

# A molecular parameter to scale the Gibbs free energies of adsorption and micellization for nonionic surfactants\*

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## Abstract

Over the past forty years, there have been many attempts to develop molecular based models for the free energy of adsorption and micellization. Historically, molecular models for  $C_iE_j$  surfactants have suggested a correlation between the length of the chemical groups and thermodynamic parameters, e.g. the critical micelle concentration (CMC), the equilibrium constant ( $K$ ), and the maximum surface concentration ( $\Gamma_\infty$ ). However, there are no current models to date that satisfactorily capture the large range of  $C_iE_j$  chemistry. In this paper, we propose a new thermodynamically consistent model that depends on two simple molecular parameters. More specifically, we combine the well-known linear dependence of CMC and  $K$  on the number of carbons,  $N_C$ , with a nonlinear dependence on the molecular hydrophobic mass fraction,  $y_{phob}$ . An extensive review of the  $C_iE_j$  literature is analyzed to parameterize and validate the models. The success of the model is demonstrated by the collapse of all experimental data onto master curves. These models represent the first successful phenomenological theory which is able to collapse a broad range of experimental data for  $C_iE_j$  surfactants. Furthermore, the free energy models are capable of predicting the maximum surface concentration at the air-water interface, thus providing a link between the physics of adsorption and self-assembly. This paper summarizes many years of experimental and theoretical understanding of surfactant structure-property relationships for  $C_iE_j$ 's, and the resulting theory quantitatively explains many hypotheses on the effect of surfactant chemistry on interfacial thermodynamics. This theory is a step towards engineering interfacial thermodynamics by chemical design which enables prediction of interfacial properties of novel surfactants without extrapolation.

**Keywords:** surfactants, micellization, adsorption, structure-property relationships

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## 1. Introduction

Understanding the link between the chemical structure of surfactants and their fundamental interfacial properties is essential for the design and selection of surfactants for industrial processes. Currently there are no comprehensive correlations between molecular parameters and experimentally determined interfacial parameters, e.g. critical micelle concentration (CMC), maximum surface

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\*This work is dedicated to the memories of Jacob N. Israelachvili (1944-2018) and Milton J. Rosen (1920-2020), whose pioneering works in the field of surfactant self-assembly [1, 2, 3, 4] and structure-property relationships [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20] laid the groundwork for the field of interfacial thermodynamics.

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concentration ( $\Gamma_\infty$ ), and the equilibrium adsorption constant ( $K_x$ ). Several attempts to correlate CMC to molecular parameters using thermodynamic models, e.g. group contribution theory [21, 22, 23, 24, 25, 26] and statistical mechanics [27, 28, 29, 30], have resulted in unsatisfactory comparisons to experimental data. In this paper we demonstrate a simple correlation of thermodynamic parameters with the length of the hydrocarbon chain and the molecular hydrophobic mass fraction.

The most studied model surfactants are poly(ethylene glycol) monoalkyl ethers ( $C_iE_j$ ) because of their simple chemistry and their use in a broad range of engineering systems [31]. The most successful attempt to correlate CMC of  $C_iE_j$ 's with molecular parameters is the scaling of CMC with the number of alkyl repeat units,  $N_C$ , i.e. the hydrophobe, with a fixed number of ethylene glycol repeat units,  $N_{EO}$ , i.e. the hydrophile [32, 33]. However, it has been shown that this scaling does not capture the relationship of CMC with constant  $N_C$  and varying  $N_{EO}$ , see Figure S1 in the supplementary materials. Note that more complicated scalings with  $N_C$  and  $N_{EO}$  have been proposed with limited success [34, 24, 4, 35, 36, 37, 38, 39, 32]. However, no scaling to date is capable of broadly capturing/predicting the change in CMC with  $N_C$  and  $N_{EO}$ .

On the other hand, many studies have attempted to find correlations between the hydophilic-lipophilic balance (HLB) and surfactant properties as a function of chemistry. The HLB is an empirical construct put forth by Griffin in the mid 1950s [40], but does not refer in general to any one specific equation. While Griffin's definition had no thermodynamic rationale, later studies focused on developing an HLB definition that is thermodynamically consistent [41]. While HLB is mostly discussed with respect to emulsion stability, it has been successfully correlated with the CMC of fluorinated and ionic surfactants [42, 43], where the CMC is argued to go as the exponential of HLB. A good resource for HLB correlations can be found in Kruglyakov [41]. The HLB was also used by Israelachvili and co-workers [4] in the derivation of the aggregation number. The majority of HLB definitions show dependencies of key thermodynamic parameters on some relative ratio of the amount of hydrophobe to hydrophile. This work aims to develop consistent equations that capture the effects of  $N_C$  and the relative balance between length of hydrophile and hydrophobe to correlate thermodynamic parameters for micellization and adsorption.

We start by defining a new parameter that quantifies the mass fraction of hydrophobe to the mass of the surfactant molecule,  $y_{phob}$ . We propose that this parameter characterizes the balance between the hydrophobe and hydrophile for a given surfactant. Note that similar parameters have been proposed in the past [44]. We acknowledge that a partial molar volume could also be used, however, this requires an independent measure of molecular density, not readily available. Historically, our definition of  $y_{phob}$  is related to Griffin's original definition of HLB, i.e.  $HLB \approx 20(1 - y_{phob})$  [40]. It is important to note that this relationship is only explicitly true for Griffin's definition. In the polymer literature,  $y_{phob}$  has been used to correlate the micellization propensity of polymers. For example, Widder et al. found that the hydration of an amphiphilic polymer goes inversely with  $y_{phob}$  such that interactions with water are favored over micellization [45].

In this paper, we propose a simple functional form of the free energies of micellization,  $\Delta G^{mic}$ , and adsorption,  $\Delta G^{ads}$ , on both  $N_C$  and  $y_{phob}$  with two fitting parameters. A wide range of experimental data for  $C_iE_j$ 's ( $N_C \geq 6$  and  $N_{EO} \geq 1$ ) is aggregated and used to parameterize the models. The models are compared to experimental data, and show a collapse of thermodynamic parameters onto master curves. We then propose a theoretical dependence of  $\Gamma_\infty$  on  $\Delta G^{mic}$  and  $\Delta G^{ads}$  considering the Langmuir isotherm. The models for the free energies are tested and validated by comparison with experimental data of  $\Gamma_\infty$ . When possible, we present comparisons of the new theory with previously reported models in the literature. Ultimately, we present a simple theory that

is capable of quantitatively explaining observed trends of thermodynamic parameters on surfactant chemistry.

