Measuring the Impact of Influence on Individuals: Roadmap to Quantifying Attitude

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***Abstract*—Influence diffusion has been central to the study of the propagation of information in social networks, where influence is typically modeled as a binary property of entities: influenced or not influenced. We introduce the notion of attitude, which, as described in social psychology, is the degree by which an entity is influenced by the information. We present an information diffusion model that quantifies the degree of influence, i.e., attitude of individuals, in a social network. With this model, we formulate and study the attitude maximization problem. We prove that the function for computing attitude is monotonic and sub-modular, and the attitude maximization problem is NP-Hard. We present a greedy algorithm for maximization with an approximation guarantee of $(1 - 1/e)$. Using the same model, we also introduce the notion of "actionable" attitude with the aim to study the scenarios where attaining individuals with high attitude is objectively more important than maximizing the attitude of the entire network. We show that the function for computing actionable attitude, unlike that for computing attitude, is non-submodular but is *approximately submodular*. We present an approximation algorithm for maximizing actionable attitude in a network. We experimentally evaluated our algorithms and studied empirical properties of the attitude of nodes in the network such as spatial and value distribution of high attitude nodes.**

I. Introduction

The proliferation of social networks and their influence in modern society led to a large body of research in several scientific domains that focus on utilizing and explaining the significance of the impact of social networks. One of the key problems investigated is to understand the diffusion of information/influence propagation in social networks. Diffusion refers to the (probabilistic) behavior of the interaction between the entities in the network describing when/how an entity is influenced by the actions of its neighbors.

Seminal works of Domingos and Richardson, and Kempe *et al.* proposed two popular models for information diffusion/influence propagation—Independent Cascade and Linear Threshold [8], [16]. In these models, a node of a network is said to be influenced if it receives the information originated at the seed set. This concept of influence is binary: an entity is either influenced or is not influenced. Real-world experience shows that not all influenced individuals are the same. I.e, some individuals are more *strongly* influenced by certain information compared to others. Thus, the *strength of influence* can vary from one individual to the other. This phenomenon has been pointed out in social sciences literature.

Within social psychology, two related concepts, attitudes and beliefs, are frequently studied to understand human behavior. Beliefs, which represent people's ideas about the way the world is or should be, are commonly conceptualized as binary in nature, present or absent [10]. Throughout their lives, people acquire new beliefs, and sometimes, new beliefs replace old beliefs. In this way, people tend to acquire a very large number of beliefs over the life course. This notion of *belief* in social psychology that is *binary* in nature can be considered similar to the notion of "influence" in computational social network analysis which is also *binary* in nature.

Attitudes, on the other hand, are "latent predispositions to respond or behave in particular ways toward attitude objects" [9]. In contrast to beliefs, which are largely cognitive in nature, attitudes, have a cognitive, affective, and a behavioral component [27]. Being subjective in nature, attitudes can vary in strength such that a person can hold a *very strong attitude or a weak attitude toward* an object or concept, and thus attitude quantifies the strength of belief [2], [10]. Individuals acquire attitudes through experiences and exposure. In the case of exposure, a body of research shows that *repeated exposure* to an object/idea increases the likelihood that a person will adopt a more favorable attitude toward it [31]. Thus *attitude* being non-binary can be thought of as the *strength of influence*. Motivated by these studies, we study the problem of arriving at a mathematical model that captures the notion of attitude resulting from information propagation in social networks.

Our first contribution is to define a mathematical model for measuring attitude. Within social networks, people are often subjected to *repeated exposures* to information such as

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an anti-vaccine message, a pro-GMO message, or gun safety messaging. It has been observed that when an individual is exposed to a large number of, say, anti-vaccine messages, this increases the probability that that person will adopt a similar anti-vaccine attitude. Based on this, we postulate that the *strength of influence or attitude* of an individual, toward an object/concept, can be captured by the number of times the individual receives the information from its neighbors. In other words, if an already influenced individual is further provided with the same/similar influencing information, then the latter reinforces the learned belief of the individual, thus shaping and increasing his/her *attitude*. We use the number of reinforcements as a way to quantify the attitude.

