Undergraduate Research in Mathematics Education: A Setting to Encourage the Use of Qualitative Data about Children's Learning to Make Decisions about Teaching

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Abstract

Undergraduate research is increasingly prevalent in many fields of study, but it is not yet widespread in mathematics education. We argue that expanding undergraduate research opportunities in mathematics education would be beneficial to the field. Such opportunities can be impactful as either extra-curricular or course-embedded experiences. To help readers envision directions for undergraduate research experiences in mathematics education with prospective teachers, we describe a model built on a design-based research paradigm. The model engages pairs of prospective teachers in working with faculty mentors to design instructional sequences and test the extent to which they support children's learning. Undergraduates learn about the nature of systematic mathematics education research and how careful analyses of classroom data can guide practice. Mentors gain opportunities to pursue their personal research interests while guiding undergraduate pairs. We explain how implementing the core cycle of the model, whether on a small or large scale, can help teachers make instructional decisions that are based on rich, qualitative classroom data.

Keywords: undergraduate research, design-based research, clinical interviews, formative assessment, classroom data analysis

Undergraduate Research in Mathematics Education: A Setting to Encourage the Use of Qualitative Data about Children's Learning to Make Decisions about Teaching

Undergraduate research is a growing trend in higher education. Currently, the Council on Undergraduate Research (CUR) has over 700 institutional members and more than 13,000 individual members (CUR, 2018a); more than 4,000 undergraduates per year participate in events they organize (CUR, 2014). Historically, university faculty in science, technology, engineering, and mathematics (STEM) have been at the forefront of encouraging undergraduates to do research in their fields (Schuster, 2018). Large-scale extra-curricular undergraduate research programs have been supported by the Research Experiences for Undergraduates (REU) program of the National Science Foundation (NSF), other federal and state funding agencies, and internal funds (Nazaire & Usher, 2015; Ramirez, McNicholas, Gilbert, Saez, & Siniawski, 2015). Smaller-scale, yet impactful, undergraduate research projects have been built into existing coursework (Chamberlain & Mendoza, 2017; Staub et al, 2016). Recently, research by undergraduates in the social sciences has become more widespread (Stanford, Rocheleau, Smith, & Mohan, 2017). Nonetheless, teacher candidates are generally under-represented among students conducting undergraduate research (DeVore & Munk, 2015; Manak & Young, 2014).

Efforts to increase the numbers of prospective teachers participating in undergraduate research are underway. CUR hosted an education institute for faculty from various institutions to discuss embedding undergraduate research experiences in teacher education courses (CUR, 2018b). *CUR Quarterly* published a special issue on undergraduate research in professional schools (Shanahan et al., 2015). NSF issued a "Dear Colleague" letter to emphasize that REU proposals for larger-scale projects may be put forth for undergraduate research in discipline-based education research in fields such as mathematics education (Ferrini-Mundy, 2011).

Although undergraduate research has traditionally been the domain of STEM majors not pursuing teaching certification, this need not be the case in the future. Prospective teachers can also benefit from doing research in their chosen field. Just as young scientists develop their craft through research so might beginning educators as they make classroom data the focus of systematic study.

Purpose and Overview

The purpose of this report is to describe a model that has been used to engage prospective teachers in mathematics education research. The model, entitled "Preparing Aspiring Teachers to Hypothesize Ways to Assist Young Students" (PATHWAYS), has been developed and refined over the course of five years. PATHWAYS came about as a result of funding from NSF for an REU program. It is offered here as one model for designing undergraduate research experiences in mathematics education.

We begin with a description of how existing literature defines undergraduate research, its essential components, and benefits. Then, we present the PATHWAYS model and assessment data on its impact when used for a summer-long extracurricular program. We then turn to an example of smaller-scale implementation of the model in a mathematics teaching methods course.

Undergraduate Research and its Potential Benefits

CUR defined undergraduate research as "An inquiry or investigation conducted by an undergraduate student that makes an original intellectual or creative contribution to the discipline" (Wentzel, 1997, p. 163). Common features of undergraduate research, included in the PATHWAYS model, include faculty mentoring (Alexander, Foertsch, Daffinrud, & Tapia, 2000; Gafney, 2005), working with a peer community (Alexander et al., 2000), and dissemination of results (Lopatto, 2009; Osborn & Karukstis, 2009). The American Association of Colleges and Universities (AACU) characterized undergraduate research as a "high-impact" practice because of the corpus of literature on how it can support learning (Kuh, 2008). In this section, we describe some documented benefits pertinent to teacher education; specifically, that undergraduate research can develop undergraduates' research skills and dispositions, develop general academic skills and dispositions, and help with career clarification and preparation. Each of these benefits can build teachers' capacity for reflective practice.

Development of Research Skills and Dispositions

The historic divide between research and practice could be narrowed by familiarizing more teachers with research. Lester and Wiliam (2002) suggested that mathematics education research will only strongly influence practice when teachers play active roles in the creation of research questions and the design of investigations. Likewise, Silver (2003) argued that more mathematics education research studies should arise from practitioner-generated questions. Cai et al. (2018) provided a detailed vision of what cooperation between researchers and teachers might look like; they proposed creating partnerships where teachers and researchers work together to identify instructional problems, create appropriate learning goals, apply research to the identified problems, create and test lessons to solve the problems, and refine the lessons they create by attending to empirical classroom data. Introducing undergraduates to this type of systematic mathematics education research could position them to make meaningful contributions to such partnerships.

Numerous studies report that undergraduate research experiences contribute to the development of undergraduates' research skills and dispositions. Such experiences tend to increase undergraduates' understanding of the research process (Lopatto, 2004) by helping them learn how to read and understand primary literature in a field (Lopatto, 2004; 2010), define a problem (Yesiclay, 2000), and make observations and collect data (Kardash, 2000). As they engage in such activities, undergraduates often report gains in their confidence and motivation to do research (Campbell & Skoog, 2004; Hunter, Laursen, & Seymour, 2006; Seymour, Hunter, Laursen, & DeAntoni, 2003). Hence, undergraduate research experiences are linked to both cognitive and affective gains in undergraduates' orientations toward research.