## 2. Literature Review and Analysis

The surfactant data used in this paper represents a comprehensive set from the scientific literature. All attempts have been made to fully represent the available data for  $C_iE_j$  surfactants at the air-water interface at 25 °C, however the authors acknowledge that some data may be inadvertently missed. Among these sources, critical micelle concentrations are by far the most commonly available values. Some works include surface tension isotherms which include the CMC and surface tension as a function of surfactant concentration. Furthermore, dynamic interfacial tension is sometimes available. In the case where isotherm parameters were not reported, the authors digitized the experimental data for the purpose of performing a nonlinear least squares regression of the appropriate isotherm parameters. Table 1 summarizes the various references for surfactant data by chemistry, including the type of available data. Interestingly, there are numerous studies on the same  $C_iE_j$ 's, while others remain uncharacterized.

The vast majority of studies determine the CMC from surface tension isotherms, i.e. the transition to a zero/small slope of  $\gamma$  versus  $\log C$ . The surface tension isotherms were measured using Wilhelmy plate [46] and du Noüy ring [47] tensiometers, drop weight method [48], pendant drop/bubble shape analysis [49, 50], and a microtensiometer [51]. Each of these methods have inherent experimental error, however, most sources report high degrees of internal reproducibility. Other methods reported include: calorimetry[52] and spectrophotometry [53]. The deviation between measurement types is expected to be small [53]. Therefore, the largest errors are likely to be derived from: (i) errors in solution concentration, (ii) system impurities/contamination, and (iii) systematic errors. An additional source of error is likely due to surfactant dispersity as a result of synthesis or imperfect purification. For instance, some groups report intensive fractionation to obtain high levels of surfactant purity [13], while others use technical grade surfactants with no purification stage [33]. We have chosen to report all datasets as individual experimental values rather than averaging or applying a filter. These decisions have been made to minimize any introduced bias into the data.

## 3. Results and Discussion

There are two free energies that govern the thermodynamics of surfactants in solution: the free energy of adsorption  $\Delta G^{ads}$  and the free energy of micellization  $\Delta G^{mic}$ . The previous scaling approaches with  $N_C$  and HLB suggest that the thermodynamics should depend on both  $y_{phob}$  and  $N_C$ , such that:

$$\Delta G^{mic} = f(N_c, y_{phob}) \quad (1)$$

$$\Delta G^{ads} = g(N_c, y_{phob}) \quad (2)$$

More specifically,  $f$  and  $g$  are both expected to depend linearly on  $N_C$  [39, 6, 32]. There is currently no proposed functional form of the dependence of  $f$  and  $g$  on  $y_{phob}$  that consistently captures experimental data for a wide range of  $C_iE_j$  chemistries.

Table 1: References to Experimental Data for Surfactant Thermodynamic Parameters

$N_{EO}$	$N_C$					
	6	8	10	12 <sup>4</sup>	14	16 <sup>4</sup>
<b>1</b>		$[48]^2$ [54] <sup>3</sup> [55] <sup>3</sup>		$[56]^3$ [54] <sup>3</sup>	$[57]^2$	
<b>2</b>				$[13]^2$ [56] <sup>2</sup>	$[57]^2$	
<b>3</b>	$[47]^2$ [58] <sup>3</sup> [59] <sup>3</sup> [54] <sup>3</sup> [60] <sup>3</sup> [55] <sup>3</sup>	$[58]^3$ [59] <sup>3</sup> [54] <sup>3</sup> [55] <sup>3</sup>	$[58]^3$ [61] <sup>3</sup> [23] <sup>3</sup> [55] <sup>3</sup>	$[13]^2$ [54] <sup>3</sup> [55] <sup>3</sup>	$[57]^2$ [55] <sup>3</sup>	[55] <sup>3</sup>
<b>4</b>	$[47]^2$ [53] <sup>3</sup> [59] <sup>3</sup> [54] <sup>3</sup>	$[59]^3$ [52] <sup>3</sup> [55] <sup>3</sup>	$[59]^3$ [63] <sup>2</sup> [64] <sup>3</sup> [54] <sup>3</sup> [24] <sup>3</sup> [49] <sup>1</sup> [50] <sup>1</sup> [65] <sup>2</sup>	$[13]^2$ [59] <sup>3</sup> [56] <sup>3</sup> [54] <sup>3</sup> [55] <sup>3</sup> [46] <sup>2</sup> [50] <sup>1</sup> [66] <sup>2</sup>	$[54]^3$ [57] <sup>2</sup>	
<b>5</b>	$[47]^2$ [54] <sup>3</sup> [60] <sup>3</sup> [55] <sup>3</sup>	$[59]^3$ [54] <sup>3</sup> [60] <sup>3</sup> [52] <sup>3</sup> [55] <sup>3</sup>	$[53]^3$ [61] <sup>3</sup> [23] <sup>3</sup> [60] <sup>3</sup> [49] <sup>1</sup> [55] <sup>3</sup> [46] <sup>2</sup>	$[13]^2$ [59] <sup>3</sup> [23] <sup>3</sup> [49] <sup>1</sup> [68] <sup>1</sup> [55] <sup>3</sup> [46] <sup>2</sup>	$[55]^3$	
<b>6</b>	$[47]^2$ [69] <sup>2</sup> [61] <sup>3</sup>	$[70]^3$ [58] <sup>2</sup> [61] <sup>3</sup> [23] <sup>3</sup> [55] <sup>3</sup>	$[53]^2$ [58] <sup>3</sup> [61] <sup>3</sup> [23] <sup>3</sup> [71] <sup>2</sup>	$[58]^3$ [61] <sup>3</sup> [59] <sup>3</sup> [64] <sup>3</sup> [63] <sup>2</sup> [23] <sup>3</sup> [55] <sup>3</sup> [72] <sup>1</sup>	$[61]^3$ [23] <sup>3</sup>	$[70]^3$ [53] <sup>2</sup> [61] <sup>3</sup> [23] <sup>3</sup> [55] <sup>3</sup>
<b>7</b>			$[73]^3$	$[67]^2$ [13] <sup>2</sup> [23] <sup>3</sup> [55] <sup>3</sup>		
<b>8</b>		$[55]^3$	$[74]^3$ [75] <sup>2</sup> [23] <sup>3</sup> [49] <sup>1</sup> [55] <sup>3</sup> [51] <sup>1</sup>	$[74]^3$ [75] <sup>2</sup> [13] <sup>2</sup> [63] <sup>2</sup> [64] <sup>3</sup> [23] <sup>3</sup> [49] <sup>1</sup> [55] <sup>3</sup> [50] <sup>1</sup> [51] <sup>1</sup>	$[75]^2$ [23] <sup>3</sup> [55] <sup>3</sup> [57] <sup>2</sup> [50] <sup>1</sup> [51] <sup>1</sup>	$[54]^3$ [55] <sup>3</sup>
<b>9</b>		$[58]^3$ [23] <sup>3</sup> [55] <sup>3</sup>	$[58]^3$ [61] <sup>3</sup> [23] <sup>3</sup> [50] <sup>1</sup>	$[67]^2$ [23] <sup>3</sup> [55] <sup>3</sup> [76] <sup>1</sup> [66] <sup>2</sup>		$[53]^2$ [23] <sup>3</sup>
<b>10</b>				$[55]^3$		$[77]^3$ [55] <sup>3</sup>
<b>12</b>				$[67]^2$ [55] <sup>3</sup>		
<b>13</b>				$[55]^3$		
<b>14</b>			$[71]^2$			