Using this model, we define the attitude of an individual and the total attitude of the network as functions from 2^V to reals (2^V denotes the powerset of nodes V of the network). We denote the function that captures the total attitude of the network with $\sigma_{Att}(.)$ We study the computational complexity of the function σ_{Att} and provide efficient algorithms to approximate it. We prove that this function is #P-hard and that it is monotone and submodular. We provide an (ϵ, δ)-approximation algorithm for computing attitude with provable guarantees. We then formulate the *attitude maximization* problem–find a seed set S of size k that will result in maximum total attitude of the network. We first prove that the attitude maximization problem is NP-hard. Based on the monotonicity and submodularity of attitude, we propose a greedy algorithm that achieves a $(1 - 1/e)$ approximation guarantee.

We further introduce the concept of *actionable* attitude. The introduction of this concept is motivated by the fact that individuals with higher attitude (strongly influenced) are likely to act according to the attitude. This is particularly important in campaigns (such as political or gun-safety messaging), where motivated and dedicated volunteers are necessary to carry and spread the message (possibly beyond the social network); and such volunteers are the ones who are strongly influenced. Our second major contribution is the study of the underlying computational problem related to actionable attitude maximization. We prove that though the function for computing actionable attitude is not submodular, it is *approximately submodular*. Based on this we design efficient approximation algorithms to maximize the actionable attitude in a network.

II. Related Work

Computational models of information diffusion in social networks is introduced and formalized in the seminal works of Domingos and Richardson [8] and Kempe, Kleinberg and Tardos [16]. There are two widely-studied probabilistic diffusion models: *Independent Cascade* (IC) model and *Linear Threshold* (LT) model. Given a seed set $S \subseteq V$, Kempe *et al.* [16] proved that the influence maximization problem is NP-hard, and also proved that a greedy algorithm achieves a $(1 - 1/e)$ approximation guarantee. The approximation guarantee of the greedy approach stems from the non-negativity, monotonicity and submodularity of the influence function. Since then several improvements have been proposed to make the greedy algorithm more practical and scalable [4], [7], [12], [15], [18], [25], [29], [30]. Several variants of the influence maximization problem have been studied in the literature, since the work of Kempe *et al.* such as topic-aware influence maximization and targeted influence maximization [3], [5], [13], [19], [20], [23], [26], [28].

Enhancements to the basic influence propagation model have been proposed that take into account the opinions of users [6], [12], [32]. Liu et al. [21], [22] introduced PageRank based diffusion model, as a generalization of the basic IC model. These models do not capture the notion of *attitude/strength of influence* that we seek to formalize. Aggarwal et al. [1] introduced a flow authority model to determine assimilation of information in a network. This model differs from the Independent Cascade and does not capture the notion of attitude due to repeated activations. In [33], the authors discussed the problem of maximizing cumulative influence in a model where the same node can repeatedly activate his/her neighbor within a given time interval. Such a model may lead to divergence in the computation of objective function, and hence, the computation is parameterized by a time interval and thus differs from our model.

Remark. Due to space constraints we omit several proofs, proof details and few experimental details. A complete version of the paper along with the source code can be found at https://github.com/madhavanrp/QuantifyingAttitude and arXiv [11].

III. Modeling Attitude

In this section, we provide a mathematical model and definition to capture the notion of *attitude*.

Definition 1. ***[Attitude-IC model (AIC)]*** *The diffusion proceeds in discrete rounds starting from some set of seed-nodes S. Initially, every seed node starts with an attitude value of 1, and all non-seed nodes have the attitude 0. At each step, each newly influenced node u tries to send information to each of its neighbor v as per the edge probability $p(u, v)$. If u succeeds, then v's attitude is incremented by 1; and its status is changed to influenced if it is not already influenced. When u succeeds in sending information v, we say that the edge $\langle u, v\rangle$ is activated. The process terminates when no new nodes are influenced in a step.*

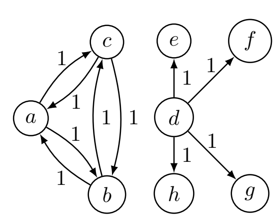

Fig. 1. Attitude

Let $\{a\}$ be the seed in Figure 1 and at step $t = 0$, its attitude is 1. At $t = 1$, a activates b, c, and the attitudes of a, b, c are 1. At $t = 2$, the newly activated nodes b, c send information to their neighbors. Node b succeeds and increments the attitude of nodes a, c. Simultaneously, c succeeds and increments the attitude of nodes a, b. After $t = 2$, the attitudes of a, b, c are $3, 2, 2$ respectively. Since no new nodes are activated in this step, the diffusion ends.