Development of General Academic Skills and Dispositions

As undergraduates engage in research, they develop general academic skills that are useful in college and later on in teaching. Gains can be observed in undergraduate researchers' critical thinking (Hunter et al., 2006; Ishiyama, 2002; Kardash, 2000), their capacity for independent study (Ishiyama, 2002; Lopatto, 2004; Yesiclay, 2000), and their ability to engage in inquiry and analysis (Lopatto, 2010). Affective benefits include developing greater intellectual curiosity (Bauer & Bennett, 2003) and tolerance for obstacles (Lopatto, 2004). When undergraduates conduct research together, as they do in the PATHWAYS model, they can develop skills in communication (Bauer & Bennett, 2003; Lopatto, 2010; Kardash, 2000; Seymour et al., 2003), leadership (Yesiclay, 2000), and teamwork (Lopatto, 2010). Collectively, such cognitive and affective benefits can help undergraduates become reflective, perseverant teachers and productive members of professional communities of practice.

Career Clarification and Preparation

Several studies indicate that undergraduate research experiences help clarify career paths and goals (Hunter et al., 2006; Lopatto, 2009; Seymour et al., 2003). In some cases, the process of clarification leads toward graduate study. Undergraduate research experiences can increase the likelihood of graduate study (Alexander et al., 2000; Bauer & Bennett, 2003; Foertsch, Alexander, & Penberthy 2000) and the likelihood of pursuing a doctoral degree (Russell, Hancock, & McCollough, 2007) at the same time they help individuals develop the abilities to succeed in such studies (Hunter et al., 2006; Lopatto, 2009; Seymour et al., 2003). In the field of mathematics education, it would be beneficial to encourage more individuals to pursue terminal degrees, as colleges and universities often find it difficult to fill mathematics education faculty positions (Reys, Reys, & Estapa, 2013). Undergraduate research in mathematics education could ultimately be a means for steering some toward such positions.

It must be noted, however, that the career preparation benefits of undergraduate research are not at all limited to those who pursue doctoral degrees. Mandinach and Gummer (2016) pointed out that all teachers can gain data literacy by engaging in undergraduate research; data literacy involves identifying problems of practice and framing questions, gathering quality data, interpreting data, making decisions from data, and evaluating outcomes. These competencies are increasingly important as teachers are increasingly asked to justify instructional decisions with data. Principals report that they highly value new teachers who can articulate and communicate problems of practice, understand the purposes of different types of data, use formative and summative assessments, understand how to analyze data, and diagnose what students need in order to adjust instruction accordingly (Hoffman & Manak, 2016). Undergraduate research experiences provide unique opportunities to foster the development of such skills.

PATHWAYS Model Components

The PATHWAYS undergraduate research model engages prospective teachers in using classroom data to inform instruction during design-based research (Smit & van Eerde, 2011). Design-based research is akin to engineering research in that a goal is to design and test structures. In PATHWAYS, undergraduate pairs work under the supervision of faculty mentors to design and test structures to support children's mathematical thinking. Next, we describe the primary components of PATHWAYS when implemented as a large-scale extra-curricular experience; later, we describe smaller-scale implementation in a methods course.

Undergraduates from our home institution and other institutions apply to participate. We require freshman and sophmore applicants to have completed at least one course offered by an education department at their institution. Junior and senior applicants need to have completed at least one mathematics teaching methods course. We balance the number of freshman/sophmore participants with the number of junior/senior participants. Each individual from the former group is paired with one from the latter, allowing for a degree of peer mentoring.

Each undergraduate pair works with a group of four children from the same grade level over the course of 10 weeks in the summer. In accord with the design-based paradigm, pairs work together to systematically gather information about the children's mathematical thinking, use the information to design instructional materials and lessons to support and develop the children's thinking, empirically test the materials during instruction, use the information from empirical trials to refine the instructional approach, and test the refined approach with the children again (Bakker & van Eerde, 2015). These activities are cyclic (Figure 1), with the aim of using data from each empirical trial to strengthen the overall instructional sequence. The core cycle shown in Figure 1 also undergirds the smaller-scale implementation of the PATHWAYS model we describe later on, which involves less trips through the same cycle. The structure of the core cycle connects directly to *noticing*, the process through which teachers manage the vast amounts of information they encounter while teaching (Sherin, Jacobs, & Philipp, 2011). Noticing entails attending to students' mathematical reasoning, interpreting it, and making decisions about future instruction (Amador, Carter, & Hudson, 2016; Jacobs, Lamb, & Phillip, 2010); all of these activities are included in the core cycle.