<sup>1</sup>Dynamic and equilibrium surface tension data

<sup>2</sup>Equilibrium surface tension data

<sup>3</sup>Critical micelle concentration only

<sup>4</sup>Further critical micelle concentrations obtained from Esumi [33] for  $N_{EO}$  ranging from 9 to 63.

We introduce a new parameter defined as the molecular mass fraction of the hydrophobic moiety,  $y_{phob}$ , such that:

$$y_{phob} = \frac{M_w^{phob}}{M_w^{phob} + M_w^{phil}} \quad (3)$$

where  $M_w^{phob}$  is the molar mass of the hydrophobic moiety, i.e. the alkyl carbons, and  $M_w^{phil}$  is the molar mass of the hydrophile, i.e. the ethylene glycol groups and terminal hydroxide. For  $C_iE_j$ ,  $M_w$  can be simply defined using the number of alkyl carbons,  $N_C$ , and the number of ethylene glycol units,  $N_{EO}$ :

$$M_w^{phob} = 14.027 N_C + 1.008 \quad (4)$$

$$M_w^{phil} = 44.053 N_{EO} + 17.007 \quad (5)$$

$$y_{phob} = \frac{14.027 N_C + 1.008}{14.027 N_C + 44.053 N_{EO} + 18.015} \quad (6)$$

$y_{phob}$  will be used to determine the appropriate scaling of the Gibbs free energy of adsorption and micellization for the collected data on  $C_iE_j$  surfactants.

### 3.1. Critical Micelle Concentration

The free energy of micellization, defined as the change in free energy required to self-assemble molecules into a micelle,  $\Delta G^{mic}$ , can be written in terms of the mole fraction of surfactant at the critical micelle concentration,  $x_{CMC}$ :

$$f = \Delta G^{mic} \approx RT \ln(x_{CMC}) \approx -RT\alpha \quad (7)$$

where  $RT\alpha$  is defined by Israelachvili as a monomer-monomer ‘bond’ energy in the aggregate relative to the free monomer in solution [4]. Specifically,  $RT\alpha \approx \mu_1^0 - \mu_N^0$ , where  $\mu_1^0$  is the standard state chemical potential of a surfactant monomer in solution, and  $\mu_N^0$  is that of a surfactant in a micelle. Note that the approximation above is in the limit of the free monomer concentration approaching  $x_{CMC} = e^{-\alpha}$  and thus does not depend on micelle dimensionality [4]. See the supplementary material for the detailed derivation. Two fundamental relationships between  $x_{CMC}$  and the size of the hydrophobe/hydrophile have been proposed based on empirical correlations: the Klevens equation [35],

$$f = A + B \cdot N_C \quad (8)$$

and the more general equations proposed by Becher and Ravey and co-workers [37, 38]

$$f = A + B \cdot N_C + C \cdot N_{EO} + D \cdot N_C \cdot N_{EO}, \quad (9)$$

for  $C_iE_j$  surfactants. Note that both equations include a term with a linear dependence on  $N_C$ .

While these equations, particularly (9) can be empirically fit to correlate  $x_{CMC}$ , there are several drawbacks to this model: (i) there are four fitting parameters which cannot be determined *a priori* and (ii) a good fit requires that the fitting parameters depend on  $N_C$  and/or  $N_{EO}$ . For example, several studies[8, 75, 36] show that the values of  $A$  and  $B$  in (8) depend nonlinearly on  $N_{EO}$ . This fact is demonstrated in Figure S2, where a single Becher-Ravey equation has been fit to all available  $x_{CMC}$  data. The equation particularly fails at large  $N_{EO}$  and would require additional parameters or coefficients which depend on  $N_C$  and/or  $N_{EO}$  to quantitatively capture the entire experimental dataset. Furthermore, this model is by definition only good for interpolation and should be fit many times to small ranges of data to ensure a good fit.