In the standard independent cascade model each activated node gets one chance to influence its *un-influenced* neighbors, while in our model, each newly activated node tries to influence all its neighbors whether they are already influenced or not. Thus, an activated node can receive information from a newly activated in-neighbor. This captures the notion of *repeated exposure* or *reinforcement*, which increases the recipient's attitude.

For any set $S \subseteq V$ of nodes, we use the random variable $\mathtt{Att}_v(S)$ to denote the final attitude of node v when the seed set is S. Let $\mathbb{E}[\mathtt{Att}_v(S)]$ denote the expectation of $\mathtt{Att}_v(S)$. We define $\mathtt{AttIn}(S)$ as $\sum_{v\in V} \mathtt{Att}_v(S)$. The *total expected attitude of the network* resulting from diffusion starting at seed S is $\sigma_{\mathtt{Att}}(S) = \mathbb{E}[\mathtt{AttIn}(S)]$. By linearity of expectation, $\sigma_{\mathtt{Att}}(S) = \sum_{v\in V} \mathbb{E}[\mathtt{Att}_v(S)]$.

By overloading notation, we often interpret G as a distribution over unweighted directed graphs, each edge $e = (u, v)$ is realized independently with probability $p(u, v)$. We write $g \sim G$ to denote that an unweighted graph g is drawn from this graph distribution G. Given a set of nodes $S \subseteq V$ and a graph g, we use

1) R_g^S to denote the set of nodes reachable from S in g.
2) $E_g^S = \{e = (u, v)|u, v \in R_g^S$ and $e \in g\}$ is the set of *activated edges* in g due to diffusion from S. Let $E_{g,v}^S$ be the set of activated edges of the form $\langle ., v\rangle$.
3) $\mathtt{AttIn}_g(S)$ to denote the attitude induced by S in graph g and is equal to $\sum_{v\in V} \mathtt{Att}_{g,v}(S)$, where $\mathtt{Att}_{g,v}(S)$ is the attitude of v in the graph g computed as the number of activated incoming edges to v.

We found that $\sigma_{\mathtt{Att}}(S)$ is determined by the expected number of activated edges.

THEOREM 1. *If $g \sim G$ then $\forall S \subseteq V$, $\sigma_{\mathtt{Att}}(S) = |S| + \sum_{g\sim G} |E_g^S| \times Pr(g \sim G)$, and $\mathbb{E}[\mathtt{Att}_v(S)] = \sum_{g\sim G} |E_{g,v}^S| \times Pr(g \sim G)$.*

THEOREM 2. *Under the AIC model, $\sigma_{\mathtt{Att}}(\cdot)$ is a monotone, non-decreasing, submodular function. Moreover both the functions $\sigma_{\mathtt{Att}}(\cdot)$ and $\mathbb{E}[\mathtt{Att}_v(\cdot)]$ are #P-hard.*

From Theorem 2, it follows that computing $\sigma_{\mathtt{Att}}(S)$ exactly is computationally infeasible. We provide efficient approximation algorithms to estimate $\sigma_{\mathtt{Att}}(S)$. Borgs et. al. [4] introduced Reverse Influence Sampling (RIS), which has been used to develop efficient Influence Maximization algorithms [14], [25], [29], [30]. Base on this, we introduce a *Reverse Attitude Sampling* (RAS) technique.

LEMMA 1. *Let $e = (x, y)$ be an arbitrary edge in G, $R_{g^T}^{\{x\}}$ be the set of nodes reachable from x in g^T, where g^T is the transpose of un-weighted graph g drawn from random distribution G. Then for any $S \subseteq V$, $P[S$ activates e in $g] = P[S \cap R_{g^T}^{\{x\}} \neq \emptyset] \cdot p(e)$*

The lemma follows because for the edge e to be activated x must be influenced and x succeeds in propagating information to y via the edge e. The latter happens with probability $p(e)$, and the probability of the former is $Pr[S \cap R_{g^T}^{\{x\}} \neq \emptyset]$ [4].