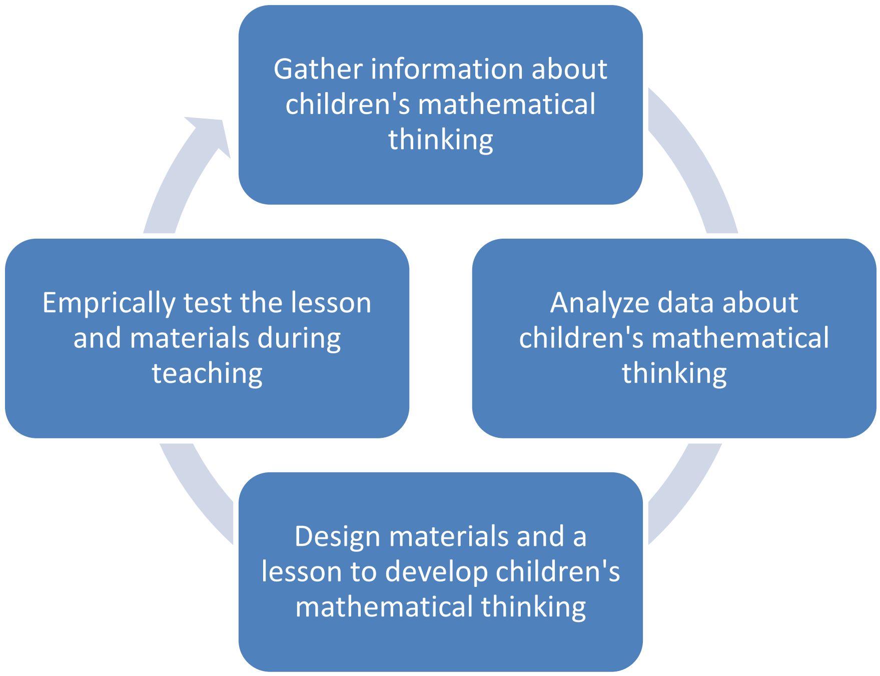


Figure 1. The core repetitive cycle for PATHWAYS design-based research

The PATHWAYS model is similar to action research (Beaulieu, 2013) and lesson study (Lewis, 2016) in its focus on directly addressing problems of classroom practice. PATHWAYS is also similar to lesson study in its prioritization of cyclic, continuous improvement that involves reflecting on instruction to decide how future lessons should be adjusted to be more responsive to students' thinking (Ricks, 2011). PATHWAYS is distinct, however, in the central role played by pre- and post-interviews of children and the methods used to gather and analyze qualitative classroom data. Details about how these activities are incorporated in the core design-based cycle for PATHWAYS (Figure 1) are given next.

Gathering and Analyzing Initial Information about Children's Thinking

In PATHWAYS, clinical interviews are used to gather baseline information about children's thinking. At the start of the summer, undergraduates conduct individual clinical interviews with the children in their groups. Faculty mentors select interview items with the potential to help yield rich information about children's mathematical thinking from sources such as NCTM journals, past items from the National Assessment of Educational Process (NAEP), and the Illustrative Mathematics website (http://www.illustrativemathematics.org). Undergraduates prepare to conduct the interviews by writing multiple possible solutions to key items and submitting them to their mentors and conducting mock interviews with one another.

Before conducting the interviews with children, undergraduates also review and discuss interview interactions from past summers to study effective and ineffective interview techniques. Ineffective techniques illustrated by interactions from past summers include quickly affirming or negating a child's response and asking only questions that require one-word or one-number responses. For example, undergraduates study the following interview interaction about an item from Illustrative Mathematics (https://www.illustrativemathematics.org/content-standards/tasks/871):

> Interviewer: Jon and Charlie plan to run together. They are arguing about how far to run. Charlie says, "I run three-sixths of a mile each day." Jon says, "I can only run half a mile." Why do you think it is silly for them to argue over this?
>
> Student: Because three-sixths is equal to one-half.
>
> Interviewer: Yeah, exactly, so it is kind of pointless.
>
> Student: Do I write that down?
>
> Interviewer: You can write that down if you want. You don't have to.

We use this excerpt to show that the interviewer missed a chance to have the student explain why the two fractions were equal. Asking for such explanation could help reveal whether the student is thinking conceptually, procedurally, or some combination of the two. Simply knowing that the student provided a correct response provides a minimal starting point for designing instruction.

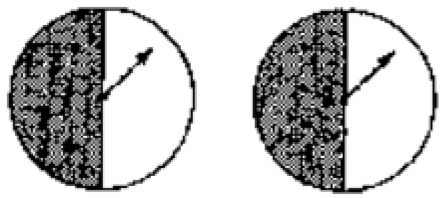
We also use some past interview interactions to illustrate "competent questioning" for undergraduates. Competent questioning requires listening to children and using their responses to construct specific probes to gain deeper understanding of their thinking (Moyer & Milewicz, 2002), as in the following interaction:

> Interviewer: A pan of brownies that is 12 inches in one direction and 2 inches in the other is cut into one-inch square pieces. Could you draw a picture or use a manipulative to show the pan of brownies once it has been cut?
>
> Student: (reaches for plastic square tiles and begins grouping them).
>
> Interviewer: So, what are you making?
>
> Student: I'm making the brownies when they've been cut.
>
> Interviewer: OK, and so how many rows did you choose to do?
>
> Student: Um, I did 12 inches this way (pointing to 12 going down vertically) and 2 inches (pointing to the 2 going across).
>
> Interviewer: Is there a particular reason why you chose to do 12 going down and then 2 across?
>
> Student: Mmm, no.
>
> Interviewer: No? OK, so without counting these one-by-one, is there a way you could find out how many total brownies there were?
>
> Student: You can count by 2s (counts by 2s to 24 using her representation).
>
> Interviewer: So you got 24?
>
> Student: Yup, or you could just do the math in your head, 12 plus 12.

In this excerpt, the interviewer questioned the student about the representation she constructed and asked about specific aspects of the student's reasoning. The excerpt shows how observations about a student's work can be used as starting points for follow-up probing questions. Undergraduates study additional examples from the Moyer and Milewicz article to further explore differences between competent questioning and other forms of interaction.

As undergraduates conduct interviews with children, they video record them, transcribe them, and retain children's written work for analysis. Each undergraduate pair then meets with their faculty mentor, who initially leads the process of coding transcripts to characterize children's thinking. The triad (faculty mentor and undergraduate pair) collaboratively devises and assigns codes to interview task responses, as exemplified in Figure 2. The codes are recorded in a codebook (DeCuir-Gunby, Marshall, & McColloch, 2011) which contains an abbreviation for each code, a brief definition, transcript examples to show when the code should be applied, reminders to know when the code should be assigned to data, and reminders about when the code should not be assigned. These codes become a language the triad uses to talk about important reasoning elements they observe throughout the research. Pre-interview code abbreviations are compiled in the cells of a matrix that has one child's name in each column and one interview item number in each row so children's pre-interview and post-interview performance can later be readily compared. Organic qualitative coding of this nature is valuable, though time-consuming; in the methods course implementation description of PATHWAYS later in this article, there is a set of guided questions undergraduates answer about the data to create a more streamlined process that fits within the usual time constraints of semester-long courses.

Interview item (NAEP 1996-12M12 #9 M070501):



The two fair spinners shown above are part of a carnival game. A player wins a prize only when both arrows land on black after each spinner has been spun once. James thinks he has a 50-50 chance of winning. Do you agree? Justify your answer.

