location of the true vacuum in the χ direction should be (recall that χ is a flat direction after inflation)

$$C = \sqrt{\frac{8}{3}} \ln \frac{M_S^i}{M_S^f}, \quad (47)$$

where M_S^f is a free parameter, which we set to be approximately 10^{-15} in Planck units.

The proposed potential $\mathcal{U}$ vanishes after inflation hence it already trivially satisfies the constraints (30). So all we need to do is to check that it also satisfies (31). Using $\mathcal{F} = e^{-3\sqrt{2/3}\chi}$, which gives $M_S^i = M_{\mathrm{pl}} = 1$ during inflation ($\chi = 0$), we first have $\mathcal{U}^{(n)}|_{\chi=0} \ll e^{-3m\sqrt{2/3}\chi}\mathcal{O}(1)|_{\chi=0} = \mathcal{O}(1)$. Using Eq. (44), we find $\partial_T = \sqrt{6}(-i\partial_\phi + e^{-\sqrt{2/3}\chi}\partial_\chi)$ and $\partial_{\bar{T}} = \sqrt{6}(i\partial_\phi + e^{-\sqrt{2/3}\chi}\partial_\chi)$. Note that $\mathcal{U}^{(n)}|_{\chi=0} \equiv \partial_T^k \partial_{\bar{T}}^l \mathcal{U}(T,\bar{T})|_{\chi=0}$ where $n = k + l$. In particular, since the functional dependence on χ does not produce any singularity at $\chi = 0$, it is sufficient to check that $\partial_\phi^n \mathcal{U} \ll \mathcal{O}(1)$. Thus, because the dependence on ϕ is solely given by the Starobinsky inflationary potential, i.e., $V \sim \alpha(1 - e^{-\sqrt{2/3}\phi})$, we will get that its derivatives are always less than the coefficient α, thanks to the decreasing exponential factor $e^{-\sqrt{2/3}\phi}$. This implies that the constraint is automatically satisfied since $\alpha \sim 10^{-10} < \mathcal{O}(1)$. So, all consistency conditions can be satisfied by a liberated supergravity potential.

ACKNOWLEDGMENTS

We would like to thank Alexios Kehagias for useful discussions. M. P. is supported in part by NSF Grant No. PHY-1915219.

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