Algorithm 1: Estimate $\sigma_{\mathtt{Att}}(S)$

Data: Graph $G = (V, E)$, $S \subseteq V$
begin
  $\mathcal{R}$ = Generate β RR Sets using **Generate RR Set**
  $X = |\{RR \in \mathcal{R} \mid S \cap RR \neq \emptyset\}|$
  return $\frac{|E| \cdot X}{\beta}$

THEOREM 3. *Given a graph $G = (V, E)$, $\forall S \subseteq V$, and $\forall v \in V$, $\mathbb{E}(\mathtt{Att}_v(S)) = |InDegree(v)| \times P_{g\sim G, e=(u,v)\in E}[S \cap R_{g^T}^{\{u\}} \mid e \in g]$ and $\sigma_{\mathtt{Att}}(S) = |S| + |E| \times P_{g\sim G, e=(x,y)\in E}[S \cap R_{g^T}^{\{x\}} \mid e \in g]$*

Consider the transpose of G, $G^T = (V, E^T)$, where the edge probabilities remain unchanged. We describe a procedure to generate *Random Reverse Reachable Sets (RR Sets)*:
Generate RR Set. Randomly pick an edge $e = (v, u) \in E^T$. With probability $p(e)$, add the node u to RR. For any u added to RR, for each edge $(u, w) \in G^T$, add w to RR with probability $p(u, w)$. The process continues till no node is added to RR.

LEMMA 2. $\sigma_{\mathtt{Att}}(S) = |S| + |E| \times P_{RR\sim\mathcal{R}}[S \cap RR \neq \emptyset]$

Lemma 2 allows us to design Algorithm 1 to estimate $\sigma_{\mathtt{Att}}(S)$ within a relative error of ϵ with probability $1 - \delta$ when $\beta \in \theta(\frac{m}{\epsilon^2 \sigma_{\mathtt{Att}}(S)} \cdot log(\frac{1}{\delta}))$.

IV. Attitude Maximization Problem

Having defined Attitude under the AIC-model, we describe the problem to find a set of users, who can maximize the attitude of the network.

PROBLEM 1. ATTITUDE MAXIMIZATION PROBLEM*: Given a graph $G = (V, E)$, a number k, find $S \subseteq V$ of size at most k such that $\sigma_{\mathtt{Att}}(S)$ is maximized.*

THEOREM 4. *Under the AIC model, the attitude maximization problem, i.e., computing $\mathtt{argmax}_{\mathtt{S}\subseteq\mathtt{V},|\mathtt{S}|\leq\mathtt{k}}\, \sigma_{\mathtt{Att}}(S)$, is NP-hard.*

We prove that influence maximization problem is different from the attitude maximization problem.

THEOREM 5. *An optimal solution to the influence maximization problem is not an optimal solution to the attitude maximization problem.*

Nemhauser et. al. [24] proved the greedy strategy to maximize a non-decreasing, monotone, and submodular function outputs a $(1 - 1/e)$-approximate solution. Motivated by this, we design a RAS-based $(1-1/e)$-approximation Algorithm 2 for the attitude maximization problem.

THEOREM 6. *When $\beta \in \theta(\frac{|E|}{\epsilon^2 \sigma_{\mathtt{Att}}(S^*)}(klogn \cdot log(1/\delta)))$, Algorithm 2 outputs a seed set S_k such that $\sigma_{\mathtt{Att}}(S_k) \geq (1 - \frac{1}{e} - \epsilon)\sigma_{\mathtt{Att}}(S^*)$ with probability at least $1 - \delta$.*

V. Attitude to Actionable Attitude

As nodes with high attitude are likely to act based on their influence, in some scenarios it is desirable to spread information that results in such highly influenced individuals.

Algorithm 2: $(1 - 1/e - \epsilon)$-approximate algorithm

Data: Graph $G = (V, E)$, k
Result: Seed Set S
begin
  $\mathcal{R}$ = Generate β RR Sets using **Generate RR Set**
  Mark all the sets in $\mathcal{R}$ as uncovered
  while $|S| \leq k$ **do**
    Find v that covers maximum uncovered sets in $\mathcal{R}$
    Mark sets covered by v as covered
    Add v to S
  return S

Motivated by this, we introduce a notion *actionable attitude* that attempts to increase the total attitude of nodes with "high enough attitude", as opposed to the total attitude of all nodes.