Student responses to interview item	**Codes assigned and rationale**
Student 1: I do not agree because there are only four different combinations for this. First you land on black and then you land on white, you land on both black, you land on both white, and then you land on another white and black. Interviewer: OK Student 1: So that makes it about a 25 or a 1 to 4 to get both to be on the black. Interviewer: OK Student 1: So I do not agree with his 50-50 because a 50-50 would mean that there are only two different combinations but here there's four. Interviewer: Ok. Student 1: With only one combination being the winner.	*Combinatorial reasoning (CR)*; the student systematically counts elements of the sample space to devise a response
Student 2: Yes, because like the black and white are both equal. Interviewer: OK Student 2: So, like, it's going to land on either one.	*Equiprobability bias (EB)*; incorrectly assumes equal probabilities in a chance process ~*CR*; the student does not systematically count elements of the sample space

Figure 2. Children's coded responses to a PATHWAYS interview item (Item source: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, NAEP, 1996 Mathematics Assessment).

Designing Instruction to Support and Develop Children's Thinking

Once the initial interviews have been coded, each undergraduate pair works with their mentor to design an appropriate lesson for their group of students. Lessons are guided by past mathematics education research and practitioner-oriented articles as well as the children's observed thinking patterns. The undergraduates reviewing the responses and codes shown in Figure 2, for example, decided to have their students play a game that involved flipping two coins in order to try to get more students to use combinatorial reasoning when analyzing such a situation. When they did a test-run of the lesson for the other PATHWAYS undergraduates and mentors, a number of suggestions emerged and were recorded for the group to consider. One suggestion was to begin by spending more time flipping one coin rather than two, since some students still struggled with simple probabilities during interviews. Such students might not be ready to move to a two-coin, compound probability situation immediately. It was also suggested that the undergraduates have children write predictions about the two-coin game before having them do coin-flipping trials and recording the results; specifically, children could first predict whether there is a 50-50 chance of having both coins land on heads. Discrepancies between predictions and empirical outcomes could then become motivation for tabulating all possible outcomes when two coins are flipped. Another suggestion was to ask students about their interpretation of the phrase "50-50 chance," since it can have different meanings to different students (Tarr, 2002; Watson, 2005). Each pair of undergraduates does a test-run of each lesson they teach over the summer with the entire group of mentors and undergraduate researchers and takes their input into account to refine each lesson before teaching it to their group of children. For smaller-scale replication of PATHWAYS, when having an entire group as a test audience is not viable, feedback from a peer or two can still yield some valuable insights.

Empirical Testing and Data Gathering

After lesson test-runs have been completed, each undergraduate pair teaches their lesson to their group of children. They use video cameras and voice recorders to capture the interactions that occur during the lesson, and they retain children's written work for analysis. We encourage undergraduates to use teaching strategies that will yield maximum information about children's thinking. In particular, we encourage brief writing activities because they give children opportunities to structure their thinking and make it more visible for the purpose of analysis (Hoffer, 2016). Throughout the summer, we also consistently encourage *focusing* rather than *funneling* patterns of classroom discourse. Herbel-Eisenmann and Breyfogle (2005) described funneling as an interaction pattern in which the teacher asks a series of step-by-step questions to guide students to a correct response. Focusing, in contrast, is a conversation pattern guided by students' thinking. During focusing, after asking a question, the teacher listens to student responses and asks follow-up questions to better understand students' thinking, much like the competent questioning (Moyer & Milewicz, 2002) that is encouraged during clinical interview interactions. Undergraduates read the Herbel-Eisenmann and Breyfogle article early in the summer, act out the examples of funneling and focusing it contains, and refer back to it throughout the project. Focusing patterns of discourse contribute to more robust student learning, and at the same time they yield more informative qualitative data for analysis (Lesseig, Casey, Monson, Krupa, & Huey, 2016). Discussions about optimal patterns of questioning also fit naturally into methods courses, where smaller-scale research projects of this nature may occur.

Undergraduates are the lead teachers during the lessons; mentors intervene only if the undergraduates request it or if they are struggling a great deal to carry out the lesson plan. For example, during one lesson, an undergraduate pair had planned to encourage students to act out word problems using manipulatives. When they struggled to relinquish control of the manipulatives to the children, the mentor intervened to encourage children to take the manipulatives and explain what they represented in the context of the problem. On another occasion, one of the undergraduates was ill for most of the week and unable to participate in collaborative planning, so the mentor served as the lead teacher for one lesson. When such interventions take place, though, responsibility for assuming the role of lead teacher is returned to undergraduates as soon as possible.

In deciding whether or not to intervene during any given lesson, we bear in mind that making mistakes when teaching can sometimes be valuable. Just as it is important for teachers to help students see mistakes they make in doing mathematics as learning opportunities (Hiebert et al., 1997), it is important for teacher educators to help prospective teachers use the mistakes they make when teaching as opportunities for professional development. Making mistakes actually stimulates brain growth, and that growth is greatest when accompanied by a mindset that one can improve with effort (Boaler, 2016). For instance, the group that had trouble relinquishing control of manipulatives also found themselves funneling students during classroom discourse on more than one occasion. As they watched their lesson videos with their mentor, they readily identified their funneling patterns and made it a goal to replace them with focusing discourse. The transition to focusing was not immediate; they were frequently disappointed with themselves when they realized they had reverted to funneling in some lessons. However, making that mistake helped them set an important professional goal that they gradually made progress toward over the course of the summer. The undergraduate pair needed to be the lead teachers in order to make the mistake and work to resolve it. So, PATHWAYS mentors seek to help undergraduates identify and address their mistakes while teaching rather than avoiding mistakes at all costs; a similar instructor mindset about learning from mistakes can enhance undergraduates' initial experiences teaching mathematics during methods courses and internships as well.