Instead, we start from the thermodynamic relationship in (7) and parameterize the free energy based on the chemical structure. We propose a superposition of the contributions from  $N_C$  and a power law scaling with  $y_{phob}$  such that  $f \sim \ln(y_{phob}^n) + k N_C$ , where  $k$  relates to the incremental

contribution of methylene groups added to the hydrophobe. Several studies have empirically determined a value of  $k$ , which will be discussed below [8, 75, 58]. In order to determine the coefficients  $n$  and  $k$ , data is grouped with constant  $N_C$  and fit by linear least squares regression. The best fit lines for constant  $N_C$  are shown in Figure S3. This approach allows for a power law to be determined without assuming a fixed value for the dependence on  $N_C$ . The best fit parameters are  $n = -1 \pm 0.35$  and  $k = -1.2 \pm 0.01$ . Note that the errors represent the standard deviation of the best fit parameters. The average coefficients are plotted along with all datasets in Figure 1a. The model is in excellent agreement with all datasets for  $i < 16$ . For  $i = 16$ , the average model parameters under predict the data, which could be due to the technical grade surfactants used in these studies [33]. Figure 1b shows a master curve for all  $C_iE_j$  for the average model parameters. The master curve clearly captures the effect of both changing  $N_C$  and  $N_{EO}$  and shows quantitative agreement for most experimental datasets.

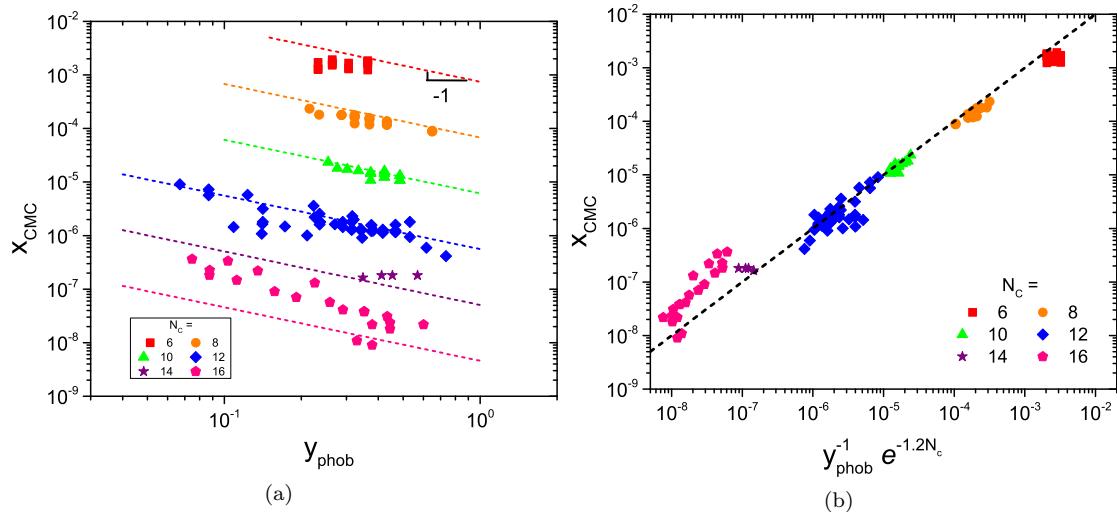


Figure 1: Critical micelle concentration as a function of (a)  $y_{phob}$  and (b) the proposed scaling. Lines in (a) represent the determined power law scaling of  $n = -1$  and  $k = -1.2$  while the line in (b) represents (10). The data are shown in log scale for clarity.

The agreement between the data and scaling suggests a functional form of:

$$x_{CMC} = y_{phob}^{-1} \exp(-1.2N_C), \quad (10)$$

$$f = \Delta G^{mic} = -RT (\ln y_{phob} + 1.2N_C) \quad (11)$$

This model incorporates the original arguments that the interaction energy scales with the length of the hydrophobe as well as the suggestions of Israelachvili and others that the interaction energy should scale with the mass (or volume) of the hydrophobe [4], including arguments for the HLB [42, 43]. The value of  $k = -1.2$  is in very good agreement with the works of Rosen and Meguro et al., who obtained a hydrophobe contribution to  $\Delta G^{mic}$  of  $-1.12RT$  and  $-1.15RT$  per methylene group, respectively [8, 75]. Corkill et al. admit that the value of  $k$  is difficult to determine due to experimental error, but that it is on the order of unity [58]. This value is slightly lower than what

is expected of a pure alkane chain, which has a factor of about 1.5 per added  $-\text{CH}_2-$  [39]. This supports the arguments of Aniansson that the free energy reduction for added  $-\text{CH}_2-$  is lowered by the proximity of the hydrophile to the micellar core [78, 4].

Equation (11) suggests that the hydrophobe and hydrophile contribute separately to the free energy of micellization, i.e. the hydrophile contributes  $-RT \ln y_{phob}$ , and the hydrophobe contributes  $-1.2RTN_C$ . However, it is not possible to change  $N_C$  without affecting both the contribution of the alkane chain and the hydrophile. In other words, an increase in  $N_C$  by unity corresponds to an increment in  $\alpha$  of 1.2. Subsequently for a constant  $N_C$ , an increase in  $\alpha$  by 1.2 corresponds to a factor of four decrease in  $N_{EO}$ . This strongly supports the arguments in the literature that suggested the hydrophile decreases the propensity of the hydrophobe to phase separate [58, 79]. From this it is clear that engineering a surfactant system for a desired  $x_{CMC}$  can be easily achieved by varying  $N_C$  compared to varying  $N_{EO}$ . This is consistent with dominance of hydrophobe interactions in the micellar core [4]. Furthermore, the dependence of  $x_{CMC}$  on  $y_{phob}$  captures the tendency for the hydrophile to resist micellization and agrees with the volume arguments for the free energy derived by Kumar and Tilton which relates  $x_{CMC}$  to the volume fraction of hydrophobe in a micelle, which is analogous to  $y_{phob}$  [80].

As discussed above, the Becher-Ravey equation was not able to fit the entire dataset. For comparison, the mean error of the Becher-Ravey equation for all  $x_{CMC}$  data is 280%, while the mean error for (10) is 35%, see supplementary for additional details. This demonstrates a significant improvement in molecular modeling for CMC.