DEFINITION 2. *[Actionable Attitude] Actionable Attitude induced by a given seed set S is $\sigma_{\mathtt{Act}}(S) = \sigma_{\mathtt{Att}}(S) - \sigma(S)$.*

PROBLEM 2. ACTIONABLE ATTITUDE MAXIMIZATION PROBLEM*: Given a graph $G = (V, E)$ and k, find $S \subseteq V$ of size at most k such that $\sigma_{\mathtt{Act}}(S)$ is maximized.*

THEOREM 7. *Under the AIC model, $\sigma_{\mathtt{Act}}(.)$ is a monotone, non-decreasing, non-submodular function function*

Even though, $\sigma_{\mathtt{Act}}(\cdot)$ is not submodular, very interestingly we show this function is *approximately submodular* [17].

DEFINITION 3. *A set function f is* Δ-approximate submodular *if for every pair of sets S and T with $S \subseteq T$ and every $x \notin T$, $f(x|S) \geq f(x|T) - \Delta$.*

THEOREM 8. *Given a graph $G = (V, E)$ let $deg_G(v)$ denote the outdegree of any $v \in V$. Then, $\forall S \subset T \subseteq V$ and $\forall v \notin T$, $\sigma_{\mathtt{Act}}(v|S) \geq \sigma_{\mathtt{Act}}(v|T) - \mathbb{E}_{g \sim G}[deg(v)]$.*

This leads to following theorem.

THEOREM 9. *The function $\sigma_{\mathtt{Act}}(\cdot)$ is Δ-approximate submodular, where Δ is the expected max degree of the graph.*

We design an efficient RR Sets-based algorithm. However, since the function $\sigma_{\mathtt{Act}}(\cdot)$ is the difference between attitude and influence, **Generate RR Set** cannot be directly used for actionable attitude maximization. We need a mechanism to generate RR sets. Instead of randomly picking an edge in the network, we generate a sufficient number of RR graphs for each vertex v. Let $F_g^S(v)$ be the number of edges from v that reaches $S \in g^T$, $\mathcal{R}_v$ be the set of RR graphs from v, and $T_g^S(v)$ be the number of edges to v that are reachable from $S \in g$.

THEOREM 10. *Given a graph $G = (V, E)$, for any $S \subseteq V$. $\sigma_{\mathtt{Act}}(S) = \sum_{v \in V} \sum_{g^T \in \mathcal{R}_v} P(g) \times max\{F_g^S(v) - 1, 0\}$, and $\sigma_{\mathtt{Act}}(u|S)$ is equal to $\sum_{v \in V} \sum_{g^T \in \mathcal{R}_v} P(g) \cdot [\max\{F_g^{S \cup \{u\}}(v) - 1, 0\} - \max\{F_g^S(v) - 1, 0\}]$*

Using Theorem 10, we can design a scalable greedy algorithm for the actionable attitude maximization problem that outputs S_k as the seed. Let S^* be the optimal solution to it.

THEOREM 11. *There is an algorithm that will generate $O(|E|k/\epsilon^2 \log n/\delta)$ RR sets, and outputs a set S_k such that*

$$\Pr[\sigma_{\mathtt{Act}}(S_k) \geq (1 - 1/e - \epsilon)\sigma_{\mathtt{Act}}(S^*) - (k-1)\Delta] \geq \delta$$

VI. EXPERIMENTAL EVALUATION

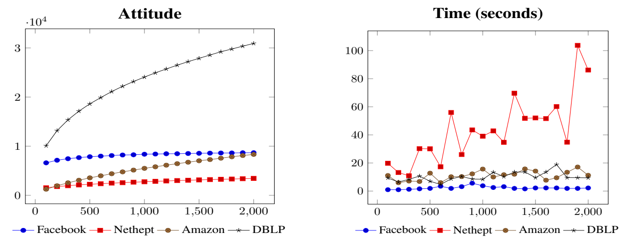

Fig. 2. Attitude results and time taken to maximize attitude.

We used datasets from the Stanford SNAP library. The size of graphs range from 4000 to $1.1M$ nodes and 88000 to $3M$ edges. Algorithms are implemented in C++ and run on Linux server with AMD Opteron 6320 CPU (8 cores and 2.8 GHz) and 128GB main memory.