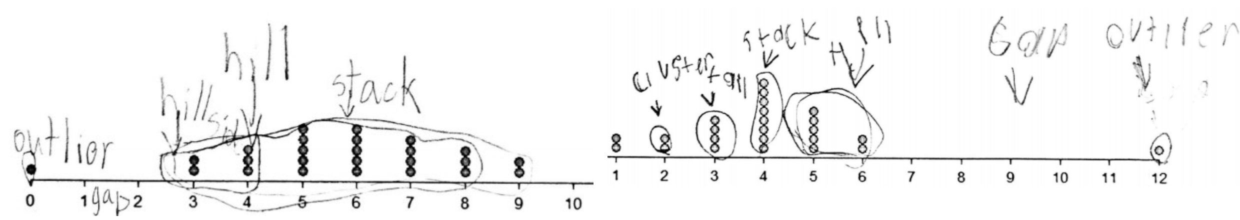
Data Analysis and Instructional Re-Design

Undergraduates transcribe videos of their lessons after teaching them. Qualitative analysis of each lesson takes place as the mentor/undergraduate triad views the lesson video while going through its transcript. Student reasoning codes developed during clinical interview coding are collaboratively assigned to relevant portions of the transcript. New codes are developed as necessary and added to the codebook. Code abbreviations assigned to students during each segment of the lesson are recorded in a matrix. A portion of a completed matrix is shown in Figure 3. The matrix allows undergraduates to track how a student's reasoning developed over the course of a lesson, compare reasoning across students, and trace students' reasoning across lessons by looking across multiple lesson matrices.

Segment	**Summary of instructional activities**	**Student reasoning codes**	
1	Introduction to data set	S1: Mark	RDP
		S2: Nate	
		S3: Emma	
		S4: Eva	
2	Answering questions about specific dots on the dot plot in TinkerPlots	S1: Mark	RDP
		S2: Nate	RDP, RCG
		S3: Emma	RDP
		S4: Eva	RDP
3	Talking about features of the dot plot and placing labels	S1: Mark	FS, FAF-G, FAF-MS
		S2: Nate	FAF-MS, FS, FAF-G
		S3: Emma	FS
		S4: Eva	FS

Figure 3. The first three segments of a coding matrix completed collaboratively by an undergraduate/mentor triad

Examining the completed matrix for a lesson provides a starting point for designing the next lesson. For example, the group creating the matrix shown in Figure 3 noticed that their students generally showed the ability to read values from dot plots (the RDP abbreviation) and had begun to focus on aggregate distribution features in dot plots (FAF) such as gaps (FAF-G) and multiple stacks clustered together (FAF-MS), as illustrated in the student work samples shown in Figure 4. One of the major goals for the summer was to help students identify typical values by looking for multiple stacks near the centers of distributions, but students persisted in focusing only on individual stacks (FS) on many occasions when they were asked to identify typical values. They usually looked only for the tallest stack in any given dot plot when asked a typical value question such as determining the typical number of red candies in a group of bags. Hence, the undergraduate/mentor triad decided to introduce dot plots with only a single point on each data value, temporarily eliminating different-sized stacks, and asked children to identify typical values in such data sets. This helped prompt the children to look horizontally across the dot plot to group together different values near the center of the distribution during typical value tasks. They continued to look horizontally across stacks near the center when different-sized stacks were later re-introduced. Each mentor/undergraduate triad looks for patterns in their data in the same manner in order to design instruction with the potential to move students' thinking toward desired learning goals.



Mark's written work　　　　Nate's written work

Figure 4. Written work indicating that students were beginning to attend to aggregate features of distributions

Retrospective Analysis

Student reasoning codes continue to accumulate as each lesson is taught and analyzed throughout the summer. As the summer progresses, undergraduates are asked to look across lessons to group codes into families and describe their relationships with one another, a process that has been called axial coding (Corbin & Strauss, 2008). As new codes are introduced, undergraduates are to continuously re-organize their axial coding document as necessary. Additionally, they are to group their lessons into three clusters that serve to tell the beginning, middle, and end of the children's mathematical reasoning journey over the summer (during a smaller-scale project in a methods course, the beginning, middle, and end can consist of three interactions with a student rather than three clusters of interactions with an entire group). At the end of the summer, each group summarizes their three clusters on a poster along with the pre- and post-interview results. A sample poster is shown in Appendix A. The posters also contain information about the group's literature review, research questions, and research methods. Posters are summarized with abstracts and oral presentation slides. Undergraduates present their research at an end-of-summer event on campus that their friends, family, and university faculty are invited to attend.

PATHWAYS Model Outcomes

We use several assessments to track undergraduates' experiences in PATHWAYS. Before undergraduates conduct post-interviews with children, we ask them to anticipate how the children will respond to key items. To obtain a snapshot of undergraduates' growth in knowledge of content and students (KCS, Hill, Ball, & Schilling, 2008), we compare their abilities to anticipate children's thinking at the end of the summer against their abilities to do so at the beginning before they have conducted pre-interviews. We administer the Survey of

Undergraduate Research Experiences (SURE; Lopatto, 2008) to gain information about participants' future career plans before and after the experience. Undergraduates take the Undergraduate Research Experience (URE) survey (Kardash, 2000) before and after as a self-assessment of growth in research skills. The Undergraduate Research Student Self-Assessment (URSSA; Weston & Laursen, 2015) serves as an additional assessment of perceived growth and future professional plans. Survey questions designed by Banilower et al. (2013) inquire specifically about participants' growth as teachers. Annual post-project follow-up surveys ask former participants to explain how PATHWAYS has influenced them professionally. We also maintain a list of the presentations undergraduates make at conferences and publications they co-author with mentors. Salient results from this assessment corpus are summarized in Table 1 and described below in terms of how the experience helped participants grow both as teachers and as researchers. We focus mainly on the eight undergraduates in the fifth PATHWAYS cohort, which included all of the program features described in this report.

	Key findings from PATHWAYS assessment data
Undergraduates' growth as teachers	• Persistence in pursuing teaching certification • Replication of PATHWAYS model components in classrooms after graduation • Improved knowledge of content and students related to mathematics content focus of summer research
Undergraduates' growth as researchers	• Increased likelihood of completing graduate study • Framing and refining research questions • Drawing upon professional literature • Disseminating research at conferences

Table 1. Summary of key areas of growth for PATHWAYS undergraduates

Growth as Teachers

When taking the URSSA, seven of the eight undergraduate researchers in the fifth cohort reported that after the experience they were either “more likely” or “extremely more likely” to continue to pursue teaching certification. Follow-up surveys from previous cohorts indicated that after receiving teaching certification, key elements of PATHWAYS continue to influence instruction; one past participant reported, “I question my students much more and push for much deeper reasoning than, I believe, I would have if I had not partaken in the PATHWAYS experience.” Another stated, “I still conduct research on my students every single day! I look at what worked and didn't work and how I can make them succeed.” On the Banilower et al. (2013) survey, undergraduates in the fifth cohort reported increases in their preparedness to teach students with learning disabilities and to provide enrichment opportunities for gifted students. Children with these exceptionalities had been part of their PATHWAYS cohort; each summer, we try to create microcosms of typical classrooms by maximizing diversity along the dimensions of race, gender, and schooling experiences to the extent possible when reading parent applications to select the children who will participate.