Another important aspect is the effect of temperature on CMC. Figure 2 shows a small dataset for  $x_{CMC}$  at various temperatures plotted versus the proposed scaling determined at 25°C. The data span a moderate range in both  $N_C$  and  $N_{EO}$  as compared to Figure 1b. It is clear from Figure 2 that the model shows good agreement with different  $x_{CMC}$  measured at different temperatures. However, there is clearly a systematic deviation with increasing temperature, which suggests that one or more parameters may depend on temperature. The most likely parameter to depend on  $T$  is the methylene interaction coefficient  $k$ . This is supported by the fact that the hydrophile is known to dehydrate with increasing temperature, which would increase the interaction energy of the hydrophobe towards that of a pure alkane chain, i.e.  $k \rightarrow -1.5$  with increasing  $T$  [32]. In any case, the change would need to be very small. The model is expected to fail at a temperature driven phase change, i.e. cloud point.

### 3.2. Adsorption Isotherm Parameters

The free energy of adsorption, defined as the change in free energy required for a molecule to adsorb to an interface, is generally written as,

$$g = \Delta G^{ads} = -RT \ln K_x \quad (12)$$

$K_x$  represents the dimensionless equilibrium constant in terms of mole fraction, e.g. for a first order kinetic model, i.e. the Langmuir rate equation,  $K_x$  becomes, [81, 82]

$$K_x = \frac{\Gamma}{x \cdot (\Gamma_\infty - \Gamma)} \quad (13)$$

where  $\Gamma$  is the surface concentration of surfactant,  $\Gamma_\infty$  is the maximum surface concentration, and  $x$  is the solution mole fraction. Note that the measure of  $K_x$  is model dependent. For simplicity,

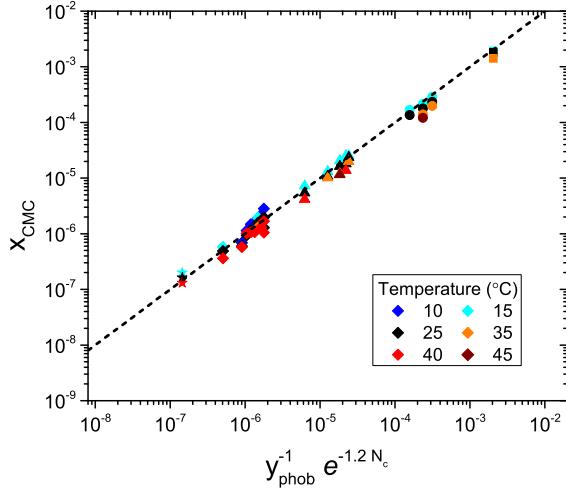


Figure 2: Critical micelle concentration for a select dataset as a function of the proposed scaling at different temperatures. Symbols correspond to different  $N_C$  as shown in the legend of Figure 1b. Data were obtained from [13, 70, 58, 74]. The dashed line represents (10).

we determine  $K_x$  from experimental data using the Langmuir-von Szyszkowski equation of state (EOS), given by

$$\gamma = \gamma_0 - RT\Gamma_\infty \ln(1 + x \cdot K_x) \quad (14)$$

where  $x$  is the bulk surfactant mole fraction,  $\gamma_0$  is the pure interfacial tension (72.01mN/m for  $T = 298.15$  K [83]), and  $\Gamma_\infty$  is the maximum surface concentration (a.k.a. the ideal monolayer concentration). While this EOS does not necessarily capture all the equilibrium adsorption physics of  $C_iE_j$ 's [50], it fits  $C_iE_j$ 's adsorption isotherms sufficiently well (see Figure S4). Furthermore, the development of more realistic physical models of interfacial kinetics has occurred over the many years since Langmuir's idealized interface [84]. However, the choice of the Langmuir equation of state is made because of: (i) its common use in describing surfactant phenomena [32, 85], (ii) its efficacy in capturing surface tension over the entire useful concentration range, and (iii) its simplicity. The Langmuir isotherm includes only two parameters, which physically correspond to the physics of adsorption. Although the Langmuir isotherm is often considered restrictive with regards to its assumptions (based on a lattice model), trends in parameters with molecular structure represent a good estimate of relative effects. More advanced isotherms could be evaluated in a similar manner, however additional parameters increase the level of uncertainty of fitting and effect the principle adsorption parameters unpredictably. Furthermore, the functional form of the free energy of adsorption for higher order isotherms depends on multiple equation of state parameters implicitly [86], making determination of  $\Delta G^{ads}$  from experimental surface tension data convoluted. By contrast, the Langmuir isotherm used in this work allows for a single measured parameter dependence of  $\Delta G^{ads}$ , i.e.  $K_x$ .

One might propose that  $K_x$  scales with either  $N_C$  or  $N_{EO}$ . However, Figure S5 shows very clearly that  $K_x$  does not scale with either. Instead, if we follow from Rosen that the free energies of micellization and adsorption should have analogous functional forms [8], then  $g \sim \ln(y_{phob}^m) + k'N_C$ , i.e.  $K_x$  should scale with some power law in  $y_{phob}$  and some linear contribution of  $N_C$ . The

dependence of  $\Delta G^{ads}$  on  $N_C$  has been determined previously by Rosen to be  $k' = -1.035$  per hydrophobe carbon for  $C_iE_j$ 's [8, 32] and by Kumar and Tilton to be  $k' = -1.375$  [80] illustrating that the contributions to the free energies of micellization and adsorption are not significantly different within the uncertainty of surface tension measurements. Note that this value may be dependent on chemistry, for example Rosen suggests a value of  $k' = -1.522$  for ionic surfactants [32]. Unfortunately due to the limited concentrations measured in the literature, there are not sufficient data to determine both  $k'$  and  $m$ . In order to reduce this uncertainty, we set the coefficient in the interaction energy of the hydrophobe equal to that of  $\Delta G^{mic}$ , i.e.  $k' = k$ . The fits are shown in Figure S6 with an average power law in  $y_{phob}$  of  $m = -4.9 \pm 0.5$ . Good fits are achieved for both datasets, however due to the restrictive number of points and narrow range of  $y_{phob}$ , the best-fit model values may change with a larger dataset.