Maximizing Attitude. The results are shown in Figure 2 (x-axis represents the seed set size and the y-axis indicates the attitude or time). The attitude results produced across a wide range of graph sizes demonstrate the scalability of RAS-based maximization. We computed the attitude maximization seed set for budgets in the range $[1, 2000]$. As expected as seed set size increases, the total attitude also increases. The time taken to compute the seed set does not increase much as the seed set size increases. For example, on $DBLP$ ($n = 317080, m = 1049866$), the time taken is less than 20 seconds for budgets ranging from $100 - 2000$. This is due to the fact that as the seed set size increases, the value of $\sigma(S^*)$ would increase thus resulting in smaller RR sets.

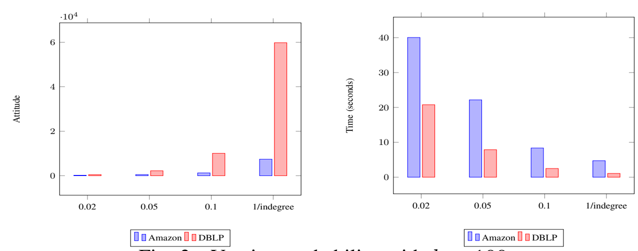

Fig. 3. Varying probability with $k = 100$

Propagation Probability and Attitude. We consider different edge probabilities such as $0.02, 0.05, 0.1$ and $1/$inDegree. The overall attitude increases as the probability increases (See Figure 3). Interestingly, the maximum attitude is observed when the probability is $1/$inDegree. This is explained by considering the fact that for each node, it is expected that one of its incoming edges is activated (if its neighbors are activated). Therefore, the overall attitude is significantly higher if $1/$inDegree is greater than 0.1, on average. We also report how time varies with probability. We observe that the time taken is least when the edge probability is $1/$inDegree and is highest when the probability is 0.02. This is again explained by observing that $\sigma_{\mathtt{Att}}(S^*)$ inversely impacts the number of RR sets required for estimating attitude.

Average Attitude. Next, we focus on the *average attitude of a node*. There are two ways to look at this number. The first is the ratio $\sigma_{\texttt{Att}}(S)/\sigma(S)$ which is the ratio of expected attitude and expected number of influenced nodes. Another measure for average attitude is to take the expectation of the following ratio: Total Attitude/Number of nodes influenced. These two quantities need not be equal, in general, as expectation of a ratio is not the ratio of expectations. We computed the former quantity by running the presented algorithms. We estimated the latter quantity by running simulations (20000). The results are shown in Table I. Interestingly both the quantities turn out

graph name	$\frac{\sigma_{\texttt{Att}}(S)}{\sigma(S)}$	$E[\frac{\texttt{Att}}{\texttt{Inf}}]$	Average indegree
ego-Facebook	3.21	3.20	21.85
Epinions	3.30	3.32	6.71
NetHept	1.34	1.38	4.12
DBLP	1.23	1.23	3.31
Youtube	1.43	1.44	2.63

budget = 100 and edge probability = 0.1

TABLE I
AVERAGE ATTITUDE

be almost the same for all the graphs. Graphs with higher average indegrees tend to achieve higher average attitudes. For example, Epinions achieves a higher average attitude than NetHept.

Maximizing Actionable Attitude. We fix the probability to 0.05. As expected, the Actionable Attitude does increase when the seed set size is increased. We observe that the Actionable Attitude grows in larger quantities for $Facebook$ than for the other graphs. This is due to the fact that $Facebook$ is denser, leading to a higher number of edges activated by the seed set. We also study how the Attitude Maximizing seed compares with the Actionable Attitude Maximizing seed. Across various graphs, we note that the Actionable Attitude Maximizing seed set activates fewer nodes when compared to the Attitude Maximizing seed. For example, on $DBLP$ with $k = 100, p = 0.05$, Attitude maximization algorithm produces Attitude of 2294 with influence 1930. In the same setting, the actionable attitude maximization algorithm produces Attitude of 870 with influence 376. We note two points. The objective function $\sigma_{\texttt{Act}}(.)$ is higher for the seed set produced by the actionable attitude maximization compared to the seed set produced by the attitude maximization problem. Very interestingly, for the attitude maximization seed set the average attitude is $2294/1930$ which is 1.19 whereas the actionable attitude maximization seed results in an average attitude of $870/376$ which is 2.31. Recall that the notion of actionable attitude attempts to maximize entities that are strongly influenced and thus should result in higher average attitude and our experiments on multiple networks (e.g., Facebook, NetHept, Amazon, DBLP) concur with this intuition.

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