Each pair of undergraduates in the fifth cohort exhibited gains in KCS. A pair that worked to teach third graders to make sense of word problems was able to anticipate how children would use number lines and hundreds charts to solve some of the problems on their interview script after seeing students use these tools over the summer. They also learned to anticipate student strategies such as acting problems out with manipulatives and student mistakes such as applying operations to numbers in a problem without regard for context. A pair working to build fourth graders’ conceptual understanding of multiplication learned to anticipate that students might add two numbers in a problem when multiplication would be appropriate, and that students might use pictures of objects involved in a problem to directly model a solution. Undergraduates working to build sixth graders' knowledge of statistical distributions better anticipated how students would focus on visual features of graphs, such as tall stacks, gaps, and clusters. Those working to support seventh graders' understanding of probability improved their anticipation of the nature of proportional and combinatorial reasoning in students' responses. Such gains in KCS are important because anticipating students' thinking helps teachers design instruction that is responsive to students' needs (Jacobs & Spangler 2017) and guide classroom discourse in productive directions (Stein, Engle, Smith, & Hughes, 2008). A participant from a past cohort affirmed, "It (PATHWAYS) also enabled me to predict common misconceptions and develop small group instruction strategies that I implement in my own classroom today."

Growth as Researchers

PATHWAYS assessment surveys suggest that the project had an impact on undergraduates' plans for graduate study and skills to engage in research. According to SURE results, before starting the project, three undergraduates from the fifth cohort had plans to pursue a master's degree; after having the experience in research, five had such plans. On the URSSA, four reported they were extremely "more likely" to enroll in a master's program after the project and two reported they were "extremely more likely" to enroll in a Ph.D. program. On the URE, undergraduates rated their research skills before and after the project; three believed they improved their use of primary research literature and three remained at the same level. Six reported that their skills in identifying a specific question for investigation improved (sample research questions from PATHWAYS appear in Appendix B). Four indicated that their ability to reformulate an original hypothesis improved, and two reported no change. Four felt an improvement in relating results to the “bigger picture” in their field, while three didn’t notice a difference. These results indicated that the project helped motivate some to pursue graduate degrees and develop the skills to do so. Notably, participants from past cohorts who pursued graduate degrees did not necessarily plan to leave their schools; one such individual, for example, wrote, “I will get my masters within the next 5 years and remain in the classroom.” Another stated “I plan to continue my career as a high school mathematics teacher, while I work towards obtaining my Masters in Administration and Leadership. After obtaining this degree, I would like to pursue being a math coordinator for my county.”

After completing the PATHWAYS summer experience and program surveys, undergraduates disseminate their research to a broader audience. Undergraduates from the first five PATHWAYS cohorts gave 13 presentations at the National Conference on Undergraduate Research, five presentations at the annual NSF/CUR undergraduate research symposium, six presentations at the National Association of Professional Development Schools Conference, and six others at various local and regional venues for presenting undergraduate research. With their mentors, the undergraduates collaboratively authored five articles that were accepted in peer-reviewed mathematics education journals. Undergraduates have reported that these dissemination experiences helped them develop identities as future teachers and researchers who share their findings and strategies in a larger professional community (Groth & McFadden, 2016). One past participant, for example, wrote, “PATHWAYS was my first experience with research and presenting at conferences. I’ve since presented my own research in both regional and national conferences using some of the skills I learned during the project.”

PATHWAYS Replication in a Methods Course

To this point in the article, we have described essential components of the entire structure of the PATHWAYS model. Readers interested in replicating the model on a large scale as an extra-curricular experience can consult Appendix C for key budget items to include. We anticipate that many readers, however, will be interested in smaller-scale replication within a course. So, next, we describe how the third author of this article used the core cycle of the model (Figure 1) as the basis for a mathematics teaching methods course assignment (Appendix D). Salient similarities and differences between large-scale PATHWAYS implementation and the small-scale implementation described below are summarized in Table 2.

Model feature	Full-scale implementation	Small-scale implementation
Pre-interviews	30 minutes per child per interview	1-3 main tasks per child per interview
Number of instructors per group of children	2	1
Number of children interviewed per group	4	1
Data gathering	Children's work, video recording, and transcription	Children's work and field notes
Qualitative data analysis	Organic coding	General guiding questions given to support analysis
Lesson demo for peers	35-40 minutes for demo and feedback	n/a

Number of lessons based on interview observations	7	3
Post-interviews	30 minutes per child	n/a
Research products	Research poster, abstract, and oral presentation	Response to set of guided analysis questions

Table 2. Similarities and differences between large-scale and small-scale PATHWAYS replication.

Some differences between the large-scale and small-scale implementation shown in Table 2 pertain to the length and number of pre-interviews and the number of tasks. Undergraduates spend 30 minutes interviewing four children at the start of the large-scale summer program, but administer just 1-3 key tasks to a single student for a methods assignment pre-interview. For these assignments, undergraduates retain children's written work and take field notes but do not video record the interaction.

To make qualitative data analysis feasible for a larger group within a reasonable amount of time, undergraduates are given a guiding set of questions to use in analyzing children's work. The guiding questions resonate with the five strands of mathematical proficiency (Kilpatrick, Swafford, & Findell, 2001), a model we used in early iterations of PATHWAYS to facilitate qualitative data analysis (Groth, 2017). Key questions given to undergraduates to analyze children's work on tasks and their connections to the five strands include:

- Does the student conceptually describe and identify the steps of the problem? (Conceptual Understanding)
- Does the student use correct procedures or invented strategies to solve the problem? (Procedural Fluency)

- Does the student use pictures, drawings, symbols, or manipulatives to represent the problem and its solution? (Strategic Competence)
- Does the student understand how to interpret the problem's solution? (Adaptive Reasoning)
- Does the student exhibit confidence and persist when solving the problem? (Productive Disposition)

In answering each of these questions, undergraduates are required to create a narrative with details from the interview to support their responses. They are then to explain steps they will take to help the child develop deeper understanding during three follow-up sessions.