Fig. 3a shows the model prediction using the average value of  $m$  for four datasets of constant  $N_C$ . The model sufficiently captures both the effect of  $N_C$  and  $y_{phob}$  for all datasets. Figure 3b shows a master curve of  $K_x$  for a small dataset of  $C_iE_j$ 's with different  $N_C$  and  $y_{phob}$ , suggesting the functional form:

$$K_x = y_{phob}^{-4.9} \exp(1.2N_C) \quad (15)$$

$$g = \Delta G^{ads} = -RT(-\ln y_{phob}^{4.9} + 1.2N_C). \quad (16)$$

It has been shown that fitting  $K_x$  to limited ranges of  $\gamma$  versus  $C$  lead to erroneous values due to the difficulty in assuring unique fits [87, 88]. Therefore, Figure 3 only considers reliably measured values of  $K_x$ , i.e. where  $\gamma$  versus  $\log C$  was measured to sufficiently low concentrations to confidently fit the value of  $K_x$ . More details are discussed in the supplementary material. Figure S9 shows comparison and good agreement of the model to the entire  $K_x$  dataset, with some expected outliers. This functional dependence on  $y_{phob}$  is in good agreement with the exponential dependence of  $\Delta G^{ads}$  on the hydrophile suggested by Rosen [32]. However, (15) disagrees with Kumar and Tilton who suggested that  $K_x$  is only a function of  $N_C$  [80].

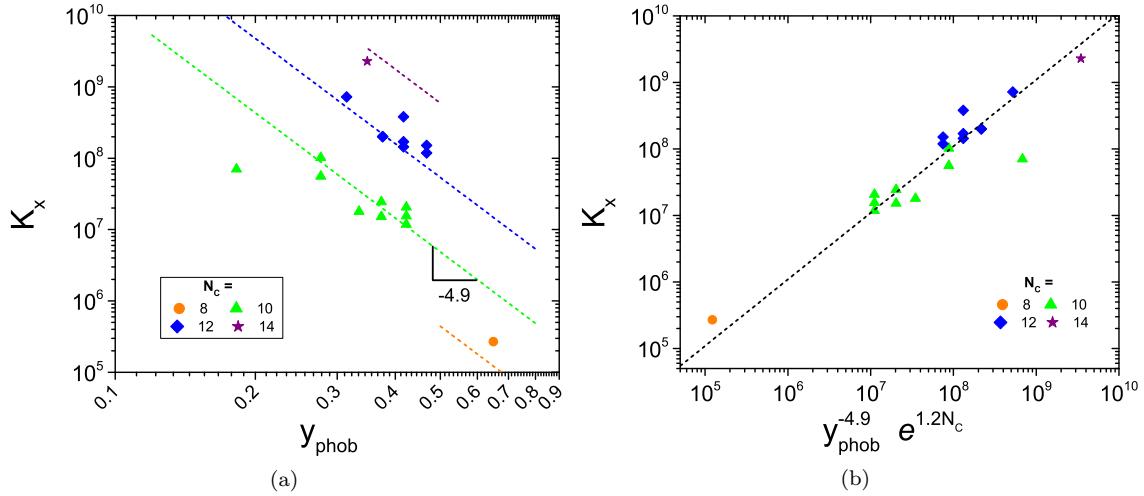


Figure 3: Adsorption equilibrium constant as a function of (a)  $y_{phob}$  and (b) the proposed scaling. In (a) lines represent power laws of  $m = -4.9$ . The dashed line in (b) represents (15).

The models described above allow for physical insight into adsorption and self-assembly thermodynamics. Equation (16) indicates that  $\Delta G^{ads}$  increases with increasing  $y_{phob}$ , while  $\Delta G^{mic}$  decreases with increasing  $y_{phob}$ . This implies that the adsorption interaction energy becomes less favorable for larger  $y_{phob}$  compared to the micelle interaction energy which becomes more favorable. Furthermore, unexpectedly, the equilibrium constant decreases with increasing  $y_{phob}$  and constant  $N_C$ , meaning desorption is more favorable for hydrophobes larger than the hydrophile. The dependence on  $N_C$  demonstrates a decreasing  $\Delta G^{ads}$  for increasing  $N_C$ . We postulate that this is due to the attractive interactions between adsorbed hydrophobes at the interface. By setting  $k' = k$ , we are suggesting that the molecular interactions in the micellar core (closely packed) are similar to that of the adsorbed (spread) state [89]. This is due to the hydrophobe's influence on the phase separation of surfactant to the interface, suggesting that the hydrophobe length equally drives the surfactant molecule to either micellize or adsorb, whichever removes the hydrophobe from solution. The hydrophile increases the propensity for adsorption with increasing  $N_{EO}$ . In other words, there is a competition between forces: the hydrophobe driving adsorption and the hydrophile remaining in solution. Our results agree with Huston and Larson, who show that in the dilute regime, a decrease in  $y_{phob}$  increases the affinity of the surfactant for the interface, i.e. makes the free energy more negative [90]. Huston and Larson suggest that this effect is solely due to increasing  $N_{EO}$ . However, Figure S7b clearly shows that both  $N_C$  and  $N_{EO}$  contribute to this affinity. Ultimately, we find that it is the relative length of  $N_{EO}$  to  $N_C$ , i.e.  $y_{phob}$ , that effectively scales the data. One physical explanation for these observed trends is that the magnitude and sign of the adsorption free energy depends on the hydration of the hydrophobe at the interface, which depends on the both  $N_{EO}$  and  $N_C$  [91]. More specifically, the smaller the  $y_{phob}$ , the more hydrated the hydrophobe remains at the interface, which requires a lower energetic penalty, i.e. smaller  $\Delta G^{ads}$ . For larger  $y_{phob}$ , the hydrophobe is significantly more dehydrated at the interface, which is less energetically favorable since it restricts the entropy of the hydrophile. These thermodynamic arguments are in line with Heusch, who hypothesized that the orientation/penetration of the interface by the surfactant is directly related to Griffin's HLB, i.e.  $y_{phob}$  [92]. These arguments would benefit from further molecular dynamic simulations.