Even though small-scale replication of this type does not provide opportunities for undergraduates to engage in more organic analyses of qualitative data or a large number of instructional follow-up sessions, the guiding questions based on the five strands do prompt reflective thinking that contributes to lesson design. For example, an undergraduate who investigated a child's thinking in multiplying 124×5 and 75×43 made observations useful for guiding instruction. During the pre-interview, the child was able to multiply 124×5 using a partial product table (Figure 5), but was not able to do so for 75×43. The child also was not able to decompose the numbers using base 10 pieces. The undergraduate conjectured that the child did not "fully grasp the ideas of the base ten system and place value," as evidenced by the child's ability to use a partial product table to multiply a one-digit number by a three-digit number but not to multiply a two-digit number with another two-digit number.

	100	+	20	+	4
5	500		100		20

Figure 5. Using a partial product table to multiply 124×5

Given her conjectures about the child's thinking about multiplication, the undergraduate set out to help the child understand how number decomposition can help solve many types of multiplication problems and that it is the main reason partial products tables work. During the first session, she prompted the child to represent multiplications of three-digit numbers by one-digit with both base 10 pieces and partial product tables. Having success in the first session with this, they did the same with multiplication of two-digit by two-digit numbers during the second and third sessions. At the end of the third session, the undergraduate observed about the child:

> She was able to identify how to decompose the two digit numbers. She was also able to place them in partial product table. She was able to identify what the numbers meant, why they were place in the spot that that they were, and what names of each place value space was called.

The undergraduate also observed that the child had not yet connected the partial product table to the conventional algorithm for multiplication taught in school, and hypothesized that making this connection would be a valuable goal for future lessons.

Conclusion

The PATHWAYS core repetitive cycle (Figure 1) provides an engine for both extra-curricular and course-embedded research experiences in mathematics education for undergraduates. In both cases, undergraduates can learn how careful qualitative analyses of classroom data contribute to improvement of instruction. Opportunities to gather and analyze qualitative data are abundant in classrooms; since such data often deal directly with how students approach specific problems, they have great potential to inform teachers' lesson design. Systematic analyses of such data, whether through guided questions or more organic means, help teachers notice key elements of children's mathematical thinking and set appropriate goals for instruction. Engaging undergraduates in systematic analysis of children's thinking thus sets the stage for improvement of teaching while simultaneously providing an introduction to formal mathematics education research. Given these benefits, the field of mathematics education is well-served to seek ways to incorporate undergraduate research in both methods courses and in larger-scale extra-curricular experiences; PATHWAYS provides a model to support both endeavors.

Acknowledgment

This article is based upon work supported by the National Science Foundation under Grants DRL-1356001 and DUE-1658968. Any opinions, findings, and conclusions or recommendations expressed in this article are those of the author and do not necessarily reflect the views of the National Science Foundation.

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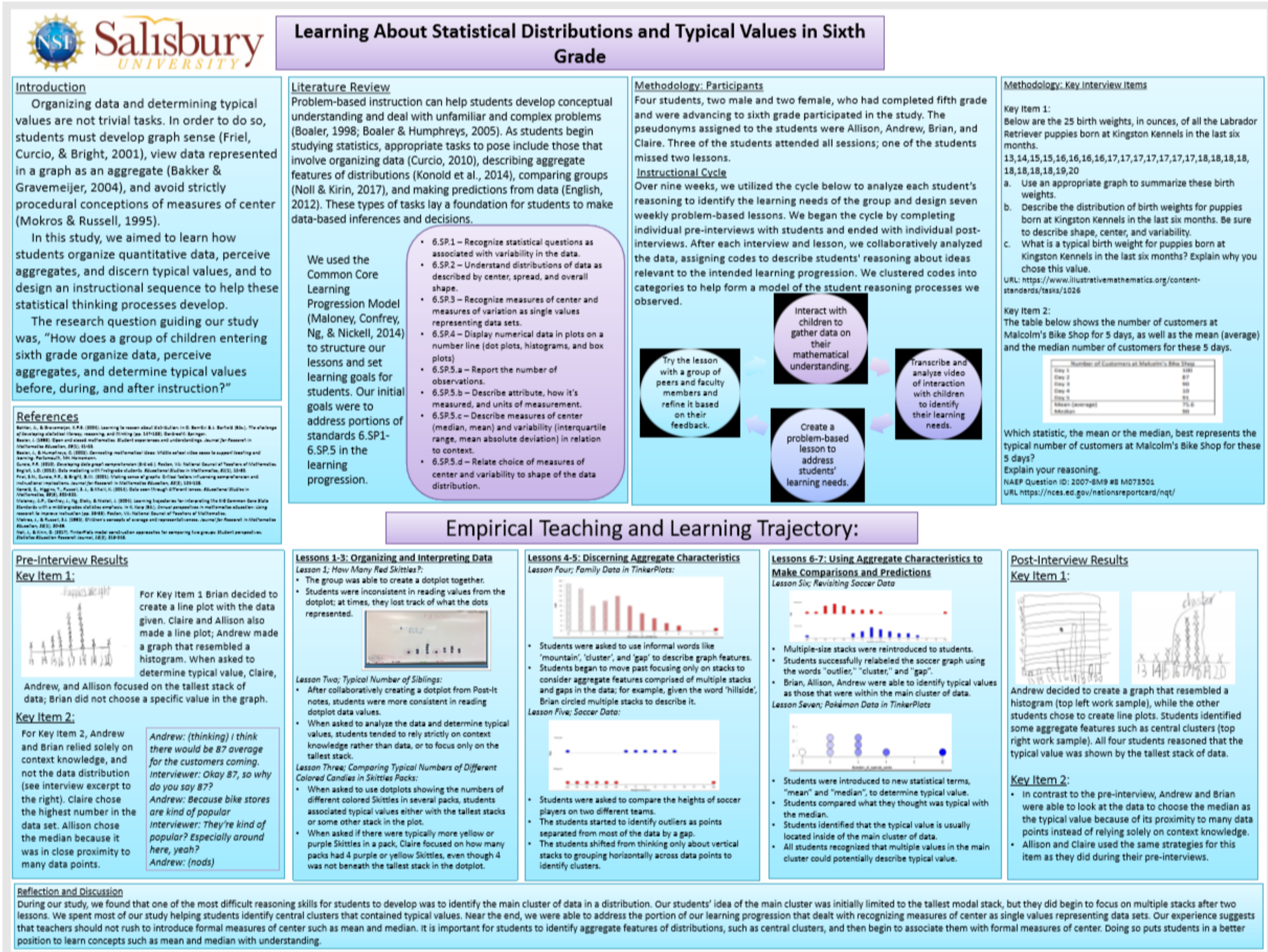
Appendix A: Sample Undergraduate Research Poster



Appendix B: Sample PATHWAYS Research Questions

Grade Level	Mathematics Content	Research Questions/Purpose Statements
3	Word problems	Which strategies do students use to solve word problems before, during, and after our instructional sequence? To what extent, and how, do the students' strategies change and develop over the course of instruction?
4	Fractions	How do students' abilities to give conceptual explanations of fraction equivalence develop?
5	Multiplication	The purpose of this study was to design a problem sequence to develop students' abilities to select strategies that are appropriate and efficient for solving problems requiring multiplicative reasoning.
6	Statistical distributions	How does a group of children entering sixth grade organize data, perceive aggregates, and determine typical values before, during, and after instruction?
7	Probability	How do children's pre-existing notions of probability influence their probabilistic problem solving? What sequence of teaching methods can develop children's thinking about empirical and theoretical aspects of compound probability situations?

Appendix C: Key Budget Items for Large-Scale Replication of the PATHWAYS Model

Item	Yearly Cost	Description
Undergraduate stipends	$5,000 per undergraduate	Undergraduates received $500 weekly stipends for completing all assigned tasks over 10 weeks.
Undergraduate meal stipends	$1,000 per undergraduate	Each undergraduate received $1,000 for the summer to cover meals.
Undergraduate mileage	$400 per undergraduate	Each undergraduate could be reimbursed up to $400 for mileage related to the project.
Undergraduate housing	$1200 per undergraduate	Non-local undergraduate researchers were provided on-campus housing.
Parent stipends	$3680	Parents of 16 children received $20 stipends for attendance at nine weekly sessions and a $50 bonus for attendance at all sessions.
Faculty mentor stipends	$4,000 per mentor	Mentors received stipends comparable to the amount paid for summer teaching.
Graduate student assistant	$2,500	Graduate student helped compile each group's codebook and manage logistics with children.
Project evaluator	$2,500	Project evaluator administered and summarized assessments of program effectiveness.
Conference travel	$3,700	Undergraduates and faculty mentors traveled to professional conferences to disseminate results.
Video and audio equipment	$3,000 (first year only)	All interviews and instructional sessions were recorded and transcribed for qualitative analysis.

Appendix D: A Methods Course Assignment that Incorporates the PATHWAYS Core Cycle

Gray sections do not need responses. All other sections do need responses.	
Example of Interview	**Watch the video** https://www.bing.com/videos/search?q=diognostic+math+interview&&view=detail&mid=63E46245EB89F27D0CB663E46245EB89F27D0CB6&&FORM=VRDGAR. After you have watched the video work below to develop your own diagnostic interview.
Goal of Assignment	The goal of the interview is to find out what they DO NOT KNOW. If in the interview your original prompt only finds out what they do know how to do, you need to continue on with prompts in order to determine what they DO NOT KNOW how to do. If the prompt is too difficult, you may need to use one of your easier prompts to find out where the line is between what they know and do not know. Your goal may be for them to add 3 digit numbers together but they have no idea what to do when you probe their thinking. You may want to explain with manipulatives how to add two 2 digit numbers. If they do not know how to do this, you may need to go to 2 one digits numbers where they have to regroup. Another example may be that the child reads the word problem and doesn't know what to do. Can they do the math in the word problem? You may want to talk and ask a similar problem but without it being in a word problem. Can the child add 126 + 8? Can the child divide 124 by 4? If they cannot do addition and division, they are not going to be able to solve the prompt provided below.
CCSS Domain and Standard	
Rationale	**Why is this mathematical concept important and how does it relate to other mathematical concepts?**
Materials Needed	
Interview Prompts	**Prompt:** **What prompt could you ask if this one was too easy?** **What prompt could you ask if this one was too hard?**
Questions	**Potential questions that will be asked to probe students' thinking (5 needed)**
Conceptual Understanding	**Have the student describe how he or she is solving the problem.**

	Does the student use pictures, drawings, symbols, or manipulatives to represent the problem and its solution? **Was the student able to interpret the language in the problem?** **Does the student connect the correct operations to the steps of the problem's solution?** **Does the student understand how to interpret the problem's solution?**
Procedural Fluency	**Does the student use correct procedures or invented strategies to solve the problem?** **Does the student conceptually describe and identify the steps of the problem?**
Disposition	**Does the student exhibit confidence and persist when solving the problem?**
Ineffective Strategies	**Does the student use tricks or specific non-thinking or nonproductive strategies to solve the problem?**
Prerequisite Knowledge or Misconceptions	**Does the student demonstrate misunderstandings in solving the problem? For example, does he or she mistake the problem for a different type of problem?** **Does the student lack prerequisite understandings and skills for solving the problem? For example, is he or she not fluent with division or working with remainders in division?**
Analysis of Interview	**What does the student understand and not understand?**
Academic Steps	**What steps will you take to help the student with this skill or a related skill?**
3 Dates of Work and progress	**Date: ___________ Today we worked on…. he/she was able to …. But still had difficulty with ….** **Date: ___________ Today we worked on…. he/she was able to …. But still had difficulty with ….** **Date: ___________ Today we worked on…. he/she was able to …. But still had difficulty with ….**

What academic steps should be taken now in order to help the child improve?	