The remaining thermodynamic parameter,  $\Gamma_\infty$ , has seldom been correlated to molecular parameters. By definition  $\Gamma_\infty$  is denoted by the asymptotic value of the Gibbs EOS, i.e. the slope of  $d\gamma/d\ln C$  near the CMC. This value of  $\Gamma_\infty$  is model independent. Note that  $\Gamma_\infty$  determined from nonlinear isotherms with additional parameters does not necessarily equal the  $\Gamma_\infty$  determined from Gibbs EOS, e.g. [50]. The Langmuir isotherm on the other hand does capture the value of  $\Gamma_\infty$  from Gibbs EOS quite well.  $\Gamma_\infty$ , as determined by the experimental slope or the Langmuir isotherm, is a function of the molecular area, the orientation of the molecules at the interface, and any interfacial interactions [32]. We are looking for a correlation between these effects on  $\Gamma_\infty$  and molecular structure.

One might correlate  $\Gamma_\infty$  from a simple molecular area argument, similar to the aggregation number in self-assembly [4], or using MD simulations [86]. One particular study, using a limited dataset of  $C_iE_j$  chemistries, showed that the change in molecular packing was related to hydrophile coiling, i.e. smaller  $\Gamma_\infty$  for higher  $N_{EO}$  [93]. This conformational change of ethylene glycol units is known to occur at  $N_{EO} \approx 9$  due to attraction between oxygen atoms [44]. The overall size of these coils, and therefore their effective area  $1/\Gamma_\infty$ , increases with increasing  $N_{EO}$ . Sedev proposes a relationship between  $\Gamma_\infty$  and  $N_{EO}$ , considering polymer scaling laws for the radius of gyration of an ethylene oxide chain with length  $N_{EO}$ , i.e.  $\Gamma_\infty \sim N_{EO}^{-1/2}$  [80, 94]. This dependence only considers the hydrophile and ignores the role of the hydrophobe. However, Sedev showed examples

of the scaling prefactor which was argued to depend on  $N_C$  [94]. Figure S7 tests these hypotheses using a large dataset. While Figure S7b shows a clear dependence on  $N_{EO}$ , Figure S7a confirms Sede's arguments of a slight dependence of  $\Gamma_\infty$  on  $N_C$ . It is evident that an appropriate model for  $\Gamma_\infty$  must depend on both  $N_C$  and  $N_{EO}$ .

Instead of using a molecular area argument, we follow the derivation proposed by Rosen [6, 32], whereby the equation of state is evaluated at a concentration which leads to a fixed surface pressure. Here, we evaluate the isotherm at a fixed concentration,  $x_{CMC}$ , which leads to a surface pressure,  $\Pi_{CMC}$ , using (14) [8]. For all  $C_iE_j$ 's reported here,  $x_{CMC} \cdot K_x \gg 1$ . Therefore, (14) can be rearranged in terms of the free energies:

$$\Gamma_\infty \approx \frac{\Pi_{CMC}}{(f - g)} \approx \frac{\Pi_{CMC}}{RT (\ln y_{phob}^{-5.9})} \quad (17)$$

where  $f$  and  $g$  are given by (11) and (16), respectively (see supplementary material for the detailed derivation). Note that this result strongly depends on the chosen isotherm, e.g. a similar, but different result was found analytically for the Brunauer–Emmett–Teller (BET) isotherm [95]. One important similarity between our result and the BET isotherm is that both result in  $\Gamma_\infty$  being a function of the difference in free energies [95].

Equation (17) depends on the value of  $\Pi_{CMC}$ , which may be a function of molecular parameters. Figure S8 shows the dependence of  $\Pi_{CMC}/RT$  versus  $y_{phob}$ . For all  $C_iE_j$ 's,  $\Pi_{CMC}/RT \approx 1.6 \times 10^{-5} \text{ mol/m}^2$ , and for the purpose of this study  $\Pi_{CMC}$  is assumed to be a constant. Thus, (17) becomes:

$$\frac{10^{-6}}{\Gamma_\infty} \approx -0.369 \ln y_{phob} \quad (18)$$

This implies that  $1/\Gamma_\infty$  goes with  $\ln y_{phob}$  only. Figure 4 shows a master curve of  $1/\Gamma_\infty$  as a function of  $\ln y_{phob}$  for all surfactants compared to the prediction of (18). Equation (18) shows very good agreement with experimental trends, validating the assumptions and the free energy models discussed above. Note that if  $k' \neq k$ , then (18) would have an explicit term for  $N_C$ . One test of the assumption that  $k' = k$  is to fit the  $k'$  values independently to the datasets and use them in the analysis, which would give  $k' = -1.5$  and  $k = -1.2$ . We found that when  $k' \neq k$ , the model does a significantly poorer job of capturing the  $\Gamma_\infty$  data. Although this is not proof that  $k' = k$ , we expect that their values are very similar and certainly within the error of the data. One limit to the applicability of the model for  $\Gamma_\infty$  is the assumption that  $\Pi_{CMC}$  is a constant for all chemistries. Relaxing this assumption and using  $\Pi_{CMC}$  as a function of  $y_{phob}$  and  $N_C$  would likely improve agreement.

Previous molecular models and hypotheses for  $\Gamma_\infty$  have argued for an explicit dependence on  $N_{EO}$  [80, 96, 94, 93]. In contrast, our model argues that the hydrophobe contributes to the maximum surface concentration of a given  $C_iE_j$  surfactant at the air-water interface. This directly contradicts the arguments of Schick that the absolute size of the hydrophobe does not contribute to the monolayer density [93]. Furthermore, we can directly compare the data presented in Figure 4 to previous models depending only on  $N_{EO}$ . Figure S7b compares  $\Gamma_\infty$  as a function of  $N_{EO}$  to the model,  $\Gamma_\infty = 1/(19N_{EO}^{1/2})$  detailed in Kumar and Tilton [80], which consistently over predicts the experimental data. The better agreement observed with  $y_{phob}$  suggests that the hydrophobe influences the packing density at the interface and thus a simple molecular area argument is not sufficient to capture the data. Instead,  $\Gamma_\infty$  more generally depends on the molecular cross-section, interactions between adsorbed molecules, and the configuration of the adsorbed molecule. For the

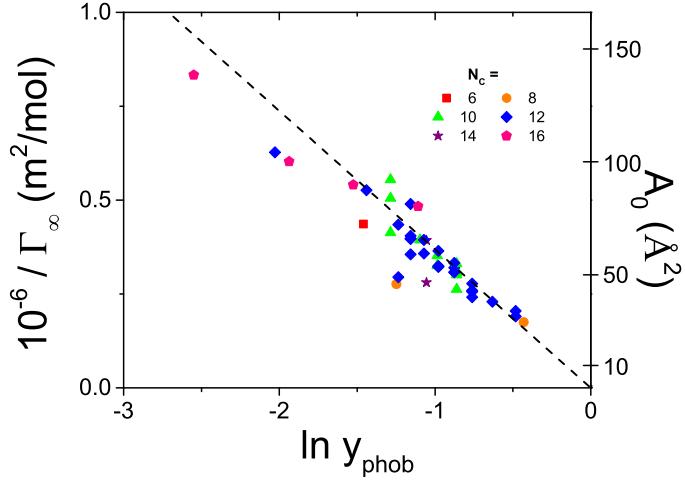


Figure 4: Maximum surface concentration as a function of  $\ln y_{phob}$ .  $A_0$  represents the effective interfacial area occupied by one surfactant molecule, i.e.  $A_0 = 1/(N_A \Gamma_\infty)$  where  $N_A$  is Avogadro's number. The dashed line represents (18).

case of  $C_iE_j$ 's, these effects are all accounted for by  $\ln y_{phob}$ . Physically, this translates to more hydrophobic molecules packing more tightly at the interface.

#### 4. Conclusions

The field of interfacial thermodynamics has always relied on empirical relationships and trial and error correlations. The literature has long proposed that the free energies of micellization and adsorption should be a function of  $N_C$  or HLB. Many attempts have been made to generate a model that can capture experimental data. In this paper, we propose models that combine HLB, via  $y_{phob}$ , and  $N_C$  to capture the effect of surfactant chemistry on adsorption and micellization thermodynamic parameters. We demonstrate that the models for  $x_{CMC}$  and  $K_x$  are capable of collapsing all surfactant data onto master curves. Furthermore, the resulting models for  $\Delta G^{mic}$  and  $\Delta G^{ads}$  are validated by their ability to predict the dependence of  $\Gamma_\infty$  on  $y_{phob}$ .

There are several advantages of the proposed models over existing empirical models. For example, the leading empirical equation for  $x_{CMC}$ , i.e. Becher-Ravey correlation, was not able to fit the entire dataset. Furthermore, the model presented here is capable of both interpolation and extrapolation to previously unmeasured surfactant chemistries. This allows for the prediction of the CMC of novel  $C_iE_j$  molecules, particularly larger  $N_{EO}$  surfactants. This is a great improvement over previous empirical models, group contribution models [24], and molecular dynamics [29] approaches, especially considering the significant reduction in parameters and computational expense.

Another advantage to the proposed models is that many hypotheses are quantitatively explained as a function of surfactant chemistry. For example, it has long been argued that the interaction of surfactants in a micelle is strongly dominated by the hydrophobe [32], however others have argued that the hydrophile contributes opposition to micellization [80]. Our model confirms the linear dependence of  $\Delta G^{mic}$  on  $N_C$  as well as the importance of the length of the hydrophile, via  $y_{phob}$ . More specifically, the model combines the contributions of the hydrophile and hydrophobe to the CMC.

The choice of surfactant for targeted  $x_{CMC}$  is controlled mostly by  $N_C$ . The model holds at various temperatures and suggests a very small temperature dependence of the interaction parameter of alkane chains, i.e. the  $N_C$  prefactor. With regards to adsorption physics, several studies propose that both the hydrophobe and hydrophile drive adsorption, which is not readily obvious [90]. Our model confirms that  $\Delta G^{ads}$  explicitly depends linearly with  $N_C$  and is also a strong function of  $N_{EO}$ , via  $y_{phob}$ . As hypothesized, the contributions of the hydrophile and hydrophobe both increase the rate of adsorption and/or decrease the rate of desorption of the molecule, i.e. larger equilibrium constant. Lastly, previous theories have suggested that the maximum packing at the interface is only determined by the cross-sectional area contributed by the hydrophile [94]. Our model shows that  $\Gamma_\infty$  is a function of only  $y_{phob}$ , and thus the length of the hydrophile and hydrophobe affect the area occupied by the molecule at the interface in a non-trivial way.

This paper summarizes years of effort toward structure-property relationships in nonionic surfactants, particularly highlighting a large collection of data and previous theoretical and empirical models for  $C_iE_j$ 's. We demonstrate that interfacial thermodynamics is capable of properties by chemical design. We have shown that the targeting of surfactant properties can be achieved by changes in both  $N_C$  and  $y_{phob}$ . The proposed model allows us to screen values of  $x_{CMC}$ ,  $K_x$ , and  $\Gamma_\infty$  based simply on chemistry, meaning the entire range of surface tensions can be predicted from molecular structure alone. This also allows for targeted selection of surfactants for desirable transport performance. One example is that experiments can be designed with different surfactants where the thermodynamics are kept constant, but the diffusion coefficient is very different. Therefore, different surfactants with the same thermodynamics can be tested in a range of applications to determine the importance of transport timescales on process efficacy [97, 98].

Our model introduces a formalism that can be easily applied to other surfactant systems, broadening the community's ability to correlate structures with properties. For example, we have already shown that for nonionic polyglycol surfactants the differences in free energy also depend on  $y_{phob}$  and  $N_C$  [99, 100]. However, it must be stated that these models are based on certain assumptions that might restrict their generality. For example, we expect that the difference in free energy of adsorption and micellization will depend on other aspects of surfactant chemistry, e.g. ionization states for charged surfactants [33]. Future studies will examine whether these simple models can be applied to surfactants with strong surface interactions and complex architecture.

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## Appendix A: Supplementary data

Supplementary material related to this article can be found, in the online version, at doi:

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